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Improved prediction model for time-dependent deformations of concrete: Part 2—Basic creep

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The second part of this series presents the formulae for the prediction of basic creep of concrete, i.e. creep at no moisture exchange. The formulae give the secant uniaxial compliance function which depends on the stress level, and, as a special case, the compliance function for linear structural analysis according to the principle of superposition. The formulae are based on the recently developed solidification theory for concrete creep which takes into account simultaneous ageing, satisfies all the basic thermodynamic requirements, and avoids divergence of creep curves. The formulae, which describe both creep and elastic properties, involve only four free material parameters. All four appear linearly, so that optimum data fits can be obtained by linear regression. For the frequent situations where no test data for the particular concrete to be used are available, empirical formulae for predicting these four parameters from the concrete mix composition and the standard compressive strength are given. These formulae, however, involve considerable error. To avoid it, one should, whenever possible, carry out measurements of the elastic modulus and, if possible, also the short-time creep of 7 to 28 days duration. With such measurements, greatly improved predictions can be achieved. The predictions are compared with 17 extensive data sets taken from the literature, and the coefficients of variation of the deviations are found to be smaller than with previous models.

1. INTRODUCTION

The time-dependent deformations of concrete consist of shrinkage and creep. Shrinkage is the strain that occurs at zero stress, and creep is the remainder, i.e. the strain produced by sustained stress. Creep needs to be subdivided into two parts: the basic creep, which occurs at constant moisture content, and the drying creep, which represents an additional creep associated with moisture content variations.

This second part of the current series will present a prediction model for basic creep which improves that given in the Bažant–Panula (BP) model. In contrast to the models for shrinkage as well as drying creep, which deal with the average response of the cross-section of a member that is in a non-uniform state, the present basic creep model can be considered as a constitutive relation of the material, since it describes test specimens that are in a state of uniform strain and moisture content.

2. MAIN FEATURES OF PROPOSED MODEL

There are five simple, experimentally well-established characteristics to which a prediction model must conform: (i) The short-time creep curves have the shape of $(t-t')^n$, where t' = age at loading, t = current age and n is roughly 1/8; (ii) the basic creep has no final asymptotic value; (iii) the long-time creep curves approach a linear function of $\log(t-t')$ of about the same slope for all t'; (iv) the higher is t', the later the transition from $(t-t')^n$ to a straight line in $\log(t-t')$ takes place; and (v) the creep for the same $t-t'$ decreases roughly as $(t')^{-1/3}$.

The foregoing simple characteristics have been adhered to in formulating the original BP model. In the absence of any physical theory which would describe the effect of ageing on creep—the most complex feature of the basic creep of concrete—the BP model formulae were chosen as the simplest ones conforming to these simple characteristics, except the straight-line logarithmic shape of the long-time creep curves. Subsequently, however, a physically based theory that explains the effect of ageing through a simplified model of the solidification process of Portland cement paste was formulated, used as the basis of a creep model, and shown to agree with the aforementioned characteristics [1]. This theory, whose derivation will not be repeated here, has several important advantages:

1. All the viscoelastic behaviour of concrete, including the elastic deformations and the ageing, can be closely described with only four free material parameters.
2. All the free material parameters can be identified from the given test data by linear regression.
3. The constitutive relation can be easily converted, by means of explicit formulae rather than some identification algorithm, to a rate-type creep model corresponding to a Kelvin chain with age-independent elastic moduli and viscosities, the age effect being totally ascribed

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to transformations of time (this simplifies the finite-element analysis of creep effects in structures).

4. The solidification theory automatically satisfies the condition that the creep curves for different ages at loading should not diverge (violation of this condition, although prohibited neither by thermodynamics nor by experimental evidence, causes various complications [2]).

5. Extension of the solidification theory to creep at variable stress correctly describes deviations from the principle of superposition (manifested by the phenomenon of adaptation), as well as the non-linearity of creep as a function of the stress.

3. PREDICTION FORMULAE FOR BASIC CREEP

The creep properties may be characterized by the secant compliance function $J(t, t', \sigma) = \varepsilon/\sigma$ where ε is the strain at time t caused by a sustained (constant) uniaxial stress σ applied at time t' (t and t' represent the ages measured from the time of initial set of concrete). According to this definition, the compliance function includes the initial instantaneous strain at age t', represented by $J(t', t', \sigma)$. The compliance function for linear structural analysis according to the principle of superposition is obtained by setting $\sigma = 0.3f_c'$, where f_c' is the standard 28-day compressive strength of concrete; that is

$$J(t, t') \leftarrow J(t, t', 0.3f_c') \tag{1}$$

This compliance function represents the strain at time t caused by a unit sustained uniaxial stress applied at time t'. The conventional elastic modulus $E(t')$ for structural analysis corresponds approximately to loading duration $\Delta = 0.1$ day and is obtained as $E(t') = 1/J(t + \Delta, t')$. The elastic modulus measured in a typical test corresponds to approximately $\Delta = 0.001$ day. For $\Delta = 10^{-7}$ day, $J(t + \Delta, t') = 1/E_{\mathrm{dyn}}(t')$ where E_{dyn} is the dynamic modulus.

Based on Bažant and Prasannan [1], the secant compliance function is recommended in the following form:

$$J(t, t', \sigma) = q_1 + F(\sigma)C_0(t, t') \tag{2}$$

$$C_0(t, t') = q_2 Q(t, t') + q_3 \ln\left[1 + (t - t')\right] + q_4 \ln\left(\frac{t}{t'}\right) \tag{3}$$

This expression contains four parameters, $q_1, \ldots q_4$, of the dimensions $(\text{stress})^{-1}$, which may be adjusted so as to obtain optimum fit of the given test data. It has been shown that these four parameters, which all appear in a linear form, suffice to achieve excellent fits of measured basic creep curves. The optimum values of these four parameters can be obtained easily by linear regression. The terms containing q_2, q_3 and q_4 represent the ageing viscoelastic compliance, the non-ageing viscoelastic compliance and the flow compliance, respectively. Furthermore, $q_1 = 10^6/E_0$, where E_0 is the asymptotic elastic modulus in psi, characterizing the strain for extremely short load duration, obtained by extrapolating the short-time creep measurements to zero time [3]. Function $C_0(t, t')$ represents the creep compliance.

The non-linear dependence on stress is characterized by the empirical function

$$F(\sigma) = \frac{1 + 3s^5}{1 - \Omega} \qquad s = \frac{\sigma}{f_c'} \tag{4}$$

which only slightly differs from that of Bažant and Prasannan [1]; Ω represents damage at high stress and is taken as $\Omega = s^{10}$. Equation 4 appears to give a very good description of the non-linearity of creep up to stress level $s = 0.6$, and an approximate description, up to the strength limit $s \rightarrow 1$. Creep in the strain-softening range is excluded from consideration.

Function $Q(t, t')$ represents the solution of a simple integral equation, which however cannot be solved exactly in a closed form. A close approximation (with error less than 0.5%) is given by the formula [1]

$$Q(t, t') = Q_f(t')\left[1 + \left(\frac{Q_f(t')}{Z(t, t')}\right)^{r(t')}\right]^{-1/r(t')} \tag{5}$$

with

$$Z(t, t') = (t')^{-1/2} \ln\left[1 + (t - t')^{0.1}\right] \tag{6}$$

$$Q_f(t') = \left[0.086(t')^{2/9} + 1.21(t')^{4/9}\right]^{-1} \tag{7}$$

$$r(t') = 1{\cdot}7(t')^{0.12} + 8 \tag{8}$$

in which t and t' must be given in days, while $J(t, t', \sigma)$ and $C_0(t, t')$ result with the dimension $10^{-6}(\text{psi})^{-1}$ (1 psi = 6895 Pa). The present expression for function $Q_f(t')$ is slightly simpler than that in Bažant and Prasannan [1] but gives equally good results.

It was shown [1] that, for short creep durations, the present formulation asymptotically approaches the double power law, while for very long creep durations, it asymptotically approaches the logarithmic law. It has also been shown that always $\partial^2 J(t, t')/\partial t\, \partial t' \geq 0$, which means that a divergence of creep curves for different ages at loading cannot occur. The flow term (viscous term), associated with q_4, becomes important only for long-time creep of concrete loaded at young age.

The parameters of a Kelvin chain model that closely approximates the present compliance function can be obtained by the explicit formulas given in Bažant and Prasannan [1] (Equations 2–4 and 17 and Table 1 of Part II of [1]).

4. PREDICTION OF MATERIAL PARAMETERS FROM COMPOSITION AND STRENGTH

In the absence of test data, parameters $q_1, \ldots q_4$ need to be estimated on the basis of mix composition and strength of concrete. The following approximate empirical prediction formulae (in which the dimensions of q_1, q_2, q_3 and q_4 are $10^{-6}\,\text{psi}^{-1}$) have been calibrated by simultaneous optimization of the fits of the test data that exist in the literature.

For instantaneous (asymptotic) strain

$$q_1 = \frac{1}{E_0} = \frac{1}{1.5E_{28}} \qquad E_{28} = 57\,000(f_c')^{1/2} \tag{9}$$

where f_c' is the 28-day cylinder strength in psi (1 psi = 6895 Pa) and E_{28} is the conventional elastic modulus at 28 days, which is taken here according to the well-known ACI formula. For ageing viscoelastic strain

$$q_2 = 0.011(w/c)^{0.8}c^{1.5}(1 - a/\rho_c)^{-0.9} \times (0.001 f_c')^{-0.5}(s/g)^{0.02} - 0.39 \quad (10)$$

where a, c, s, g and w are the specific contents in lb ft^{-3} (1 lb $\mathrm{ft}^{-3} = 16.02\,\mathrm{kg\,m^{-3}}$) of aggregate, cement, sand, gravel and water, respectively, and ρ_c is the unit mass of concrete in lb ft^{-3}. For non-ageing viscoelastic strain

$$q_3 = \alpha q_2 \qquad \alpha = \begin{cases} 0.0003c + 0.0125 & \text{for } c \geq 26\,\mathrm{lb\,ft^{-3}}\ (416\,\mathrm{kg\,m^{-3}}) \\ 0.001c - 0.005 & \text{for } 15 \leq c \leq 26\,\mathrm{lb\,ft^{-3}} \\ 0.01 & \text{for } c \leq 15\,\mathrm{lb\,ft^{-3}}\ (240\,\mathrm{kg\,m^{-3}}) \end{cases} \quad (11)$$

For ageing viscous strain (flow)

$$q_4 = 0.072(w/c)^{2.3}c^{0.2}(1 - a/\rho_c)^{0.39} \times (0.001 f_c')^{-0.46}(s/g)^{-0.73} \quad (12)$$

When s/c or g/c is less than unity, one must reset s/c or g/c as 1 in Equations 10–12.

According to Equations 9–12, the instantaneous (asymptotic) compliance characterized by q_1 depends only on the strength of concrete, as in the ACI formula. The ageing viscoelastic compliance characterized by q_2 reflects the fact that an increase of the water/cement ratio or the specific cement content engenders an increase of creep, particularly of the ageing viscoelastic strain. The non-ageing viscoelastic compliance characterized by q_3 exhibits the same trends, except for a somewhat different influence of the specific cement content. The flow compliance characterized by q_4 reflects the fact that an increase of strength tends to increase the flow component of creep. According to Equations 9–12, creep increases with an increase of the water/cement ratio and of the specific cement content. Also, creep generally decreases with an increase of strength and of the weight fraction of the aggregate a/ρ_c. The influence of the sand/gravel ratio is complicated but relatively small, as revealed by data fitting.

It must be also emphasized that the aforementioned influences are not independent of each other; for example, a change in the water/cement ratio of course causes a change in the strength of concrete. Such interrelations are captured by the foregoing formulae only in a very crude manner. It is interesting that empirical data fitting indicated for parameter q_4 (characterizing flow) an opposite influence of strength to that of q_2 and q_3, but this is only true for a constant water/cement ratio; if one recognizes that an increase of strength requires a decrease in the water/cement ratio, then the influence of concrete strength on flow appears to be the same as on the other creep components because the exponent of w/c in Equation 12 is larger than that in Equation 10.

5. PREDICTION IMPROVEMENT BASED ON SHORT-TIME DATA

As documented by previous studies, complete prediction of concrete creep from the mix composition and strength of concrete inevitably involves very large errors. The predictions are greatly improved if at least some short-time measurements are available.

A significant improvement of prediction is achieved if at least the elastic modulus is measured. In that case, one first predicts parameters $q_1, \ldots q_4$ from the foregoing formulae, and then replaces them with

$$\alpha_1 q_1, \alpha_1 q_2, \alpha_1 q_3, \alpha_1 q_4 \quad (13)$$

where the multiplier α_1 is determined so as to match the measured elastic strain.

If short-time creep measurements of 7 to 28 days are available, a still greater improvement of prediction is possible. In that case, one again calculates parameters $q_1, \ldots q_4$ from the foregoing formulae, and then replaces them with

$$\alpha_1 q_1, \alpha_2 q_2, \alpha_2 q_3, \alpha_2 q_4 \quad (14)$$

where the multipliers α_1 and α_2 are two unknown parameters which must be determined so as to give the best fit of the measured short-time creep data. This is a problem of linear regression, which is easy to carry out.

6. COMPARISON WITH TEST DATA

The foregoing formulae have been used to fit collectively 17 different comprehensive data sets for basic creep under uniaxial compression, existing in the literature [4–20]. The method of optimizing the data fits was the same as that used in Part 1 for shrinkage and explained in detail in Part VI of [21]. However, the optimization was much easier to carry out because, in contrast to [21], the unknown parameters $q_1, \ldots q_4$ appear linearly in the present prediction model, so that the optimization consists of linear regression.

The predictions based on concrete strength and composition are shown in Figs 1–4 as the solid curves, and the test results as the data points. The statistical scatter of the deviations of the present predictions (the solid curves) from the (hand-smoothed) measured creep curves is characterized by the value of the coefficient of variation of these deviations, which is listed for each item of data in Table 1.

For comparison, Figs 5–7 also show the optimum fits with the present compliance function when the present formulae for the effect of strength and composition are ignored. These fits are excellent, which confirms that the mathematical form of the present compliance function is correct, and that the greatest error in the prediction arises from the influence of the mix composition and strength of concrete.

It may be noted that much of the disagreement with the data of L'Hermite *et al.* [17] seen in Fig. 1 is due to the poor prediction of the elastic modulus of this particular

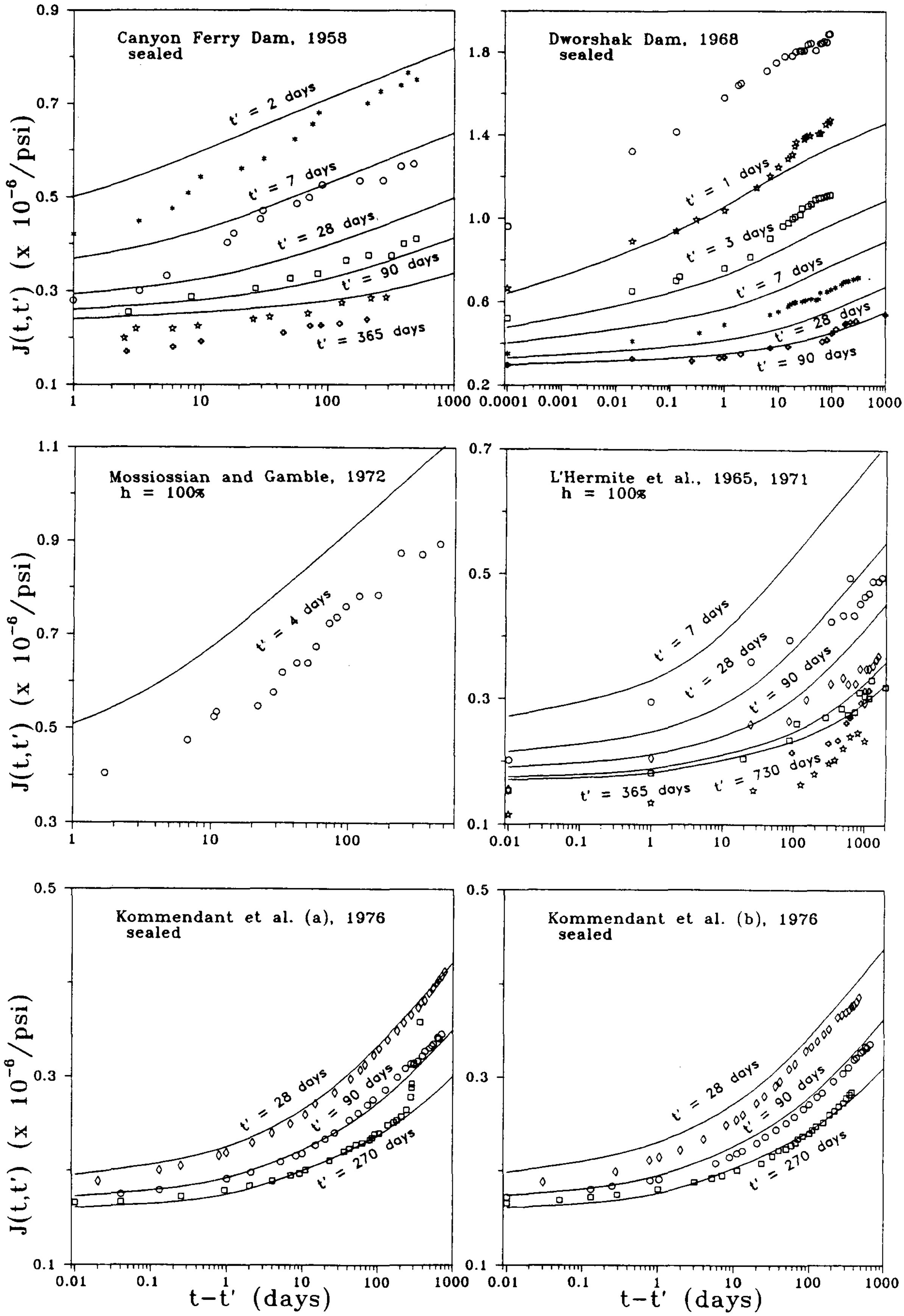

Fig. 1 Predictions of basic creep and data by Hansen and Harboe (Canyon Ferry Dam), Pirtz (Dworshak Dam), Mossiossian and Gamble, L'Hermite *et al.*, and Kommendant *et al.*

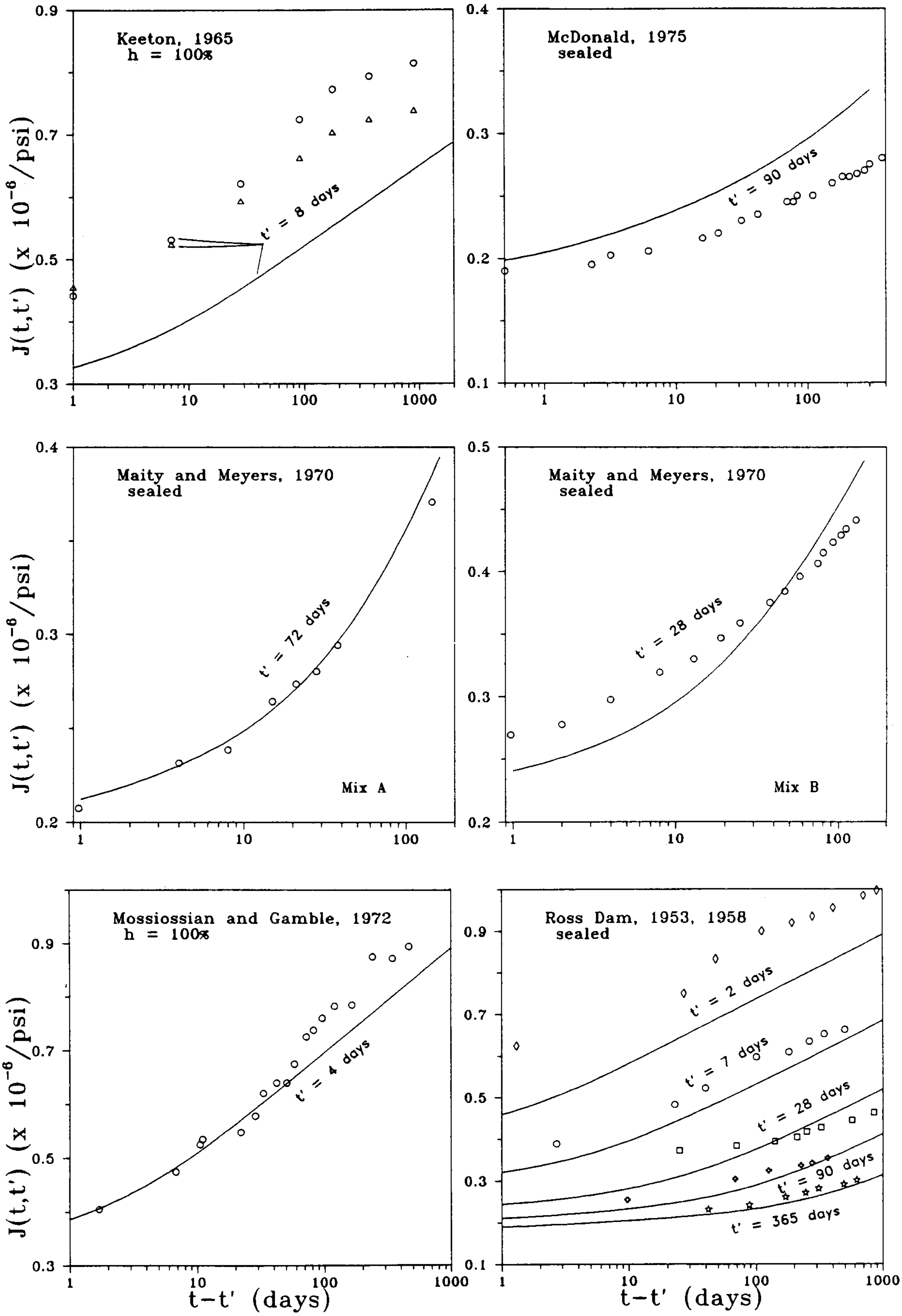

Fig. 2 Predictions of basic creep and data by Keeton, McDonald, Maity and Meyers, Mossiossian and Gamble, and Hansen and Harboe (Ross Dam).

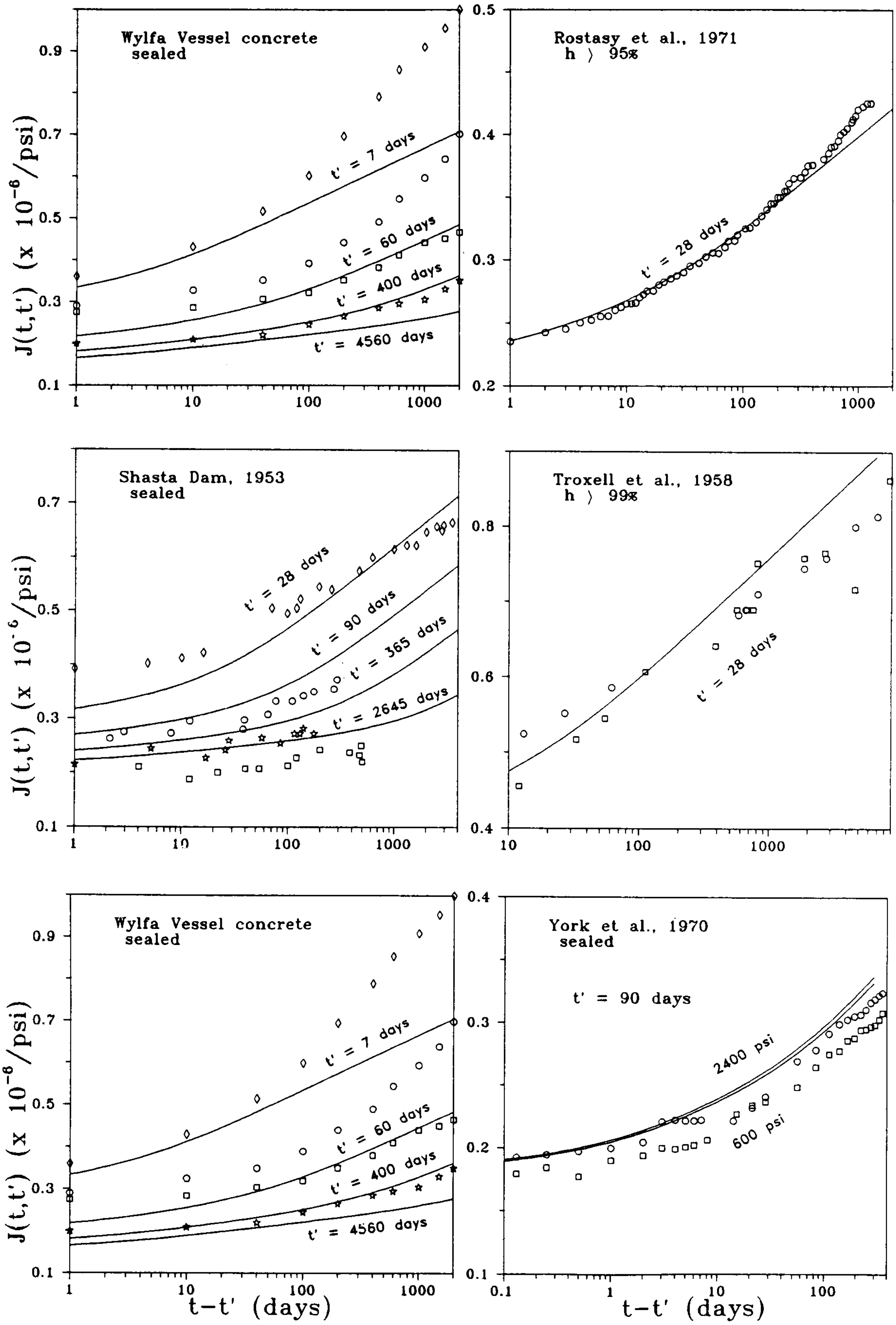

Fig. 3 Predictions of basic creep and data by Ross, Rostasy *et al.*, Hansen and Harboe (Shasta Dam), Browne and co-workers (Wylfa Vessel) and York *et al.*

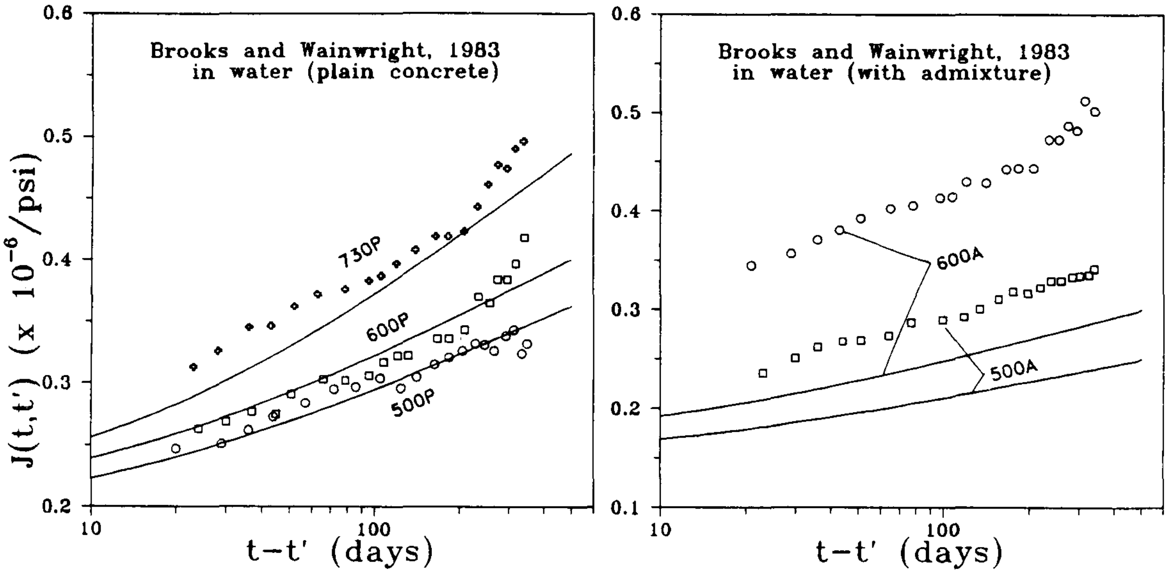

Fig. 4 Predictions of basic creep and data by Brooks and Wainwright.

concrete from concrete strength and composition by ACI formula (Equation 9). The reason that the early age creep curves in Fig. 1 for Dworshak Dam [13] are fitted poorly is probably due to the fact that pozzolan caused a slower-than-usual strength development, especially for times $t' = 1$ and 3 days (the effect of pozzolan is not captured by the Equations 9–12). It may further be noted that in the data of Brooks and Wainwright [22] (Fig. 4) only the basic creep of normal concrete was predicted well, while the creep of concrete with admixtures was underestimated. Since again Equations 9–12 do not give the effect of admixtures, the data for concrete with admixtures was excluded from the calculation of the coefficient of variation. Similarly, the effects of fly ash and silica fume are not covered by Equations 9–12, and further research is needed. However, the basic formulations (Equations 1–8) are valid for concretes with admixtures, pozzolans, fly ash or silica fume and give good results if the adjustment in Equation 14 is made, on the basis of short-time tests.

Table 1 Coefficients of variation for deviation of formulae from hand-smoothed data for basic creep

Test data	$\bar{\omega}$
Browne and co-workers [4–6] (Wylfa Vessel)	23.7
Brooks and Wainwright [22]	4.8
Hansen [8] and Harboe *et al.*, [9] (Canyon Ferry Dam)	18.2
Hansen [8] and Harboe *et al.* [9] (Shasta Dam)	12.6
Hansen [8] and Harboe *et al.* [9] (Ross Dam)	10.7
Gamble and Thomass [7]	26.8
Keeton [16]	24.5
Kommendant *et al.* [10]	3.8
L'Hermite *et al.* [17]	34.0
Maity and Meyers [11]	6.6
McDonald [18]	17.9
Mossiossian and Gamble [12]	6.9
Pirtz [13] (Dworshak Dam)	25.5
Ross [14]	10.5
Rostasy *et al.* [15]	3.5
Troxell *et al.* [19]	5.3
York *et al.* [20]	10.9
All data	$\bar{\omega}_{\text{all}} = 17.2$

One difficulty in interpreting test data as reported by various authors is the lack of a clear and uniform definition of the initial 'elastic' (or 'instantaneous') strain that is substracted from the total strain measurements to get the creep part. The value of Δ (below Equation 1) in various tests reported in the literature probably ranged from 0.1 s to 0.1 day. Private correspondence with some of the authors helped to resolve the question, but in many cases this trivial problem rendered potentially valuable test data useless. For structural creep analysis, it is only the sum of elastic and creep strains which matters, and test results should always be reported in this manner.

7. APPROXIMATE EQUIVALENCE WITH LOG-DOUBLE-POWER LAW

A very short formula for the compliance function is the log-double-power law [23]:

$$J(t,t') = \frac{1}{E_0} + \frac{\psi_0}{E_0} \ln\{1 + \psi_1[(t')^{-m} + \alpha](t-t')^n\} \quad (15)$$

where E_0, ψ_0, ψ_1, m, n and α are material parameters. This law is nearly as good in data-fitting as the present formulation but has several disadvantages (the possibility of a limited divergence of creep curves, the unavailability of explicit formulae for conversion to the rate-type creep law, and the impossibility of identifying the material parameters by linear regression). The short form of Equation 15 is nevertheless appealing. Therefore, a Leverberg–Marquardt optimization subroutine has been used to calculate Table 2, presenting the values of parameters q_2, q_3, q_4 that give (for the ranges 5 days $\le t' \le$ 5000 days, 1 day $\le (t-t') \le$ 10 000 days) the least-squares approximations of Equation 15 for various combinations

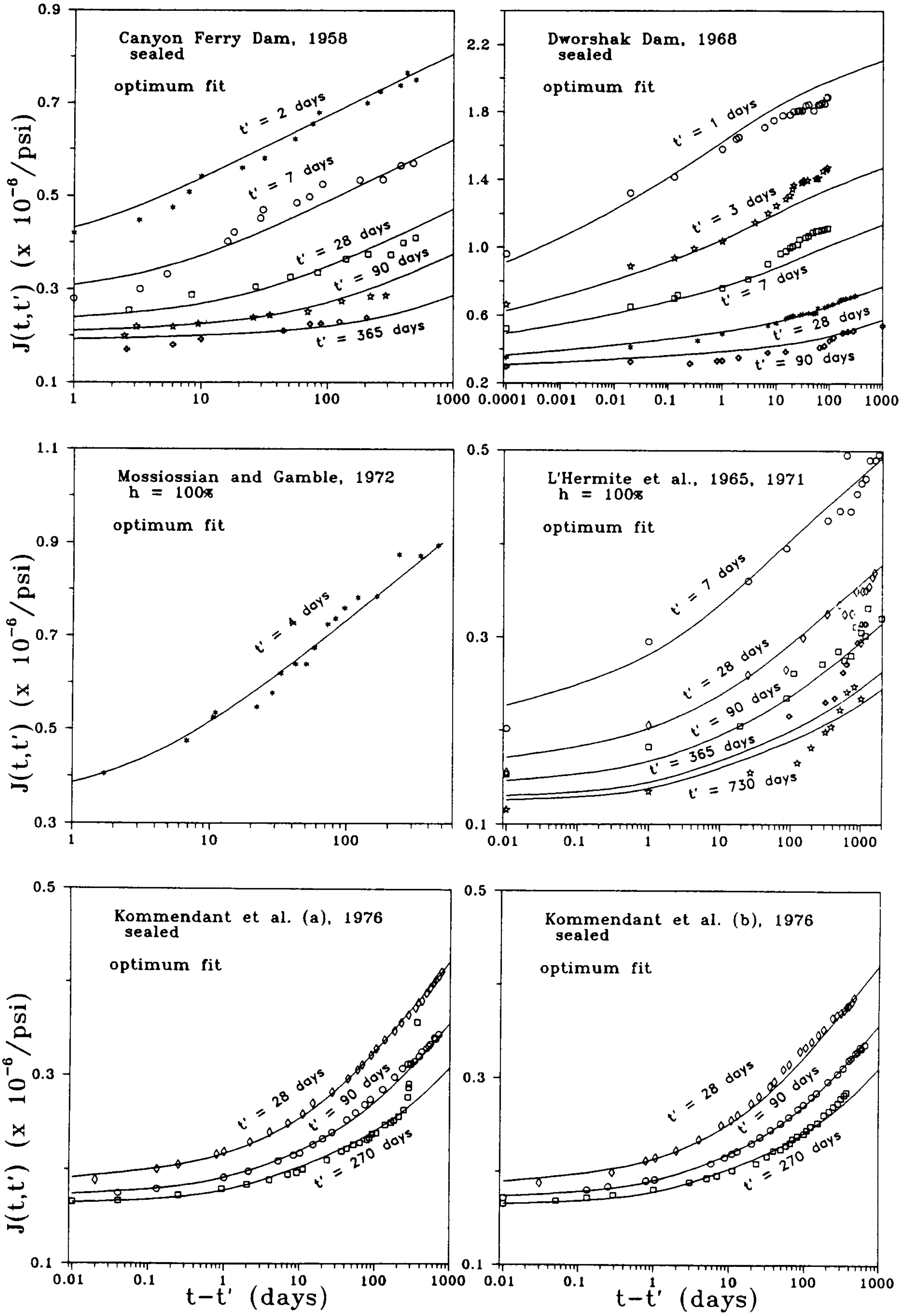

Fig. 5 Optimum fits of basic creep and data by Hansen and Harboe (Canyon Ferry Dam), Pirtz (Dworshak Dam), Mossiossian and Gamble, L'Hermite *et al.*, and Kommendant *et al.*; four parameters are optimized by linear regression.

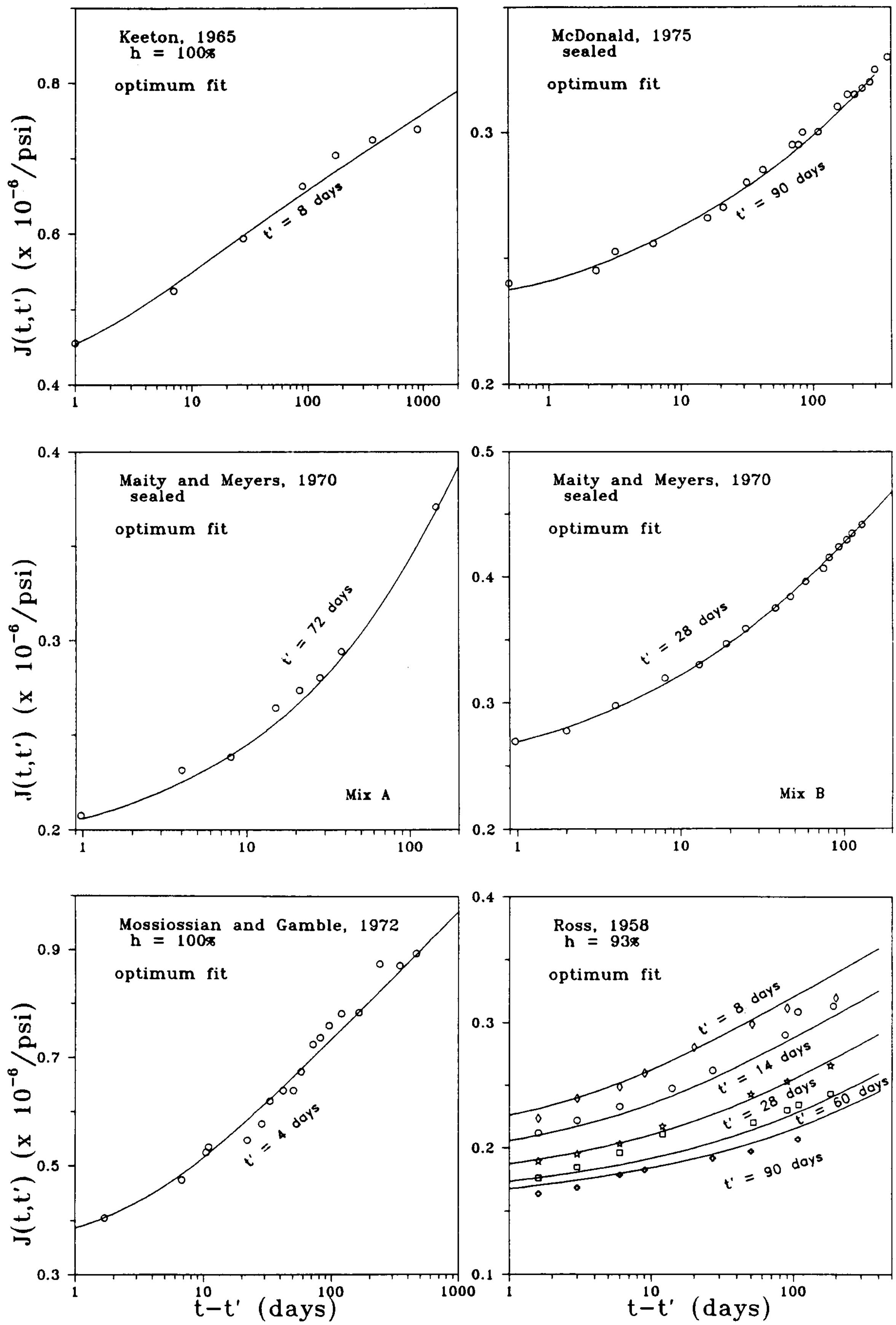

Fig. 6 Optimum fits of basic creep and data by Keeton, McDonald, Maity and Meyers, Mossiossian and Gamble, and Hansen and Harboe (Ross Dam); four parameters are optimized by linear regression.

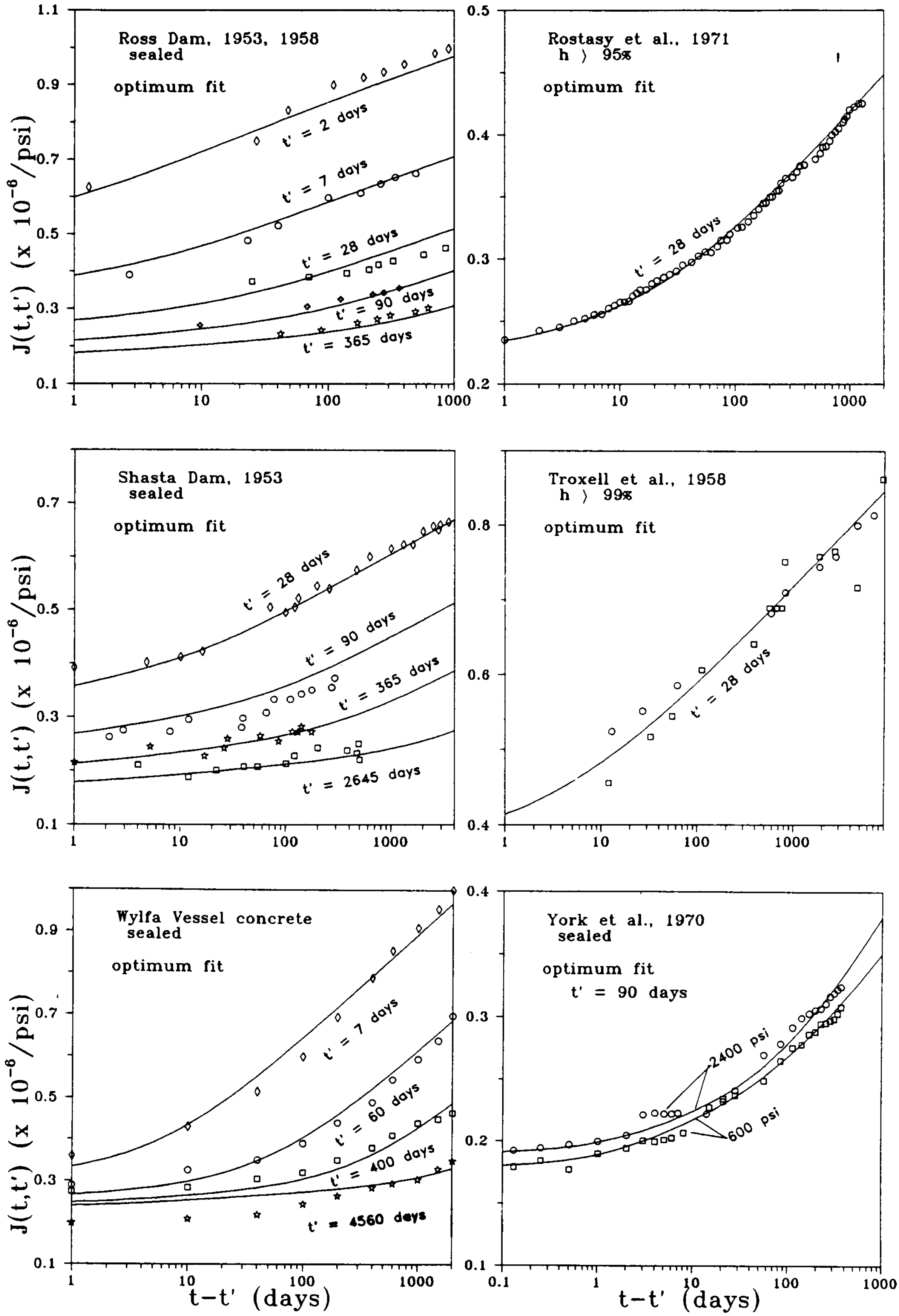

Fig. 7 Optimum fits of basic creep and data by Ross, Rostasy *et al.*, Hansen and Harboe (Shasta Dam), Browne and co-workers (Wylfa Vessel) and York *et al.*; four parameters are optimized by linear regression.

Table 2 Coefficients q_2, q_3 and q_4 of the present compliance function that gives the least-squares approximation of log-double-power law, and coefficients of variation of errors

	$q_2(\times 10^{-6})$					$q_3(\times 10^{-6})$				
ψ_2 \ ψ_0	1.0	1.5	2.0	2.5	3.0	1.0	1.5	2.0	2.5	3.0
0.1	0.35	0.41	0.46	0.49	0.52	0.013	0.014	0.015	0.015	0.015
0.2	0.30	0.37	0.41	0.45	0.48	0.010	0.012	0.013	0.013	0.014
0.3	0.26	0.33	0.38	0.42	0.45	0.009	0.010	0.011	0.012	0.012
0.4	0.23	0.30	0.35	0.39	0.42	0.007	0.009	0.010	0.011	0.011
0.5	0.21	0.27	0.32	0.36	0.39	0.006	0.008	0.009	0,010	0.010
0.6	0.19	0.25	0.30	0.34	0.37	0.006	0.007	0.008	0.009	0.009
0.7	0.17	0.23	0.28	0.32	0.35	0.005	0.006	0.007	0.008	0.009

	$q_4(\times 10^{-6})$					Coefficient of variation (%)				
ψ_2 \ ψ_0	1.0	1.5	2.0	2.5	3.0	1.0	1.5	2.0	2.5	3.0
0.1	0.028	0.043	0.056	0.069	0.080	3.1	3.4	4.0	4.7	5.5
0.2	0.028	0.042	0.055	0.067	0.078	2.8	3.4	4.1	4.9	5.8
0.3	0.028	0.042	0.054	0.066	0.077	2.7	3.5	4.3	5.2	6.1
0.4	0.028	0.041	0.053	0.064	0.075	2.7	3.6	4.5	5.5	6.4
0.5	0.028	0.040	0.052	0.063	0.073	2.7	3.7	4.7	5.7	6.7
0.6	0.027	0.039	0.051	0.062	0.072	2.8	3.9	4.9	5.9	6.9
0.7	0.026	0.039	0.050	0.060	0.070	2.9	4.1	5.1	6.2	7.1

of the values of parameters ψ_0 and ψ_1 ($m = 0.5$, $n = 0.3$ and $\alpha = 0.03$ have been assumed). For both cases q_1 and $1/E_0$ have also been assumed as 0.16×10^{-6}. The range of ψ_0 is between 1 and 3. As for the value of ψ_1, the following simple function has been verified:

$$\psi_1 = (0.31\psi_0 + \psi_2)^{-1} \tag{16}$$

where ψ_2 = empirical parameter which is between 0.1 and 0.7.

The errors of these approximations within the ranges 5 days $\leq t' \leq 5000$ days, 1 day $\leq (t - t') \leq 10\,000$ days are characterized by their coefficients of variation listed also in the table. As we see, the errors are small, but not very small.

APPENDIX: Basic information on basic creep test data used

Browne and co-workers [4–6] (for Wylfa Vessel concrete). Cylinders 6 in. × 12 in. (152 mm × 305 mm), sealed, at 20°C; water:cement:sand:gravel ratio 0.42:1:1.45:2.95. Ordinary Portland cement and crushed limestone aggregate of max. size 1.5 in. (38 mm); 28-day average cube (6 in., 152 mm) strength 7250 psi (50 N mm^{-2}). Measured were also creep curves for $t' = 28$ and 180 days which were excluded from analysis because they exhibit an increase rather than a decrease of creep with increasing t'. Axial compressive stress 2116 psi (14.6 N mm^{-2}).

Brooks and Wainwright [22]. Cylinders 76 mm × 255 mm. After demoulding at the age of 1 day, specimens cured in water at 20 ± 2°C and tested at the age of 28 days in water. Ordinary Portland cement and North Nottinghamshire quartzite coarse aggregate of max. size 10 mm. Initial stress/strength ratio = 0.25 of the creep specimen cylinder strength. Five mixes with designations 500P, 500A, 600P, 600A, 730P were used, with cement contents 520, 535, 608, 628, 725 kg m^{-3}; aggregate/cement ratios 3.3, 3.3, 2.6, 2.6, 2.0 (by weight); contents of fines 28, 28, 22, 22, 10%; water/cement ratios 0.36, 0.27, 0.34, 0.27, 0.3; and admixture contents 0, 1.5, 0, 1.3, 0% of weight of cement, respectively (admixture trade name: Irgament Mighty 150).

Hansen [8] and *Harboe* et al. [9] (for Canyon Ferry Dam). Cylinders 6 in. × 16 in. (152 mm × 406 mm) sealed at 70°F (21°C), 28-day cylinder strength = 2920 psi (20.1 N mm^{-2}); cement type II; max. size of aggregate 1.5 in. (38 mm); water:cement:sand:coarse aggregate ratio = 0.5:1:2.87:10.37. Axial compressive stress ≤ 1/3 of 28-day cylinder strength.

Hansen [8] and *Harboe* et al. [9] (for Shasta Dam). Cylinders 6 in. × 26 in. (152 mm × 660 mm) sealed at 70°F (21°C), 28-day cylinder strength = 3230 psi (22.3 N mm^{-2}); cement type IV; max. size of aggregate 0.75 to 1.5 in. (19–38 mm); water:cement:sand:coarse aggregate ratio = 0.58:1:2.5:7.1. Also measured was short-time creep for $t' = 2$ days: $J(t,t') = 1.362$, 1.386×10^{-6} psi at $t - t' = 12.7$ and 19 days, respectively, and for $t' = 7$ days: $J(t,t') = 0.712$, 0.718, 0.783, 0.735, 0.798, 0.754, 0.810, 0.824, 0.843, 0.819×10^{-6} psi^{-1} at $t - t' =$ 2.8, 17.5, 18, 25, 27, 30, 42, 52, 67, 79 days, respectively (1 psi^{-1} = 145 N^{-1} m^2). These data were not fitted because the early strength development was unusually slow (cement type IV). Axial compressive stress $\doteq$ 1/3 of 28-day cylinder strength.

Hansen [8] and *Harboe* et al. [9] (for Ross Dam). Cylinders 6 in. × 16 in. (152 mm × 406 mm) sealed at 70°F (21°C), 28-day cylinder strength = 4970 psi (34.3 N mm^{-2}); cement type I, content 221 kg m^{-3}, max. size of aggregate 1.5 in. (38 mm); water:cement:sand: coarse aggregate ratio = 0.56:1:2.73:7.14. Axial compressive stress = 1/3 of 28-day cylinder strength.

Gamble and Thomass [7]. Cylinders 4 in. × 10 in. (102 mm × 254 mm) tested at 94% relative humidity and 75°F (24°C), 28-day cylinder strength = 4850 psi (33.4 N mm^{-2}); cement type I; max. size of aggregate 3/16 in. (4.76 mm); water:cement:sand:coarse aggregate ratio = 0.7:1:2.04:3.06. Axial compressive stress = 0.36 of 28-day cylinder strength.

Keeton [16]. Cylinders 3 in. × 9 in. (76 mm × 229 mm) and 6 in. × 18 in. (152 mm × 457 mm) at 100% relative humidity and 73°F (23°C), 28-day cylinder strength = 6550 psi (45.2 N mm^{-2}); Portland cement type III, content 451.2 kg m^{-3}, max. size of aggregate 0.75 in. (19 mm); water:cement:sand:coarse aggregate ratio = 0.457:1:1.66:2.07. Axial compressive stress = 30% of the compressive strength of the specimens.

Kommendant et al. [10]. Cylinders 6 in. × 16 in. (152 mm × 305 mm) sealed at 73°F (23°C), 28-day cylinder strengths = 6590 psi (45.4 N mm^{-2}) and 6700 psi (46.2 N mm^{-2}); cement Medusa type II; max. size of aggregate 1.5 in. (38 mm). Water:cement:sand:coarse aggregate ratios = 0.38:1:1.73:2.61 and 0.38:1:1.65:2.38, respectively. Axial compressive stress = 32% of 28-day strength.

L'Hermite et al. [17]. Prisms 7 cm × 7 cm × 28 cm cured in water. French type 400/800 cement, content 350 kg cm^{-3}; water:cement:sand:coarse aggregate ratio = 0.49:1:1.75:3.07, 28-day strength 370 kg cm^{-2} (34.8 N mm^{-2}), Seine gravel (siliceous calcite). Axial compressive stress 1315 psi (9.1 N mm^{-2}).

Maity and Meyers [11]. Mix A: prisms 14 in. × 3.5 in. × 3.5 in. (356 mm × 89 mm × 89 mm), sealed. 70°F (21°C). 13-day prism strength 4350 psi (30 N mm^{-2}). Portland cement of type III. Applied load ~40% of prism strength. Water:cement:sand:gravel ratio = 0.85:1:3.81:3.81 by weight. Crushed limestone aggregate; local quartz sand (from different batches for mixes A and B). Mix B: same as mix A except: 12-day cylinder strength 5200 psi (35.9 N mm^{2}). Applied load ~35% of cylinder strength.

McDonald [18]. Cylinders 6 in. × 16 in. (152 mm × 406 mm), sealed at 73°F (23°C), 28-day cylinder strength = 6300 psi (43.4 N mm^{2}); cement type II, content 404 kg m^{-3}, limestone, max. size of aggregate 0.75 in. (19 mm); water:cement:sand:coarse aggregate ratio = 0.425:1:2.03:2.62. Axial compressive stress 2400 psi (16.6 N mm^{-2}).

Mossiossian and Gamble [12]. Cylinders 6 in. × 12 in. (152 mm × 305 mm). At 100% relative humidity and 70°F (21°C), 29-day cylinder strength = 7160 psi (49.4 N mm^{-2}); content 418 kg m^{-3} of cement type III, max. size of aggregate 1 in. (25.4 mm); water:cement sand: coarse aggregate ratio = 0.49:1.35:2.98. Axial compressive stress 1/3 of cylinder strength.

Pirtz [13] (for Dworshak Dam). Cylinders 6 in. × 18 in. (152 mm × 457 mm) sealed at 70°F (21°C), 28-day cylinder strength = 2080 psi (14.33 N mm^{-2}); mix with 196.7 kg m^{-3} of cement type II and 68 kg m^{-3} of pozzolan. Granite–gneiss aggregate with max. size 1.5 in. (38 mm); water:(cement $\pm$ pozzolan):sand:coarse aggregate ratio = 0.56:1:2.79:4.42. Axial compressive stress $\leq$ 1/3 of cylinder strength.

Ross [14]. Cylinders 4.63 in. × 12 in. (118 mm × 305 mm), stored at 17°C and 93% relative humidity (not exactly basic creep, but close to it, especially for short times), 28-day cube strength = 9600 psi (66.1 N mm^{-2}); rapid-hardening Portland cement. Water:cement:sand: coarse aggregate ratio = 0.375:1:1.6:2.8.

Rostasy et al. [15]. Cylinders 20 cm × 140 cm at relative humidity ≥ 95% and 20°C; 28-day cube strength = 455 kg cm^{-2} (44.6 N mm^{-2}); Rhine gravel and sand, max. size of aggregate 30 mm; water:cement:sand;coarse aggregate ratio = 0.41:1:2.43:3.15. Axial compressive stress 94.7 kg cm^{2} (9.3 N mm^{-2}).

Troxell et al. [19]. Cylinders 4 in. × 14 in. (102 cm × 356 cm) at 70°F (21°C), 28-day cylinder strength = 2500 psi (17.2 N mm^{-2}); granite aggregate, max. size of aggregate 1.5 in. (38 mm); cement type I; water:cement: sand:coarse aggregate ratio = 0.59:1:2:3.67. Axial stress 32% of 28-day cylinder strength.

York et al. [20]. Cylinders 6 in. × 16 in. (152 mm × 406 mm), sealed, 75°F (24°C); 28-day cylinder strength = 6200 psi (42.8 N mm^{-2}); content 404 kg m^{-3} of Portland cement type II; max. size of aggregate 0.75 in. (19 mm); water:cement:sand:coarse aggregate ratio = 0.425:1: 2.03:2.62. Axial compressive stress 2400 psi (16.6 N mm^{-2}).

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RESUME

Modèle amélioré de prédiction des déformations du béton en fonction du temps: 2ème partie – Fluage de base

Dans le second volet de cette série on présente les formules de prédiction du fluage de base du béton, c'est à dire du fluage en confinement. Les formules donnent la fonction sécante de compliance uniaxiale qui dépend du niveau de contrainte, et comme cas particulier, la fonction de compliance pour l'analyse structurelle linéaire suivant le principe de superposition. Les formules s'appuient sur une théorie de la solidification récemment établie pour le fluage du béton, qui prend en compte le vieillissement simultané, satisfait à toutes les exigences thermodynamiques de base, et évite la divergence des courbes de fluage. Les formules, qui décrivent aussi bien le fluage que les propriétés élastiques, comprennent seulement quatre paramètres indépendants du matériau libre. Les quatre apparaissent de façon linéaire, en sorte qu'on peut obtenir les ajustements de données optimaux par régression linéaire. Dans les situations fréquentes où l'on ne dispose pas de données d'essai pour un béton particulier à utiliser, on donne des formules empiriques de prédiction de ces quatre paramètres à partir de la composition du mélange de béton et de la résistance à la compression normale. Cependant, ces formules entraînent une erreur importante. Pour l'éviter, il convient, autant que possible, de réaliser des mesures du module d'élasticité et, si possible, du fluage à court terme d'une durée de 7 à 28 jours. Avec ces mesures, les prédictions se trouvent considérablement améliorées. On compare les prédictions avec 17 séries importantes de données prises dans la littérature, et on trouve que les coéfficients de variation des déviations sont plus petites qu'avec les modèles précédents.