# Improved Test of the Equivalence Principle for Gravitational Self-Energy

S. Baeßler, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, U. Schmidt, and H. E. Swanson
*Department of Physics, University of Washington, Seattle, Washington 98195-1560*
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The lunar-ranging test of the equivalence principle for gravitational self-energy is ambiguous. Although the Earth has more gravitational self-energy than the Moon, its sizable Fe/Ni core also gives it a different composition than the Moon. We removed this ambiguity by comparing, in effect, the accelerations of "miniature" earths and moons toward the Sun. Our composition-dependent Earth-Moon acceleration, $\Delta a_{\rm CD}/a_{\rm s} = (+0.1 \pm 2.7 \pm 1.7) \times 10^{-13}$, and lunar-ranging data provide an unambiguous test at the $1.3 \times 10^{-3}$ level.

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Although general relativity, Einstein's elegant theory of gravity, has passed all experimental tests [1,2], it is a classical theory that cannot be quantized, and much theoretical effort is now devoted to developing a realistic quantum theory that would reduce to Einstein's theory in the appropriate limit [3]. General relativity has an exact symmetry, the equivalence principle (EP), that is expected to be violated to some degree by any quantum theory of gravity and by many alternative classical theories as well. The violation is linked to the fact that these theories have scalar gravitational fields in addition to the usual tensor field. Precision tests of the EP could therefore provide direct evidence for new gravitational phenomena. Recent cosmological data on distant Type Ia supernovae [4,5] that suggest a repulsive gravitational effect provide additional motivation for subjecting the theory to stringent experimental tests.

The most precisely tested manifestation of the EP is the universality of free fall (UFF), the prediction that *all* bodies in a uniform gravitational field have exactly the same gravitational acceleration. The UFF has been tested to roughly 1 part in $10^{12}$ [6–8] using laboratory test bodies in the gravitational fields of the Earth and the Sun. But, because gravitational self-energy is negligible for any laboratory-scale object, these experiments cannot address a crucial issue, whether gravitational self-energy obeys the EP. The importance of this issue was emphasized by Nordtvedt [9], who showed that theories with more than one metric field (which naturally respect the EP for laboratory-size bodies) nevertheless predict violations of the EP for gravitational self-energy. Nordtvedt noted that this could be tested by using lunar laser-ranging (LLR) to compare the accelerations of the Earth and Moon toward the Sun; the Earth and Moon are sufficiently massive that gravitational self-energy reduces their masses by 4.6 and 0.2 parts in $10^{10}$, respectively.

However, as noted by Nordtvedt, the LLR measurement of $\Delta a_{\rm LLR} = a_{\rm e} - a_{\rm m}$ ($a_{\rm e}$ and $a_{\rm m}$ are the accelerations of Earth and Moon toward the Sun) is ambiguous because, from the point of view of the EP test, the Earth and Moon "test bodies" differ in two significant ways. The Earth has a greater fraction of gravitational self-energy than the Moon, and it also has a different composition (its core gives the Earth a larger Fe/Ni content than the Moon). The first difference allows LLR to probe the EP for gravitational self-energy (the strong EP), while the composition difference makes LLR sensitive also to the EP for nongravitational forms of energy (the weak EP). The EP parameter associated with gravitational self-energy alone is

$$\eta_{\rm SEP} = \Delta a_{\rm SEP}/a_{\rm s} = (\Delta a_{\rm LLR} - \Delta a_{\rm CD})/a_{\rm s}\,, \quad (1)$$

where $\Delta a_{\rm SEP}$ and $\Delta a_{\rm CD}$ are differential accelerations due to violation of the strong EP and to composition-dependent interactions, respectively, and $a_{\rm s} = (a_{\rm e} + a_{\rm m})/2 = 0.593\ {\rm cm/s^2}$. The best current values for $\Delta a_{\rm LLR}/a_{\rm s}$ and their "realistic uncertainties" come from two analyses of the same data which yield

$$\begin{aligned} \Delta a_{\rm LLR}/a_{\rm s} &= (+3.2 \pm 4.6) \times 10^{-13} \quad (\text{Ref. [10]})\,, \\ \Delta a_{\rm LLR}/a_{\rm s} &= (-3.6 \pm 4.0) \times 10^{-13} \quad (\text{Ref. [11]})\,, \end{aligned} \quad (2)$$

implying an upper bound of *about 1 cm* on the amplitude of the EP-violating distortion of the Moon's orbit. An upper bound on $\Delta a_{\rm CD}$ [8] has been deduced under the assumption of a specific (and now theoretically disfavored) model of a *vector* EP-violating interaction. This paper reports a "model-independent" upper limit on $\Delta a_{\rm CD}$ valid for arbitrary (including *scalar*) EP-violating interactions.

We removed the ambiguity in the LLR test by using a torsion balance to compare, in effect, the accelerations of a "miniature" earth and "miniature" moon toward the Sun. By miniature moon, for example, we mean a test body with a composition very close to that of the actual Moon. A violation of the weak EP would affect the the acceleration of this body just as it would the Moon, but a violation of the strong EP would not contribute to the test body acceleration. Because the Earth's mantle has a composition similar to that of the Moon (see Table I) the Earth-Moon composition difference is dominated by the difference between earth-core and mantle materials. We enhanced our sensitivity by comparing accelerations of "earth's core" (EC) and "moon/mantle" (MM) test bodies. In this case the

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TABLE I. Comparison of EC and MM test-body compositions to those of the Earth's core, Earth's mantle, and Moon. Earth and Moon data taken from Ref. [12]. Percentages refer to mass fractions.

| Element | Earth core | EC body | Mantle | Moon | MM body |
|---|---|---|---|---|---|
| Fe | 89% | 72% | 6.1% | 10% | ... |
| Ni | 5.7% | 9% | ... | ... | ... |
| Cr | ... | 17% | ... | ... | ... |
| Ca | ... | ... | 3.0% | 2.7% | ... |
| S | 5.1% | ... | ... | ... | ... |
| Si | ... | 0.4% | 22% | 21% | 39% |
| Mg | ... | ... | 21% | 20% | 16% |
| O | ... | ... | 45% | 43% | 44% |
| $Z_{\mathrm{mean}}$ | 25.6 | 25.8 | 10.9[a] | 10.8[a] | 11.1 |
| $N_{\mathrm{mean}}$ | 29.2 | 29.6 | 11.0[a] | 10.9[a] | 11.3 |

[a]The Fe/Ni content has been subtracted.

composition-dependent differential acceleration of Earth and Moon toward the Sun is

$$\Delta a_{\mathrm{CD}} = (f_{\mathrm{e}} - f_{\mathrm{m}})\Delta\tilde{a}_{\odot}\,; \tag{3}$$

$f_{\mathrm{e}} = 0.382$ [12] and $f_{\mathrm{m}} = 0.101$ [12] are the fractions of the Earth's and Moon's masses that reside in the Fe/Ni content of their cores and mantles, and $\Delta\tilde{a}_{\odot} = a_{\mathrm{EC}} - a_{\mathrm{MM}}$ where $a_{\mathrm{EC}}$ and $a_{\mathrm{MM}}$ are the accelerations toward the Sun of the EC and MM test bodies.

We performed our test using an upgraded version of the Eöt-Wash II continuously rotating torsion balance. This device, which was described in Ref. [8], uses a co-rotating optical system to monitor the twist of a freely suspended torsion pendulum. The EC test bodies were made from a stainless steel alloy, and were carefully demagnetized before installing them in the pendulum. The MM test bodies contained both quartz and a magnesium alloy. Table I compares the compositions of the test bodies to those of the Earth's core and the Moon. The EC and MM test bodies have the same mass and outer dimensions, and have negligible $\ell = 2$ moments and identical $\ell = 4$ moments. Two EC bodies and two MM bodies were mounted in a two-sided beryllium tray forming (see Fig. 1) a torsion pendulum with vanishing $\ell = 1$, $\ell = 2$, and $\ell = 3$ mass moments. The test bodies, pendulum tray, and mirrors used to detect the pendulum twist were all coated with gold to minimize electrostatic effects.

The performance of the balance was recently enhanced by a number of upgrades including the following. The vacuum was improved to $<10^{-6}$ Torr by adding an ion pump to the rotating instrument. The readout of pendulum twist was improved by replacing the light-emitting diode on the autocollimator with a 780 nm laser and replacing the beam splitter with a polarization-sensitive unit. The gain bandwidth of the turntable rate controller was increased to improve the constancy of the rotation rate. The most recent data were taken with an improved upper attachment of the suspension fiber that substantially reduced the "tilt effect" described below.

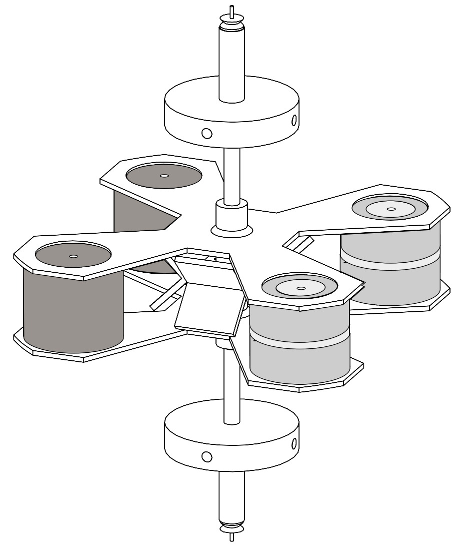

FIG. 1. Torsion pendulum used to measure $\Delta a_{\mathrm{CD}}$. The four cylindrical test bodies are held by the four-bladed arms. The heavily shaded cylinders are the EC bodies; the Mg ($SiO_2$) portions of the MM bodies are lightly (moderately) shaded. The arms and the central axle, are beryllium. The objects on the ends of the axle minimize the mass quadrupole moment of the entire pendulum; they contain small screws that were used to tune out stray mass moments arising from machining imperfections. Any of the four right-angle mirrors situated between the test bodies could be used by the optical system that monitored the pendulum twist.

We rotated the balance at periods ranging from about 1 h to about 36 min (higher frequencies were employed as we improved the turntable). Any differential acceleration of the EC and MM test bodies toward the Sun would produce an approximately sinusoidal torque on the pendulum as a function of turntable angle, and the Sun's daily movement would cause the amplitude and phase of this torque signal to track the Sun's azimuthal projection in the horizontal plane. We acquired equal amounts of data with two opposite orientations of the pendulum in the vacuum vessel by periodically rotating the upper fiber attachment by 180° (this suppressed systematic errors from diurnal temperature fluctuations).

We extracted $\Delta a_{\mathrm{CD}}$ from our data by dividing the data into segments containing exactly 2 revolutions of the apparatus. Three sample segments are shown in Fig. 2. The torque signal in each segment was found by fitting the filtered [13] pendulum twist, $\theta$, as a function of turntable angle, $\phi$, with

$$\theta = \sum_{n=1}^{5}[c_n^{\sin}\sin(n\phi) + c_n^{\cos}\cos(n\phi)] + b_0 + b_1\phi\,, \tag{4}$$

where $b_0$ and $b_1$ accounted for the steady unwinding ($\approx$0.1 $\mu$rad/h) of the suspension fiber. The $n = 1$ harmonic coefficients contain the EP-violating signal, while lab-fixed gravity gradients contribute to all the $c_n$ coefficients. The $c_n^{\sin}$ and $c_n^{\cos}$ coefficients were corrected for tilt, and for attenuation and phase shift from pendulum inertia, electronic time constants and discrete sampling. Tilt was continuously monitored by electronic level sensors mounted on the rotating torsion balance. By deliberately misleveling the apparatus we found that a given tilt twisted the torsion pendulum by an amount proportional to the tilt angle; the proportionality constant was 0.03 (0.006) before (after) the new fiber attachment was installed. Reference [8] gives a detailed description of the tilt correction procedure.

The EP-violating acceleration, $\Delta\tilde{a}_\odot$, was extracted by fitting the corrected coefficients $\tilde{c}_1^{\sin}$ and $\tilde{c}_1^{\cos}$ in terms of a solar signal $\Delta\tilde{a}_\odot$, a quadrature signal pointing 90° away from the Sun $\Delta\tilde{a}_\perp$, and a constant, lab-fixed signal from local gravity gradients and turntable imperfections.

$$\kappa/ms[\tilde{c}_1^{\sin}\sin(\phi) + \tilde{c}_1^{\cos}\cos(\phi)] = \cos\theta_\odot[\Delta\tilde{a}_\odot\sin(\phi_\odot - \phi) + \Delta\tilde{a}_\perp\cos(\phi_\odot - \phi)] + \gamma\sin(\phi_g - \phi), \quad (5)$$

where $\kappa = 0.0316$ erg/rad is the torsional constant of the suspension fiber, $m = 10.0448$ g is a test-body mass, $s = 4.885$ cm is the distance between the centers of adjacent test bodies, and $\gamma$ and $\phi_g$ characterize the signal from local gravity gradients and turntable imperfections. The Sun's altitude, $\theta_\odot$, and azimuth, $\phi_\odot$, were computed at the midpoint of each segment and taken to be constant during that segment. The quantities $\Delta\tilde{a}_\odot$, $\Delta\tilde{a}_\perp$, $\gamma$, and $\phi_g$ were treated as free parameters in each data set. Figures 3 and 4 show the results of measurements extending from June 1998 to April 1999.

Combining the results in Fig. 4, we obtain $\Delta\tilde{a}_\odot = (0.3 \pm 5.6) \times 10^{-13}$ cm/s$^2$ and $\Delta\tilde{a}_\perp = (-0.7 \pm 5.4) \times 10^{-13}$ cm/s$^2$, where the errors are $1\sigma$ statistical uncertainties. This corresponds to

$$\Delta a_{\rm CD}/a_{\rm s} = (+0.1 \pm 2.7 \pm 1.7) \times 10^{-13}, \quad (6)$$

where the first uncertainty is statistical; the second, systematic, uncertainty is discussed below. Note that our combined statistical and systematic errors are slighly smaller than those of the LLR results.

Our systematic error budget, shown in Table II, is dominated by the instrument's sensitivity to gravity gradients. The error from diurnal variations in the local gravity gradient was obtained by multiplying the measured sensitivity to the leading-order $Q_{21}$ gradient, which averaged to $\partial(\Delta a_{\rm CD}/a_{\rm s})/\partial Q_{21} = 6.3 \times 10^{-11}$ cm$^3$/g, by the mean diurnal gradient amplitude measured over two periods totaling three weeks, $Q_{21} < 0.0017$ g/cm$^3$. The techniqes for measuring the sensitivities and gradients are described in Ref. [8].

The sensitivity of our instrument to temperature changes was found by slowly modulating the temperature of the apparatus and observing the induced pendulum twist. We found that while temperature changes had a negligible effect on the twist *calibration,* they did induce a twist

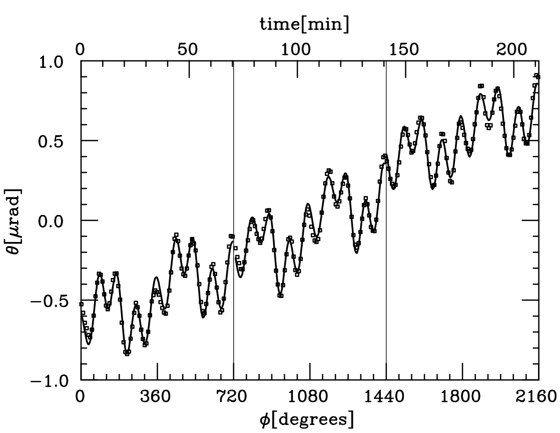


FIG. 2. Sample 2-revolution data segments showing filtered pendulum twist, $\theta$, as a function of turntable angle, $\phi$. The turntable period equaled 2.5 periods of the free torsional oscillation. Data points were taken at a uniform rate giving 100 points per turntable revolution. The smooth curves show the best fits using Eq. (4).

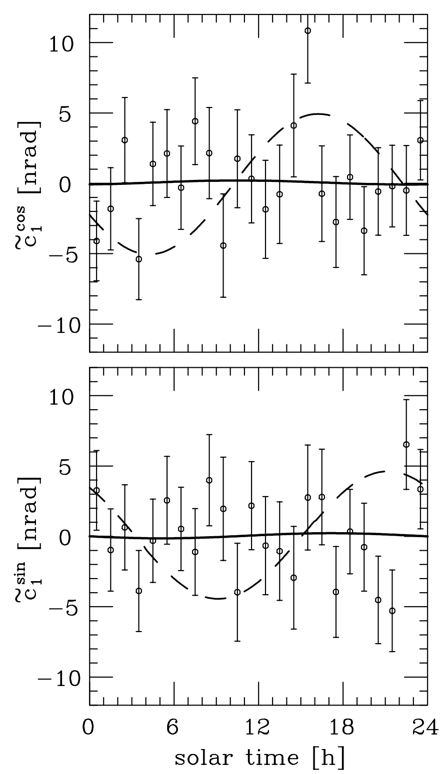


FIG. 3. Cumulative results for $\tilde{c}_1^{\sin}$ and $\tilde{c}_1^{\cos}$ histogrammed as a function of time of day. The solid curves show the mean result of fitting the individual data segments with Eq. (5). Note that the $\tilde{c}_1^{\sin}$ and $\tilde{c}_1^{\cos}$ results are not fitted independently; Eq. (5) gives oscillations in $\tilde{c}_1^{\sin}$ and $\tilde{c}_1^{\cos}$ that have the same amplitude and a 90° phase difference. This is shown by the dashed curves which illustrate the hypothetical signal for $\Delta a_{\rm CD}/a_{\rm s} = 20 \times 10^{-13}$.

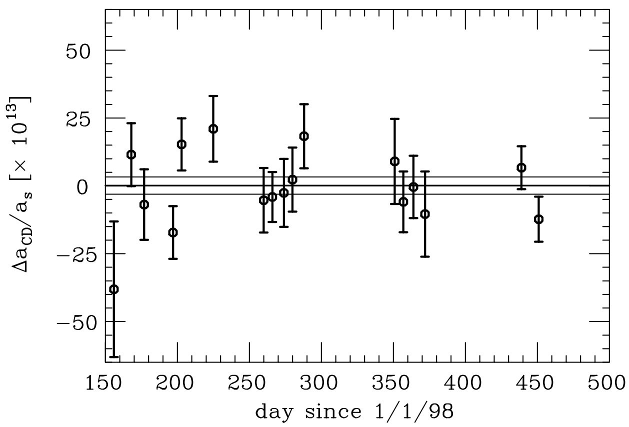


FIG. 4. Each point represents a 4-day measurement of $\Delta a_{\rm CD}$. The periods with no points were spent checking for systematic errors or making improvements to the apparatus. The horizontal lines show the $1\sigma$ statistical plus systematic limits from the combined data.

$\Delta\theta \propto \Delta T$. The thermal systematic error was inferred from the diurnal modulation of the temperature that tracks the Sun seen by two independent sensors closely coupled to the rotating apparatus; this averaged to $-1.3 \pm 4.8$ $\mu$K. The temperature signals from the individual data sets were multiplied by the measured temperature sensitivities for that particular data set to obtain the thermal effect on the pendulum twist. The twist that tracked an EP-violating signal was $\Delta a_{\rm CD}/a_{\rm s} = (0.45 \pm 0.57) \times 10^{-13}$.

Our instrument is sensitive to "tilt." We found that changes in the tilt of the laboratory floor (which ranged from $\approx 1$ $\mu$rad/d in the winter to $\approx 4$ $\mu$rad/d in the summer) had a diurnal component with an average amplitude of $\pm 0.74$ $\mu$rad. The net tilt correction to $\Delta a_{\rm CD}/a_{\rm s}$ was $(-31.3 \pm 0.6) \times 10^{-13}$; the systematic uncertainty follows because our correction procedure applied to deliberately tilted runs reduced the tilt feedthrough by a factor of 55.

The magnetic systematic error was obtained by monitoring the ambient $B$ field with a 3-axis flux-gate magnetometer. The magnetic systematic error is the product of the upper limit on diurnal fluctuations of $B$ (the least restrictive limit, $12 \pm 16$ nT, was on vertical fields) times the instrument's sensitivity to such fields, $\partial(\Delta a_{\rm CD}/a_{\rm s})/\partial B < 1.4 \times 10^{-7}$ T$^{-1}$. The total systematic error was obtained by combining the individual errors in quadrature.

TABLE II. Systematic error budget for $\Delta a_{\rm CD}/a_{\rm s}$.

| Source | Error ($\times 10^{13}$) |
|---|---|
| Diurnal gravity gradients | 1.1 |
| Diurnal temperature variations | 1.0 |
| Diurnal tilt | 0.6 |
| Diurnal magnetic effects | 0.05 |
| Total | 1.7 |

When the LLR results are combined with our data, the resulting, "loop-hole free" values for the strong EP parameter are

$$\eta_{\rm SEP} = (+3 \pm 6) \times 10^{-13}, \qquad \eta_{\rm SEP} = (-4 \pm 5) \times 10^{-13}, \tag{7}$$

from the LLR analyses of Refs. [10] and [11], respectively. A "maximal" SEP violation would give $\eta_{\rm SEP} = 4.4 \times 10^{-10}$. We combine the results in Eqs. (7) to obtain $1\sigma$ confidence intervals

$$\eta_{\rm SEP} = (-0.4 \pm 5.5) \times 10^{-13} \quad \text{or} \quad |\eta_{\rm SEP}| \leq 5.5 \times 10^{-13}$$

which correspond to an unambiguous $1\sigma$ test of the EP for gravitational self-energy with a precision

$$|\eta_{\rm grav}| = \frac{|\eta_{\rm SEP}|}{4.4 \times 10^{-10}} \leq 1.3 \times 10^{-3}. \tag{8}$$

The uncertainty in $\Delta a_{\rm LLR}$ is expected to drop in the near future [14]. We are currently testing additional improvements to the Eöt-Wash II balance that should allow us to make a corresponding improvement in our sensitivity.

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