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Improving Response Characteristics of Split Ring Resonators Using Experimental Designs

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IMPROVING RESPONSE CHARACTERISTICS OF SPLIT RING RESONATORS USING

EXPERIMENTAL DESIGNS

by

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ABSTRACT

IMPROVING RESPONSE CHARACTERISTICS OF SPLIT RING RESONATORS USING EXPERIMENTAL DESIGNS

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The purpose of this thesis is to investigate the application of the design-of-experiments approaches in the analysis of split ring resonators, and to compare two different approaches. The design parameters of the meta-material device are examined in order to study all of the main effects, the two factor interactions, and the curvature effects. A full factorial design and a central composite design is utilized for the study. The results in improving the split ring resonator design are discussed.

This study shows that the design-of-experiments approaches can effectively be utilized to examine the effects on the response values of a specific type of meta-material. The optimal values for each specific design parameter to maximize the response values are determined.

Another important aspect of the thesis is to demonstrate the existence of a tradeoff between the efficiency and accuracy of different experimental design models. This study demonstrated that a full factorial design gives more dependable results than a central composite design, even though more experiments are required. This result is due to the more extensive coverage of the design space using full factorial designs.

This thesis is dedicated to my family, who have always supported me.

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I am very thankful to the Turkish Military Academy and to Turkish Land Forces for granting me this opportunity.

NOMENCLATURE

CCD	Central Composite Design
DOE	Design-of-experiments
FDTD	Finite-Difference Time-Domain
FFD	Full Factorial Design
nm	Nanometer

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CHAPTER 1

INTRODUCTION

2.1 BACKGROUND

Meta-materials, as an area of study, are open to new development. Meta-materials are artificial structures that have a negative refraction index. Because of this feature, which differentiates them from ordinary materials, meta-materials have distinct characteristics against electromagnetic and acoustic waves. Meta-materials have a negative index as a consequence of their shapes and arrangements. Their characteristics differ, based on the changes in their shapes, sizes, materials, fabrication methods, and scales (Aydin, Cubukcu, Ozbay, & Bayindir, 2003; Aydin & Ozbay, 2007; Guven, Caliskan, & Ozbay, 2006).

Meta-materials have a variety of application areas that includes antennas, lenses, electromagnetic and optical cloaking, and energy harvesting (Alù & Engheta, 2010; Cubukcu, Kort, Crozier, & Capasso, 2006; Ozbay, Guven, & Aydin, 2007; Wood, 2009). The scale of the meta-materials is determined, depending on the application of the device. While the targeted frequency range of the meta-material device changes from megahertz to terahertz level, the scale of the device varies from millimeters to nanometers.

Research on meta-materials has increased in the last decade (Lourtioz, 2008). Most studies have been based upon empirical approaches in this area. Multidisciplinary studies are crucial, and include electronics, materials, and nanoscience.

2.2 PROBLEM STATEMENT

This study examines whether the application design-of-experiments approaches can improve meta-material design optimization. This study also examines whether a full factorial design approach provides a better understanding of a process, compared to a more efficient central composite design.

2.3 PURPOSE OF THE STUDY

The purpose of the study is to investigate whether the response characteristics of split ring resonators can be improved by using different design-of-experiments approaches.

2.4 SIGNIFICANCE OF THE STUDY

The study aims to apply the design-of-experiments approaches to the split ring resonator design parameters (variables). The study is intended to provide information about important factors of design parameters that affect the response value of absorption.

Studies on meta-materials are generally based on research that monitors the effect of only one parameter. Most of the studies that test multiple factors are based on one-variable-at-a-time methods (Cakmakyapan, Caglayan, & Ozbay, 2014; Geng, Guo, Cao, Yang, & Chen, 2015; Kocer et al., 2015; Li, Zhang, Xiong, & Shen, 2014). However, this method is not capable of revealing the interaction and curvature effects of input variables. This study aims to investigate whether the design parameters of split ring resonators have interaction and curvature effects on the absorption characteristics of the structure. Appropriate design-of-experiments methods have the ability to reveal these effects and to provide researchers with important information that can accordingly be used in their design efforts and optimization studies.

The expertise on the design-of-experiments approaches and statistical foundations is crucial to success in designing, conducting, and analyzing experiments. Expertise in these areas may be gained through the Master of Science in Engineering Management Program. The Engineering Management skills, techniques, and knowledge are used to improve the quality characteristics of a recently developing area of research. In this interdisciplinary study, engineering management methods provide guidance to improve the efficiency of the process.

CHAPTER 2

LITERATURE REVIEW

2.5 SIMPLE METHODS

Progress in many fields depends on experimental research. Before complex experimental design approaches were developed, the "best-guess approach" was used; in fact, it is sometimes still used today (Montgomery, 2013). Based on this approach, the values of design parameters are adjusted and new experiments are conducted, depending on the outcome of the previous experiments. It is possible to provide some progress using this method. However, while complexity increases, it becomes harder to be certain that any more progress can be made. At best, this approach may be beneficial at the very beginning of a process as a screening study.

A more systematic way is the one-variable-at-a-time approach. This method suggests changing the values of a design parameter, while keeping them constant at their best values, and doing this for each variable sequentially. However, this method studies only the main factor effects and disregards the interaction effects, which are commonly significant for the response of process (Montgomery, 2013; Unal, Stanley, & Joyner, 1993). Therefore, the one-variable-at-a-time approach does not provide sufficient information about the process.

2.6 DESIGN-OF-EXPERIMENTS

Design-of-experiments (DOE) was introduced by Sir Ronald Fisher in the 1920s in order to increase efficiency in agricultural applications (Montgomery, 2013). DOE designs may be classified as full factorial designs and as fractional factorial designs.

2.6.1 FULL FACTORIAL DESIGNS

Among the design-of-experiments approaches, the full factorial design is the simplest method to construct. All of the interaction effects, as well as all of the main factors, are studied. All of the curvature effects can also be studied by increasing the level from two to three for each parameter. The main disadvantage of the full factorial design is that the required number of experiments can grow exponentially as the number of parameters in the study increases. For instance, for a three level model, with each additional factor to be studied, the required number of experiments triples, and this may increase to prohibitive numbers easily (Stanley, Unal, & Joyner, 1992). For example, for five factors, a three level model requires 243 experiments, which is not generally considered to be feasible.

2.6.2 FRACTIONAL FACTORIAL DESIGN

When there are more than four factors, a full factorial design, which examines all possible combinations, may generally not be necessary (Montgomery, 2013). Instead, fractional factorial designs may used in these types of situations, when higher level interactions are considered negligible (Cornell, 1990).

A disadvantage of fractional factorial designs is the existence of confounding patterns. This causes an inability to separate the effects of the different orders of parameters and interactions. For example, when two factor interactions confound with the main effects, it can cause a serious problem, because two factor interactions are generally shown to be significant.

2.7 TAGUCHI'S ORTHOGONAL ARRAYS

Dr. Genichi Taguchi provided several models to study multiple factors with a lesser number of experiments (Unal et al., 1993). These are some orthogonal arrays that researchers can use, depending on their fitness to the situation under focus (İç & Yildirim, 2012; Karakurt, 2014; Maghsoodloo, Ozdemir, Jordan, & Huang, 2004; Unal et al., 1993). Both two- and three-level orthogonal arrays exist.

This method is also prone to the disadvantage of confounding patterns. However, Taguchi provided linear graphs in order to easily adjust confounding patterns according to the attributes of the process (Taguchi, Chowdhury, & Wu, 2005). This usually comes at the expense of an increased number of experiments.

2.8 RESPONSE SURFACE METHODOLOGY

Response surface methodology is used to optimize response; this consists of mathematical and statistical techniques (Montgomery, 2013). Response surface methodology can study second order approximation models, as well as first order models.

2.8.1 CENTRAL COMPOSITE DESIGNS

Central composite design was developed by Box and Wilson (1951). It is an efficient method to study curvature effects, in addition to the main factor and interaction effects, with a significantly lower number of experiments, as compared to full factorial designs. It uses a full or fractional factorial design as a base. The star points, or axial points, and the center point are added to the base design (NIST/SEMATECH, 2013). This approach usually examines factors at five levels, even though three levels may also be used (face centered designs).

A disadvantage of the central composite design is that the model may require that parameters have continuous values instead of discrete values. Depending on the number of factors in the model, the values of star points vary and can be a decimal value (Cornell, 1990).

2.8.2 FACE CENTERED DESIGNS

The face centered design is similar to the central composite design. The star points at face centered design are at the same distance with the corner points of the base factorial design. This model studies factors at three levels, instead of five as in the central composite design. This fact creates the ability to study factors with discrete values at three points as an advantage (NIST/SEMATECH, 2013).

CHAPTER 3

METHODOLOGY

2.9 DESIGN OVERVIEW

The most suitable DOE approach is determined, conducted, and analyzed, to improve the design of the split ring resonators.

2.10 SPECIFIC DESIGN

The study is conducted by determining the specific response characteristic and main effects that affect this response. The next step is to determine the appropriate designs in order to analyze the process. Then, a simulation model is constructed to conduct the experiments. Finally, the results of the analysis are studied.

2.10.1 SPLIT RING RESONATORS

The split ring resonator is a particular type of meta-material. It consists of two concentric rings that have splits on different sides. This device shows the different responses to electromagnetic waves with different frequencies. First, the scale of the device is determined, because the size of the device is determined based on the electromagnetic waves to be studied. A nanoscale split ring resonator, which is the focus in this research, shows the distinct characteristics against infrared electromagnetic waves.

2.10.1.1 Determining the Response Characteristic

There are several response characteristics for meta-materials, such as manufacturing cost, absorption value, effective wave range, and observation frequency of the maximum absorption value. The response characteristic studied is the absorption value. This characteristic is the first step for the research in this field. Other response characteristics may be considered after having a high level of absorption.

2.10.1.2 Determining the Design Parameters

Multiple factors affect the response characteristics of a meta-material. As a type of meta-material, split ring resonators have several factors that affect response, such as materials that are used as components, thickness of the device, shape, temperature of the environment, usage of more concentric rings, and design parameters.

In this study, the main parameters examined are the four different design parameters of the split ring resonator. These parameters are the diameter of the outer ring (d), the width of the rings (w), the gap between the rings (g), and the central angle of splits (s) (Please see Figure 1).

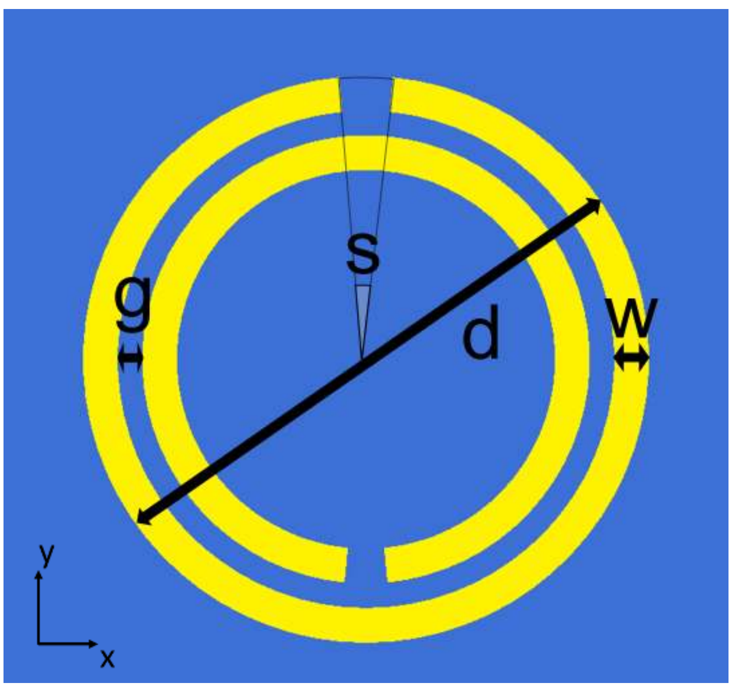

Figure 1. Top View of Split Ring Resonator with Design Parameters

After the determination of the main factors, the next step is to specify their upper and lower limit values. The unit for diameter, width, and gap is nanometers (nm) and the angle unit is degree (See the Table 1 for design parameter limits).

Parameter		Lower Limit	Upper Limit
d	Diameter of Outer Ring (nm)	140	300
w	Width Rings (nm)	2	25
g	Gap Between Rings (nm)	2	17
s	Central Angle of Split (degree)	2	20

Table 1. Design Parameter Limits

2.10.1.3 Building the Simulation Model:

In order to generate the response values required for the design-of-experiment models, Lumerical FDTD Solutions software is used. This simulation can measure the reflected amount of electromagnetic waves from the split ring resonator.

The simulation model consists of three main entities. The FDTD simulation region intersects with the reflection monitor, the plane wave source, and the split ring resonator (see Figure 2).

The source emits plane electromagnetic waves with a wavelength range from 1000 to 8000 nm (i.e. with a frequency range from approximately 27 to 300 terahertz). The electromagnetic waves propagate toward the split ring resonator along the –z direction.

The reflection monitor measures the reflected electromagnetic waves from the split ring resonator. The response values, which are the absorption value to be used in analysis, are derived from the measurement of this reflection monitor. The absorption value ($A(f)$) is calculated by subtracting the reflection ($R(f)$) and transmission ($T(f)$) values from 1, as follows:

$$A(f) = 1 - T(f) - R(f) \qquad \text{(Equation 1)}$$

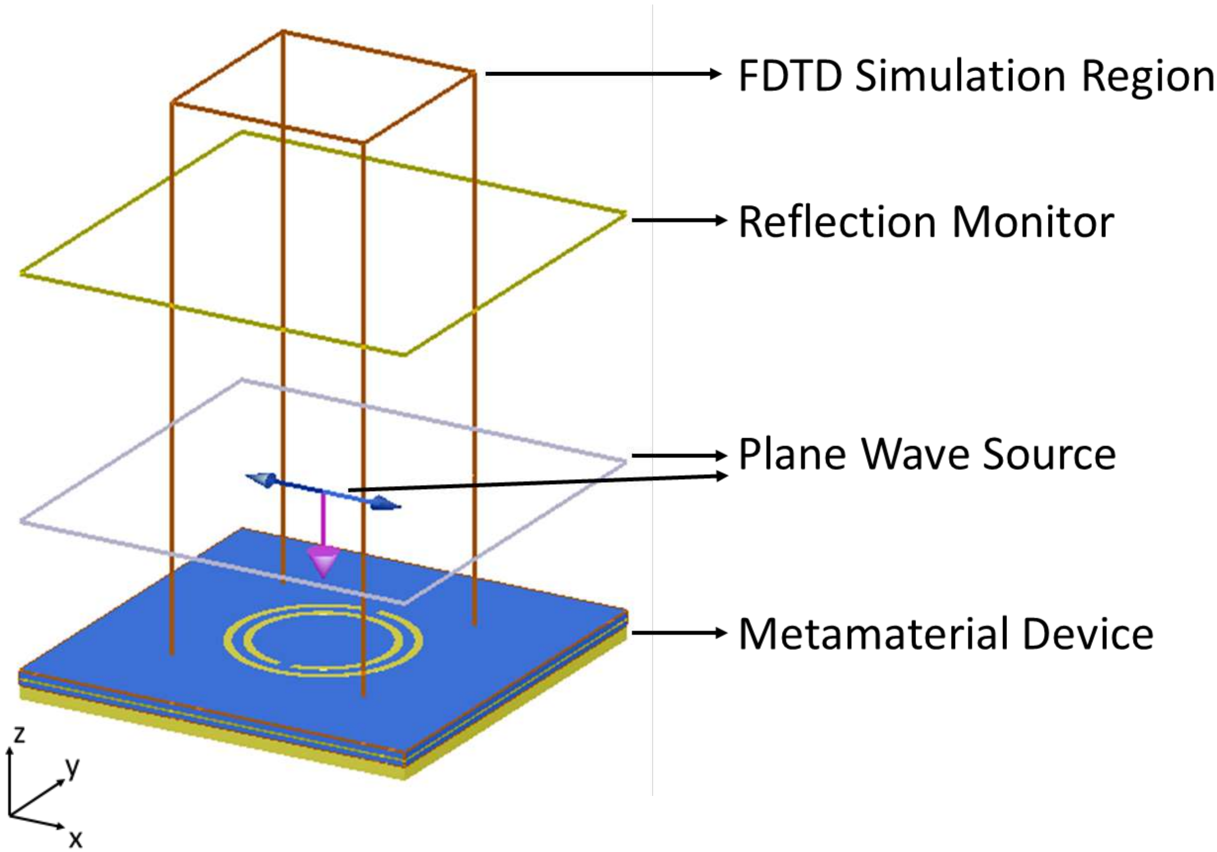

Figure 2. Simulation Model

The split ring resonator, which consists of two gold rings, is embedded in vanadium dioxide (VO_2) as the insulator material, and it has a golden ground (see Figure 3). Each layer has a thickness of 100 nm. Because the ground of the meta-material device is of a sufficiently thick metal, the entire amount of the waves that propagate through the split ring resonator is reflected back. Therefore, the transmission of plane waves is always equal to zero for this meta-material device. When this term is removed from the equation, the absorption value is:

$$A(f) = 1 - R(f) \qquad \text{(Equation 2)}$$

Figure 3. Cross-section View of the Meta-material Device

The simulation model is constructed as explained. Each experiment run is conducted by changing the values of design parameters of the split ring resonator.

2.10.2 BUILDING THE DESIGN-OF-EXPERIMENT MODELS

By the design-of-experiments approach, the absorption value of the split ring resonator is to be maximized. This approach aims to reveal all of the main factor effects, all of the two-factor interactions, and all of the curvature effects. In order to fulfill this goal, a two level model is not appropriate, because studying curvature effects requires at least three levels.

After analyzing the output data of design-of-experiments approach, a mathematical model, which includes all main effects, all two factor interaction effects, and all curvature effects, is intended to be constructed, as follows:

$$Y = \beta_0 + \sum \beta_i x_i + \sum \sum \beta_{ij} x_i x_j + \sum \beta_{ii} {x_i}^2 \quad \text{(Equation 3)}$$

Where:

Y : Quality Characteristic to be optimized (absorption)

Xi, Xj : Input variable for factors (A, B, C, and D for d, w, g, and s)

$\beta_0, \beta_i, \beta_{ij}$: Estimated regression coefficients

At the end of the design-of-experiments approach, the mathematical model will be used to predict the absorption of the nanoscale split ring resonator. After that step, prediction of the absorption value of a new design, without conducting either simulation or real experiments, would be possible.

In this study, a three level full factorial design (FFD) and a central composite design (CCD) are conducted separately and their results are compared. Another method used to study this process is to utilize Taguchi's orthogonal arrays that are fractions of full factorial designs (Unal et al., 1993). To determine which orthogonal array is suitable for this case, the total number of degrees of freedom needs to be computed as follows:

Total Degrees of Freedom		
Mean		1
Main Effects	4*(3-1)	8
Two-factor Interactions	6*(3-1)*(3-1)	24
		33

Table 2. Total Number of Degrees of Freedom

Taguchi's L(81) orthogonal array is an appropriate design for this situation; it requires 81 experiments. Therefore, this design requires as many experiments as a full factorial design in order to study all of the main effects and two-factor interactions. Therefore, it is not more efficient than a full factorial design.

2.10.3 FULL FACTORIAL DESIGN

A full factorial design is beneficial when all of the main effects, all of the interaction effects, and all of the curvature effects need to be studied. However, when the number of factors and levels increases, the number of experiments increases significantly as well. Therefore, a full factorial design may not be considered as an efficient option in most situations.

2.10.3.1 Building the Full Factorial Design Model:

In this study, there are four factors at three levels. Thus, this model requires $3^4 = 81$ experiments for a full factorial design. Although it demands much time and effort, this model is used and analyzed so that its effectiveness can be compared to a central composite design.

This model has three levels and requires an appropriate coding for the parameter values. The codes -1, 0, +1 are used for lower limit, middle value, and upper limit, respectively (see Table 3 for the code values of the main parameters).

		Lower Limit		Upper Limit
		-1	0	+1
d	A	140	220	300
w	B	2	13.5	25
g	C	2	9.5	17
s	D	2	11	20

Table 3. Code Values Representing Parameter Values for FFD

The full factorial design is constructed using the Yates Algorithm and is randomized using Minitab software. The complete model consisting of 81 experiments can be seen in Table 4. The randomly conducted experiments based on the run order are shown in Table 4. For each run, the parameter values are changed, using the coded values at each row.

StdOrder	RunOrder	A	B	C	D
1	51	-1	-1	-1	-1
2	71	-1	-1	-1	0
3	68	-1	-1	-1	1
4	5	-1	-1	0	-1
5	1	-1	-1	0	0
6	80	-1	-1	0	1
7	78	-1	-1	1	-1
8	46	-1	-1	1	0
9	70	-1	-1	1	1
10	66	-1	0	-1	-1
11	6	-1	0	-1	0
12	4	-1	0	-1	1
13	67	-1	0	0	-1
14	7	-1	0	0	0
15	12	-1	0	0	1
16	28	-1	0	1	-1
17	8	-1	0	1	0
18	43	-1	0	1	1
19	35	-1	1	-1	-1
20	56	-1	1	-1	0
21	73	-1	1	-1	1
22	74	-1	1	0	-1
23	21	-1	1	0	0
24	36	-1	1	0	1
25	65	-1	1	1	-1
26	50	-1	1	1	0
27	64	-1	1	1	1
28	30	0	-1	-1	-1
29	15	0	-1	-1	0
30	52	0	-1	-1	1
31	2	0	-1	0	-1
32	72	0	-1	0	0
33	23	0	-1	0	1
34	47	0	-1	1	-1
35	32	0	-1	1	0
36	20	0	-1	1	1
37	48	0	0	-1	-1
38	3	0	0	-1	0
39	61	0	0	-1	1
40	39	0	0	0	-1
41	34	0	0	0	0

StdOrder	RunOrder	A	B	C	D
42	37	0	0	0	1
43	19	0	0	1	-1
44	42	0	0	1	0
45	29	0	0	1	1
46	33	0	1	-1	-1
47	76	0	1	-1	0
48	40	0	1	-1	1
49	77	0	1	0	-1
50	57	0	1	0	0
51	41	0	1	0	1
52	31	0	1	1	-1
53	18	0	1	1	0
54	10	0	1	1	1
55	25	1	-1	-1	-1
56	45	1	-1	-1	0
57	13	1	-1	-1	1
58	54	1	-1	0	-1
59	49	1	-1	0	0
60	44	1	-1	0	1
61	55	1	-1	1	-1
62	53	1	-1	1	0
63	27	1	-1	1	1
64	38	1	0	-1	-1
65	63	1	0	-1	0
66	69	1	0	-1	1
67	24	1	0	0	-1
68	79	1	0	0	0
69	17	1	0	0	1
70	62	1	0	1	-1
71	81	1	0	1	0
72	11	1	0	1	1
73	22	1	1	-1	-1
74	14	1	1	-1	0
75	16	1	1	-1	1
76	9	1	1	0	-1
77	26	1	1	0	0
78	60	1	1	0	1
79	58	1	1	1	-1
80	75	1	1	1	0
81	59	1	1	1	1

Table 4. Full Factorial Design for Four Factors at Three Levels

2.10.3.2 Results of the Full Factorial Design Model:

The raw simulation output data includes the percentage-based reflection value. For example, according to experiment 1, the split ring resonator has a minimum reflection value of 1.96% for 1651 nm wavelength (see Figure 4 for output data graph).

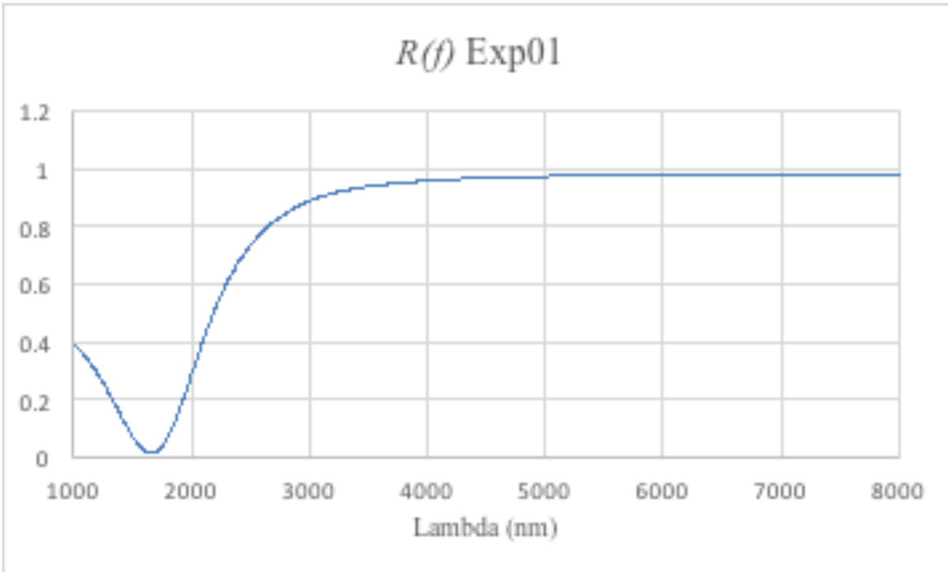

Figure 4. *R(f)* output Graph for Experiment 1 of FFD

In order to calculate the maximum absorption value, based on equation 3, the specific split ring resonator used in experiment 1 has a maximum absorption value of 98.04% at 1651 nm wavelength.

Equation 2 is applied to all experiment runs. and $A(f)$ values are calculated (see Table 5).

In order to conduct a multiple regression analysis, the columns for interactions and curvature effects should be generated. The values of these columns are calculated by multiplying each value at the related main effect columns for each row. For example, AB interaction value at the first row is calculated by multiplying values at A and B columns ($(-1) * (-1) = 1$). This action is repeated for each cell at the interaction and for the curvature columns (see Table 6).

Exp#	Reflection *R(f)*	Absorption *A(f)*	Exp#	Reflection *R(f)*	Absorption *A(f)*
1	1.95796%	98.04204%	41	0.28457%	99.71543%
2	1.95796%	98.04204%	42	0.53968%	99.46032%
3	1.95796%	98.04204%	43	0.14131%	99.85869%
4	2.08186%	97.91814%	44	0.14131%	99.85869%
5	2.08186%	97.91814%	45	0.59970%	99.40030%
6	2.08186%	97.91814%	46	0.04958%	99.95042%
7	1.58910%	98.41090%	47	0.04958%	99.95042%
8	1.58910%	98.41090%	48	0.04059%	99.95941%
9	1.58910%	98.41090%	49	0.14677%	99.85323%
10	0.55430%	99.44570%	50	0.14677%	99.85323%
11	0.55430%	99.44570%	51	0.43166%	99.56834%
12	0.47862%	99.52138%	52	0.10304%	99.89696%
13	0.90295%	99.09705%	53	0.10304%	99.89696%
14	0.90295%	99.09705%	54	0.12818%	99.87182%
15	0.97873%	99.02127%	55	1.55950%	98.44050%
16	0.98095%	99.01905%	56	1.55950%	98.44050%
17	0.98095%	99.01905%	57	1.55950%	98.44050%
18	1.27932%	98.72068%	58	1.12771%	98.87229%
19	0.63745%	99.36255%	59	1.12771%	98.87229%
20	0.63745%	99.36255%	60	1.12771%	98.87229%
21	0.54163%	99.45837%	61	0.86391%	99.13609%
22	0.58168%	99.41832%	62	0.86391%	99.13609%
23	0.58168%	99.41832%	63	0.86391%	99.13609%
24	1.04965%	98.95035%	64	0.00055%	99.99945%
25	1.12542%	98.87458%	65	0.12895%	99.87105%
26	1.12542%	98.87458%	66	0.09799%	99.90201%
27	1.09343%	98.90657%	67	0.00060%	99.99940%
28	1.91576%	98.08424%	68	0.00225%	99.99775%
29	1.91576%	98.08424%	69	0.00275%	99.99725%
30	1.91576%	98.08424%	70	0.06034%	99.93966%
31	1.99369%	98.00631%	71	0.00466%	99.99534%
32	1.99369%	98.00631%	72	0.00177%	99.99823%
33	1.99369%	98.00631%	73	0.16535%	99.83465%
34	1.89453%	98.10547%	74	0.15316%	99.84684%
35	1.89453%	98.10547%	75	0.00307%	99.99693%
36	1.89453%	98.10547%	76	0.17731%	99.82269%
37	0.20341%	99.79659%	77	0.00070%	99.99930%
38	0.20341%	99.79659%	78	0.03393%	99.96607%
39	0.33471%	99.66529%	79	0.11192%	99.88808%
40	0.28457%	99.71543%	80	0.03510%	99.96490%
			81	0.00194%	99.99806%

Table 5. Reflection and Absorption Values of Experiments (FFD)

Exp#	A	B	C	D	AB	AC	AD	BC	BD	CD	AA	BB	CC	DD	Absorption
1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1	1	0.980420400
2	-1	-1	-1	0	1	1	0	1	0	0	1	1	1	0	0.980420400
3	-1	-1	-1	1	1	1	-1	1	-1	-1	1	1	1	1	0.980420400
4	-1	-1	0	-1	1	0	1	0	1	0	1	1	0	1	0.979181400
5	-1	-1	0	0	1	0	0	0	0	0	1	1	0	0	0.979181400
6	-1	-1	0	1	1	0	-1	0	-1	0	1	1	0	1	0.979181400
7	-1	-1	1	-1	1	-1	1	-1	1	-1	1	1	1	1	0.984109000
8	-1	-1	1	0	1	-1	0	-1	0	0	1	1	1	0	0.984109000
9	-1	-1	1	1	1	-1	-1	-1	-1	1	1	1	1	1	0.984109000
10	-1	0	-1	-1	0	1	1	0	0	1	1	0	1	1	0.994457020
11	-1	0	-1	0	0	1	0	0	0	0	1	0	1	0	0.994457020
12	-1	0	-1	1	0	1	-1	0	0	-1	1	0	1	1	0.995213810
13	-1	0	0	-1	0	0	1	0	0	0	1	0	0	1	0.990970500
14	-1	0	0	0	0	0	0	0	0	0	1	0	0	0	0.990970500
15	-1	0	0	1	0	0	-1	0	0	0	1	0	0	1	0.990212660
16															
17															

Table 6. Regression Input Data for FFD

This table only shows the first 15 experiments. To conduct the entire analysis, this table is prepared for all experiments. This portion of the table is offered as an example.

The three- and four-factor interactions are not included in the model, as these interaction effects are generally negligible (Cornell, 1990), and they are removed from the model after the first step of the regression analysis.

The regression analysis is conducted using Excel Analysis Toolpak Add-in. The input data is analyzed and the output is examined (see Table 7 for output of the first regression step).

Regression Statistics	
Multiple R	0.963630649
R Square	0.928584028
Adj. R Sq.	0.913435186
Std. Error	0.002138448
Observations	81

ANOVA

	df	*SS*	*MS*	*F*
Regression	14	0.003924345	0.00028031	61.29735854
Residual	66	0.000301815	4.57296E-06	
Total	80	0.00422616		

	Coefficients	*Std. Error*	*t Stat*	*P-value*
Intercept	0.995994256	0.000712816	1397.267039	2.7961E-149
A	0.003747764	0.000291006	12.87865203	1.13051E-19
B	0.006610478	0.000291006	22.71596	6.57615E-33
C	1.35819E-05	0.000291006	0.046672337	0.962915316
D	-0.000261085	0.000291006	-0.897181809	0.372882992
AB	0.000127207	0.000356408	0.356915161	0.722293961
AC	0.001248684	0.000356408	3.503523897	0.00082959
AD	0.000281456	0.000356408	0.789700652	0.432530652
BC	-0.001335179	0.000356408	-3.746209689	0.000379569
BD	-6.2655E-05	0.000356408	-0.175795686	0.860992722
CD	-0.00019316	0.000356408	-0.541964031	0.589668419
AA	-0.00015218	0.000504037	-0.301921941	0.763660918
BB	-0.006095618	0.000504037	-12.09359237	2.12422E-18
CC	0.000578482	0.000504037	1.147697429	0.255236677
DD	-0.000331913	0.000504037	-0.658509843	0.512501279

Table 7. Output of the First Step of Regression Analysis for FFD

The output of the analysis shows a considerable R square and adjusted R square value. When the main factor, interaction, and curvature effects are examined on the basis of their coefficients and P-values, the main factors A and B, the two-way interactions AC and BC, and the curvature of B are seen to be significant and the other terms to be insignificant. In order to increase the quality of the model, these insignificant terms should be removed from the model in accordance with their hierarchy. Removing insignificant terms starts from the highest order effects and ends at the main factor effects. If there is an insignificant main effect while its interaction or curvature effect is significant, this main effect should not be removed from the model. For this particular situation, even if the main effect C is insignificant, it cannot be removed from the model because it exists in significant AC and BC interactions.

Another important point is that the gradual removal of terms. The insignificant terms are removed at a rate of just one order of terms at a time, instead of removing all insignificant terms at the same time, regardless of their different hierarchical position.

This analysis is completed in four steps. Table 7 is the first step of the analysis. The regression outputs of the second and third steps are shown at Appendices 1 and 2, respectively. The final step of the analysis is given in Table 8.

Regression Statistics	
Multiple R	0.961538336
R Square	0.924555971
Adjusted R Square	0.918438888
Standard Error	0.002075724
Observations	81

ANOVA

	df	*SS*	*MS*	*F*
Regression	6	0.003907321	0.00065122	151.1432671
Residual	74	0.000318839	4.30863E-06	
Total	80	0.00422616		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.996057182	0.000399473	2493.426726	5.8116E-184
A	0.003747764	0.00028247	13.26781905	2.98696E-21
B	0.006610478	0.00028247	23.40239074	6.20036E-36
C	1.35819E-05	0.00028247	0.048082681	0.961779885
AC	0.001248684	0.000345954	3.609393361	0.000555271
BC	-0.001335179	0.000345954	-3.85941263	0.000240992
BB	-0.006095618	0.000489253	-12.45903647	7.57741E-20

Table 8. Output of the Final Step of Regression Analysis for FFD

Based on the final regression output, the main factors A and B, A*C and B*C interactions, and the B*B curvature effects are significant, having P-values less than α = 0.05 (as highlighted yellow in Table 8). The R Square value is 92.46% and the adjusted R-Square value is 91.84% Results are not perfect but are acceptable.

By using coefficients calculated by the regression analysis, the math model for the full factorial design is constructed (see Equation 4).

$$Y = 0.996057 + 0.003748 * A + 0.00661 * B + 0.0000135 * C + 0.001249 * A * C - 0.001335 * B * C + 0.006096 * B^2 \quad \text{(Equation 4)}$$

In order to test the validity of the math model, the absorption prediction for each of the main factor values of the 81 experiments is compared with the real simulation values. These error values have an average of -3.5E-16 and a standard deviation of 0.001996. In addition, according to the normal probability plot, one fails to reject that these values are normally distributed (see Figure 5). This can be another indication of the fitness of this math model for this process, in addition to the R square and adjusted R square values.

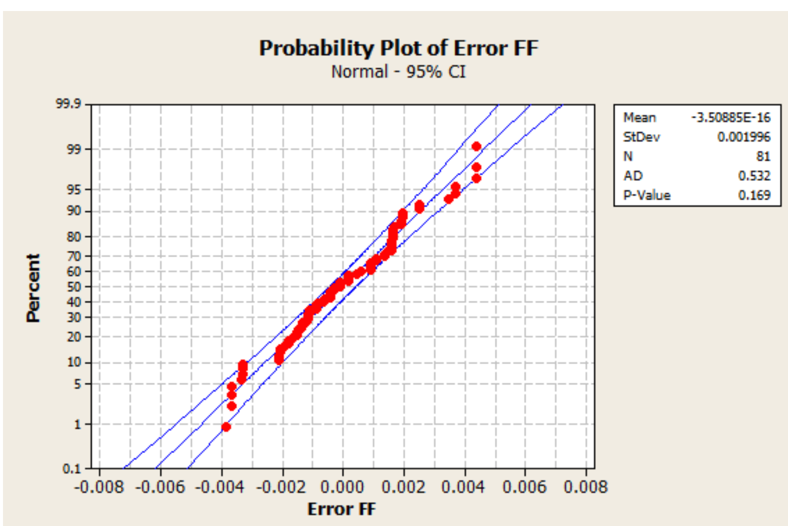

Figure 5. Normal Probability Plot of Math Model Prediction Error for FFD

2.10.4 CENTRAL COMPOSITE DESIGN

Response Surface Methodology with Central Composite Design is a more efficient approach. For four factors, by conducting only 25 $(2^4 + 2 * 4 + 1)$ experiments, all of the main effects, all of the two factor interactions, and all of the second order interactions can be studied. This model is an efficient option with respect to the full factorial design.

2.10.4.1 Building the Central Composite Design Model:

The model consists of two blocks; one is for factorial design, and the other is for axial (α) points. The value for α can be calculated by using the following formula:

$$\alpha = [2^k]^{1/4} = [2^4]^{1/4} = 2 \quad \text{(Equation 5)}$$

Based on Equation 5, the axial points have -2 and 2 code values, which contribute the lower and upper limits, respectively. In this situation, the code values (-2, -1, 0, +1, +2) and their representing parameter values are as follows:

		Lower Limit				Upper Limit
		-2	-1	0	+1	+2
d	A	140	180	220	260	300
w	B	2	7.75	13.5	19.25	25
g	C	2	5.75	9.5	13.25	17
s	D	2	6.5	11	15.5	20

Table 9. Code Values Representing Parameter Values for CCD

The factorial part of the Central Composite Design is constructed by using Yates algorithm as a two level Full Factorial Design for four factors, which requires $2^4 = 16$ experiments. There are two axial points for each factor, which require 2 * 4 = 8 experiments, and there is a center point in the model which requires one experiment. In total, this central composite design requires 25 experiments (16 + 8 + 1). The experiment runs are conducted in random order, based on the run order and using the codes given in Table 10.

StdOrder	RunOrder	Blocks	A	B	C	D	
1	18	1	-1	-1	-1	-1	Factorial Design
2	10	1	1	-1	-1	-1	
3	20	1	-1	1	-1	-1	
4	13	1	1	1	-1	-1	
5	17	1	-1	-1	1	-1	
6	14	1	1	-1	1	-1	
7	16	1	-1	1	1	-1	
8	25	1	1	1	1	-1	
9	21	1	-1	-1	-1	1	
10	24	1	1	-1	-1	1	
11	12	1	-1	1	-1	1	
12	11	1	1	1	-1	1	
13	23	1	-1	-1	1	1	
14	22	1	1	-1	1	1	
15	19	1	-1	1	1	1	
16	15	1	1	1	1	1	
17	6	2	-2	0	0	0	Axial Points
18	1	2	2	0	0	0	
19	8	2	0	-2	0	0	
20	4	2	0	2	0	0	
21	5	2	0	0	-2	0	
22	9	2	0	0	2	0	
23	7	2	0	0	0	-2	
24	3	2	0	0	0	2	
25	2	2	0	0	0	0	Center Point

Table 10. Central Composite Design for Four Factors

2.10.4.2 Results of the Central Composite Design Model:

Similar to the full factorial design model, the central composite design model uses the data from the simulation. As an example, according to experiment 1, the split ring resonator has a minimum reflection value of 1.85% for 1651 nm wavelength (see Figure 6 for output data graph).

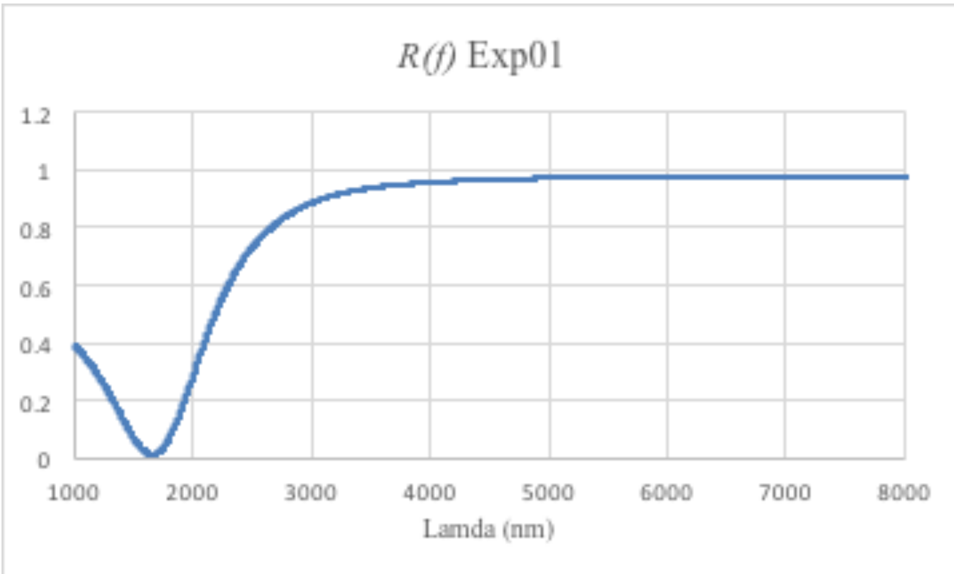

Figure 6. Output Graph for Experiment 1 of CCD

In order to calculate the maximum absorption value, based on equation 3, the specific split ring resonator used in experiment 1 has a maximum absorption value of 98.15% at 1651 nm wavelength.

Equation 2 is applied to all experiment runs and $A(f)$ values are calculated (see Table 11).

In order to conduct a multiple regression analysis, the columns for the interactions and the curvature effects should be generated in a similar fashion to the full factorial design. The values of these columns are calculated by multiplying each value at the related main effect columns for each row. This is the same convention that is used in the full factorial design, and it is shown in Table 6.

Exp #	Reflection *R(f)*	Absorption *A(f)*
1	1.853710%	98.146290%
2	1.436180%	98.563820%
3	0.453967%	99.546033%
4	0.000361%	99.999639%
5	1.650280%	98.349720%
6	0.437633%	99.562367%
7	0.478484%	99.521516%
8	0.006325%	99.993675%
9	1.853710%	98.146290%
10	1.436180%	98.563820%
11	0.428486%	99.571514%
12	0.016578%	99.983422%
13	1.650280%	98.349720%
14	0.437633%	99.562367%
15	0.654245%	99.345755%
16	0.073480%	99.926520%
17	0.902950%	99.097050%
18	0.002250%	99.997750%
19	1.993690%	98.006310%
20	0.146770%	99.853230%
21	0.203409%	99.796591%
22	0.141306%	99.858694%
23	0.284573%	99.715427%
24	0.539679%	99.460321%
25	0.284573%	99.715427%

Table 11. Reflection and Absorption Values of Experiments (CCD)

The regression analysis is conducted by using Excel Analysis Toolpak Add-in. The input data is analyzed and the output is examined (see Table 12 for output of the first regression step).

Regression Statistics	
Multiple R	0.967675671
R Square	0.936396204
Adj. R Square	0.847350891
Std. Error	0.002662101
Observations	25

ANOVA

	df	*SS*	*MS*	*F*
Regression	14	0.00104334	7.45243E-05	10.51595153
Residual	10	7.08678E-05	7.08678E-06	
Total	24	0.001114208		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.99715427	0.002662101	374.5741437	4.52461E-22
A	0.002908414	0.000543399	5.352260268	0.000322564
B	0.005140634	0.000543399	9.460143211	2.6372E-06
C	0.000922924	0.000543399	1.698427192	0.120272652
D	-0.000309943	0.000543399	-0.570378299	0.581009776
AB	-0.000838697	0.000665525	-1.260203258	0.236209064
AC	0.001111027	0.000665525	1.669398655	0.125996237
AD	4.18179E-05	0.000665525	0.062834419	0.951136569
BC	-0.001698185	0.000665525	-2.551646767	0.02878256
BD	-0.000146032	0.000665525	-0.219423824	0.830733949
CD	-0.000157612	0.000665525	-0.23682388	0.817574206
AA	-0.000882388	0.000792134	-1.113938531	0.29136284
BB	-0.002426463	0.000792134	-3.063199486	0.011978934
CC	-0.000181782	0.000792134	-0.229483798	0.823118561
DD	-0.000781203	0.000792134	-0.986201216	0.347294461

Table 12. Output of the First Step of Regression Analysis of CCD

Based on the output of the analysis, it can be stated that R square and adjusted R square values are considerable. Some of the main factors, interaction, and curvature effects are not significant based on their coefficients and P-values. The main factors A and B, the two-factor interaction BC, and the curvature of B are significant; the other terms are insignificant. In order to increase the quality of the model, these insignificant terms should be removed from the model in regard to their hierarchy. Removing insignificant terms starts from the highest order effects and ends in the main factor effects. For this particular situation, even if the main effect C is insignificant, it cannot be removed from the model. This is because it exists in significant BC interaction.

The terms are removed gradually – just one order of terms at a time, instead of removing all insignificant terms at the same time – regardless of their different hierarchical position.

This analysis is completed at four steps. Table 12 is the first step of the analysis. The regression outputs of the second and third steps are shown at Appendix 3 and 4, respectively. The final step of the analysis is given in Table 13.

Regression Statistics	
Multiple R	0.94475386
R Square	0.892559856
Adjusted R Square	0.864286134
Standard Error	0.002510091
Observations	25

ANOVA

	df	*SS*	*MS*	*F*
Regression	5	0.000994497	0.000198899	31.56853044
Residual	19	0.000119711	6.30056E-06	
Total	24	0.001114208		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.994883042	0.000696174	1429.072206	2.87703E-49
A	0.002908414	0.00051237	5.676390316	1.79627E-05
B	0.005140634	0.00051237	10.03304447	4.99051E-09
C	0.000922924	0.00051237	1.801283044	0.087551327
BC	-0.001698185	0.000627523	-2.706173145	0.014001518
BB	-0.001905973	0.00050242	-3.793582748	0.001227622

Table 13. Output of the Final Step of Regression Analysis for CCD

Based on the final regression output, the main factors A and B, the B*C interaction, and the B*B curvature effects are significant, having P-values less than $\alpha = 0.05$ (as highlighted yellow in

Table 13). The R Square value is 89.26% and the adjusted R-Square value is 86.43%, results which are not perfect but are acceptable.

By using coefficients calculated by the regression analysis, the math model for the central composite design is constructed (see Equation 6)

$$Y = 0.994883 + 0.002908 * A + 0.00514 * B + 0.0009229 * C - 0.001698 * B * C - 0.001906 * B^2 \quad \text{(Equation 6)}$$

The absorption prediction for each of the main factor values of the 25 experiments are compared with the real simulation values, in order to test validity of the math model. These error values have an average of -2.2E-17 and a standard deviation of 0.002233. In addition, according to the normal probability plot, these values can be said to be normally distributed (see Figure 7). This can be another indication of the fitness of this math model for this process, in addition to the R square and adjusted R square values.

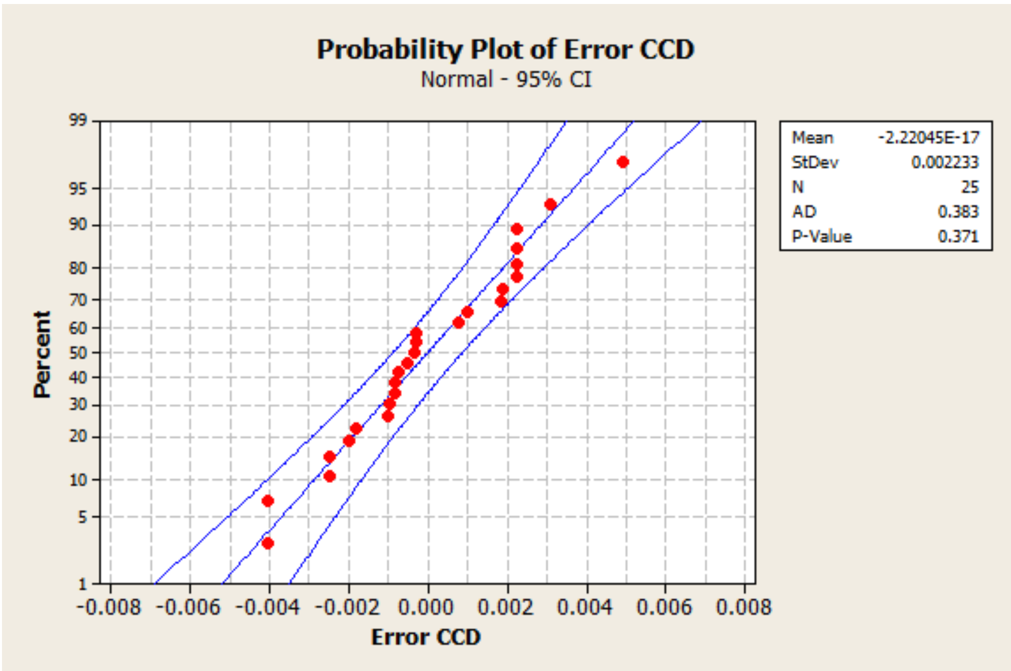

Figure 7. Normal Probability Plot of Math Model Prediction Error for CCD

2.11 ASSUMPTIONS AND LIMITATIONS

The scope of this study is limited to the absorption characteristics of split ring resonators. There are numerous types of meta-materials beyond the split ring resonators, and each type has different characteristics, such as manufacturing cost, absorption value, effective wave range, and observation frequency of the maximum absorption value. Studying all of these types and characteristics is beyond the scope of this particular thesis.

The split ring resonators have several factors that affect response, such as the materials that are used as components, the thickness of the device, the shape, the temperature of the environment, the use of more concentric rings, and the design parameters.

Future studies can focus on these different characteristics and factors by employing other design-of experiment methods.

The limitation of this study is that it was not conducted through real experiments by manufacturing, testing, and measuring response, due to the lack of required hardware to conduct real experiments by fabricating nano-scale devices and by measuring responses. Instead, the simulation software is used to build the model and obtain results in this study.

An assumption of this study is that electromagnetic waves propagate perpendicular to the meta-material device, instead of having different angle of approach.

CHAPTER 4

RESULTS AND COMPARISON

2.12 INTERPRETATION OF RESULTS

2.12.1 INTERPRETATION OF THE FULL FACTORIAL DESIGN MODEL RESULTS

2.12.1.1 Optimization of the Parameters:

In this study, the maximization of the absorption value is intended. In order to optimize the parameter values to reach this goal in accordance with the math model, the Microsoft Excel Solver Add-in is used. Solving the model for the maximization gives optimal values for each statistically significant factor (see Table 14).

Factor		Codes	Value
A	d	1	300
B	w	0.432712362	18.47619216
C	g	1	17

Table 14. Optimal Values of Factors (FFD)

Factor D does not exist because of its insignificance. No simulation run consisting of these values was conducted during the design-of-experiments approach. Therefore, a validation experiment is performed using the simulation software by keeping factor D as code zero value. The mathematical model predicts the absorption value as 1.00220856, which is practically impossible since the absorption value cannot take a value greater than 1. However, the mathematical model function can have values greater than 1. In simulation and in real experiments, it is not possible for this value exceed the limit of 100%, but it can be expected to be a higher value with respect to the average.

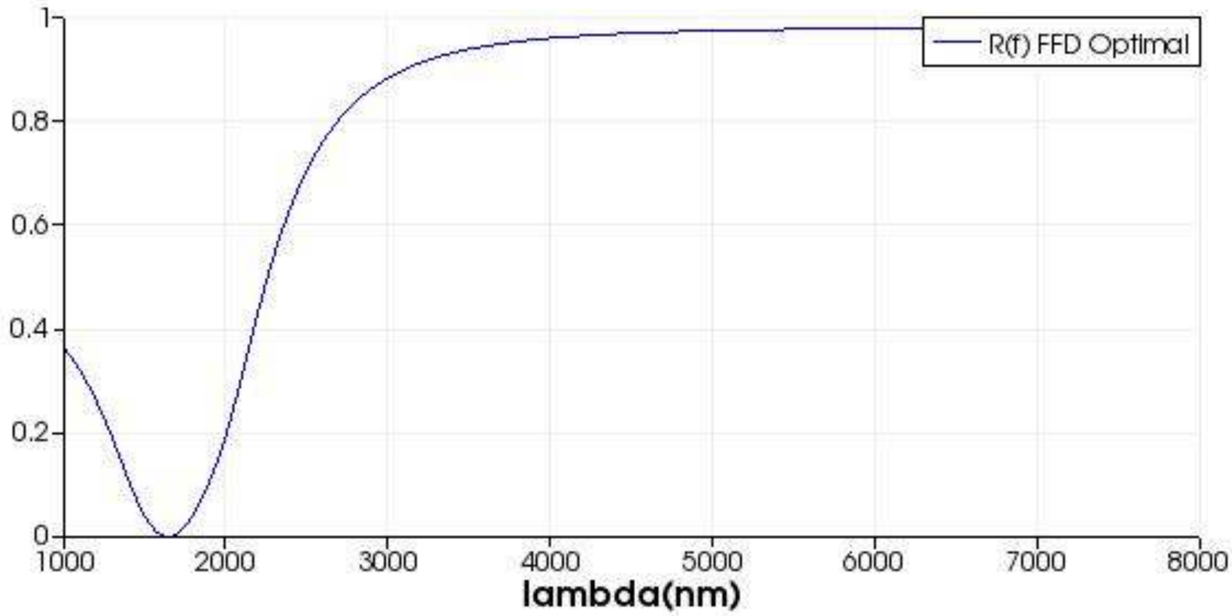

Figure 8. Simulation Output for Optimal Values (FFD)

According to the results of the validation run, the absorption value is 0.999975572. This value is not equal to the math model value and is less than 1, as expected. This value is also not the greatest value among the other 81 simulation runs; it is the seventh value in the order of these 81 values.

2.12.1.2 Plots of Significant Effects:

The main effects and interaction effects are plotted using Minitab software. The main effect plots consist of an average of all of the related response values for each code value. For this specific design, each of three points at each main effect plot illustrates the average value of the related 27 response values at which the factor is at the code value indicated by horizontal axis.

According to the main effect plot of factor A, it can be considered significant (see Figure 9. Main Effects Plot for Full Factorial Design). The statistical results support this fact with a coefficient value of 0.0037 and a P-value of 2.99E-21.

The effect of factor B is considered to be significant, according to the statistical values. The coefficient of factor B is the highest value of 0.0066 with the lowest P-value of 6.2E-36. It can also be inferred that factor B has a significant curvature effect, while factor A has a linear effect, as the slope of the factor B plot is lower for positive code values than for negative values.

Based on the plot, factor C is not significant. According to the statistics, its effect is much lower than the significant terms. Coefficient factor C is 1.358E-05 with a P-value of 0.9618, which is higher than $\alpha = 0.5$.

The effect of factor D is higher than that of factor C; however, it is not high enough to be considered as significant. Its coefficient value is -0.00026 and P-value is 0.3589. Factor D is also removed from the model, while factor C is not. This is because factor C remains in the significant AC and BC interactions, and it cannot be removed according to the model hierarchy. However, factor D does not exist in any other significant interaction or curvature effects. Therefore, it is removed from the model.

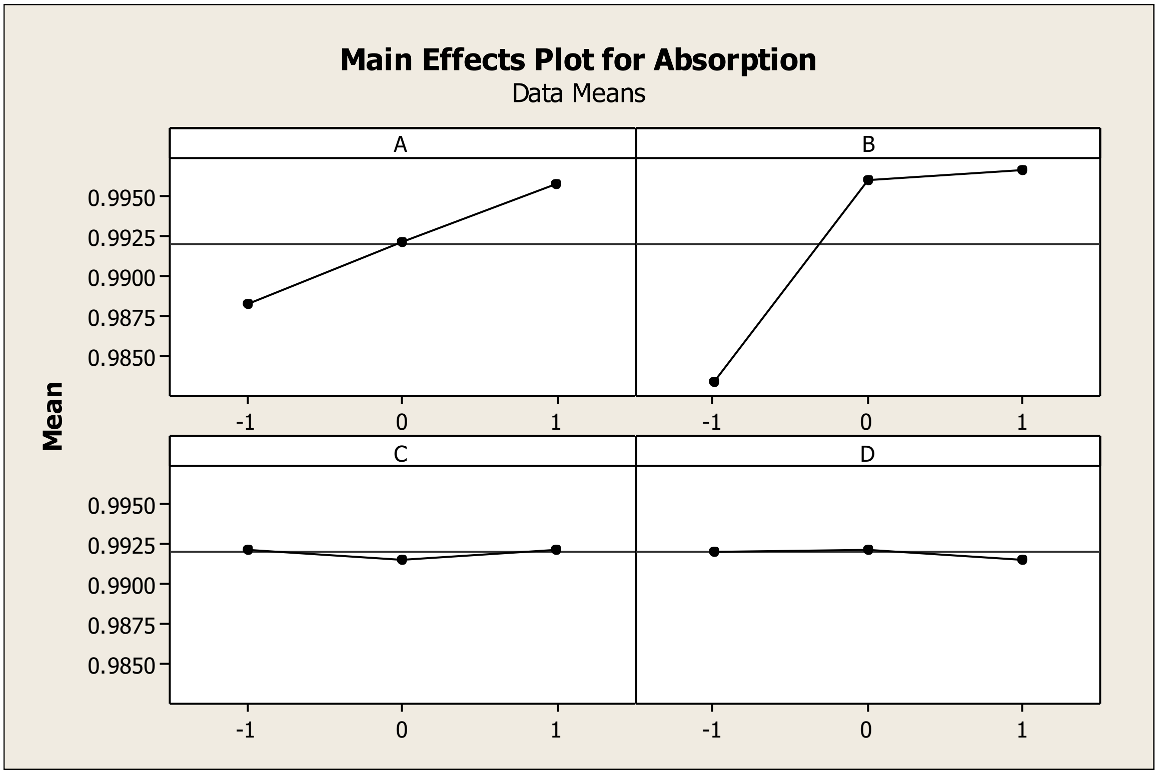

Figure 9. Main Effects Plot for Full Factorial Design

In summary, the main effects plots show that factors A and B have a significant effect on the response value, absorption. On the other hand, factors C and D are not significant. It can also be inferred that factor A has a linear effect, while factor B has a significant curvature effect.

According to the interaction effects plot (see Figure 10. Interaction Effects Plots for Full Factorial Design), AC and BC interactions can be said to be significant because of being unparalleled. AB interaction also seem to have a significant effect, based on its plot. However, according to the statistics, it is not a significant effect. The interactions AD, BD, and CD are not significant because plots of these effects are almost parallel.

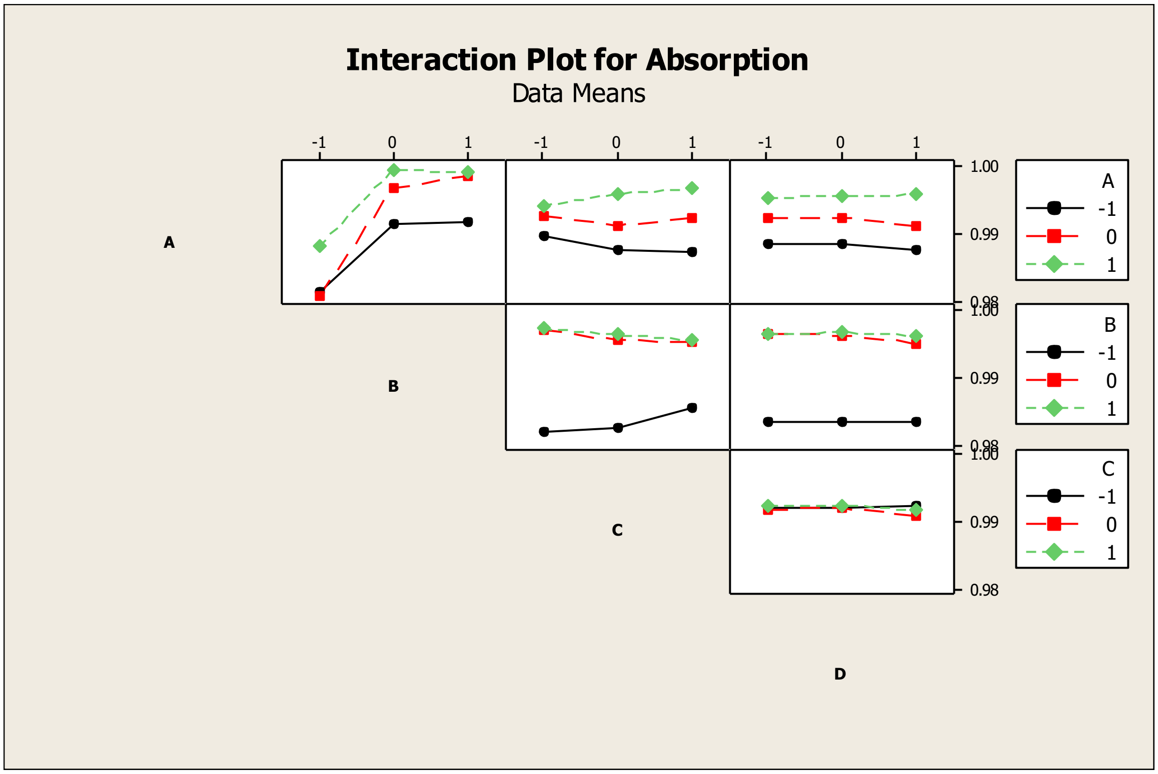

Figure 10. Interaction Effects Plots for Full Factorial Design

2.12.1.3 Validity of the Full Factorial Design Model:

The approach has an issue with the math model; it can give values that greater than one. However, the complete approach provides good information about the design parameters and the absorption response. The results provide us with an idea of how the process responses to the changes in the values of the parameters.

Based on the normal probability plot and the histogram for Residuals (see Figure 11. Normal Probability Plot for Residuals (FFD) and Figure 12. Histogram for Residuals (FFD), the residuals can be said to be normally distributed. It can also be seen that the data is not skewed to one side; instead, it is symmetrically distributed. However, there might be a problem at the positive and negative ends of the data as it starts to drift apart.

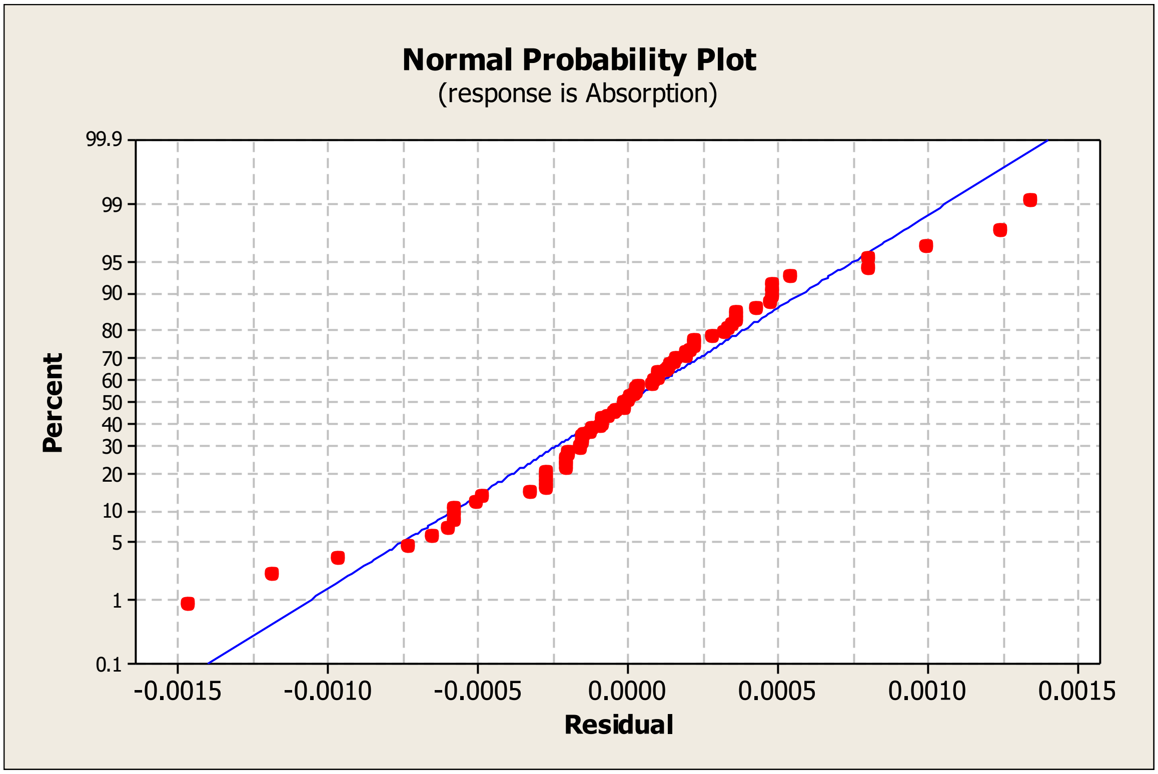

Figure 11. Normal Probability Plot for Residuals (FFD)

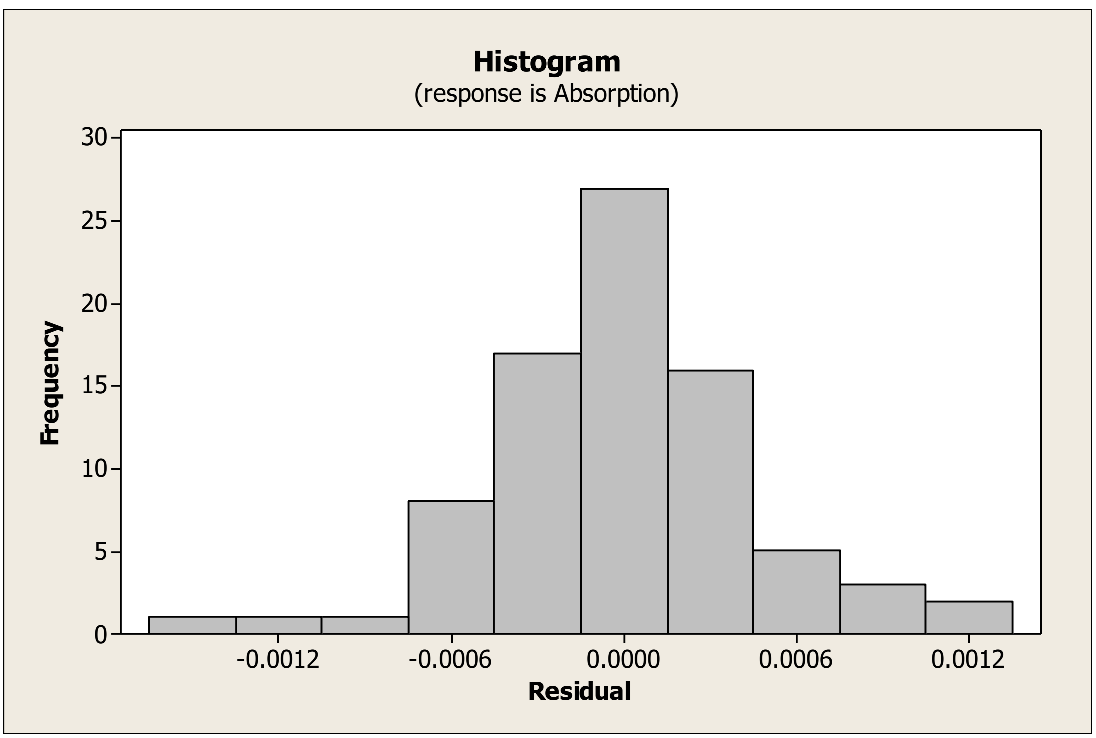

Figure 12. Histogram for Residuals (FFD)

According to the residuals versus fitted values plot, it can be inferred that the data points are randomly distributed, with a few outliers (see Figure 13. Residuals versus Fitted Values Plot for FFD). These points may require attention.

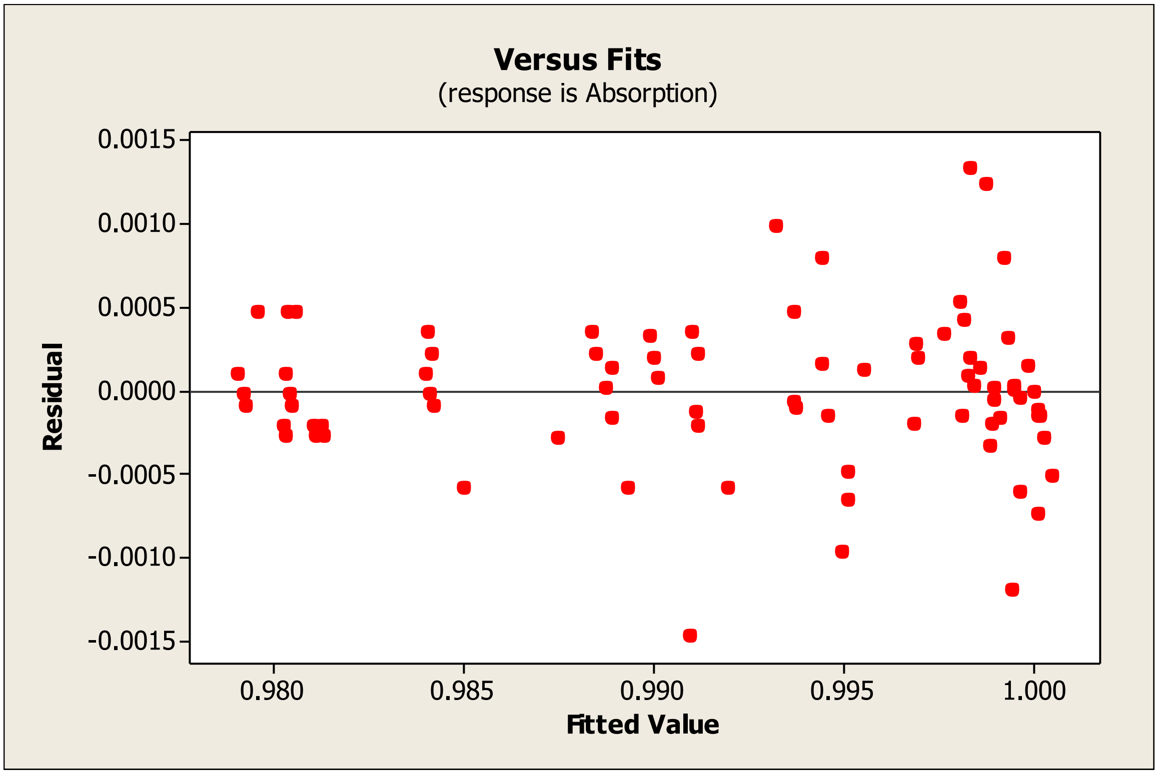

Figure 13. Residuals versus Fitted Values Plot for FFD

Based on the residuals versus order plot, it can be concluded that the process is independent of the time order (see Figure 14. Residuals versus Order Plot for FFD). This fact indicates that there is not any other significant factor related to the order of the experiments.

Based on these plots, it is clear that, overall, the regression model for the full factorial design fits to the process.

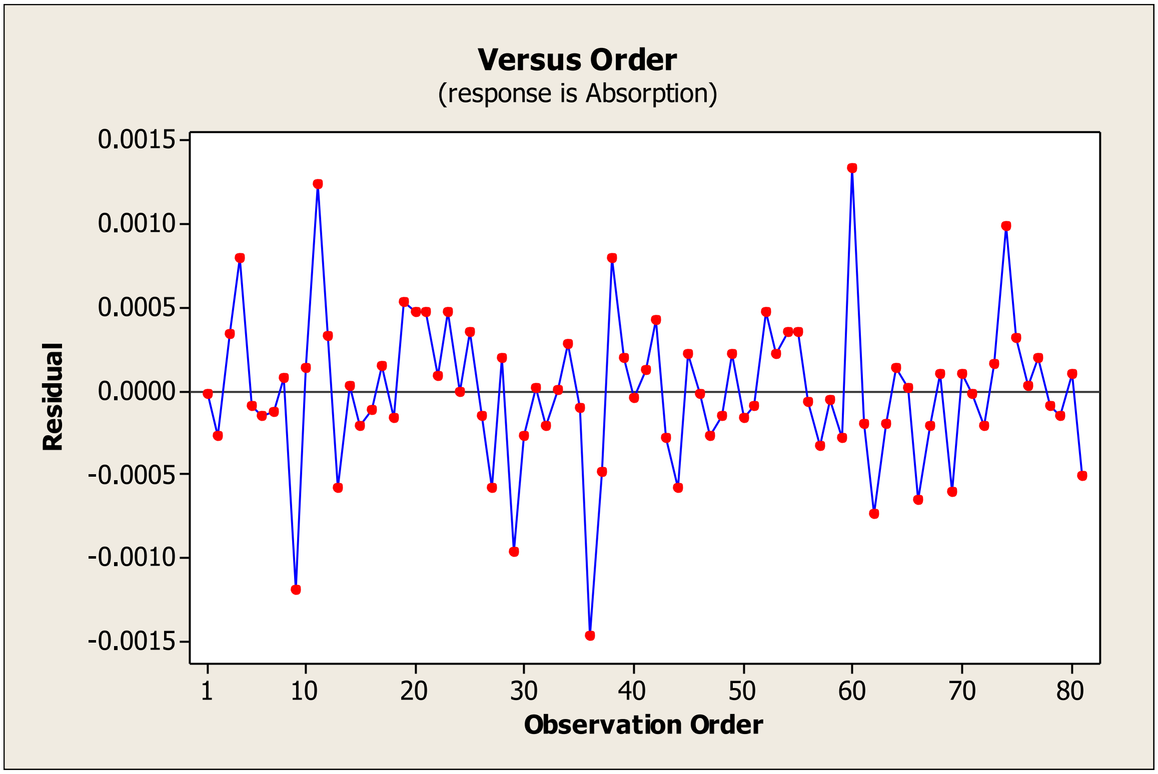

Figure 14. Residuals versus Order Plot for FFD

2.12.2 INTERPRETATION OF THE CENTRAL COMPOSITE DESIGN MODEL RESULTS

2.12.2.1 Optimization of the Parameters:

In order to maximize the absorption value, the parameter values are optimized according to the math model, using the Microsoft Excel Solver Add-in. Solving the model for maximization gives optimal values for each statistically significant factor (see Table 15).

Factor		Codes	Value
A	d	2	300
B	w	2	25
C	g	-2	2

Table 15. Optimal Values for Factors (CCD)

No simulation run consisting of these values was conducted during the design-of-experiments approach. As a consequence, factor D does not exist in this optimization analysis. Therefore, a validation experiment is conducted by using simulation software, keeping factor D as code zero value. The mathematical model predicts the absorption value as 1.008304137, which is not practically possible because the absorption value cannot take a value greater than 1. However, the mathematical model function can have values greater than 1. In simulations, and in real experiments, this value is not possible to exceed the limit of 100%, but it can be expected to be a higher value with respect to the average.

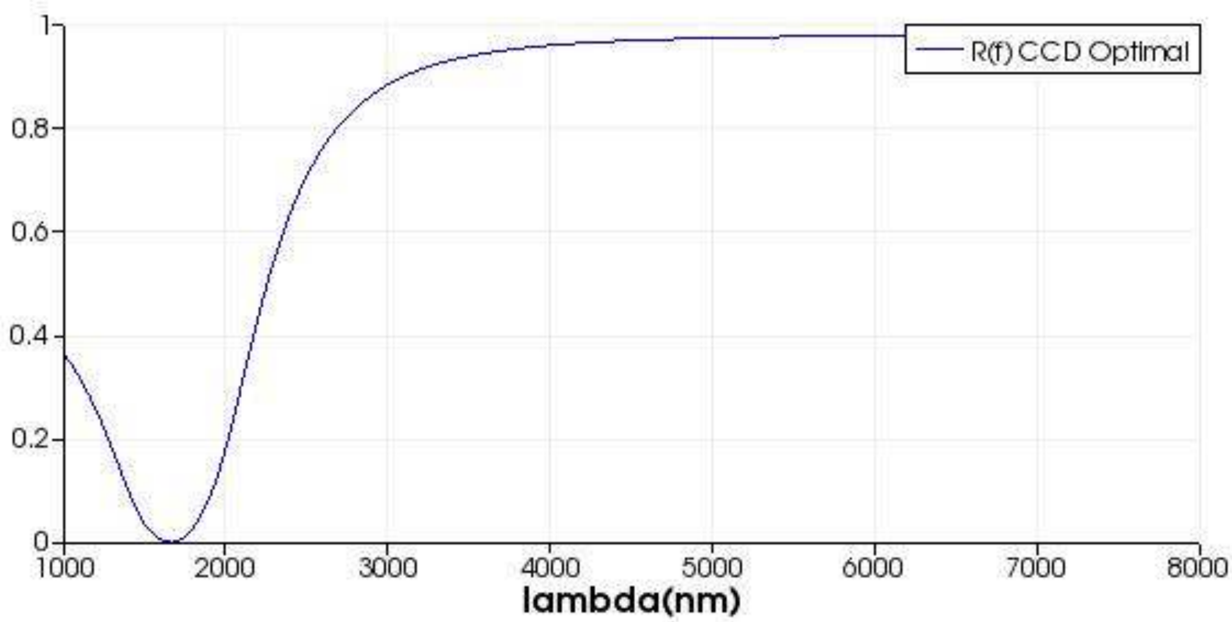

Figure 15. Simulation Output for Optimal Values (CCD)

According to the results of the validation run, the absorption value is 0.99846837. This value is not equal to the mathematical model value and is less than 1, as expected. In addition, this value is not the greatest value among the other 25 simulation runs; it is the eighth value in the order of these 25 values.

2.12.2.2 Plots of Significant Effects:

The main effects and interaction effects are plotted using Excel software. The main effects plots show that factors A and B have significant effect on response value, absorption. On the other hand, factors C and D are not significant (see Figure 16. Main Effects Plot for Central Composite Design). The basic criterion for this decision is the vertical difference in these plots. The central composite design analyzes factors at five levels. This is an advantage of the central composite design. However, the response values at the star points come only from one experiment for each star point of each factor. The values of the other parameters are kept at the center point. That is why the interpretation of the results, only looking to the main factor effects, is not sufficient to reach dependable conclusions. Plots could be different if more experiments were conducted for each star point with different combinations of values for other factors.

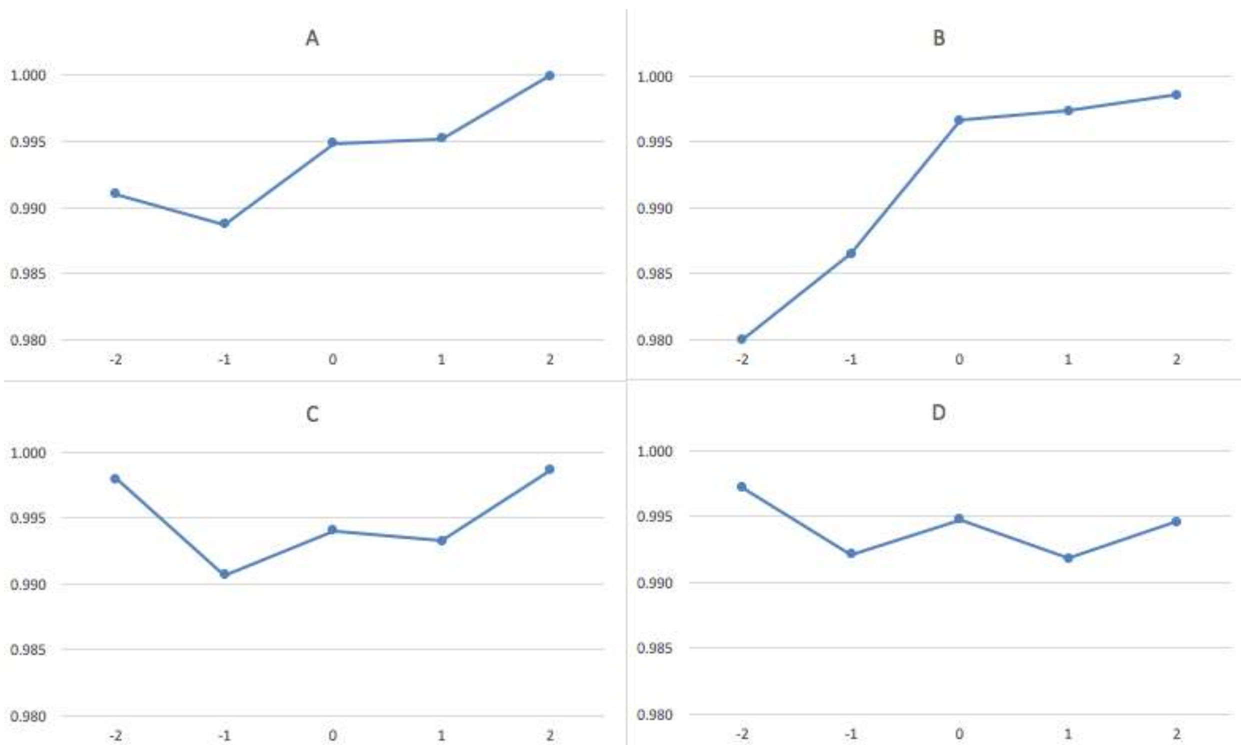

Figure 16. Main Effects Plot for Central Composite Design

The plot of factor B is the easiest one to interpret. It can confidently be said that the most significant effect on the response value is formed by factor B, since its effect on the average response can be observed as differing from almost 0.98 to 1, which is the greatest among other main factors. The statistical results support this fact, with the highest coefficient value of 0.0051 and the lowest P-value of 1.1075E-08.

It also can be inferred that factor B has a significant curvature effect, while factor A has a linear effect. That is because the slope of the factor B plot is lower for positive code values than for negative values.

The effect of factor A is also considered as significant, according to the statistical values. The coefficient of factor A is 0.002908, with a P-value of 2.6994E-05.

Based on the plot, factor C also seems to have a significant effect; however, according to the statistics, its effect is relatively lower than others. The coefficient factor C is 0.000923 with a P-value of 0.87551, which is higher than $\alpha = 0.5$.

The effect of factor D is less than factor C. Its coefficient value is -0.0003099, and its P-value is 0.5599551. Factor D is also removed from the model, while factor C is not. The reason is that the factor C remains in the significant BC interaction and it cannot be removed according to the model hierarchy. However, factor D does not exist in any other significant interaction or curvature effects. Therefore, it is removed from the model.

The BC interaction effect, which has a coefficient value of -0,001698 and a P-value of 0.0012, is considered significant. However, the AC interaction effect is not considered significant, based on the statistical values. It has a coefficient value of 0.00111 and a P-value of 0.126. Both interaction effects are plotted (Please see Figure 17).

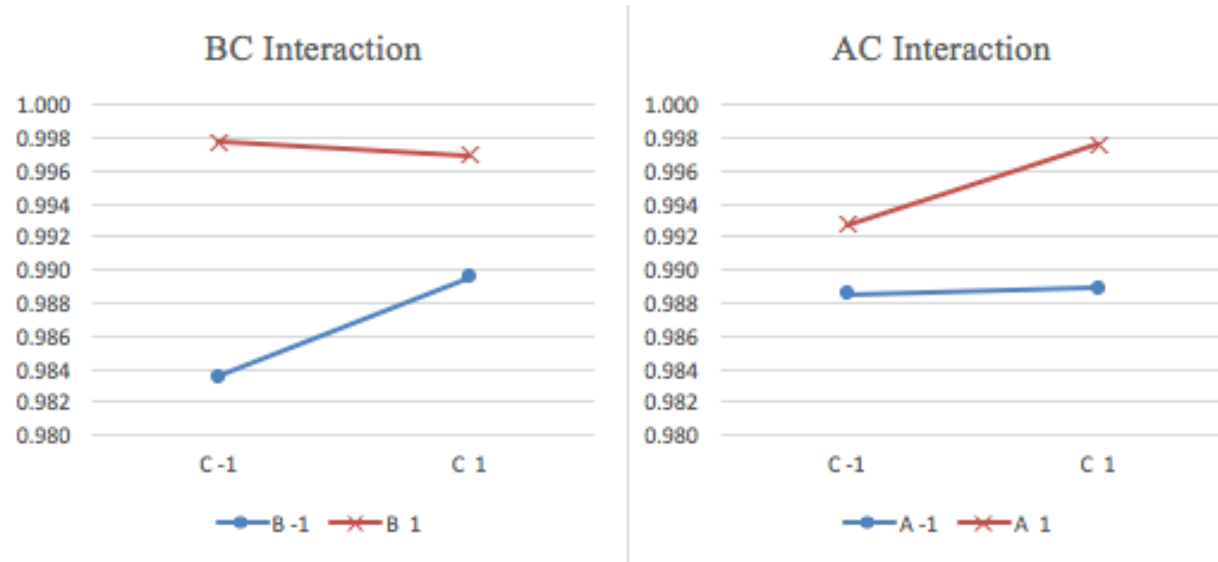

Figure 17. BC and AC Interaction Effect Plots for Central Composite Design

Based on the plots, it can be said that the significance of the BC and AC interaction effects are statistically close to each other. However, the angle between the two lines at the BC interaction plot is slightly higher than the angle at the AC interaction plot. This indicates that the effect of the BC interaction on the process is greater than the AC interaction effect.

Using Minitab software, the surface plots are generated. These plots demonstrate the interaction effects and the curvature effects in a better way than the main effect and the interaction effect plots.

The surface plot of absorption versus B, A shows the effect of different values of factors A and B on the response value (see Figure 18). It is easy to see the effect of both factors individually using this plot, since the AB interaction effect is not significant. An increase at the value of A causes a linear increase on the value of absorption. On the other hand, the increase at the value of factor B causes a curvature effect on the value of absorption. It can also be seen that the effect of factor B is stronger than effect of factor A.

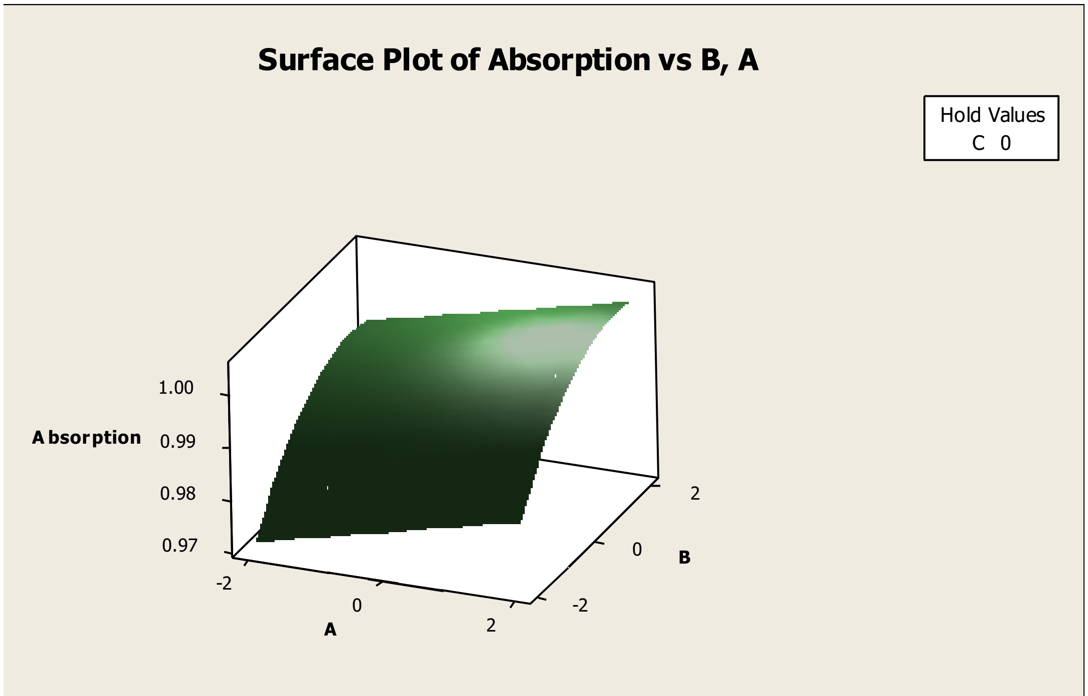

Figure 18. Surface Plot for B, A (CCD)

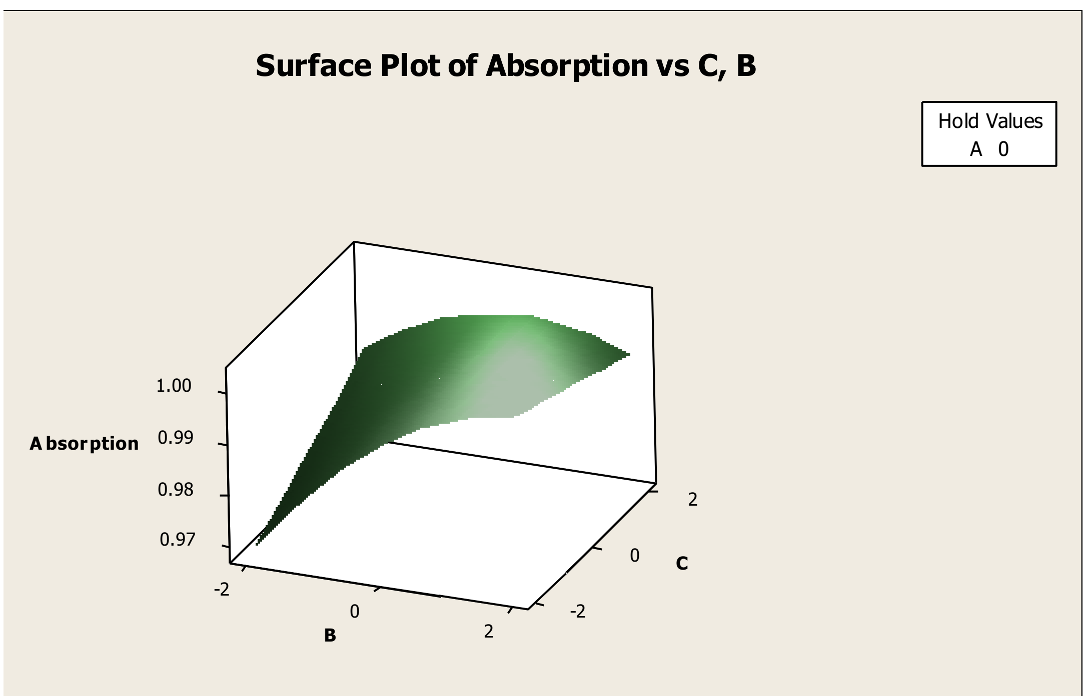

Figure 19. Surface Plot for C, B (CCD)

The surface plot of absorption value versus factors C and B shows the effect of the significant BC interaction as well as the significant effect of the main factor B (see Figure 19). As in the surface plot for B, A, this plot shows the highly significant B curvature effect. The surface also bends, while value of factor C increases. The reason for this bend is the significant BC interaction effect.

The surface plot of absorption value versus Factors C and A shows the AC interaction effect, which is not considered statistically significant (see Figure 20). However, at first glance, this plot seems to be as significant as the BC interaction effect. The reason for that is the different scale of the vertical axis indicating absorption value. This axis shows values from 0.97 to 1 at surface plots for B, A and for C, B. On the other hand, at the surface plot for C, A, this axis starts from 0.99. This causes the exaggeration of the bend on the surface, but the effect of the difference of the values of the factors can still be observed.

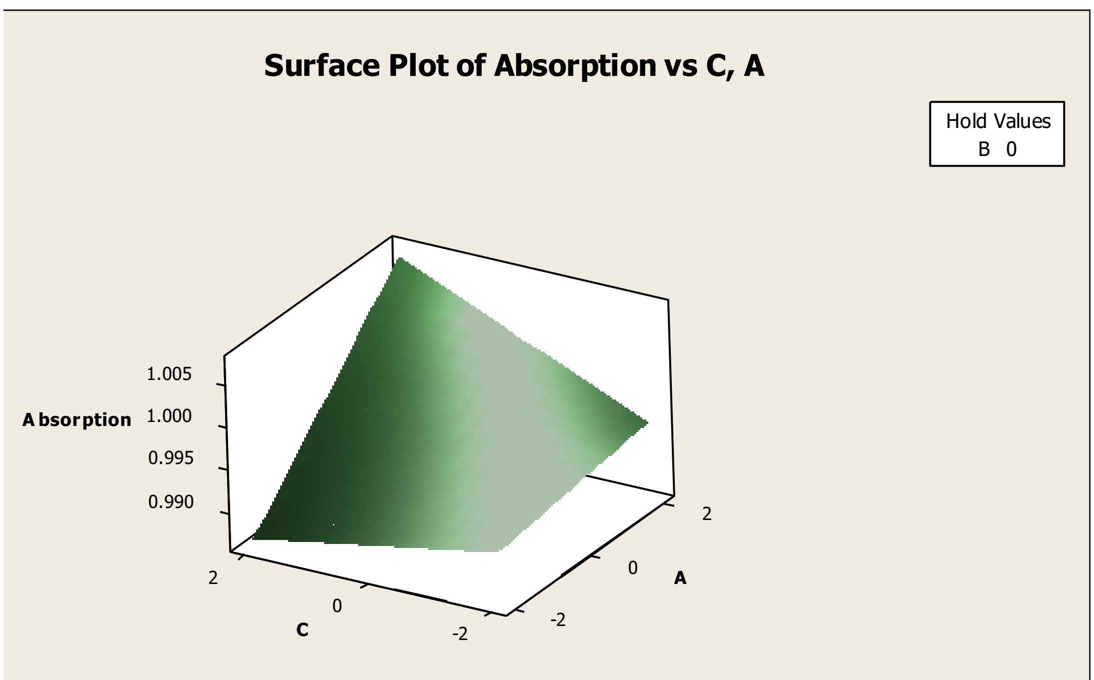

Figure 20. Surface Plot for C, A (CCD)

In Appendix 4, the contour plots of absorption value versus main factors can be seen. The contour plots show the same information in a different way than the surface plots so. These plots are beneficial tools in the interpretation of the outputs of analyses.

2.12.2.3 Validity of the Central Composite Design Value:

The approach has an issue with the math model that can give values that greater than one, 100%. However, the complete approach provides good information about the design parameters and the absorption response. The results provide an idea of how the process responds to the changes in the values of the parameters. Specifically, it can be inferred that keeping the diameter of the outer ring and the width of the rings high, and the gap and split angle low, gives better solutions in having a high absorption value.

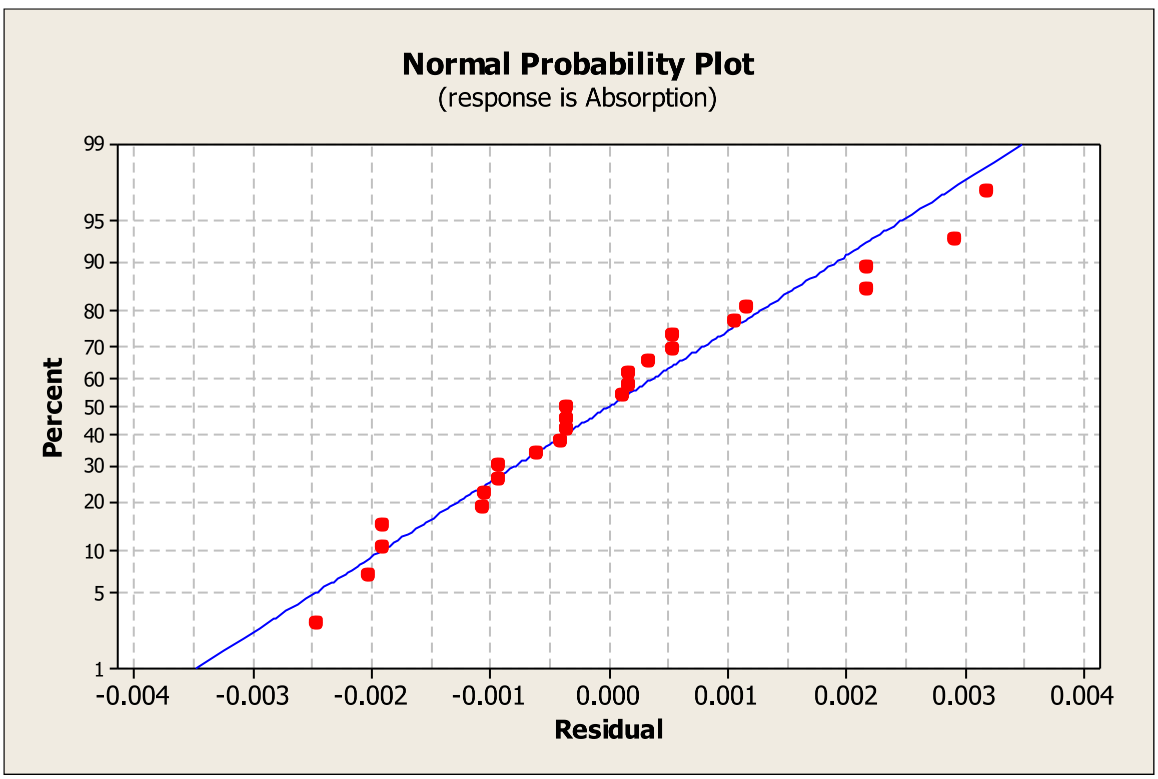

Figure 21. Normal Probability Plot for Residuals (CCD)

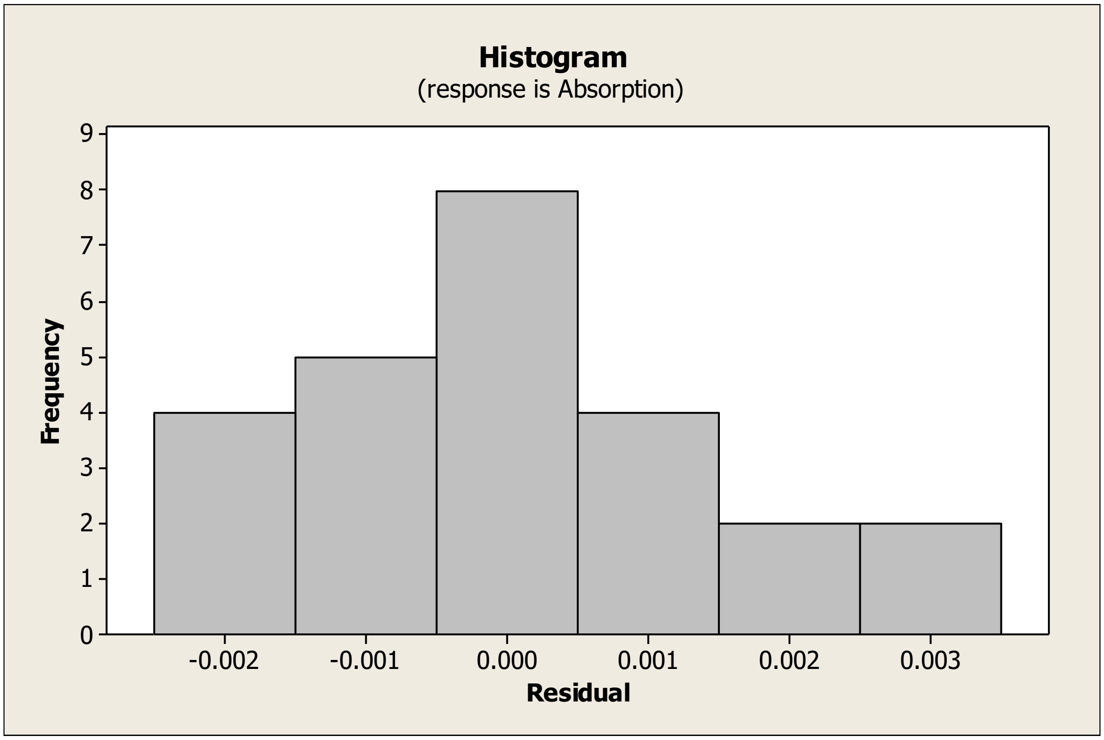

Figure 22. Histogram for Residuals (CCD)

Based on the normal probability plot and histogram for Residuals (see Figure 21 and Figure 22), it can be concluded that the residuals are distributed normally. It can also be seen that the data is slightly skewed to the positive side. However, since this is a minor finding, it is not a critical issue.

According to the residuals versus fitted values plot, it can be inferred that the data points are randomly distributed, with a few outliers (see Figure 23). These points may require attention; however, the data is mainly distributed randomly, which indicates that the regression model fits to the process.

Based on the residuals versus order plot, it can be said that the process is independent of the time order (see Figure 24), which indicates that there is not any other significant factor related to the order of the experiments. Based on these plots, it can be seen that, overall, the regression model for the full factorial design fits to the process.

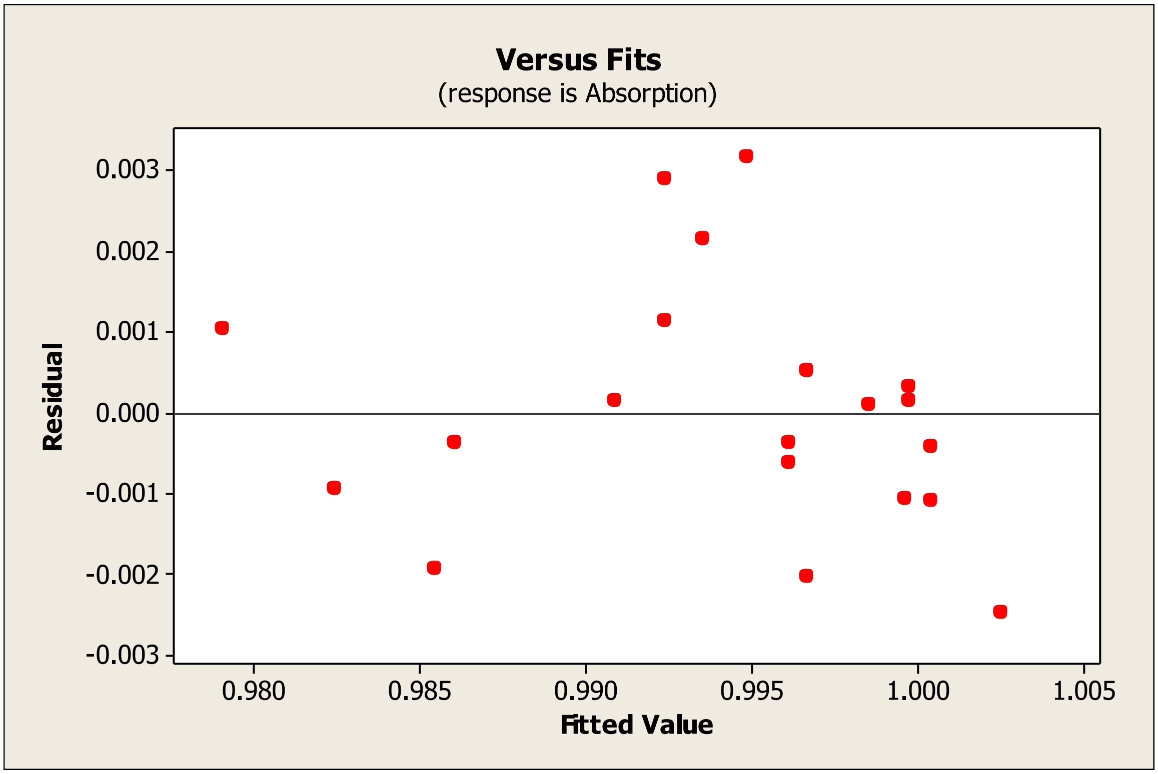

Figure 23. Residuals versus Fitted Values Plot for CCD

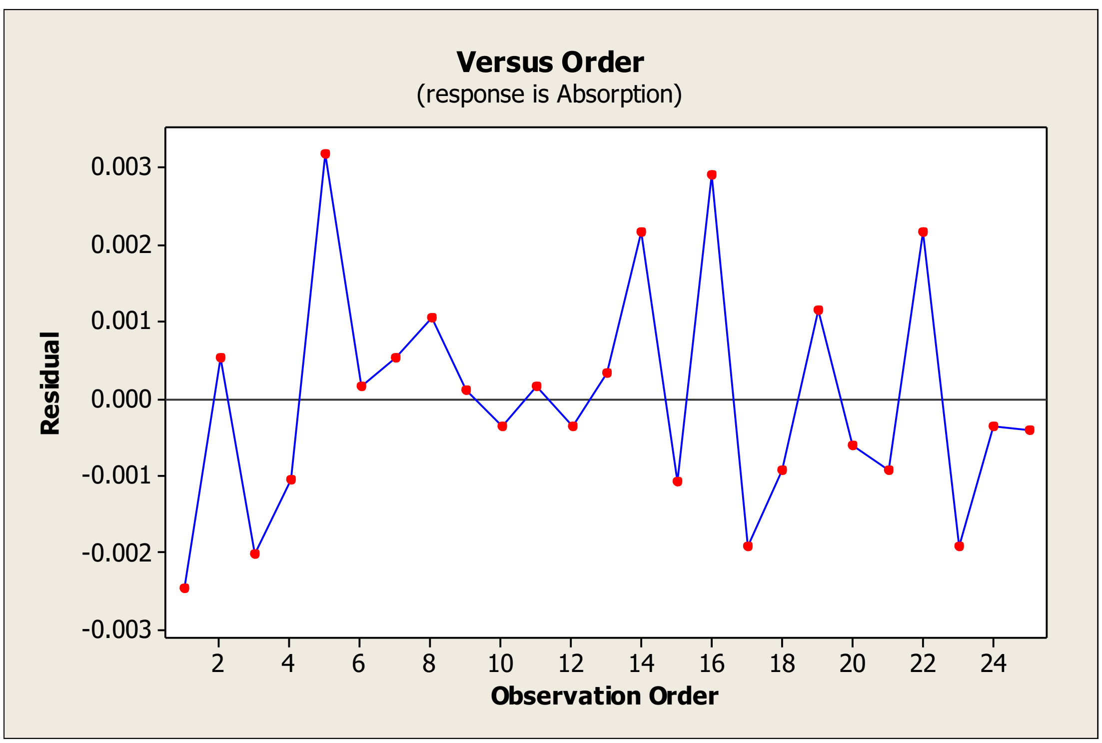

Figure 24. Residuals versus Order Plot for CCD

2.13 COMPARISON OF THE DESIGN-OF-EXPERIMENT APPROACHES

In order to compare and discuss the advantages and disadvantages of the full factorial design model and the central composite design model, three criteria are used. These criteria are efficiency, regression and math model, and validity and analysis of residuals.

2.13.1 REGRESSION ANALYSIS AND MATH MODEL

In this study, regression analyses for the full factorial design and the central composite design give different outputs. Because input parameters have different code values in models, it is expected that there will be different coefficient values for terms of the mathematical models. However, because of studying the same process, it is also expected that there will be the same significant terms in the math models. However, the central composite design analysis at the final form suggests that the AC interaction effect is insignificant, while the full factorial design shows that this effect is significant (see Table 16).

Full Factorial Design			Central Composite Design		
	Coefficients	*P-value*		*Coefficients*	*P-value*
Intercept	0.996057182	5.8116E-184	Intercept	0.994883042	2.87703E-49
A	0.003747764	2.98696E-21	A	0.002908414	1.79627E-05
B	0.006610478	6.20036E-36	B	0.005140634	4.99051E-09
C	1.35819E-05	0.961779885	C	0.000922924	0.087551327
AC	0.001248684	0.000555271			
BC	-0.001335179	0.000240992	BC	-0.001698185	0.014001518
BB	-0.006095618	7.57741E-20	BB	-0.001905973	0.001227622

Table 16. Comparison of Regression Outputs

The central composite design examines the effect of the AC interaction as well as the full factorial design. However, based on the statistical values of the regression analysis output, it is not considered significant enough to be included in the math model. This interaction effect is removed from the model because it has a P-value of 0.125996, which is greater than a $\alpha = 0.05$ confidence level.

At this point, it should be determined which model fits the process better.

2.13.2 VALIDITY AND ANALYSIS OF RESIDUALS

It is important to investigate the fitness of model for the process. First, the values of R square and adjusted R square should be taken into consideration.

R square, basically, is the percentage of the degree to which the mathematical model that uses the coefficient values for each independent variable explains the value of dependent variable, which is absorption in this study. Adjusted R square is a value that is adjusted depending on the size of the model (Montgomery, 2013). This value increases when the insignificant terms are removed from the model.

The R values of the full factorial design are higher than those of the central composite design. The R square and adjusted R square values of the full factorial design are 0.924556 and 0.9184389, respectively. These values of the central composite design are 0.89256 and 0.864286, respectively. These numbers indicate that the regression model from the full factorial design fits to this process better.

In addition to the R square values, an examination of residuals gives a good understanding of the fitness of the model. The histogram and normal probability plots for residuals, residual versus fitted values, and residual versus observation graphs for each model were investigated and interpreted. In comparing these results, it is hard to suggest that one of the models is far better than the other. Therefore, the full factorial design can be said to have a better fit to the model, based on the R values.

2.13.3 EFFICIENCY

Efficiency is an important criterion because conducting experiments usuallly requires resources, such as money, time, materials, and facilities. The more the shortage of these resources increases, the more the importance of the efficiency increases, as well.

One indicator of the efficiency of a design-of-experiment method is the number of experiments required to obtain results. In order to study the main factor effects, two factor interaction effects, and curvature effects, the full factorial design model required 81 experiments while the central composite design model required only 25 experiments. Based on these numbers, the central composite design model is highly efficient with respect to the full factorial design model.

On the other hand, the quality of the outputs of the model is another aspect of efficiency. There is a trade-off between conducting fewer experiments and getting more information about the process. The effort to decrease the number of experiments might cause a significant effect to be missed and might lead to the misinterpretation of the process. As the results indicate, the central composite design considers a two factor interaction effect insignificant, while the full factorial design shows that it is significant.

In this specific study, the full factorial design required 2.24 times more experiments. However, its results are more accurate and reliable based on the comparison of their fitness. Therefore, conducting more experiments in this study has given better results.

2.14 RECOMMENDATIONS

2.14.1 RECOMMENDATIONS

When considering split ring resonators specifically among all types of meta-materials, there are some recommendations for future studies. The absorption level in this study was considered without regarding the wavelength of the related specific electromagnetic wave. The bandwidth of the high absorption values can be another response characteristic. Because it doesn't have a limit of 100% as in the absorption level, the bandwidth is a characteristic that is the larger the better. The cost may also be another response value that needs to be studied in addition to the absorption value. Different materials, thickness of the metal, different parameters for rings, using more rings, and temperature of the environment can be other input variables.

2.14.2 ISSUES IDENTIFIED

Without a metal ground layer, the meta-material device would also have transmission value. This value was disabled in this project by adding a golden ground layer. However, this increases the cost of the device and decreases its affordability. Alternative methods to solve this issue should be considered.

VO_2 used as an insulator material in this model has different characteristics at high temperatures. This feature of this material is a good area of study that should be considered while improving meta-materials.

From the design-of-experiment point of view, this study has the issue of limiting the regression model to give values in boundaries, such as between zero and one. It may require the addition of another constraint to the model that limits the function to give bounded estimates.

2.14.3 IMPLICATIONS

Meta-materials are a demanding and developing area of research. The design-of-experiments methods are not commonly used in this area. The recommendations for split ring resonators are also valid for other meta-material applications which are very broad. This study shows that the design-of-experiments approaches can be applied to this area of research in order to examine the process. It also shows that different methods of design-of-experiments can provide different results.

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APPENDICES

APPENDIX 1 Regression output of the second step of the full factorial design analysis

Regression Statistics	
Multiple R	0.96302654
R Square	0.927420116
Adjusted R Square	0.917051561
Standard Error	0.002093303
Observations	81

ANOVA

	df	*SS*	*MS*	*F*
Regression	10	0.003919426	0.000391943	89.44545598
Residual	70	0.000306734	4.38192E-06	
Total	80	0.00422616		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.995994256	0.000697768	1427.401033	5.4643E-158
A	0.003747764	0.000284862	13.15639795	1.33571E-20
B	0.006610478	0.000284862	23.20586105	1.35709E-34
C	1.35819E-05	0.000284862	0.04767889	0.96210793
D	-0.000261085	0.000284862	-0.916530774	0.362535457
AC	0.001248684	0.000348884	3.579082229	0.000631396
BC	-0.001335179	0.000348884	-3.827001875	0.000279115
AA	-0.00015218	0.000493396	-0.308433304	0.75866866
BB	-0.006095618	0.000493396	-12.35440739	2.95117E-19
CC	0.000578482	0.000493396	1.172449109	0.244991478
DD	-0.000331913	0.000493396	-0.672711517	0.503345557

APPENDIX 2 Regression output of the third step of the full factorial design analysis

Regression Statistics	
Multiple R	0.961991143
R Square	0.92542696
Adjusted R Square	0.91827612
Standard Error	0.002077794
Observations	81

ANOVA

	df	*SS*	*MS*	*F*
Regression	7	0.003911002	0.000558715	129.4151493
Residual	73	0.000315158	4.31723E-06	
Total	80	0.00422616		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.996057182	0.000399872	2490.942435	1.11E-181
A	0.003747764	0.000282752	13.25459985	4.09247E-21
B	0.006610478	0.000282752	23.3790741	1.24436E-35
C	1.35819E-05	0.000282752	0.048034775	0.961819705
D	-0.000261085	0.000282752	-0.923371937	0.358855454
AC	0.001248684	0.000346299	3.605797193	0.000565949
BC	-0.001335179	0.000346299	-3.855567359	0.000246397
BB	-0.006095618	0.000489741	-12.44662308	1.00525E-19

APPENDIX 3 Regression output of the second step of the central composite design analysis

Regression Statistics	
Multiple R	0.952828132
R Square	0.907881448
Adjusted R Square	0.852610317
Standard Error	0.002615839
Observations	25

ANOVA

	df	*SS*	*MS*	*F*
Regression	9	0.001011568	0.000112396	16.42596112
Residual	15	0.000102639	6.84261E-06	
Total	24	0.001114208		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.99715427	0.002615839	381.1986826	2.56897E-31
A	0.002908414	0.000533956	5.446917779	6.74522E-05
B	0.005140634	0.000533956	9.627450772	8.21519E-08
C	0.000922924	0.000533956	1.728464762	0.104427169
D	-0.000309943	0.000533956	-0.580465736	0.570218915
BC	-0.001698185	0.00065396	-2.59677397	0.020228473
AA	-0.000882388	0.000778368	-1.133639115	0.274740013
BB	-0.002426463	0.000778368	-3.117373765	0.007062046
CC	-0.000181782	0.000778368	-0.233542338	0.81849753
DD	-0.000781203	0.000778368	-1.003642698	0.33146755

APPENDIX 4 Regression output of the third step of the central composite design analysis

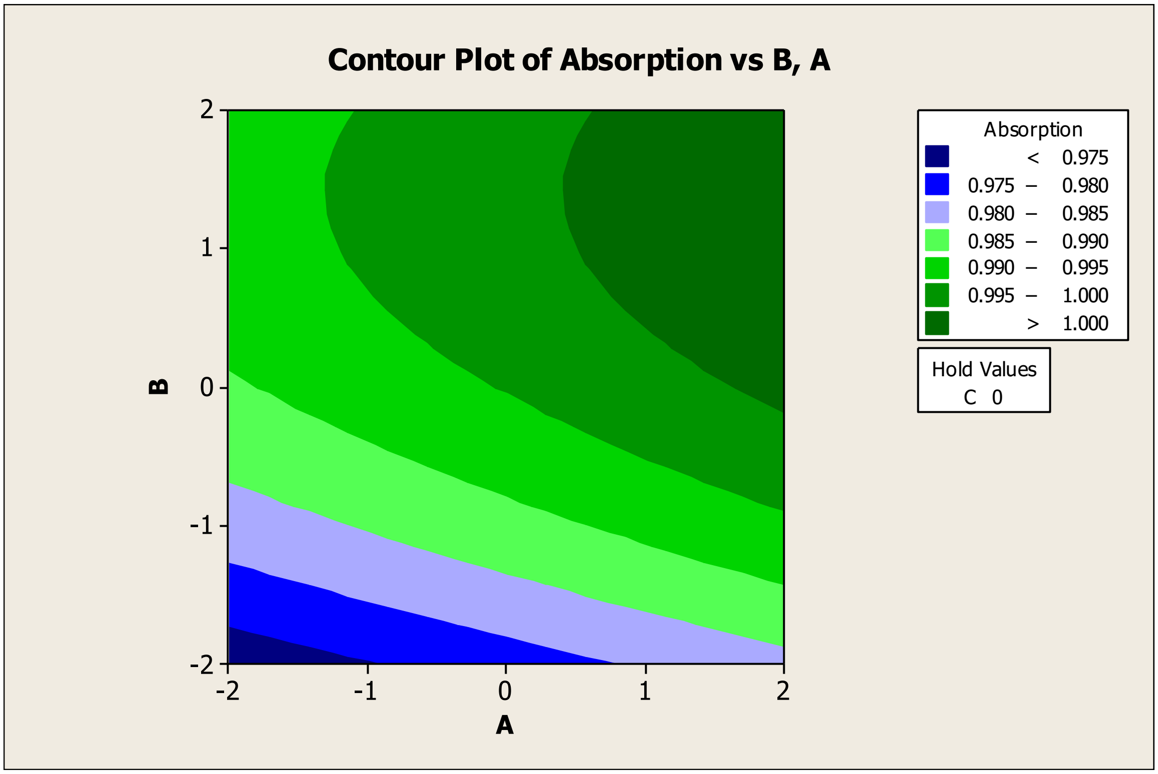
Regression Statistics	
Multiple R	0.945848343
R Square	0.894629087
Adjusted R Square	0.85950545
Standard Error	0.002553919
Observations	25

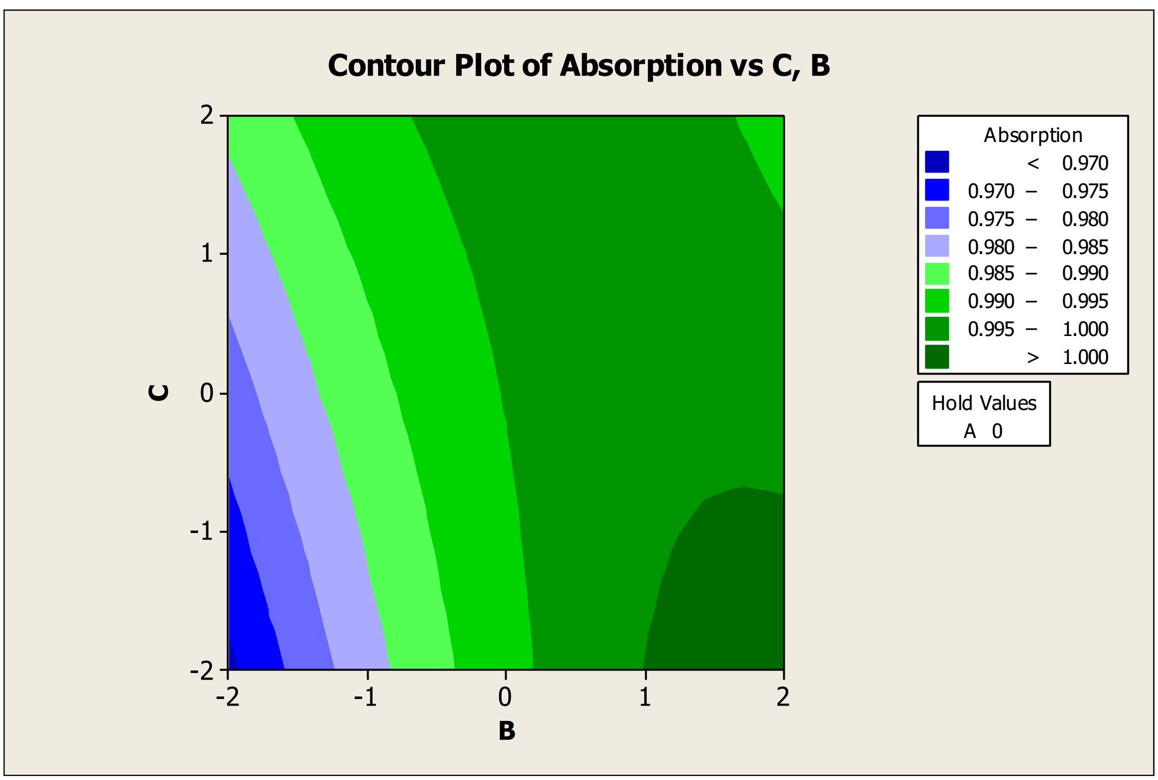
ANOVA

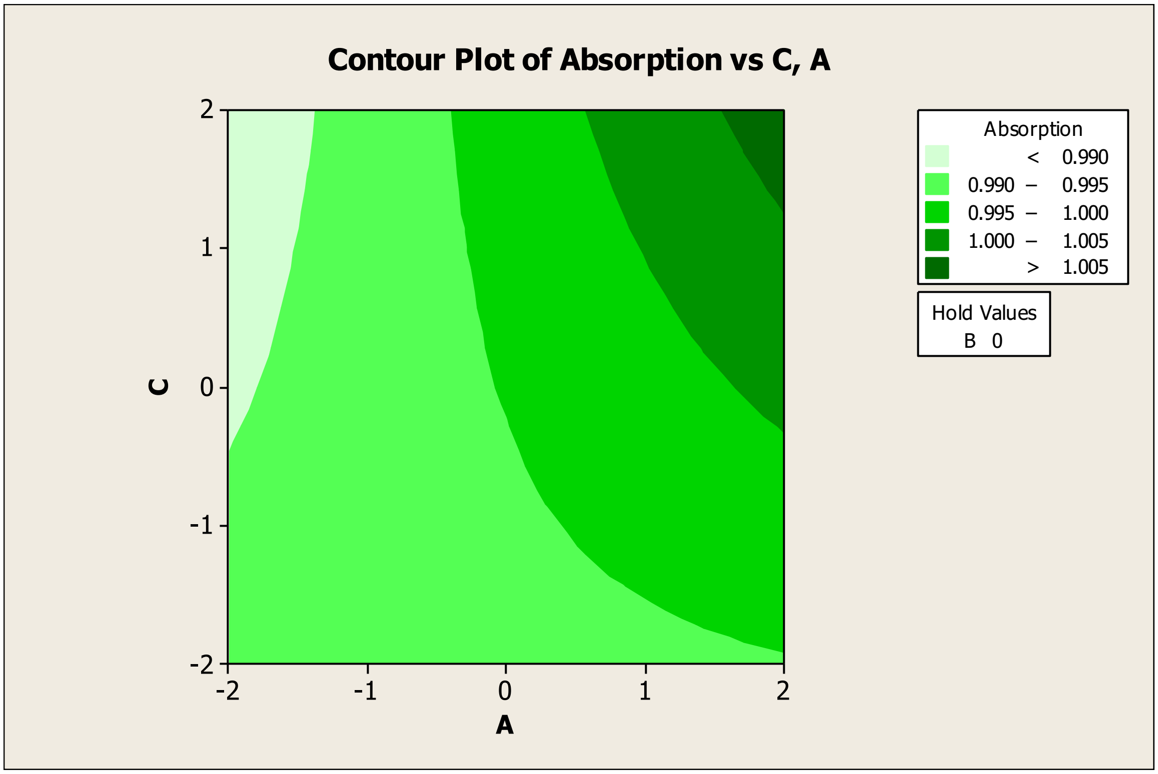
	df	*SS*	*MS*	*F*
Regression	6	0.000996802	0.000166134	25.47085522
Residual	18	0.000117405	6.5225E-06	
Total	24	0.001114208		

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.994883042	0.00070833	1404.547868	8.12898E-47
A	0.002908414	0.000521317	5.578977659	2.69942E-05
B	0.005140634	0.000521317	9.860867182	1.10752E-08
C	0.000922924	0.000521317	1.770371187	0.09360005
D	-0.000309943	0.000521317	-0.594539059	0.559551415
BC	-0.001698185	0.00063848	-2.659732449	0.015957115
BB	-0.001905973	0.000511193	-3.728480992	0.001537933

APPENDIX 5 Contour Plots for Absorption versus Factors (Central Composite Design)






Contour Plot of Absorption vs C, A
Absorption
< 0.990
0.990 – 0.995
0.995 – 1.000
1.000 – 1.005
> 1.005
Hold Values
B 0
C
A
2
1
0
-1
-2
-2
-1
0
1
2

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