

# IMPROVING THE COVARIATIONAL THINKING ABILITY OF SECONDARY SCHOOL STUDENTS

Keywords: Functions, Calculus, Covariation, Representations

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*The aim of this intervention study is to improve secondary school students' ability of thinking in terms of functions with regard to the aspect of covariation. The aspect of covariation focuses on a particular aspect of the concept of function: the direction and scale of the change of the y-value induced by a certain increase of the x-value. In Germany, the aspect of covariation is usually taught in grades 11-13 when dealing with infinitesimal calculus where the derivative is introduced as the instantaneous rate of change. From a more general point of view covariational thinking can be considered as a sub-category of dynamical thinking. Dealing with problems where dynamical thinking is involved, the person will have to predict the status of a system if a certain change in the system occurs (Ossimitz 2000). The intention of this poster (70 cm x 90 cm) is to display the training sessions and the design of the study.*

## THEORETICAL BACKGROUND

A significant number of students and even adults have difficulties to solve simple mathematical tasks involving dynamical thinking. Studies e.g. in the context of functions (Monk 1992), stock-and-flow-problems (Sweeney & Sterman 2000) and in geometrical configurations (Roth 2008) have proven this lack of competence.

Vollrath (1989) considers three aspects of the concept of function to be essential. Firstly, in the point-wise view every value of the domain corresponds to a precise value of the range. Secondly, the aspect of covariation requires taking the neighborhood of the point into account: In which way will the function value vary if the x-value is varied? Thirdly, the global view contemplates the function as a whole. This will be necessary if e.g. statements about symmetry are made.

Researchers point out that the aspect of covariation is not sufficiently implemented in the mathematical curricula (Malle 2000). Instead, school teaching mainly focuses on the point-wise aspect of functions even if it is hardly possible to construct adequate mental models of the concept of function without considering the covariational aspect. Despite the researchers' consensus on the importance of this aspect, there is a lack of concepts how to train it within secondary education.

## AIMS AND RESEARCH QUESTIONS

Developing empirically based training sessions in covariational thinking, we hope to better prepare students for the infinitesimal calculus but also to show a way to

improve a neglected aspect of mathematical literacy in secondary grades. A research hypothesis is that covariational thinking is not restricted to students of grades 11-13 and that students from grade 6 onwards are able to make substantial improvements in this kind of functional thinking. Furthermore, we suppose that the pupils' performance in tasks with discrete domain is better than in tasks with continuous domain. Finally, we conjecture that a quantitative exploration of the covariational behaviour of a function is best done with t-charts as external representations whereas graphs as external representations are more beneficial in a qualitative exploration.

## **METHOD**

A training session in 6th grade will start with a material-based analysis of the covariation of different functional dependencies (linear vs. nonlinear) with discrete domain. The pupils will be asked to produce t-charts and graphs in a paper-and-pencil work. They will explore the covariation in qualitative and quantitative manner based on their previously produced representations. A second treatment group will do a comparable training session on functional dependencies with continuous domain. The effects of the variables (discrete vs. continuous, t-charts vs. graphs) will be examined through pre- and post-tests.

In 8th and 10th grade comparable trainings at an advanced level will be conducted. As these pupils have already been introduced to the concept of function as well as some function classes the training will be expanded with exercises aiming at connecting their newly acquired knowledge of covariational thinking to their conception of function.

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