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Improving VWAP strategies : A dynamical volume approach

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Abstract

In this paper, we present a new methodology for modeling intraday volume which allows for a reduction of the execution risk in VWAP (Volume Weighted Average Price) orders. The results are obtained for the all stocks included in the CAC40 index at the beginning of September 2004. The idea of considered models is based on the decomposition of traded volume into two parts: one reflects volume changes due to market evolutions, the second describes the stock specific volume pattern. The dynamics of the specific part of volume is depicted by ARMA, and SETAR models. The implementation of VWAP strategies imposes some dynamical adjustments within the day.

Keywords: Intraday Volume, VWAP Strategies, Principal Component Analysis, Arbitrage.

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1 Introduction

In financial literature, when considering perfect markets, volume is often ignored. But it is an important market characteristic for practitioners who aim at lowering the market impact of their trades. This impact can be measured by comparing the execution price of an order to a benchmark. The larger this price difference, the higher the market impact. One such benchmark is known as the Volume Weighted Average Price, or VWAP. Informally, the VWAP of a stock over a period of time is just the average price paid per share during that period. The VWAP benchmark is then the sum of every transaction price paid, weighted by its volume. The goal of any trader, tracking VWAP benchmark, is to find and define ex ante strategies, which ex post lead to an average trading price being as close as possible to the VWAP price. Hence, VWAP strategies are defined as buying and selling a fixed number of shares at an average price that tracks the VWAP.

VWAP execution orders represent around 50% of all the institutional investors' trading. The simplicity of such strategies explain its growing success. First, investors who ask for VWAP execution accept they will postpone or sequence their trades in order to reduce their trading cost when selling or buying large amounts of shares. Doing so, they reduce their market impact, and thus increase the profitability of their transactions by accepting a risk in time. Likewise, VWAP orders allow foreign investors to avoid the high risk related to the fact that their orders have to be placed before the market opens. Secondly, it is a common practice to evaluate the performance of trades based on their ability to execute the orders at a price better or equal to VWAP. In this case, VWAP can be seen as an optimal benchmark[1]. Finally, VWAP is a better benchmark than any price at a fixed time in the future as it cannot be manipulated. Consequently, it improves both market transparency and efficiency [see Cuching and Madhavan (2001)].

To implement VWAP strategies, we first need to model the intraday evolution of the relative volume and as we will see below, we don't need to model the intraday price evolution. It is now common knowledge that intraday volume moves around a U-shape seasonal

[1]Berkowitz, Logue and Noser (1988) show that VWAP is a good proxy for the optimal price attainable by passive traders.

pattern [see for example Biais, al. (1995), Gouriéroux et al. (1999) for the French stock market.]. These seasonal fluctuations have hampered volume modeling. One way to circumvent this problem is to work on a transaction or market time scale instead of calendar time scale [see Engle (2000), Gouriéroux and Le Fol (1998) for example]. However, this transformation is useless when working on strategies which are defined on a calendar time scale [Le Fol and Mercier (1998) suppose that the time transformation is fixed and use this hypothesis to pass from one time scale to the other]. Other approaches correct volume on a stock by stock time varying average volume [Engle (1998), Easley, O'Hara (1987)], and others take the time varying across stock and average volume [See Mc Culloch (2004)]. In all this work, seasonal variation is just a problem that they adequately and empirically dispose. On the contrary, in our work we do not have to eliminate seasonal considerations as we use it to arrive at the common component and thus, to construct our volume benchmark for VWAP strategies. Here, we want to discriminate between the seasonal and static part of volume from the dynamic one. The identification of such components of volume comes from the observation that seasonal fluctuation is common across stocks whereas dynamics is a stock by stock feature.

If volume has been analyzed in the financial market literature, it has often been used for a better understanding of other financial variables, like price [Easley, O'Hara (1987), Foster, Wisvanathan (1990) for example] or volatility [Tauchen, Pitts (1983), Karpoff (1987), Anderson (1996) and Manganelli (2002) for example]. Moreover most of these studies use daily or even lower data frequency [one exception is Darrat, Rahman, and Zhong (2003) who examine intraday data of stocks from Dow Jones index, and reported significant lead-lag relations between volume and volatility]. The rare papers that concentrate on volume are Kaastra and Boyd (1995), Darolles and Le Fol (2003).

This paper is in the line with the methodology proposed by Darolles and Le Fol (2003) for volume decomposition. The main contributions are first to work on intraday data, second to propose some dynamically updated predictions of volume and finally to use VWAP strategies to test the accuracy of the approach. Basically, volume is decomposed into two components: the first describes the size of volume on ordinary days and is extracted from

the stocks included in the CAC40 index. The second component measures the abnormal or unexpected changes of volume.

The CAPM is one of the most famous models for returns that is based on such techniques. Lo and Wang (2000) were the first ones to transpose this model to volumes, also used by Darolles and Le Fol (2003). This study is a natural extension of this work on high frequency data relating to the problem of optimal executions of VWAP orders. Furthermore, it is worth highlighting that, by separating the market part from observed volume, two additional goals were obtained. First, the specific component, as a measure of liquidity for a particular company, is a much more reliable indicator of arbitrage activity than the observed volume. Secondly, this decomposition allows us to accurately remove seasonal variations, without imposing any particular form.

The paper is organized as follows: Section 2 starts with a simple example showing why volume is the only important variable when tracking VWAP. We then provide a description of the models for a market component, and a specific component of volume. Section 3 contains data description and summary statistics of the data, as well as in and out sample estimations results. Applications to VWAP strategies are presented in section 4. Section 5 concludes the paper.

2 The volume trading model

In this section, we explain why we don't need to predict price to achieve the goal of tracking VWAP. We then introduce the volume statistical model which includes the decomposition of volume method and the intraday volume dynamics. As mentioned before, the major problem of intraday volume is its high intraday seasonal variation. Two approaches have been considered to deal with this problem. The first takes an historical average of volume for any stock as its seasonal pattern or normal volume [Easley, O'Hara (1987)]. The second takes the average volume across stocks to get this normal volume [McCulloch (2004)]. Here, we propose another method to extract the seasonal, or normal, volume based on principal component analysis. Such a method allows us to get a normal non stationary volume

component, which is common across stocks, and a specific stationary component. Next, we propose to model the dynamics of the aforementioned components taken separately.

2.1 Predicting volume to track the VWAP

As we said before, the goal of any trader tracking the VWAP is to define ex ante strategies, which ex post lead to an average trading price as close as possible to the VWAP. In fact, as soon as we know the future sequence of intra day trading volume and we can adapt our trading scheme accordingly, we are good. As a consequence, the problem resumes to adequately forecast the intra day volume. To see this, let us consider a simple financial market where trades can only occur every hour $i = 1, ..., 9$, and a broker who wants to trade 100 000 shares at the VWAP. If she knows what will be the hourly pattern of volume (V_i) or equivalently the hourly turnover[2], $x_i = \frac{V_i}{N_i}$, she can calculate the hourly traded volume percentage $\left(\widetilde{x}_i = \frac{x_i}{\sum x_i} = \frac{V_i}{\sum V_i}\right)$ and trade her 100 000 shares accordingly, $\underline{V}_i = 100000\widetilde{x}_i$, at price P_i.

We give in table 1 the hourly traded volume (column 5) which give the hourly percentage traded volume (column 6). Applying this splitting scheme to 100 000 shares, we get the hourly volume to trade (column 7). At the end of the day, we collect the hourly price and calculate, for three different price evolutions, the true VWAP of the day and of the portfolio. The VWAP of this specific day is:

$$\begin{aligned} VWAP &= \frac{\sum_{i=1}^{9} P_i V_i}{\sum_{i=1}^{9} V_i} = 161.7070 = \sum_{i=1}^{9} P_i \left(\frac{V_i}{\sum_{i=1}^{9} V_i} \right) \\ &= \sum_{i=1}^{9} P_i \left(\frac{x_i}{\sum_{i=1}^{9} x_i} \right) = \sum_{i=1}^{9} P_i \widetilde{x}_i. \end{aligned}$$

[2]The turnover is the number of traded shares V_i divided by the number of floated shares N_i.

The VWAP of the portfolio is:

$$\begin{aligned} VWAP = \frac{\sum_{i=1}^{9} P_i \underline{V}_i}{\sum_{i=1}^{9} \underline{V}_i} = 161.7070 &= \sum_{i=1}^{9} P_i \left(\frac{\underline{V}_i}{\sum_{i=1}^{9} \underline{V}_i} \right) \\ &= \sum_{i=1}^{9} P_i \left(\frac{100.000 x_i}{100.000 \sum_{i=1}^{9} x_i} \right) \\ &= \sum_{i=1}^{9} P_i \widetilde{x}_i. \end{aligned}$$

As we can see with this simple example, the only need when tracking the VWAP is the volume shape. Up to here, we have considered that the hourly volume was know. When this is not the case, one needs to predict the turnover time series x_i at the very beginning of the day, and of course, the better the prediction, the closer we get to the VWAP. We consider that a prediction of the hourly volume is given in column 8, which leads to the hourly percentage of traded volume in column 9. Applying this percentage to the same 100 000 shares portfolio as before, we get the strategy to implement. Here again, we collect the hourly price at the end of the day and calculate ex post VWAP of the day and of the portfolio. As we can see, the only case where we reach the VWAP we were tracking is the one where price are constants where the error is 0.4% to compare with 16% when it is not the case. As a consequence, errors in the volume predictions are negligible when prices are stable but can really become dramatic during trendy price periods. These prediction errors represent an execution risk which cannot be ignored particularly since volume surprises are usually linked to price surprises. This example stresses the importance in finding a good volume trading model which is the aim of the following sections.

2.2 Intraday volume decomposition section

The chosen methodology comes from asset management practices, where any portfolio can be decomposed into a market and arbitrage portfolios. A similar process can be applied to intraday volume: the trading volume has a market and a specific components [Darolles and Le Fol (2003) propose a theoretical model to explain such a decomposition of volume as well as a link with market practices]. Any stock volume or stock turnover, at any date,

depends on an average term and a deviation term. The average part corresponds to trading volume coming from market portfolio adjustments. Our interpretation is that the deviation element is due to the opening and closure of arbitrage positions. In order to get the two components of volume, we conduct a principal component analysis.

Let $x_{it} = \frac{V_{it}}{N_i}$, $i = 1, \ldots, I$, $t = 1, \ldots, T$ denotes the turnover series for stock i at date t, i.e. the number of traded shares V_{it} divided by the number of floated shares N_{it}. As shown in Darolles and Le Fol (2003), the market turnover x_t^I can be written as:

$$x_t^I = \frac{\sum_i P_{it} V_{it}}{\sum_k P_{kt} N_k} = \frac{\sum_i P_{it} N_i \frac{V_{in}}{N_i}}{\sum_k P_{kt} N_k} = \sum_i w_{it} x_{it}, \tag{1}$$

where P_{it} is the transaction price for stock i at date t, and w_{it} is the stock relative capitalization. In fact, all the series should also be indexed by day. It would become x_{it}^j, denoting the turnover for stock i and date t, and day j. However, we will ignore this last index, unless explicitly needed, for ease of the demonstration. Since the aim of principal component analysis is to explain the variance-covariance structure of the data through a few linear combinations of the original data, the first step is to calculate the $I \times I$ dimension variance-covariance matrix of the data. The spectral decomposition of this matrix leads to I orthogonal vectors, $C_t^k = x_{it}' u_k$, with dimension T, where u_k is the k^{th} eigenvector. Each eigenvector is associated with a positive eigenvalue λ_k such that:

$$Cov(C_t^k, C_t^l) = \lambda_k \delta_{kl}, \tag{2}$$

where δ_{kl} stands for Kroneker symbol. The standardized turnover times series can be decomposed as:

$$\frac{x_{it} - \overline{x}_i}{\sigma_i} = \sum_k u_k^i C_t^k.$$

Since the correlation is $corr(x_{it}, C_t^k) = \sqrt{\lambda_k} u_k^i$, the previous equation can be rewritten as:

$$\begin{aligned} x_{it} - \overline{x}_i &= \sigma_i \sum_k \frac{corr(x_{it}, C_t^k)}{\sqrt{\lambda_k}} C_t^k, \\ &= \sigma_i \sum_k \frac{corr(x_{it}, C_t^k)}{\sqrt{var\left(C_t^k\right)}} C_t^k, \\ &= \sum_k \frac{Cov(x_{it}, C_t^k)}{var\left(C_t^k\right)} C_t^k. \end{aligned}$$

Finally, we get the centered turnovers :

$$x_{it} - \overline{x}_i = \sum_k \frac{Cov(x_{it}, C_t^k)}{var\left(C_t^k\right)} C_t^k, \tag{3}$$

$$= \sum_k \frac{1}{\lambda_k} Cov(x_{it}, C_t^k) C_t^k. \tag{4}$$

Isolating the first factor, we get:

$$x_{it} - \overline{x}_i = \frac{1}{\lambda_1} Cov(x_{it}, C_t^1) C_t^1 + \sum_{k>1} \frac{1}{\lambda_k} Cov(x_{it}, C_t^k) C_t^k. \tag{5}$$

The first component is the larger variant and captures the seasonal changes. The others are stationary. In the following, we use this decomposition to predict future volume.

From equation (5), we get :

$$x_{it} = c_{i,t} + y_{i,t},$$

where

$$\begin{aligned} c_{i,t} &= \overline{x}_i + \frac{1}{\lambda_1} Cov(x_{it}, C_t^1) C_t^1, \\ y_{i,t} &= \sum_{k>1} \frac{1}{\lambda_k} Cov(x_{it}, C_t^k) C_t^k. \end{aligned}$$

The volume turnover $x_{i,t}$ at time t, is the sum of a common - or market - turnover $c_{i,t}$ and a specific turnover $y_{i,t}$. On the one hand, the market component of intraday volume is expected to capture all volume seasonal fluctuations and represents the long term volume

of the stock. On the other hand, the specific component should feature no seasonal pattern and represents the short term volume of the stock. It depends on the inflow of information about important events for the company's shareholders.

2.3 Intraday volume dynamics

In order to incorporate the features mentioned above into the model for intraday volume $x_{i,t}$ where $t = 1, \ldots, T$, we proposed the following framework:

$$\hat{x}_{i,t} = \bar{c}_{i,t} + y_{i,t}, \tag{6}$$

$\bar{c}_{i,t}$ represents the common component historical average of intraday volume over the last L-trading days. As said above, $c_{i,t}$ depends on the trading day and should be written as $c^j_{i,t}$ for day j. Hence, $\bar{c}_{i,t}$ is equal to:

$$\bar{c}_{i,t} = \frac{1}{L} \sum_{l=1}^{L} c^{j-l}_{i,t}. \tag{7}$$

This modeling choice seems accurate as the common component for short period (no longer than 3 months) is assumed to be static. Note that, in our empirical study, the size of the interval δt is equal to 20 minutes. The second term $y_{i,t}$ represents intraday specific volume for each equity and is modeled considering two specifications. The first on is an $ARMA(1,1)$ with white noise, defined as:

$$y_{t,i} = \psi_1 y_{t-1,i} + \psi_2 + \epsilon_{t,i}. \tag{8}$$

The alternative considered specification is a SETAR (self-extracting threshold autoregressive model) which allows for changes in regime in the dynamics. We get :

$$y_{t,i} = \big(\phi_{11} y_{t-1,i} + \phi_{12}\big)\mathbf{I}(y_{t-1,i}) + \big(\phi_{21} y_{t-1,i} + \phi_{22}\big)\big[1 - \mathbf{I}(y_{t-1,i})\big] + \epsilon_{t,i}. \tag{9}$$

where

$$\mathbf{I}(x) = \begin{cases} 1 & x \leq \tau, \\ 0 & x > \tau. \end{cases}$$

Therefore, we assume that when the specific part of intraday volume exceeds a threshold value of τ its dynamics is described by a different set of parameters.

In contrast to the above framework, the classical approach takes the simple volume average over the past L-trading days. Hence, intraday volume $x_{i,t}$ is approximated by:

$$\hat{x}_{i,t} = \frac{1}{L} \sum_{l=1}^{L} x_{i,t}^{j-l}. \tag{10}$$

Undoubtedly, the advantage of this classical approach is its simplicity. However, it ignores the dynamics of intraday volume, what has a negative impact on the quality of volume forecast.

3 Empirical analysis

3.1 The data

The empirical results are based on the analysis of the all securities included in CAC40 index at the beginning of September 2004. We use the turnover as a measure of (relative) volume. The turnover is defined as the traded volume divided by the outstanding number of shares. A similar measure was used by Lo and Wang (2000). Tick-by-tick volume and prices were obtained from the Euronext historical data base. We consider one year sample, ranging from the beginning of September 2003 to the end of August 2004. The data is adjusted for the stock's splits and dividends. The 24 and 31 of December 2003 were excluded from the sample. For any 20 minute interval, volume is the sum of the traded volumes whereas the price is the average price, both over that period.

This study focuses on modeling volume during the day with continuous trading, therefore we consider transaction between $9:20$ and $17:20$, and exclude pre-opening trades. As the result, there are twenty five 20-minute intervals per day. In addition to high-frequency

data from EURONEXT, volume weighted average prices, with a daily horizon for each company, were used.

We give in table 2, intraday volume summary statistics for securities from the CAC40 index. The comparison of the mean with the 5% and 95% quantiles, gives clear indications of the large dispersion of volume stock by stock. For companies like SODEXHO ALLIANCE, SANOFI-AVENTIS, and CREDIT AGRICOLE, the mean is around three times lower than the 95%-quantile. On average this ratio is equal to 2.7. In turn, 5%-quantiles are five to nine times smaller than the mean. This strong dispersion comes from the strong intraday seasonal variation. It is worth noting that the table also shows large dispersion across equities, where the average volume is ranging from 0.006 for DEXIA up to 0.438 for CAP GEMINI. The explanation comes from equities' particular events such as earning announcements, dividend payments, changes in management board etc., which have direct influence on the price and volume of their stock. These observations encourage the application of a model such as the one we propose, which is based on volume decomposition in the market and its specific components.

3.2 Estimation results

The first step of our methodology is to run a principal component analysis (PCA) on the intraday volumes for all companies included in CAC40. Table 3 shows that the longer the period the lower the dispersion explained by the first three components. For a one month period these components explain 48.5% of the dispersion. It falls to 35.6% when we extend the decomposition to a one year period. Since principal component analysis is a static method, it has to be applied to short periods of time. Over long periods PCA fails to capture the dynamical links which prevail. Therefore, we choose to work on a one month period to decompose volume. Next, we calculate the autocorrelation (ACF) and partial autocorrelation functions (PACF) for common and specific parts which are plotted in Figure 1 for TOTAL equity. The upper graphs in the Figure show typical characteristics of the intraday volume, namely seasonal variations. From the middle figures, one recognizes the ability of common component to capture seasonal variations. The last graphs illustrate ACF

and PACF for the specific part of volume. The fast decay of the autocorrelation suggests that the ARMA type model is suitable to depict this time series. The results of stationarity tests are presented in Table 4. The null hypothesis of unit root is rejected by the Augmented Dickey-Fuller and Philips-Perron tests, for the specific volume. Finally, the inspection of residuals confirmed that ARMA and SETAR models are accurate to describe the dynamics of the specific volume. Figure 2 shows classical white noise properties. The conclusions drawn from these autocorrelation function plots are confirmed by the results of Portmanteau tests.

Figure 3 shows the result of our decomposition for two succeeding days, for TOTAL company. The upper graphs give the intraday evolution of volume where we can see a stochastic evolution around a seasonal U-shape pattern. The middle graphs give the intraday evolution of the common component. This part of the volume is the same for any day of the sample. Finally, the lower graph represents the evolution of the specific component. This component is responsible for the stochastic behavior around the seasonal pattern and changes from day to day.

The final stage to evaluate the accuracy of the models is to use two error measures, such as the mean absolute percentage error (MAPE) and mean squared percentage error (MSPE) for the daily horizon. Tables from 5 to 10 show the outcome of this analysis for all equities, for the classical approach, ARMA and SETAR model respectively. The results reported in the tables are obtained by calculating the MAPE and MSPE for each day. Note that the statistics have been computed over all trading days for the period from September 2 to December 16, 2003. The summary for all examined companies is given in Table 11. The outcomes indicate that both models based on principal component decomposition outperform the classical approach to predict the daily U-shape of volume. Moreover, the SETAR model better fits the daily volume dynamics than the ARMA model. In fact, there are three of the thirty nine companies for which ARMA slightly surpasses SETAR model. Further arguments in favor of the decomposition concept comes from the fact that the standard deviation for both models is significantly smaller than the one observed in the classical approach. The same applies to the maximum, and the 95%-quantile, that confirms

the dominance of our approach.

To summarize, we have demonstrated that models based on decomposition are better in modeling intraday volume than those assuming the calculation of simple averages from historical data. The importance of this outcome will be discussed in the next section which focus on the problem of reducing the cost of VWAP orders.

4 Application to VWAP strategies

VWAP analysis works best under particular conditions. As we explain in the following section, VWAP analysis may be misleading and self-fulfilling under every day institutional trading conditions, such as rapidly changing market conditions, trades motivated by current news and recommendations, trade dominating daily volume, principal trades and trades whose execution stretches out over several days. We detail these below and argue the set of assumptions used to ensure the accuracy of VWAP benchmarking.

4.1 VWAP strategies : an overview

Trends in algorithmic trading An actual trend observed in financial markets is the increasing use of computer trading, or, shortly speaking, electronic trading versus a specific benchmark. Measurability is one of the more obvious benefits of benchmarking. Indeed, when trading performance is measured in comparison to a benchmark (meaning that if my benchmark is an index, the performance of a portfolio is the extra performance compared to the index), you easily obtain an execution quality measure. Two main factors explain this phenomena. On the one hand, the computer trading offer is now easily accessible. If sell-side firms execution systems have been used internally by traders for years, these systems become recently available directly to clients via electronic platforms. A steady drop in transaction rates is forcing sell-side firms to become more efficient in processing trades and more reliant on automation and computer power to cut costs. At the same time, firms are looking for ways to outsource their trading desks to increase their capacity to execute more volume. Major brokerage houses are then franchising their computer trading

strategies to smaller firms which in turn are pressured to offer the service. Small and midsize broker-dealers that lack resources and time to invest in developing VWAP engines and other quantitative strategies can then offer the proprietary benchmark trading to their buy-side customers. In return, the source firms are paid a percentile per share based on the volume that is pumped through their models. Even if the franchisee broker puts it own name on the algorithm strategy, the execution occurs on major brokerage houses, virtually invisible to the institutional firm. The originating broker-dealer gets credit for the volume since it represents the order at the exchange and still preserves the execution clearing relationship with the buy-side client.

On the other hand, buy-side customers are asking for the algorithms. There are numerous reasons for buy-side firms to ask for this type of trading. The buy side is being more closely monitored and scrutinized for its execution quality. Algorithmic trading offers a less expensive option to full service brokers, while providing a way to complete a complex order type. In general, pre- trade analytic tools are readily and easily available. The execution environment allows clients to obtain analysis relevant to the context in which they make trades. Moreover, market fragmentation drives traders to use electronic tools to access the market in different ways. Quant fund traders began to have more to be a larger part of the market liquidity and need flexible and easy access to the market. For small brokers, access to big brothers' algorithms is far from cheap. But if a customer needs better execution, it's incumbent on them to provide it.

VWAP benchmark Several benchmarks are proposed in the field of algorithmic trading (These prices are based on market close, percentage of volume, opportunistic model for small-cap stocks, ...), but the most common and popular one is VWAP. The main reason is obvious: the computation of daily VWAP is straightforward for anyone with access to daily stock transactions records. Moreover the use of VWAP is simple in itself: if the price of a buy trade is lower than VWAP, it is a good trade; if the price is higher, it is a bad trade (and conversely for sell trades). In general, brokers propose several ways to reach VWAP benchmark. Agency and guaranteed VWAP execution services are the two main possibilities. In the guaranteed case, the execution is guaranteed at VWAP for a fixed

commission per share, and the broker dealer ensures the entire risk of failing to meet the benchmark. In the agency trading case, the order is sent to a broker-dealer, to trade on an agency basis, with the aim of obtaining the VWAP or better. Obviously, the transaction costs are not the same depending on the chosen method and the larger the client residual risk, the smaller the cost.

Timing dimension VWAP strategies introduce a time dimension in the order execution process. If the trader loses control of whether the trade will be executed during the day, VWAP strategies allow it to dilute the impact of orders through the day. To understand the immediacy and good price trade-off, let's take the two examples of action and investor traders. Action traders go where the action is, meaning that they don't care about the firm stock they are trading. Investor traders lack that flexibility. Since their job represents the final task in a sequential decision process, they are expected to trade specific stocks, even if the action is over. Of course, trade information cannot remain proprietary for long and trade delays resulting in trade process that can defer greatly from the manager's original decision price. VWAP strategies ensure investor traders' good participation during the day, and then trade completion at the closing time.

Size effect Under particular conditions VWAP evaluation may be misleading and even harmful to portfolio performance. Most institutional trading occurs filling orders that exceed the daily volume. When large numbers of shares must be traded, liquidity concerns are against price goals. Then trade evaluation becomes more complicated. Action traders watch the market for this reason and try to benefit from those trades. A naive investor could indiscreetly reveal her interest for the market or a particular stock. Action traders can then cut themselves in by capturing available liquidity and reselling it to an unskilled trader. On the other hand, skilled traders will deal amounts below or beyond the action trader's radar screen to avoid such behavior. Using automatic participation strategies as VWAP may be dangerous in these cases. Since it pays no attention to the full size of the trade, trading costs are biased by VWAP benchmark since the benchmark itself depends on the trades.

For this reason, some firms offer multi-days VWAP strategies to respond to customers

requests. To further reduce the market impact of large orders, customers can specify their own volume participation by limiting the volume of their orders on days when a low volume is expected. As a first step each order is spread over several days and then sent to the VWAP engine for the corresponding days.

To avoid this first limitation, we make the assumption concerning the order size sent to VWAP engine. We assume that any considered VWAP execution order is low compared to the daily volume.

Trade motivation Most trading observed on the market, such as balancing or inflow trading, is not price sensitive and evaluation by a VWAP analysis will not be misleading. However, some trades and hence trading prices reflect objectives that cannot be captured by a VWAP analysis. To see this, we must look deeper into trading motivations to discriminate whether a particular price represents a good or bad execution. Let us consider two types of traders: value and growth managers. Value managers are looking for under priced situations. They buy the stock and wait to sell it until good news raises its price. Growth managers react to good news and hope that it portends to more good news. Thus, while growth managers buy on good news value managers sell. Consequently growth managers have a clear trading disadvantage because they buy when the buying interest dominates the market. They are frequently lower ranked than value traders. If the skilled traders can understand the motivations beyond the decisions, they will try to adjust their strategy accordingly. Automatic participation algorithms cannot take into account such a dimension in trading.

The second assumption we make in our empirical study is to only consider low motivation trading. In such a case the VWAP benchmark can be used without bias.

Benchmarking arbitrage In the case of VWAP trading, any price is a good price if the size of the trade dominates the daily volume implying that the trading price dominates the VWAP. Trading dominating VWAP is evaluated as good trade, no matter how expensive the price might be, compared to a manager's decision targets. Hence, VWAP makes the trader insensitive to price since any price becomes as good as any other price. This denigrates

trader's skills and can destroy the value of research. Moreover, VWAP is very beneficial for screening people who don't know that it is used to evaluate them. Anyone who knew they were going to be evaluated by this measure would be a combination of stupid, incompetent, or corrupt, depending on how they behaved. Even though you know you can play this method as a game you don't.

As third assumption, we assume that traders have no strategic behavior.

4.2 VWAP dynamic implementation

We propose in this section to implement VWAP strategies. The main issue here is to use the dynamic model to enhance execution. The implementation can take three ways depending on the volume shape predictions we make. The first one, that we call the theoretical VWAP execution, is based on one-step ahead predictions of the specific part of the volume. In the second one, the prediction of the specific part of volume is predicted at once for the entire day (1 to 25 step ahead predictions). As the predictions are done once and never revised during the day, we call it the static execution. The last one consists in predicting first the specific part of volume for the entire day and then to adjust the predictions as the day goes on and the information increases. We call it the dynamic VWAP execution as prediction are dynamically adjusted during the day.

4.2.1 Theoretical VWAP execution

Recall that the time series model is based on 20mn by 20mn specific turnovers, which are our measures of intraday volumes. For any interval $i = 1, ..., 25$ of the day, we can easily predict $\hat{x}_{i+1}$ form the observation of x_i. However, on a practical point of vue, in addition to this prediction we need to know what will be the total volume of the day to exactly know what part should be traded at time $i+1$, $\frac{\hat{x}_{i+1}}{\sum_{k=1}^{K} x_k}$. Hence, such a strategy is just impossible to implement without knowing the K turnovers of the day or equivalently the total volume of the day. Obviously, this value is unknown before the market closes.

The implementation is then theoretical as it takes the unknown daily volume as perfectly known. However, it remains interesting to test such strategy as it gives the upper VWAP

execution improvement of the method.

4.2.2 Static VWAP execution

As mentioned above, traders cannot use the theoretical execution since they don't know, at the beginning of the day, what is the daily volume. However, they can use the dynamic model of x_i to predict $\hat{x}_1, \hat{x}_2, ..., \hat{x}_{25}$ and calculate the proportions to trade at each i interval $\frac{\hat{x}_i}{\sum_{i=1}^{25} x_i}$.

The simplicity of such strategy is offset by the poor quality of the long horizon estimations given by the ARMA models. Quickly, the specific volume prediction will be zero and the dynamic part of the model will have no effect on the VWAP implementation. In such a scheme, we just add one step to the classical approach, where we do a rolling cross sectional decomposition before taking an historical average. This strategy will for sure be worse than the classical approach since the specific volume plays almost no part and the average of common volume contains less than the average of volume.

4.2.3 Dynamic VWAP execution

However, our decomposition can help to improve the execution by taking advantage of the dynamic part of the model even when the daily volume is unknown. The idea is to incorporate after each step, all the information about volume that one knows.

The prediction $\hat{x}_{i+1}$, $i = 1, ..., 25$, is still the one-step ahead prediction of the dynamic model as in the theoretical execution. And we use the same model to get all the $\hat{x}_{i+l}, l \geq 1$ until the end of the day. The proportion $\frac{\hat{x}_{i+1}}{\sum_{l=1}^{25-i} x_{i+l}}$ is applied only on the remaining volume to trade after interval i.

As a consequence, at the very beginning of the day, we trade without information and we are in the static execution case. Then, at each new interval, we improve our prediction about future volume to trade including the intraday realized past volume. Finally, the last trade corresponds to the theoretical case.

4.3 Empirical results

In this section, the question about the usefulness of the above discussed models for the prediction of volume weight average price (VWAP) is addressed. Obviously, the answer has an important meaning for brokers, who are supposed to execute VWAP orders, and whose trades are evaluated according to benchmarks based on VWAP.

This empirical study focuses on VWAP orders with a one day horizon. The examination is organized as follows: the proxy of volume weighted price is computed based on twenty five time points during a trading day. The first point corresponds to the time 9:20 a.m. and the last to the time 5:20 p.m. The time interval between two succeeding time points is 20 minutes. The equity price for each of the twenty five points were computed as an arithmetic average of the price of the transaction which took place in the previous twenty minutes. The prediction of volume is carried out using on the one hand, our models based on principal component decomposition and an ARMA or a SETAR model, and using, on the other one, the classical approach to describe daily pattern of intraday volume.

We examine VWAP predictions errors in three different ways. First, we make in sample stock-by-stock VWAP predictions for a period between September, 2 and December, 16, 2003 (75 trading days), and substract the true VWAP to get the in-sample prediction errors for each day. Second, we examine the out-sample case. Each time, we make a one-day out-sample prediction. For example, estimating from September 2, 2003 to October 7, 2003 (25 trading days), we get the first VWAP prediction for following day, namely October 8, 2003. Again, the true VWAP is subtracted from the predicted one to get the first out-sample error. Then, we move our estimation window by one day, thus estimating from September 3, 2003 to October 8, 2003 and predicting for october, 9, 2003 and so forth. As a result, for out-sample prediction, we obtain VWAP predictions errors for 50 days for all stocks included in CAC40.

Finally, we calculate the cost of execution of VWAP of a portfolio made of all the stocks in the CAC40 index.

4.3.1 Single stock in-sample results

Tables from 12 to 17 are comparisons of in-sample performances for all models based on mean absolute percentage error (MAPE) and mean square percentage error (MSPE). The examination is carried out for the period ranging from September 2 to December, 16 2003. As we are in-sample, we only have to focus on the theoretical VWAP execution.

In all cases, the PCA-ARMA model out performs the classical approach. In 25 cases, this decomposition model reduces the error measure MAPE by more than 3 basis points or $bips$ (1 bp = 0.01%). For 8 equities the reduction exceeds 5 bp in comparison to the classical approach. The major decrease of the error measure is observed for ALCATEL, where it reaches 9 bp. On average, this reduction is around 4 bp and there are only 2 cases where the reduction can be considered as negligible, since it is below 1 bp. These two cases are DANONE (0.6 bp) and TF1 (0.6 bp).

The modeling of the specific part by a SETAR allows for further decline of the mean absolute prediction error in comparison with the classical approach. In fact, a reduction of more than 3 bp is observed for 33 equities. For 13 equities the reduction exceeds 5 bp. The most substantial decrease of the prediction error is again obtained for the Alcatel equity where it is around 10 bp. On average, the application of the decomposition model allows for an improvement in the quality of VWAP forecasts by almost 5 bp. The only exception is TF1, where the SETAR model fails to improve the risk reduction and the classical approach beats the PCA-SETAR by only 0.4 bp and can hence be considered as non significant.

All together, the decomposition models outperform the classical approach. If the PCA-ARMA model does a very good job already, the PCA-SETAR model allows for an additional reduction of more than 1 bp on average for 29 of the stocks. For 8 of the stocks, the ARMA model is better but the improvement is lower than 1 bips and hence neglectable. In the last 2 of stocks left, the ARMA model out-perform the SETAR model by almost 1 bips: LAFARGE and TF1.

From a broker's perspective the 95%-quantile contains important informations about the risk of applying one particular model. The 95% quantile has much smaller value for the decomposition models than for the classical approach. Furthermore, the SETAR model

seems to be better than ARMA to describe the specific part of the intraday volume. This is due the SETAR ability to discriminate between turbulent and flat periods in the market. The 95% quantiles for the classical approach and the model with an ARMA specific part are ranging from 19 bp to 78 bp, and from 11 bp to 49 bp respectively. In the SETAR case, the 95% for all companies range from 8 bp to 39 bp.

As result of in-sample performance comparisons, we show that decomposition models can be successfully used to predict the volume weight of average price (VWAP). Furthermore, a broker who exploits our approach to forecast VWAP, compared to the classical one, is lowering his risk.

Moreover, the in-sample results are confirmed by out-of-sample ones. This analysis is carried out by applying a twenty days moving window. Thus, the decomposition is performed using the twenty trading days preceeding the day where the execution of the VWAP order takes place. The average common part of intraday volume is computed and known in the evening of the day preceding VWAP trade. In turn, the specific part is forecasted with a twenty minute delay, on the considered day.

4.3.2 Single stock out-sample results

The out-of-sample performance of models under consideration for the period from September 2 to December 16, 2003 is summarized in tables 18 to 25.

Before starting the analysis of the results two comments must be made. First, it is fundamental here, and unlike in the in-sample part, to present the results of the models based on the volume decomposition for static, dynamic and theoretical VWAP execution algorithms (See section 4.2 for a description). If this distinction is useless in the in-sample study, you cannot get away from it in the out-sample analysis. In fact, all the approaches need a prediction of the intra-daily and daily volumes to implement the strategies but the theoretical one which takes the latter as known. As a consequence, the theoretical approach is not implementable but the results are still interesting as they give an idea of the upper improvement limit of our approach. As expected, the static method gives very poor results and are not presented in the paper for succinctness, but are available upon request. Second, still

for succinctness, we only comment the SETAR specification results which out-performs the ARMA ones.

We start analysing the results of the theoretical approach comparing tables 18- 19 to tables 26- 25 and tables 31 and 32 for a summary. Over the 39 stocks of our sample, the decomposition model outperforms the classical approach. For all companies, the use of the classical approach results in a higher risk of execution of VWAP orders. The gains in basis points are greater than 1 bp for 30 out of the 39 stocks of the sample (77% of the stocks). CAP GEMINI and THOMSON are the stocks for which the gains are the most important with a mean absolute percentage error (MAPE) falling from 23 bp to 14 bp (-9 bp) and from 15 bp to 8 bp (-7 bp), respectively. Conversely, for 9 stocks, the gain is below 1 bp and can be considered as non significant. If these results are promising, recall that these gains are theoretical since they correspond to a non realistic VWAP execution in practice.

The analysis of the dynamic VWAP execution is the implementable version of the theoretical VWAP execution and allows us to check if the above theoretical can be reached. The results (tables 22 - 27 to be compared to tables 18 - 19 and tables 31 and 32 for a summary) of course more mitigated. We see in tables tables 31 and 32, that over our sample, only 30 stocks shows a lower execution error when the classical algorithm is replaced by the dynamic VWAP one. However, over these 9 stocks presenting a deteriorated execution, 7 correspond to a deterioration smaller than 1 bp, hence non significant. Only two, LAGADERE (1.3 bp) and SCHNEIDER (1.6 bp), present significant, although limited, deterioration. Conversely, for the 30 well-behaving stocks, the improvement can reach high levels : -8 bp for CAP GEMINI, -5 bp for EADS. All in all, 14 stocks show a decrease of the VWAP execution risk larger than 1 bp.

The comparison of the theoretical and the dynamic executions gives some insight concerning the loss we bear due to the fact that we don't have access to the overall information at the very beginning of the day neither we can erase nor modify the trades we already made even if the information we get as time goes by showes us that we did wrong. In fact, we can update our strategy as we get more information about volume by adapting the rest of the day strategy, but we cannot modify past trades. This loss is calculated as the difference in

MAPE between the theoretical and the dynamic VWAP execution models. As we can see in the tables 31 and 32, the loss can vary a lot from one stock another. It is not significant (lower than 10%) for 13 stocks whereas it can be greater than 50% for two stocks. In fact, the error on ARCELOR is rising from 6.6 bp to 10.6 bp (60%) and from 8 bp to 14 bp for THOMSON (78%). On average, the loss is larger than 1 bp.

Finally, we can conduct one more analysis of our method by studying the link between the improvement gained by our method and the classical approach error. The idea here is to see if our method is able or not to correct the largest errors made when applying the classical approach. To do this, we present in Figure 4, the scatter plot of the classical approach tracking error on the x-axis against the gain or loss observed by applying our dynamical strategy on the y-axis. Here again, the gain or loss of our strategy is measured by the difference in of the Mean of MAPE between the dynamic PCA-SETAR model and the classical approach. When this difference is positive we suffer a loss, when it is negative we gain by applying our strategy instead of the classical one. Having a look to the scatter plot and the regression line, we can see that the larger the error, the larger the gain. In fact, when the classical approach is efficient (the tracking error is below 10%), the incorporation of the intraday volume dynamic has a limited impact (or no impact). On the contrary, in cases where the classical approach is worse tracking the VWAP (CAP GEMINI and EADS), the improvement is the largest. This result is confirming that our dynamic VWAP execution is a real improvement since if it is efficient in mean, the worse execution provided by the classical approach, the larger the correction allowed by our model.

4.3.3 Portfolio in and out sample results

The obtained results advocate the approach based on principal component decomposition. In order to summarize the results, we estimate the cost of the VWAP order execution when the subject of transaction are all stocks included in index CAC40. Therefore, we compute the VWAP for the whole index as weighed average of VWAP over equities. We use the same weights as were used for the construction of the index at the beginning of September 2004. Tables 28 present the summary of the model's performance comparison in case of

VWAP order for the whole index.

The application of the decomposition model with the specific part described by SETAR induce a portfolio risk fall greater than $4\ bp$ (a drop of around 40%) in the in-sample comparison. The out-sample results are comfirming the superiority of our method. In fact, the trading tracking error of the CAC basket using the classical approach is on average $10\ bp$ which falls to approximatively $8\ bp$ when using the theoretical VWAP execution, diminishing the error by 20%. Recall that this is the upper improvement limit of our method. To compare with an implementable strategy, we need to focus on the dynamic VWAP execution results. Here again, the tracking error is lower ($8\ bp$) and the use of our method allows for a reduction of the error of 10%. Note that to use our methodology in practice, we should not use means of MAPE but rather calculate the errors on the basket and the calculate the MAPE of the error. However, this remark does not question our conclusions as the results would even be better in that case. In fact, the individual stocks errors could then compensate which is not possible using means of the MAPE.

The above outcomes show that using the decomposition of volume into market and specific parts reduces the cost of execution of VWAP orders. From the perspective of brokerage houses, which are directly engaged in the process of VWAP orders execution, an additional issue of "beating the VWAP" seems crucial. It is clear, that the primary aim of a broker is to keep the execution price of orders, as close as possible to the VWAP price, and in this manner, to generate profits from the commissions paid by investors who asked for execution of VWAP orders. Nevertheless, there is another potential source of profit. An additional gain can be made when brokers manage to execute the sale of a VWAP-order at a higher price, higher than the observed end of the day volume weighed against average price. The same applies to a buy VWAP-order at a lower price than the observed volume weighed average price. To verify the possibility of beating the VWAP by applying our methodology, we present in table 30, separate statistics for situations, where the predicted VWAP is lower and higher than observed at the end of the day. The results indicate that the difference between the predicted VWAP and the observed one can be either positive or negative with the same probability. Roughly, the average of mean absolute percentage error

average over the period ranging from September 2 to December 16, 2003, for the SETAR, the ARMA and the classical approach are equal to 7 bp, 8 bp, and 11 bp respectively.

4.3.4 Robustness check

As a robustness check of our results, we conduct the same analysis on two other time periods running from January 2 to April 20, and from April 21 to August 3 2004. For succinctness of the presentation[3], we only report the summary results of the comparison of VWAP predictions in table 29 to be compared to table 28 which give the same summary results for the period running from September 2, to December 16, 2003. On both periods, the decomposition models beat the classical approach by more than 1 bp. Moreover, this method allows for a reduction of the larger error when tracking the VWAP by more than 10 bp, on either period.

5 Conclusion

In this paper, we present a new methodology for modeling the dynamics of intraday volume which allows for a significant reduction of the execution risk in VWAP (Volume Weighted Average Price) orders. The models are based on the decomposition of traded volume into two parts: one reflecting volume changes due to market evolutions, the second describing the stock specific volume pattern. The first component of volume is taken as a static cross historical average whereas the dynamics of the specific part of volume is depicted by ARMA, and SETAR models.

This methodology allows us to propose an accurate statistical method of volume predictions. These predictions are then used in a benchmark tracking price framework.

The following results are obtained through our analysis. Not only do we get round the problem of seasonal fluctuations but we use it to propose a new price benchmark. We also show that some simple time-series models give good volume predictions. Also, applications of our methodology to VWAP strategies reduce the VWAP tracking error, and thus the

[3]The detailed results are available upon request from the authors.

execution risk due to the use of such order type and so the associated cost. On average, and depending on the retained strategy, the reduction is greater than 10% and can even reach 50% for some stocks.

However, in order to beat the VWAP, our price adjusted-volume model is not sufficient and it is essential to derive a bivariate model for volume and price.

References

[1] Andersen, T., 1996, *Return volatility and trading volume: An information flow interpretation of stochatic volatility*, Journal of Finance 51(1), 169-204.

[2] Berkowitz, S., D., Logue, and E. Noser, 1988, *The Total Cost of Transactions on the NYSE*, Journal of Finance 41, 97-112.

[3] Biais, B., Hillion, P. and C. Spatt, 1995, *An empirical analysis of the limit order book and the order flow in the Paris bourse*, Journal of Finance 50(5), 1655-1689.

[4] Cuching, D., and Madhavan, A., 2000, *Stock Returns and Institutional Trading at the Close*, Journal of Financial Markets 3, 45-67.

[5] Darolles, S., and G., Le Fol,2003, *Trading volume and Arbitrage*, Working paper, CREST.

[6] Darat, A.,Rahman, S., and, M. Zhong, 2003, *Intraday Trading Volume and Return Volatility of the DJIA Stocks: A Note*, Journal of Banking and Finance 27(10), 2035-2043.

[7] Easley, D., and M., O'Hara, 1987, *Price; Trade Size, and Information in Securities Markets* , Journal of Financial Economics 19, 69-90.

[8] Engle, R. and J. Russel, 1998, *Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data*, Econometrica (1998) 66: 1127-1162.

[9] Engle, R., 2000, *The Econometrics of Ultra High Frequency Data*, Econometrica, (2000) 68: 1-22.

[10] Foster, D., and S. Viswanathan, 1990, *A Theory of Intraday Variations in volume, variance and trading costs in securities market*, Review of Financial Studies 3, 593-624.

[11] Gouriéroux, C., Jasiak, J., and G. Le Fol, 1999, *Intra-day market activity*, Journal of Financial Markets 2, 193-226.

[12] Gouriéroux, C. And G. Le Fol, 1998, *Effet des Modes de Négociation sur les Echanges*, Revue Economique, Vol. 49, 3, 795-808.

[13] Kaastra, I., Boyd, M., 1995, *Forecasting Futures Trading Volume Using Neural Networks*, Journal of Futures Markets 15, 953-970.

[14] Karpoff, J., Boyd, M., 1987, *The Relationship between Price Changes and Trading volume: a Survey*, Journal of Financial and Quantitative Analysis 22(1), 109-126.

[15] Le Fol, G. and L. Mercier, 1998, *Time Deformation : Definition and Comparisons*, Journal of Computational Intelligence in Finance 6, 5, 19-33.

[16] Lo, A., Wang J., 2000, *Trading Volume: Definition, Data Analysis, and Implication of Portfolio Theory*, Review of Financial Studies 13, 257-300.

[17] Madhavan, A., 2002, *VWAP strategies*, Transaction Performance: The Changing Face of Trading Investment Guides Series, Institutional Investor Inc., 32-38.

[18] Manganelli, S., 2002, *Duration, Volume, and Volatility Impact of Trades*, European Central Bank Working Papers Series No. 125.

[19] McCulloch, J., 2004, *Relative Volume as a Doubly Stochastic Binomial Point Process*, Working Papers Series.

[20] Tauchen, G., Pitts, M.,1983,*The Price Variability-Volume Relationship on Speculative Markets*, Econometrica 51, 485-505.

Table 1: Hourly VWAP and VWAP strategies for a 100000 shares portfolio when the volume is known or predicted, with different (unknown) price evolutions (increasing, decreasing or constant).

	Price evolution			Known volume			Predicted volume		
Hours	Decreasing Price P_i	Increasing Price P_i	Constant Price P_i	Traded volume V_i	% of traded volume x_i	Volume to trade $\underline{V}_i$	Traded volume V_i	% of traded volume x_i	Volume to trade $\underline{V}_i$
09:00	162.84	159.17	162.84	$164 \cdot 10^4$	0.0905	9050	$1207 \cdot 10^3$	0.0695	6950
10:00	163.02	160.54	162.83	$220 \cdot 10^4$	0.1214	12140	$181 \cdot 10^4$	0.1042	10420
11:00	162.93	161.66	162.83	$250 \cdot 10^4$	0.1380	13800	$210 \cdot 10^4$	0.1209	12090
12:00	162.69	161.57	162.85	$180 \cdot 10^4$	0.0993	9930	$160 \cdot 10^4$	0.0921	9210
13:00	162.09	162.09	162.87	$140 \cdot 10^4$	0.0773	7730	$120 \cdot 10^4$	0.0691	6910
14:00	161.57	162.69	162.85	$148 \cdot 10^4$	0.0817	8170	$150 \cdot 10^4$	0.0864	8640
15:00	161.66	162.93	162.84	$210 \cdot 10^4$	0.1159	11590	$240 \cdot 10^4$	0.1382	13820
16:00	160.54	163.02	162.86	$210 \cdot 10^4$	0.1159	11590	$235 \cdot 10^4$	0.1353	13530
17:00	159.17	162.84	162.84	$290 \cdot 10^4$	0.1600	16000	$320 \cdot 10^4$	0.1843	18430
		Sum		$1812 \cdot 10^4$	1.0000	100000	$17367 \cdot 10^3$	1.0000	100000
		Decreasing price			161.7070			161.5436	
VWAP		Increasing price			161.9007			162.0577	
		Constant price			162.8439			162.8443	

Table 2: Summary statistics for the intraday aggregated volume over 20 minute intervals, September 2, 2003 to August 31, 2004

Companies	*Mean*	*Std*	*Q5*	*Q95*
ACCOR	0.0191	0.0273	0.0028	0.0523
AGF-ASS.GEN.FRANCE	0.0076	0.0087	0.0010	0.0212
AIR LIQUIDE	0.0120	0.0182	0.0022	0.0314
ALCATEL	0.0381	0.0383	0.0062	0.1064
ARCELOR	0.0234	0.0241	0.0034	0.0648
AXA	0.0166	0.0220	0.0034	0.0404
BNP PARIBAS	0.0147	0.0350	0.0034	0.0338
BOUYGUES	0.0129	0.0264	0.0019	0.0344
CAP GEMINI	0.0438	0.0514	0.0058	0.1241
CARREFOUR	0.0132	0.0232	0.0025	0.0317
CASINO GUICHARD	0.0106	0.0118	0.0013	0.0312
CREDIT AGRICOLE	0.0083	0.0120	0.0012	0.0233
DANONE	0.0149	0.0310	0.0024	0.0381
DEXIA	0.0055	0.0069	0.0006	0.0164
EADS	0.0092	0.0092	0.0015	0.0265
FRANCE TELECOM	0.0123	0.0115	0.0025	0.0312
L'OREAL	0.0069	0.0120	0.0014	0.0177
LAFARGE	0.0188	0.0307	0.0035	0.0477
LAGARDERE S.C.A.	0.0163	0.0385	0.0020	0.0423
LVMH	0.0105	0.0185	0.0018	0.0276
MICHELIN	0.0167	0.0238	0.0024	0.0450
PERNOD-RICARD	0.0157	0.0303	0.0022	0.0427
PEUGEOT	0.0205	0.0454	0.0035	0.0515
PIN.-PRINT.REDOUTE	0.0149	0.0210	0.0020	0.0426
RENAULT	0.0165	0.0414	0.0024	0.0412
SAINT GOBAIN	0.0154	0.0332	0.0030	0.0382
SANOFI-AVENTIS	0.0151	0.0228	0.0020	0.0444
SCHNEIDER ELECTRIC	0.0145	0.0264	0.0021	0.0378
SOCIETE GENERALE	0.0155	0.0205	0.0031	0.0390
SODEXHO ALLIANCE	0.0172	0.0318	0.0016	0.0518
STMICROELECTRONICS	0.0223	0.0230	0.0030	0.0604
SUEZ	0.0162	0.0182	0.0032	0.0418
TF1	0.0198	0.0449	0.0026	0.0531
THALES	0.0120	0.0134	0.0016	0.0336
THOMSON (EX:TMM)	0.0270	0.0465	0.0035	0.0776
TOTAL	0.0150	0.0277	0.0031	0.0373
VEOLIA ENVIRON.	0.0120	0.0158	0.0017	0.0333
VINCI (EX.SGE)	0.0261	0.0687	0.0034	0.0689
VIVENDI UNIVERSAL	0.0215	0.0203	0.0044	0.0543
Overall	0.0166	0.0265	0.0026	0.0445

Table 3: Correlation matrix decomposition of intraday volume for CAC40 index stocks.

Est.period	*Rank*	*Eigenvalue*	*Difference*	*Proportion*	*Cumulative*
from 1 September to 30 September 2003	1	12.93	10.96	0.392	0.392
	2	1.967	0.281	0.050	0.442
	3	1.686	0.256	0.043	0.485
from 1 September to 31 October 2003	1	12.95	11.21	0.371	0.371
	2	1.740	0.197	0.044	0.411
	3	1.543	0.243	0.039	0.450
from 1 September to 30 November 2003	1	12.41	10.93	0.358	0.358
	2	1.484	0.151	0.038	0.396
	3	1.333	0.052	0.034	0.430
from 1 September to 28 February 2003	1	11.16	9.893	0.286	0.286
	2	1.267	0.126	0.032	0.318
	3	1.141	0.027	0.029	0.347
from 1 September 2003 to 31 August 2004	1	8.614	5.737	0.221	0.221
	2	2.877	0.502	0.074	0.295
	3	2.375	0.868	0.061	0.356

Table contains the highest eigenvalues of the correlation matrix, differences between successive eigenvalues, the portion of variance explained by each eigenvalue, and the cumulative proportion of the variance.

Table 4: Results of test on unit root for series defined as difference between intraday volume and its common component obtained from principal component analysis.

	ADF			PP		
	Mean	Min	Max	Mean	Min	Max
Zero mean	-7.98	-11.14	-5.18	-10.83	-16.37	-6.53
Single mean	-15.92	-19.66	-11.37	-22.28	-28.59	-14.93
Trend	-16.14	-19.71	-11.45	-22.57	-28.66	-15.80

Outcomes of Augmented Dickey-Fuller (ADF), Philips-Perron (PP). For all examined time series the null hypothesis was rejected at 1%.

Table 5: Comparison of intraday volume models performance, for period September 2, 2003 to October 6, 2003, classical approach.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	1.15E-02	1.64E-02	3.69E-06	1.98E-01	2.88E-02	4.01E-04	2.40E-03	1.36E-11	3.93E-02	8.27E-04
AGF-ASS.GEN.FRANCE	3.78E-03	4.35E-03	1.07E-06	5.09E-02	1.01E-02	3.32E-05	1.45E-04	1.15E-12	2.59E-03	1.02E-04
AIR LIQUIDE	6.99E-03	7.75E-03	4.80E-06	6.66E-02	1.86E-02	1.09E-04	3.60E-04	2.31E-11	4.44E-03	3.47E-04
ALCATEL	2.30E-02	2.39E-02	2.85E-05	2.88E-01	6.13E-02	1.10E-03	4.01E-03	8.14E-10	8.32E-02	3.76E-03
ARCELOR	1.18E-02	1.18E-02	2.54E-05	8.71E-02	3.39E-02	2.78E-04	7.10E-04	6.48E-10	7.59E-03	1.15E-03
AXA	9.97E-03	9.97E-03	3.14E-05	8.33E-02	2.69E-02	1.99E-04	5.35E-04	9.86E-10	6.94E-03	7.23E-04
BNP PARIBAS	6.59E-03	7.06E-03	1.98E-06	6.65E-02	1.68E-02	9.32E-05	3.22E-04	3.90E-12	4.42E-03	2.83E-04
BOUYGUES	5.50E-03	6.67E-03	1.33E-05	9.57E-02	1.63E-02	7.46E-05	4.11E-04	1.77E-10	9.15E-03	2.65E-04
CAP GEMINI	2.40E-02	2.40E-02	4.35E-05	2.55E-01	6.56E-02	1.16E-03	3.62E-03	1.89E-09	6.51E-02	4.31E-03
CARREFOUR	4.70E-03	5.36E-03	1.58E-06	6.17E-02	1.29E-02	5.08E-05	2.15E-04	2.49E-12	3.81E-03	1.65E-04
CASINO GUICHARD	8.25E-03	8.69E-03	7.70E-06	9.99E-02	2.12E-02	1.43E-04	5.08E-04	5.93E-11	9.97E-03	4.50E-04
CREDIT AGRICOLE	5.35E-03	5.20E-03	1.26E-05	4.39E-02	1.44E-02	5.56E-05	1.49E-04	1.60E-10	1.93E-03	2.06E-04
DANONE	1.16E-02	1.45E-02	1.67E-06	1.25E-01	3.66E-02	3.43E-04	1.27E-03	2.78E-12	1.55E-02	1.34E-03
DEXIA	4.88E-03	8.25E-03	2.15E-05	9.77E-02	1.23E-02	9.18E-05	6.52E-04	4.64E-10	9.54E-03	1.52E-04
EADS	4.82E-03	5.02E-03	3.33E-05	5.77E-02	1.29E-02	4.84E-05	1.75E-04	1.11E-09	3.32E-03	1.66E-04
FRANCE TELECOM	7.81E-03	7.93E-03	2.60E-05	6.53E-02	2.07E-02	1.24E-04	3.33E-04	6.76E-10	4.27E-03	4.27E-04
L'OREAL	3.73E-03	5.10E-03	2.88E-06	6.79E-02	1.03E-02	3.99E-05	2.18E-04	8.30E-12	4.61E-03	1.07E-04
LAFARGE	1.16E-02	1.33E-02	3.49E-06	1.40E-01	2.93E-02	3.11E-04	1.20E-03	1.22E-11	1.95E-02	8.57E-04
LAGARDERE S.C.A.	1.05E-02	1.34E-02	1.97E-05	1.34E-01	2.68E-02	2.89E-04	1.35E-03	3.87E-10	1.79E-02	7.20E-04
LVMH	6.17E-03	7.55E-03	4.19E-06	9.06E-02	1.62E-02	9.49E-05	4.39E-04	1.75E-11	8.22E-03	2.61E-04
MICHELIN	9.35E-03	1.02E-02	2.17E-05	1.31E-01	2.76E-02	1.92E-04	8.00E-04	4.71E-10	1.71E-02	7.62E-04
PERNOD-RICARD	9.15E-03	1.16E-02	6.30E-06	1.81E-01	2.37E-02	2.19E-04	1.39E-03	3.97E-11	3.26E-02	5.60E-04

Table 6: (Continued) Comparison of intraday volume models performance, for period September 2, 2003 to October 6, 2003, classical approach.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	1.28E-02	1.51E-02	3.28E-05	1.66E-01	3.71E-02	3.89E-04	1.55E-03	1.08E-09	2.76E-02	1.38E-03
PIN.-PRINT.REDOUTE	1.09E-02	1.16E-02	2.94E-06	1.14E-01	3.03E-02	2.53E-04	8.26E-04	8.65E-12	1.29E-02	9.15E-04
RENAULT	1.14E-02	1.36E-02	5.40E-06	1.31E-01	3.31E-02	3.16E-04	1.21E-03	2.92E-11	1.73E-02	1.09E-03
SAINT GOBAIN	8.29E-03	9.26E-03	1.51E-06	9.11E-02	2.42E-02	1.54E-04	5.28E-04	2.27E-12	8.30E-03	5.84E-04
SANOFI-AVENTIS	4.95E-03	6.21E-03	1.32E-06	7.96E-02	1.43E-02	6.29E-05	3.12E-04	1.75E-12	6.33E-03	2.04E-04
SCHNEIDER ELECTRIC	7.43E-03	1.02E-02	9.83E-06	1.56E-01	1.84E-02	1.59E-04	1.12E-03	9.66E-11	2.43E-02	3.38E-04
SOCIETE GENERALE	7.33E-03	7.52E-03	2.65E-05	6.03E-02	2.10E-02	1.10E-04	3.15E-04	7.05E-10	3.63E-03	4.42E-04
SODEXHO ALLIANCE	9.11E-03	1.89E-02	8.60E-06	3.89E-01	2.29E-02	4.40E-04	6.25E-03	7.40E-11	1.51E-01	5.23E-04
STMICROELECTRONICS	1.23E-02	1.34E-02	1.87E-06	1.61E-01	3.40E-02	3.31E-04	1.30E-03	3.49E-12	2.61E-02	1.16E-03
SUEZ	8.87E-03	1.05E-02	3.08E-05	1.41E-01	2.74E-02	1.89E-04	9.28E-04	9.51E-10	2.00E-02	7.48E-04
TF1	1.12E-02	1.41E-02	1.29E-05	2.13E-01	3.05E-02	3.23E-04	1.98E-03	1.68E-10	4.52E-02	9.33E-04
THALES	9.45E-03	1.28E-02	3.07E-06	1.18E-01	2.75E-02	2.54E-04	1.09E-03	9.44E-12	1.40E-02	7.57E-04
THOMSON (EX:TMM)	1.13E-02	1.33E-02	1.06E-05	1.97E-01	3.21E-02	3.04E-04	1.69E-03	1.12E-10	3.87E-02	1.03E-03
TOTAL	6.12E-03	7.17E-03	4.71E-06	8.42E-02	1.75E-02	8.88E-05	3.70E-04	2.22E-11	7.09E-03	3.06E-04
VEOLIA ENVIRON.	1.19E-02	1.95E-02	1.22E-05	2.83E-01	3.45E-02	5.20E-04	3.61E-03	1.49E-10	8.01E-02	1.19E-03
VINCI (EX.SGE)	1.38E-02	1.57E-02	1.87E-05	1.71E-01	3.65E-02	4.34E-04	1.71E-03	3.50E-10	2.93E-02	1.33E-03
VIVENDI UNIVERSAL	1.26E-02	1.89E-02	2.27E-05	2.14E-01	3.20E-02	5.13E-04	2.92E-03	5.16E-10	4.59E-02	1.03E-03

Table 7: Comparison of intraday volume models performance, for period September 2, 2003 to October 6, 2003, theoretical PCA-ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	1.08E-02	1.61E-02	1.97E-05	1.99E-01	2.83E-02	3.76E-04	2.40E-03	3.87E-10	3.94E-02	8.01E-04
AGF-ASS.GEN.FRANCE	3.59E-03	3.93E-03	4.73E-07	4.17E-02	9.79E-03	2.83E-05	1.06E-04	2.24E-13	1.73E-03	9.58E-05
AIR LIQUIDE	6.62E-03	7.49E-03	1.1E-05	6.33E-02	2.05E-02	1.00E-04	3.21E-04	1.21E-10	4.01E-03	4.19E-04
ALCATEL	2.10E-02	2.31E-02	9.71E-05	2.84E-01	6.19E-02	9.72E-04	3.85E-03	9.43E-09	8.05E-02	3.83E-03
ARCELOR	1.08E-02	1.10E-02	1.51E-06	9.74E-02	3.26E-02	2.38E-04	6.43E-04	2.28E-12	9.49E-03	1.06E-03
AXA	8.97E-03	9.29E-03	7.87E-06	7.48E-02	2.38E-02	1.67E-04	4.75E-04	6.19E-11	5.59E-03	5.68E-04
BNP PARIBAS	6.35E-03	7.04E-03	2.69E-05	6.87E-02	1.67E-02	8.97E-05	3.39E-04	7.24E-10	4.72E-03	2.80E-04
BOUYGUES	5.23E-03	6.91E-03	4.13E-06	1.02E-01	1.43E-02	7.49E-05	4.65E-04	1.71E-11	1.04E-02	2.05E-04
CAP GEMINI	2.17E-02	2.19E-02	3.7E-05	2.49E-01	5.86E-02	9.51E-04	3.20E-03	1.37E-09	6.20E-02	3.43E-03
CARREFOUR	4.48E-03	5.48E-03	3.61E-05	6.14E-02	1.21E-02	5.00E-05	2.20E-04	1.31E-09	3.77E-03	1.46E-04
CASINO GUICHARD	7.63E-03	8.46E-03	9.75E-06	9.57E-02	2.31E-02	1.30E-04	4.64E-04	9.50E-11	9.16E-03	5.33E-04
CREDIT AGRICOLE	4.89E-03	4.99E-03	5.7E-06	3.90E-02	1.40E-02	4.88E-05	1.33E-04	3.25E-11	1.52E-03	1.95E-04
DANONE	1.01E-02	1.24E-02	1.79E-05	1.08E-01	3.00E-02	2.55E-04	8.85E-04	3.19E-10	1.17E-02	8.97E-04
DEXIA	4.18E-03	5.72E-03	4.48E-06	6.69E-02	1.11E-02	5.01E-05	2.62E-04	2.00E-11	4.48E-03	1.23E-04
EADS	4.63E-03	4.97E-03	1.82E-05	5.95E-02	1.21E-02	4.60E-05	1.78E-04	3.31E-10	3.54E-03	1.45E-04
FRANCE TELECOM	6.76E-03	6.98E-03	2.36E-05	6.55E-02	1.71E-02	9.43E-05	2.88E-04	5.55E-10	4.29E-03	2.92E-04
L'OREAL	3.38E-03	4.84E-03	1E-05	6.88E-02	9.20E-03	3.48E-05	2.14E-04	1.01E-10	4.74E-03	8.46E-05
LAFARGE	1.04E-02	1.27E-02	4.05E-05	1.35E-01	2.95E-02	2.70E-04	1.08E-03	1.64E-09	1.81E-02	8.71E-04
LAGARDERE S.C.A.	9.64E-03	1.33E-02	2.6E-05	1.42E-01	2.86E-02	2.71E-04	1.39E-03	6.75E-10	2.01E-02	8.18E-04
LVMH	5.80E-03	7.30E-03	3.94E-06	9.12E-02	1.63E-02	8.69E-05	4.18E-04	1.55E-11	8.32E-03	2.67E-04
MICHELIN	8.69E-03	1.02E-02	9.86E-05	1.37E-01	2.53E-02	1.80E-04	8.63E-04	9.72E-09	1.88E-02	6.42E-04
PERNOD-RICARD	8.49E-03	1.07E-02	2.02E-05	1.66E-01	2.40E-02	1.87E-04	1.17E-03	4.09E-10	2.74E-02	5.75E-04

Table 8: (Continued) Comparison of intraday volume models performance, for period September 2, 2003 to October 6, 2003, theoretical PCA-ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	1.18E-02	1.40E-02	7.18E-06	1.66E-01	3.27E-02	3.34E-04	1.48E-03	5.15E-11	2.77E-02	1.07E-03
PIN.-PRINT.REDOUTE	8.76E-03	9.68E-03	2.08E-05	9.08E-02	2.62E-02	1.70E-04	5.48E-04	4.34E-10	8.24E-03	6.89E-04
RENAULT	1.03E-02	1.23E-02	6.54E-05	1.39E-01	3.07E-02	2.56E-04	1.05E-03	4.27E-09	1.92E-02	9.44E-04
SAINT GOBAIN	7.77E-03	9.11E-03	2.92E-05	9.39E-02	2.42E-02	1.43E-04	5.27E-04	8.55E-10	8.81E-03	5.88E-04
SANOFI-AVENTIS	4.73E-03	6.13E-03	5.43E-06	8.27E-02	1.35E-02	5.99E-05	3.23E-04	2.95E-11	6.85E-03	1.82E-04
SCHNEIDER ELECTRIC	7.24E-03	1.02E-02	5.27E-06	1.55E-01	1.90E-02	1.56E-04	1.12E-03	2.78E-11	2.39E-02	3.59E-04
SOCIETE GENERALE	7.05E-03	7.45E-03	2.38E-05	6.29E-02	2.13E-02	1.05E-04	3.22E-04	5.64E-10	3.96E-03	4.56E-04
SODEXHO ALLIANCE	7.90E-03	1.75E-02	1.17E-05	4.01E-01	2.06E-02	3.67E-04	6.45E-03	1.38E-10	1.61E-01	4.25E-04
STMICROELECTRONICS	1.18E-02	1.26E-02	1E-05	1.23E-01	3.24E-02	2.98E-04	1.01E-03	1.00E-10	1.52E-02	1.05E-03
SUEZ	7.88E-03	1.00E-02	8.84E-06	1.31E-01	2.05E-02	1.63E-04	8.47E-04	7.82E-11	1.72E-02	4.21E-04
TF1	1.09E-02	1.43E-02	5.99E-07	2.23E-01	3.04E-02	3.23E-04	2.17E-03	3.58E-13	4.99E-02	9.23E-04
THALES	8.85E-03	1.27E-02	1.59E-05	1.23E-01	2.69E-02	2.40E-04	1.11E-03	2.51E-10	1.50E-02	7.25E-04
THOMSON (EX:TMM)	1.06E-02	1.30E-02	2.09E-05	1.84E-01	2.91E-02	2.83E-04	1.51E-03	4.37E-10	3.38E-02	8.49E-04
TOTAL	5.93E-03	7.01E-03	1.82E-05	8.51E-02	1.67E-02	8.42E-05	3.73E-04	3.32E-10	7.25E-03	2.78E-04
VEOLIA ENVIRON.	8.15E-03	1.40E-02	5.2E-06	2.12E-01	2.66E-02	2.62E-04	2.03E-03	2.70E-11	4.51E-02	7.07E-04
VINCI (EX.SGE)	1.22E-02	1.42E-02	3.68E-06	1.42E-01	3.46E-02	3.49E-04	1.32E-03	1.36E-11	2.01E-02	1.20E-03
VIVENDI UNIVERSAL	1.11E-02	1.51E-02	1.49E-05	1.51E-01	2.88E-02	3.50E-04	1.74E-03	2.23E-10	2.29E-02	8.30E-04

Table 9: Comparison of intraday volume models performance, for period September 2, 2003 to October 6, 2003, theoretical PCA-SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	1.04E-02	1.46E-02	5.08E-05	1.96E-01	2.63E-02	3.21E-04	2.22E-03	2.58E-09	3.85E-02	6.91E-04
AGF-ASS.GEN.FRANCE	2.77E-03	3.21E-03	1.39E-05	3.85E-02	7.44E-03	1.80E-05	8.21E-05	1.93E-10	1.48E-03	5.53E-05
AIR LIQUIDE	6.57E-03	7.25E-03	7.41E-06	6.40E-02	2.00E-02	9.56E-05	3.04E-04	5.49E-11	4.10E-03	4.00E-04
ALCATEL	1.85E-02	2.13E-02	4.30E-05	2.70E-01	5.35E-02	7.94E-04	3.41E-03	1.85E-09	7.29E-02	2.86E-03
ARCELOR	7.32E-03	7.14E-03	5.44E-06	6.70E-02	1.98E-02	1.04E-04	2.77E-04	2.96E-11	4.49E-03	3.91E-04
AXA	9.12E-03	9.08E-03	1.97E-05	7.54E-02	2.41E-02	1.66E-04	4.66E-04	3.88E-10	5.69E-03	5.81E-04
BNP PARIBAS	5.17E-03	6.24E-03	2.00E-07	6.11E-02	1.41E-02	6.55E-05	2.61E-04	3.99E-14	3.74E-03	1.99E-04
BOUYGUES	4.18E-03	5.84E-03	7.67E-06	9.43E-02	1.10E-02	5.16E-05	3.83E-04	5.88E-11	8.88E-03	1.22E-04
CAP GEMINI	2.08E-02	2.14E-02	4.99E-05	2.43E-01	5.48E-02	8.87E-04	3.06E-03	2.49E-09	5.93E-02	3.00E-03
CARREFOUR	3.85E-03	5.07E-03	3.32E-07	6.23E-02	9.98E-03	4.05E-05	2.10E-04	1.10E-13	3.89E-03	9.96E-05
CASINO GUICHARD	5.36E-03	5.79E-03	7.21E-06	7.45E-02	1.43E-02	6.22E-05	2.57E-04	5.19E-11	5.56E-03	2.04E-04
CREDIT AGRICOLE	3.64E-03	4.14E-03	1.77E-06	3.48E-02	1.07E-02	3.04E-05	9.31E-05	3.15E-12	1.21E-03	1.15E-04
DANONE	7.17E-03	8.60E-03	6.18E-06	8.67E-02	2.28E-02	1.25E-04	4.80E-04	3.82E-11	7.52E-03	5.20E-04
DEXIA	3.85E-03	4.73E-03	1.06E-06	5.57E-02	1.05E-02	3.72E-05	1.86E-04	1.12E-12	3.10E-03	1.09E-04
EADS	3.29E-03	4.07E-03	5.04E-06	5.49E-02	9.23E-03	2.73E-05	1.40E-04	2.54E-11	3.01E-03	8.51E-05
FRANCE TELECOM	6.53E-03	6.43E-03	1.27E-05	6.49E-02	1.59E-02	8.38E-05	2.56E-04	1.60E-10	4.22E-03	2.53E-04
L'OREAL	3.30E-03	4.03E-03	1.59E-06	3.25E-02	9.13E-03	2.71E-05	9.88E-05	2.54E-12	1.06E-03	8.34E-05
LAFARGE	8.34E-03	9.96E-03	1.93E-06	1.12E-01	2.18E-02	1.69E-04	7.23E-04	3.74E-12	1.26E-02	4.75E-04
LAGARDERE S.C.A.	7.14E-03	1.07E-02	3.30E-05	1.23E-01	1.99E-02	1.64E-04	9.90E-04	1.09E-09	1.51E-02	3.98E-04
LVMH	4.72E-03	5.60E-03	7.24E-07	7.43E-02	1.23E-02	5.35E-05	2.59E-04	5.24E-13	5.52E-03	1.52E-04
MICHELIN	8.34E-03	9.82E-03	2.72E-06	1.34E-01	2.36E-02	1.66E-04	8.13E-04	7.40E-12	1.79E-02	5.55E-04
PERNOD-RICARD	7.48E-03	1.03E-02	1.54E-05	1.63E-01	2.09E-02	1.62E-04	1.13E-03	2.36E-10	2.67E-02	4.36E-04

Table 10: (Continued) Comparison of intraday volume models performance, for period September 2, 2003 to October 6, 2003, theoretical PCA-SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	8.80E-03	1.13E-02	9.78E-06	1.42E-01	2.30E-02	2.05E-04	1.04E-03	9.57E-11	2.03E-02	5.27E-04
PIN.-PRINT.REDOUTE	6.67E-03	7.83E-03	2.17E-05	8.96E-02	1.82E-02	1.06E-04	4.31E-04	4.71E-10	8.03E-03	3.30E-04
RENAULT	8.18E-03	7.64E-03	4.01E-05	8.32E-02	2.19E-02	1.25E-04	3.55E-04	1.61E-09	6.93E-03	4.81E-04
SAINT GOBAIN	7.74E-03	8.73E-03	1.03E-05	9.21E-02	2.34E-02	1.36E-04	4.99E-04	1.05E-10	8.49E-03	5.47E-04
SANOFI-AVENTIS	4.21E-03	5.53E-03	6.97E-07	7.89E-02	1.10E-02	4.83E-05	2.86E-04	4.85E-13	6.23E-03	1.22E-04
SCHNEIDER ELECTRIC	6.89E-03	9.80E-03	1.61E-05	1.56E-01	1.62E-02	1.43E-04	1.12E-03	2.59E-10	2.45E-02	2.64E-04
SOCIETE GENERALE	6.99E-03	7.24E-03	4.30E-06	6.28E-02	2.02E-02	1.01E-04	3.12E-04	1.85E-11	3.94E-03	4.07E-04
SODEXHO ALLIANCE	7.55E-03	1.00E-02	2.83E-05	1.47E-01	2.08E-02	1.58E-04	9.88E-04	8.02E-10	2.15E-02	4.31E-04
STMICROELECTRONICS	1.12E-02	1.19E-02	5.85E-05	1.20E-01	2.82E-02	2.67E-04	9.17E-04	3.42E-09	1.44E-02	7.97E-04
SUEZ	7.93E-03	1.00E-02	5.01E-05	1.40E-01	2.00E-02	1.64E-04	9.11E-04	2.51E-09	1.96E-02	4.01E-04
TF1	8.10E-03	1.23E-02	5.71E-06	2.07E-01	2.05E-02	2.16E-04	1.84E-03	3.26E-11	4.29E-02	4.21E-04
THALES	6.46E-03	8.03E-03	2.89E-06	8.61E-02	1.87E-02	1.06E-04	4.58E-04	8.38E-12	7.42E-03	3.49E-04
THOMSON (EX:TMM)	8.04E-03	8.70E-03	2.38E-06	1.37E-01	2.02E-02	1.40E-04	7.93E-04	5.66E-12	1.88E-02	4.10E-04
TOTAL	6.21E-03	6.93E-03	1.19E-05	8.47E-02	1.58E-02	8.65E-05	3.65E-04	1.41E-10	7.18E-03	2.51E-04
VEOLIA ENVIRON.	7.90E-03	1.44E-02	2.72E-05	2.27E-01	2.18E-02	2.69E-04	2.28E-03	7.37E-10	5.15E-02	4.75E-04
VINCI (EX.SGE)	9.35E-03	1.28E-02	1.70E-06	1.48E-01	2.66E-02	2.51E-04	1.22E-03	2.90E-12	2.19E-02	7.09E-04
VIVENDI UNIVERSAL	1.10E-02	1.47E-02	1.02E-06	1.48E-01	3.00E-02	3.37E-04	1.69E-03	1.04E-12	2.19E-02	9.00E-04

Table 11: Summary of comparison for intraday volume model performance for period September 2, 2003 to October 6, 2003.

	MAPE					MSPE				
	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
SETAR	7.52E-3	8.69E-3	1.43E-6	1.00E-1	2.01E-2	1.59E-4	6.88E-4	4.26E-10	1.25E-2	4.95E-4
ARMA	8.29E-3	9.73E-3	1.78E-6	1.08E-1	2.33E-2	1.94E-4	8.17E-4	5.56E-10	1.46E-2	6.53E-4
Classical approach	9.05E-3	1.05E-2	1.45E-5	1.14E-1	2.49E-2	2.32E-4	9.48E-4	3.69E-10	1.66E-2	7.58E-4

Note:The volume is defined as percentage of total number of shares on the stock market. The values presented in Table are calculated as weight averages of values reported in Tables 5 - 10. The used weights are equal to those for composition of CAC40 index in September 2004

Table 12: Summary of in-sample estimated costs of execution on VWAP order for period from September 2, 2003 to December 16, 2003, classical approach

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.1161	0.1024	2.23E-03	0.4429	0.3618	0.0080	0.0134	1.72E-06	0.0656	0.0447
AGF-ASS.GEN.FRANCE	0.1305	0.1270	1.78E-03	0.5670	0.387263	0.0144	0.0272	1.27E-06	0.1417	0.0693
AIR LIQUIDE	0.0878	0.0973	8.95E-04	0.4966	0.301276	0.0214	0.0506	1.06E-06	0.3028	0.1142
ALCATEL	0.1800	0.1813	1.28E-03	0.7605	0.546576	0.0071	0.0134	1.87E-07	0.0659	0.0309
ARCELOR	0.1443	0.1545	1.22E-03	0.6068	0.531083	0.0051	0.0099	1.85E-07	0.0421	0.0341
AXA	0.1425	0.2487	1.38E-03	1.5999	0.513325	0.0132	0.0606	3.00E-07	0.4251	0.0411
BNP PARIBAS	0.0952	0.1138	1.57E-03	0.5683	0.32196	0.0096	0.0235	1.11E-06	0.1366	0.0472
BOUYGUES	0.1767	0.1454	5.23E-03	0.7857	0.486319	0.0126	0.0239	6.56E-06	0.1470	0.0570
CAP GEMINI	0.1964	0.2767	2.21E-03	1.2944	0.775938	0.0444	0.1295	2.06E-06	0.6666	0.2291
CARREFOUR	0.0876	0.1119	8.65E-04	0.6629	0.252594	0.0088	0.0275	3.36E-07	0.1906	0.0290
CASINO GUICHARD	0.1023	0.0849	3.31E-03	0.4390	0.241523	0.0137	0.0255	8.48E-06	0.1520	0.0450
CREDIT AGRICOLE	0.1650	0.1843	3.70E-04	1.1034	0.453169	0.0106	0.0314	2.37E-08	0.2167	0.0347
DANONE	0.0763	0.0657	3.52E-03	0.3286	0.190152	0.0133	0.0247	1.60E-05	0.1439	0.0484
DEXIA	0.1291	0.2939	5.02E-04	2.0266	0.388591	0.0124	0.0701	3.30E-08	0.4956	0.0201
EADS	0.1745	0.1858	1.33E-04	1.0620	0.507217	0.0097	0.0263	2.72E-09	0.1760	0.0359
FRANCE TELECOM	0.1139	0.1657	1.91E-03	0.9721	0.345296	0.0084	0.0300	7.85E-07	0.2011	0.0254
L'OREAL	0.0980	0.1044	1.90E-03	0.4776	0.31271	0.0124	0.0269	2.27E-06	0.1375	0.0632
LAFARGE	0.1461	0.1767	1.61E-04	0.7172	0.665107	0.0306	0.0730	1.60E-08	0.3159	0.2544
LAGARDERE S.C.A.	0.1263	0.1245	6.82E-03	0.7049	0.348792	0.0133	0.0329	1.97E-05	0.2181	0.0498
LVMH	0.0893	0.1075	2.41E-03	0.4778	0.319032	0.0110	0.0259	3.23E-06	0.1320	0.0579
MICHELIN	0.1401	0.1266	4.09E-03	0.5544	0.442438	0.0118	0.0208	5.55E-06	0.0991	0.0653
PERNOD-RICARD	0.0920	0.1151	2.44E-03	0.6983	0.291567	0.0177	0.0584	4.82E-06	0.4041	0.0688

Table 13: (Continued) Summary of in-sample estimated costs of execution on VWAP order for period from September 2, 2003 to December 16, 2003, classical approach.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.1035	0.1084	9.21E-04	0.5013	0.287562	0.0083	0.0171	3.13E-07	0.0914	0.0300
PIN.-PRINT.REDOUTE	0.1373	0.1394	2.99E-03	0.7602	0.424165	0.0295	0.0676	6.55E-06	0.4099	0.1397
RENAULT	0.1497	0.1301	1.63E-03	0.5811	0.39367	0.0213	0.0366	1.43E-06	0.1921	0.0832
SAINT GOBAIN	0.1238	0.1338	1.83E-03	0.7419	0.319529	0.0110	0.0276	1.14E-06	0.1745	0.0361
SANOFI-AVENTIS	0.1063	0.1420	1.31E-04	0.8494	0.352434	0.0165	0.0563	8.88E-09	0.3864	0.0657
SCHNEIDER ELECTRIC	0.0991	0.0943	3.00E-03	0.5345	0.234314	0.0088	0.0196	4.41E-06	0.1293	0.0281
SOCIETE GENERALE	0.0939	0.0981	2.61E-03	0.4271	0.393898	0.0112	0.0250	4.17E-06	0.1100	0.0897
SODEXHO ALLIANCE	0.1386	0.1733	6.83E-04	0.9847	0.472744	0.0117	0.0339	1.17E-07	0.2283	0.0522
STMICROELECTRONICS	0.0989	0.1176	2.00E-03	0.5661	0.350686	0.0052	0.0123	8.79E-07	0.0674	0.0286
SUEZ	0.1365	0.1143	9.01E-04	0.5169	0.338701	0.0045	0.0068	1.22E-07	0.0369	0.0172
TF1	0.1070	0.1009	1.53E-03	0.5220	0.272837	0.0058	0.0129	6.06E-07	0.0792	0.0197
THALES	0.1320	0.1724	1.62E-03	0.7725	0.621652	0.0115	0.0285	6.62E-07	0.1432	0.0942
THOMSON (EX:TMM)	0.1762	0.2763	9.81E-04	1.6851	0.562888	0.0172	0.0671	1.65E-07	0.4518	0.0510
TOTAL	0.0683	0.0753	6.82E-04	0.3208	0.208425	0.0137	0.0284	6.22E-07	0.1380	0.0588
VEOLIA ENVIRON.	0.1071	0.1001	1.43E-04	0.4116	0.297758	0.0040	0.0067	3.88E-09	0.0321	0.0162
VINCI (EX.SGE)	0.0720	0.0744	2.38E-03	0.3527	0.219507	0.0066	0.0136	3.59E-06	0.0777	0.0298
VIVENDI UNIVERSAL	0.1529	0.1523	6.51E-04	0.7945	0.448559	0.0076	0.0161	6.69E-08	0.1015	0.0315

Table 14: Summary of in-sample estimated costs of execution of VWAP order for period from September 2, 2003, theoretical PCA-ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.0952	0.0871	1.78E-03	0.3792	0.2903	0.0056	0.0103	1.06E-06	0.0522	0.0282
AGF-ASS.GEN.FRANCE	0.0985	0.0922	6.84E-04	0.5101	0.2351	0.0078	0.0168	2.05E-07	0.1126	0.0244
AIR LIQUIDE	0.0668	0.0706	7.07E-05	0.3700	0.2162	0.0118	0.0285	6.34E-09	0.1681	0.0596
ALCATEL	0.0919	0.0939	4.56E-04	0.4378	0.3043	0.0019	0.0042	2.18E-08	0.0204	0.0100
ARCELOR	0.1142	0.1261	1.49E-03	0.4826	0.4629	0.0033	0.0066	2.73E-07	0.0276	0.0245
AXA	0.1014	0.2357	1.98E-03	1.6533	0.2720	0.0107	0.0640	6.51E-07	0.4540	0.0121
BNP PARIBAS	0.0599	0.0487	4.06E-04	0.2168	0.1553	0.0026	0.0040	7.40E-08	0.0209	0.0105
BOUYGUES	0.1296	0.1026	9.17E-03	0.5991	0.3062	0.0067	0.0135	2.03E-05	0.0920	0.0228
CAP GEMINI	0.1403	0.1833	4.80E-03	1.1443	0.3913	0.0205	0.0714	9.94E-06	0.4989	0.0564
CARREFOUR	0.0639	0.0605	8.30E-04	0.2468	0.2276	0.0034	0.0062	3.05E-07	0.0274	0.0224
CASINO GUICHARD	0.0732	0.0483	4.38E-05	0.2164	0.1646	0.0060	0.0078	1.39E-09	0.0370	0.0217
CREDIT AGRICOLE	0.1059	0.1300	6.26E-03	0.8630	0.2361	0.0049	0.0187	6.95E-06	0.1326	0.0102
DANONE	0.0700	0.0712	1.45E-03	0.4283	0.1724	0.0130	0.0354	2.77E-06	0.2432	0.0393
DEXIA	0.0810	0.0677	4.23E-03	0.3215	0.2471	0.0014	0.0026	2.46E-06	0.0133	0.0082
EADS	0.1433	0.1478	4.94E-03	0.7029	0.4665	0.0062	0.0133	3.83E-06	0.0710	0.0304
FRANCE TELECOM	0.0781	0.1430	7.28E-03	0.9949	0.2501	0.0055	0.0297	1.16E-05	0.2107	0.0126
L'OREAL	0.0553	0.0399	2.87E-03	0.1985	0.1214	0.0028	0.0041	5.17E-06	0.0234	0.0089
LAFARGE	0.0863	0.0788	1.38E-03	0.3153	0.2573	0.0080	0.0126	1.11E-06	0.0585	0.0381
LAGARDERE S.C.A.	0.0956	0.0876	4.11E-04	0.4197	0.2609	0.0071	0.0135	7.36E-08	0.0773	0.0292
LVMH	0.0518	0.0538	2.26E-03	0.2684	0.1527	0.0031	0.0069	2.82E-06	0.0416	0.0126
MICHELIN	0.1172	0.0816	1.33E-03	0.4158	0.2416	0.0069	0.0095	6.28E-07	0.0576	0.0194
PERNOD-RICARD	0.0784	0.0809	2.54E-03	0.4079	0.2096	0.0104	0.0233	5.35E-06	0.1379	0.0356

Table 15: (Continued) Summary of in-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, theoretical PCA-ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.0731	0.0652	2.42E-03	0.2749	0.2085	0.0036	0.0059	2.15E-06	0.0274	0.0169
PIN.-PRINT.REDOUTE	0.0793	0.0813	3.62E-03	0.3642	0.2589	0.0102	0.0209	1.08E-05	0.1163	0.0543
RENAULT	0.0753	0.0597	1.29E-03	0.2633	0.2319	0.0051	0.0082	8.95E-07	0.0394	0.0312
SAINT GOBAIN	0.1002	0.0815	4.90E-04	0.3259	0.2651	0.0057	0.0081	8.37E-08	0.0337	0.0234
SANOFI-AVENTIS	0.0749	0.0877	1.02E-03	0.5433	0.1974	0.0070	0.0226	5.67E-07	0.1581	0.0204
SCHNEIDER ELECTRIC	0.0877	0.0721	4.73E-04	0.2982	0.2661	0.0062	0.0098	1.00E-07	0.0402	0.0362
SOCIETE GENERALE	0.0466	0.0367	1.31E-03	0.1637	0.1023	0.0022	0.0032	1.16E-06	0.0177	0.0066
SODEXHO ALLIANCE	0.0953	0.1012	6.09E-04	0.5330	0.3103	0.0047	0.0115	9.01E-08	0.0743	0.0216
STMICROELECTRONICS	0.0612	0.0540	9.73E-04	0.2394	0.1562	0.0015	0.0024	2.22E-07	0.0121	0.0054
SUEZ	0.0911	0.0765	3.57E-03	0.3406	0.2312	0.0020	0.0032	1.92E-06	0.0162	0.0081
TF1	0.1011	0.0967	2.38E-03	0.6163	0.2082	0.0053	0.0156	1.67E-06	0.1104	0.0112
THALES	0.1175	0.1395	8.50E-03	0.5471	0.4870	0.0082	0.0177	1.82E-05	0.0718	0.0586
THOMSON (EX:TMM)	0.0908	0.0990	2.79E-04	0.6205	0.2116	0.0029	0.0087	1.31E-08	0.0613	0.0075
TOTAL	0.0388	0.0368	8.22E-04	0.1533	0.1138	0.0038	0.0063	8.99E-07	0.0308	0.0176
VEOLIA ENVIRON.	0.0772	0.0707	9.98E-04	0.2832	0.2202	0.0020	0.0033	1.91E-07	0.0146	0.0091
VINCI (EX.SGE)	0.0492	0.0390	1.20E-04	0.1833	0.1233	0.0024	0.0037	9.12E-09	0.0210	0.0094
VIVENDI UNIVERSAL	0.0818	0.0708	3.17E-03	0.3719	0.1927	0.0019	0.0036	1.66E-06	0.0228	0.0069

Table 16: Summary of in-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, theoretical PCA-SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.0777	0.0823	4.36E-03	0.4442	0.2359	0.0042	0.0104	6.79E-06	0.0660	0.0180
AGF-ASS.GEN.FRANCE	0.0841	0.0977	8.45E-06	0.5449	0.2490	0.0072	0.0197	3.06E-11	0.1285	0.0288
AIR LIQUIDE	0.0666	0.0705	1.49E-04	0.4144	0.1771	0.0117	0.0316	2.85E-08	0.2108	0.0393
ALCATEL	0.0811	0.0980	3.48E-03	0.5847	0.2432	0.0018	0.0059	1.39E-06	0.0390	0.0064
ARCELOR	0.0900	0.0797	1.43E-04	0.3147	0.2459	0.0017	0.0026	2.45E-09	0.0113	0.0074
AXA	0.1030	0.2367	5.16E-04	1.6533	0.2823	0.0108	0.0641	4.19E-08	0.4540	0.0131
BNP PARIBAS	0.0601	0.0534	1.04E-03	0.2265	0.1663	0.0029	0.0045	4.86E-07	0.0228	0.0125
BOUYGUES	0.1240	0.0970	5.16E-03	0.4038	0.2876	0.0060	0.0086	6.20E-06	0.0393	0.0201
CAP GEMINI	0.1047	0.1732	1.59E-03	1.0626	0.3224	0.0157	0.0643	9.84E-07	0.4302	0.0383
CARREFOUR	0.0492	0.0464	4.39E-05	0.2017	0.1352	0.0020	0.0035	8.93E-10	0.0176	0.0084
CASINO GUICHARD	0.0646	0.0439	3.38E-04	0.1611	0.1530	0.0047	0.0055	8.33E-08	0.0200	0.0181
CREDIT AGRICOLE	0.0939	0.1032	9.31E-04	0.6922	0.1817	0.0034	0.0120	1.47E-07	0.0853	0.0056
DANONE	0.0536	0.0336	1.22E-03	0.1683	0.1135	0.0052	0.0066	1.91E-06	0.0373	0.0172
DEXIA	0.0763	0.0784	8.44E-04	0.3936	0.2565	0.0015	0.0035	9.00E-08	0.0205	0.0087
EADS	0.1265	0.1324	3.12E-03	0.6988	0.3869	0.0049	0.0110	1.44E-06	0.0701	0.0234
FRANCE TELECOM	0.0792	0.1502	6.01E-03	1.0195	0.2927	0.0060	0.0313	7.80E-06	0.2212	0.0172
L'OREAL	0.0463	0.0432	2.51E-04	0.2178	0.1425	0.0025	0.0051	3.91E-08	0.0305	0.0127
LAFARGE	0.1001	0.1033	6.04E-03	0.4688	0.2875	0.0120	0.0257	2.10E-05	0.1293	0.0467
LAGARDERE S.C.A.	0.0765	0.0647	2.03E-03	0.3079	0.2157	0.0042	0.0073	1.66E-06	0.0407	0.0197
LVMH	0.0517	0.0473	8.40E-04	0.2438	0.1348	0.0027	0.0053	4.03E-07	0.0344	0.0101
MICHELIN	0.1101	0.0830	2.75E-03	0.3588	0.2658	0.0064	0.0084	2.75E-06	0.0429	0.0235
PERNOD-RICARD	0.0707	0.0678	8.53E-04	0.3101	0.2397	0.0078	0.0151	6.02E-07	0.0797	0.0465

Table 17: (Continued)Summary of in-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, theoretical PCA-SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.0610	0.0500	1.79E-04	0.2504	0.1407	0.0023	0.0038	1.16E-08	0.0236	0.0072
PIN.-PRINT.REDOUTE	0.0782	0.0785	2.76E-03	0.3978	0.2471	0.0096	0.0209	6.65E-06	0.1283	0.0433
RENAULT	0.0808	0.0616	2.21E-03	0.2286	0.2133	0.0057	0.0076	2.65E-06	0.0297	0.0240
SAINT GOBAIN	0.0701	0.0521	2.33E-03	0.2509	0.1623	0.0026	0.0038	1.90E-06	0.0215	0.0095
SANOFI-AVENTIS	0.0640	0.0697	5.67E-04	0.3293	0.2291	0.0047	0.0110	1.71E-07	0.0562	0.0278
SCHNEIDER ELECTRIC	0.0844	0.0790	3.19E-03	0.3975	0.2287	0.0064	0.0124	4.80E-06	0.0715	0.0262
SOCIETE GENERALE	0.0526	0.0451	3.12E-03	0.1870	0.1459	0.0029	0.0044	6.38E-06	0.0203	0.0127
SODEXHO ALLIANCE	0.0852	0.1020	3.29E-03	0.5126	0.3249	0.0042	0.0115	2.80E-06	0.0687	0.0237
STMICROELECTRONICS	0.0597	0.0546	3.59E-04	0.2058	0.1929	0.0015	0.0024	2.94E-08	0.0091	0.0086
SUEZ	0.0976	0.0827	4.54E-03	0.3293	0.2602	0.0023	0.0036	2.90E-06	0.0150	0.0095
TF1	0.1109	0.1204	1.09E-02	0.7697	0.2645	0.0073	0.0244	3.51E-05	0.1723	0.0185
THALES	0.0961	0.1098	2.34E-03	0.4640	0.3503	0.0053	0.0114	1.30E-06	0.0517	0.0305
THOMSON (EX:TMM)	0.0941	0.0899	5.36E-03	0.4480	0.2728	0.0028	0.0054	5.38E-06	0.0319	0.0126
TOTAL	0.0397	0.0371	4.12E-04	0.1627	0.1164	0.0039	0.0067	2.35E-07	0.0347	0.0179
VEOLIA ENVIRON.	0.0723	0.0656	1.26E-04	0.3145	0.2007	0.0018	0.0032	2.84E-09	0.0180	0.0079
VINCI (EX.SGE)	0.0355	0.0259	2.49E-03	0.1209	0.0830	0.0012	0.0018	3.87E-06	0.0093	0.0043
VIVENDI UNIVERSAL	0.0661	0.0685	2.85E-03	0.3587	0.1749	0.0015	0.0033	1.27E-06	0.0212	0.0059

Table 18: Summary of out-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, classical approach.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.10473	0.120887	9.63E-05	0.551475	0.364007	0.008667	0.019298	3.31E-09	0.106835	0.047337
AGF-ASS.GEN.FRANCE	0.131635	0.143419	0.004299	0.780154	0.420724	0.016356	0.040833	7.97E-06	0.257852	0.081774
AIR LIQUIDE	0.080098	0.07862	0.000213	0.347809	0.267794	0.016298	0.030737	6.04E-08	0.151459	0.096209
ALCATEL	0.13355	0.121244	0.00334	0.470753	0.436743	0.003573	0.005914	1.17E-06	0.024805	0.020877
ARCELOR	0.117065	0.133359	0.001586	0.613585	0.351661	0.003782	0.008209	2.95E-07	0.042996	0.014443
AXA	0.092991	0.134509	0.00521	0.672537	0.386335	0.004239	0.013041	4.34E-06	0.073352	0.024652
BNP PARIBAS	0.078249	0.06503	0.000919	0.312203	0.208593	0.004732	0.007629	3.81E-07	0.04441	0.019698
BOUYGUES	0.171501	0.100734	0.000489	0.503438	0.308118	0.009813	0.011386	5.6E-08	0.061117	0.025105
CAP GEMINI	0.23231	0.295336	0.003557	1.30021	1.138396	0.054896	0.141549	4.96E-06	0.672608	0.479306
CARREFOUR	0.062836	0.059824	0.000457	0.249051	0.191973	0.003346	0.005899	8.99E-08	0.028191	0.016761
CASINO GUICHARD	0.146471	0.219818	0.005503	1.472293	0.439929	0.053252	0.236859	2.41E-05	1.675549	0.152666
CREDIT AGRICOLE	0.138865	0.197163	0.002775	1.078061	0.542391	0.010325	0.032196	1.44E-06	0.206887	0.050779
DANONE	0.054773	0.049043	1.06E-05	0.200395	0.156687	0.006984	0.011398	1.45E-10	0.051469	0.032055
DEXIA	0.109919	0.224349	0.001054	1.443772	0.536146	0.008178	0.039745	1.45E-07	0.277381	0.038698
EADS	0.194675	0.23971	0.007875	1.266802	0.593885	0.016546	0.04886	9.58E-06	0.29084	0.068889
FRANCE TELECOM	0.139777	0.211803	0.001373	1.138462	0.502473	0.013507	0.047594	4.05E-07	0.27759	0.051672
L'OREAL	0.086626	0.092238	0.004831	0.460046	0.276497	0.009811	0.0224	1.5E-05	0.130733	0.045345
LAFARGE	0.107592	0.137086	0.001946	0.659896	0.425454	0.018543	0.050934	2.36E-06	0.26746	0.120901
LAGARDERE S.C.A.	0.100285	0.083665	0.008624	0.394801	0.275215	0.00736	0.013494	3.25E-05	0.068864	0.033642
LVMH	0.113146	0.115475	0.001387	0.571628	0.325866	0.01497	0.034401	1.11E-06	0.18723	0.060437
MICHELIN	0.154091	0.206228	0.004786	1.109848	0.556597	0.022093	0.067374	7.74E-06	0.40964	0.103156
PERNOD-RICARD	0.077535	0.074666	0.003623	0.292375	0.245627	0.009856	0.016423	1.09E-05	0.072868	0.051538

Table 19: (Continued) Summary of out-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, classical approach.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.07616	0.091633	0.001125	0.448031	0.292859	0.00534	0.013486	5.03E-07	0.074993	0.031624
PIN.-PRINT.REDOUTE	0.138913	0.127963	0.000861	0.455642	0.416876	0.029041	0.045141	6.11E-07	0.174501	0.14263
RENAULT	0.140574	0.12552	0.001126	0.561398	0.364709	0.019575	0.031702	7.29E-07	0.179263	0.075486
SAINT GOBAIN	0.097856	0.085823	0.000143	0.291448	0.252967	0.006039	0.008351	7.75E-09	0.03071	0.023002
SANOFI-AVENTIS	0.099893	0.108369	0.000635	0.477165	0.379653	0.011763	0.025023	2.26E-07	0.128397	0.080006
SCHNEIDER ELECTRIC	0.086546	0.131574	0.003445	0.873692	0.225114	0.012561	0.055929	6.17E-06	0.394043	0.025897
SOCIETE GENERALE	0.069867	0.068663	0.000897	0.41165	0.208138	0.006245	0.016314	5.25E-07	0.108959	0.030047
SODEXHO ALLIANCE	0.123303	0.13401	0.001181	0.589764	0.414888	0.007619	0.016537	3.19E-07	0.08303	0.039809
STMICROELECTRONICS	0.090614	0.090484	0.001886	0.381984	0.258905	0.003741	0.006951	7.77E-07	0.033773	0.015468
SUEZ	0.096785	0.098772	0.004347	0.530998	0.28356	0.002744	0.006571	2.63E-06	0.043443	0.01122
TF1	0.110343	0.100563	0.002831	0.475552	0.290891	0.005941	0.010768	2.2E-06	0.060747	0.022388
THALES	0.09586	0.139949	0.000712	0.67571	0.45149	0.007064	0.02128	1.26E-07	0.112826	0.051969
THOMSON (EX:TMM)	0.145994	0.158754	0.013218	0.790626	0.424288	0.007884	0.016793	3.22E-05	0.104628	0.031645
TOTAL	0.052786	0.053217	0.001484	0.211851	0.163156	0.00753	0.013916	2.9E-06	0.060592	0.037718
VEOLIA ENVIRON.	0.12997	0.162395	0.000275	0.822174	0.408336	0.00836	0.023107	1.46E-08	0.138172	0.032558
VINCI (EX.SGE)	0.077354	0.108837	0.000834	0.595414	0.250262	0.011363	0.035806	4.46E-07	0.229369	0.039174
VIVENDI UNIVERSAL	0.109508	0.101303	0.00145	0.491998	0.288309	0.004099	0.007506	4.02E-07	0.047049	0.013952

Table 20: Summary of out-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, theoretical PCA-ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.101032	0.125263	3.65E-05	0.656141	0.406081	0.008812	0.023944	4.76E-10	0.151236	0.055117
AGF-ASS.GEN.FRANCE	0.100311	0.133961	7.15E-05	0.780385	0.260686	0.012014	0.038906	2.13E-09	0.258005	0.030916
AIR LIQUIDE	0.073266	0.073724	0.003349	0.339292	0.256931	0.01399	0.027697	1.4E-05	0.144132	0.087253
ALCATEL	0.098674	0.096642	0.001016	0.435166	0.25912	0.002094	0.004222	1.19E-07	0.020726	0.007399
ARCELOR	0.090767	0.090891	0.000691	0.382858	0.30878	0.00199	0.003707	5.71E-08	0.017965	0.011092
AXA	0.069464	0.091932	0.001602	0.472202	0.250792	0.002106	0.006311	4.1E-07	0.036161	0.010481
BNP PARIBAS	0.070067	0.059323	0.000462	0.297555	0.187281	0.003848	0.006829	9.54E-08	0.040341	0.015806
BOUYGUES	0.165915	0.082428	0.023921	0.41475	0.318636	0.008515	0.008201	0.000152	0.041817	0.025001
CAP GEMINI	0.178651	0.219114	0.002489	1.174109	0.561131	0.031315	0.081067	2.41E-06	0.52526	0.116454
CARREFOUR	0.06174	0.059977	0.000472	0.245889	0.193687	0.003294	0.00584	1.02E-07	0.027479	0.017062
CASINO GUICHARD	0.123652	0.179234	0.008428	1.247152	0.240706	0.036171	0.169248	5.37E-05	1.202284	0.043829
CREDIT AGRICOLE	0.104677	0.15018	0.001983	0.937612	0.327376	0.005932	0.022765	7.2E-07	0.156492	0.019785
DANONE	0.051885	0.04278	0.001511	0.184403	0.141413	0.005841	0.009211	2.94E-06	0.044261	0.02611
DEXIA	0.094336	0.199633	0.003382	1.415669	0.237838	0.006381	0.03761	1.51E-06	0.266688	0.007577
EADS	0.157575	0.206163	0.004654	1.316431	0.430686	0.011847	0.044937	3.2E-06	0.314075	0.033562
FRANCE TELECOM	0.107563	0.18937	0.002274	1.114558	0.252288	0.009981	0.041812	1.09E-06	0.266055	0.013434
L'OREAL	0.078233	0.087448	0.000789	0.431261	0.255377	0.008444	0.020237	3.82E-07	0.114885	0.041165
LAFARGE	0.083574	0.099491	0.000576	0.46409	0.347965	0.010337	0.024396	2.11E-07	0.132285	0.079173
LAGARDERE S.C.A.	0.100623	0.085864	0.00189	0.393571	0.316833	0.007572	0.013574	1.49E-06	0.06343	0.045744
LVMH	0.087971	0.095328	0.000482	0.510107	0.281609	0.009605	0.023348	1.36E-07	0.149097	0.045984
MICHELIN	0.141631	0.182732	0.002296	0.997271	0.423821	0.017847	0.05263	1.82E-06	0.330751	0.059811
PERNOD-RICARD	0.074763	0.073995	0.001046	0.342696	0.194887	0.00934	0.018008	8.97E-07	0.095102	0.034075

Table 21: (Continued) Summary of out-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, theoretical PCA-ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.065013	0.076633	0.00058	0.363543	0.246957	0.003806	0.009907	1.24E-07	0.049376	0.022488
PIN.-PRINT.REDOUTE	0.093109	0.093481	0.000849	0.3926	0.241447	0.014251	0.024458	6.17E-07	0.135122	0.048965
RENAULT	0.109862	0.096077	0.003059	0.45923	0.284489	0.011809	0.019894	5.01E-06	0.119953	0.043966
SAINT GOBAIN	0.09635	0.073971	0.00505	0.311034	0.246172	0.005266	0.007628	9.65E-06	0.034976	0.021783
SANOFI-AVENTIS	0.080981	0.085475	0.000925	0.470069	0.234014	0.007541	0.018552	4.61E-07	0.124607	0.029378
SCHNEIDER ELECTRIC	0.089963	0.137223	0.000771	0.919957	0.276185	0.013613	0.061775	2.83E-07	0.43688	0.038981
SOCIETE GENERALE	0.05898	0.059823	0.000929	0.329148	0.191882	0.004573	0.011581	5.4E-07	0.069661	0.024269
SODEXHO ALLIANCE	0.090263	0.085281	0.006608	0.386401	0.333731	0.003547	0.007206	1E-05	0.03453	0.026587
STMICROELECTRONICS	0.077424	0.082216	0.0011	0.354715	0.258909	0.002905	0.006086	2.83E-07	0.029251	0.015468
SUEZ	0.081059	0.066647	0.0007	0.265047	0.225211	0.001557	0.002317	6.74E-08	0.009764	0.00749
TF1	0.100125	0.08724	0.000237	0.411404	0.261296	0.00469	0.008132	1.45E-08	0.045463	0.018064
THALES	0.090614	0.119713	0.001196	0.544583	0.373197	0.00555	0.015036	3.57E-07	0.073285	0.035508
THOMSON (EX:TMM)	0.088386	0.098192	0.001182	0.577237	0.260797	0.002953	0.008321	2.38E-07	0.055772	0.01154
TOTAL	0.046162	0.048694	0.001695	0.216688	0.145632	0.006017	0.011929	3.84E-06	0.06339	0.02866
VEOLIA ENVIRON.	0.107108	0.097232	0.000563	0.437048	0.306249	0.004014	0.007298	6.09E-08	0.035858	0.019171
VINCI (EX.SGE)	0.069448	0.082488	0.000468	0.413568	0.195252	0.007453	0.019448	1.48E-07	0.11066	0.023845
VIVENDI UNIVERSAL	0.082903	0.064668	6.57E-05	0.235136	0.209637	0.002027	0.002635	8.05E-10	0.010507	0.007376

Table 22: Summary of out-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, theoretical PCA-SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.090626	0.137886	0.002348	0.649917	0.536377	0.009189	0.029316	1.93E-06	0.144187	0.096162
AGF-ASS.GEN.FRANCE	0.102291	0.133633	0.000582	0.779412	0.286914	0.012129	0.039268	1.41E-07	0.257362	0.03677
AIR LIQUIDE	0.072565	0.066163	0.000213	0.300349	0.176441	0.012488	0.021743	6.07E-08	0.112945	0.041765
ALCATEL	0.084513	0.090354	0.000423	0.457669	0.278512	0.00167	0.003795	2.06E-08	0.022925	0.008392
ARCELOR	0.066488	0.062096	0.005302	0.275934	0.179569	0.001007	0.001882	3.76E-06	0.008858	0.003864
AXA	0.072036	0.105963	0.001963	0.634953	0.220673	0.002605	0.009807	5.93E-07	0.065383	0.007983
BNP PARIBAS	0.07099	0.058753	0.001241	0.324374	0.184274	0.003878	0.007371	6.93E-07	0.04794	0.015302
BOUYGUES	0.162325	0.083088	0.015894	0.362491	0.310375	0.008194	0.007476	5.86E-05	0.031943	0.023393
CAP GEMINI	0.144827	0.195543	0.001048	1.066173	0.573595	0.023025	0.068357	4.88E-07	0.433125	0.121685
CARREFOUR	0.053692	0.04869	0.000588	0.173051	0.162094	0.00233	0.003645	1.57E-07	0.013611	0.01148
CASINO GUICHARD	0.105362	0.187273	0.001999	1.319837	0.205416	0.035152	0.189768	3.02E-06	1.346507	0.032469
CREDIT AGRICOLE	0.090236	0.132885	2.55E-05	0.838342	0.186043	0.004565	0.01835	1.18E-10	0.125109	0.005974
DANONE	0.045923	0.040887	0.003067	0.200447	0.122816	0.00486	0.009038	1.21E-05	0.052298	0.019593
DEXIA	0.084817	0.184903	0.001923	1.320052	0.207549	0.005415	0.032714	4.86E-07	0.231879	0.00577
EADS	0.143373	0.197349	0.003053	1.31882	0.353095	0.010522	0.044449	1.52E-06	0.315216	0.024352
FRANCE TELECOM	0.100619	0.190161	0.001971	1.098809	0.2805	0.009736	0.041666	8.27E-07	0.25859	0.01709
L'OREAL	0.0698	0.084434	0.002911	0.428982	0.208267	0.007348	0.020075	5.27E-06	0.113674	0.027378
LAFARGE	0.096378	0.13112	7.31E-05	0.637217	0.473039	0.016177	0.044913	3.48E-09	0.249392	0.131625
LAGARDERE S.C.A.	0.081552	0.070788	0.001943	0.307987	0.254365	0.005055	0.008835	1.57E-06	0.038843	0.028124
LVMH	0.091313	0.12251	0.000786	0.766689	0.285854	0.013293	0.048341	3.72E-07	0.336811	0.048951
MICHELIN	0.13799	0.174418	0.002542	0.914142	0.573479	0.016481	0.047011	2.17E-06	0.277909	0.109736
PERNOD-RICARD	0.053228	0.055811	0.000956	0.281365	0.182382	0.004956	0.010665	7.61E-07	0.064108	0.026978

Table 23: (Continued) Summary of out-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, theoretical PCA-SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.059045	0.06552	0.000591	0.31294	0.225685	0.002938	0.007139	1.35E-07	0.036587	0.019138
PIN.-PRINT.REDOUTE	0.077759	0.077514	0.000343	0.237852	0.223485	0.009731	0.014472	9.18E-08	0.045605	0.04205
RENAULT	0.107648	0.093682	0.001188	0.41558	0.266571	0.011348	0.017524	7.85E-07	0.098234	0.040327
SAINT GOBAIN	0.089499	0.064233	0.001668	0.305529	0.225959	0.004355	0.006521	9.98E-07	0.034689	0.018459
SANOFI-AVENTIS	0.070683	0.081308	0.000302	0.468029	0.21672	0.006318	0.018496	4.91E-08	0.123527	0.025925
SCHNEIDER ELECTRIC	0.078796	0.128382	0.003367	0.863407	0.219359	0.011501	0.05438	5.79E-06	0.38482	0.024058
SOCIETE GENERALE	0.065305	0.068449	0.009275	0.37611	0.199134	0.005834	0.014646	5.41E-05	0.090957	0.027321
SODEXHO ALLIANCE	0.080608	0.085728	0.001775	0.380575	0.323014	0.003182	0.007198	7.18E-07	0.033497	0.02476
STMICROELECTRONICS	0.080173	0.099279	0.002199	0.585146	0.246765	0.003717	0.011863	1.11E-06	0.079598	0.014051
SUEZ	0.072454	0.066261	0.00277	0.325311	0.206004	0.001351	0.002569	1.06E-06	0.014708	0.005891
TF1	0.089901	0.080429	0.001446	0.370885	0.260495	0.003856	0.006832	5.38E-07	0.036949	0.018702
THALES	0.078236	0.086654	0.000947	0.364021	0.333167	0.003388	0.007474	2.24E-07	0.032745	0.027787
THOMSON (EX:TMM)	0.078428	0.062133	0.003129	0.26672	0.218878	0.001717	0.002593	1.54E-06	0.01207	0.008421
TOTAL	0.049591	0.05375	0.000242	0.222192	0.178904	0.007152	0.01504	7.68E-08	0.066651	0.04288
VEOLIA ENVIRON.	0.089907	0.092122	0.000975	0.416781	0.309455	0.003187	0.006369	1.76E-07	0.033918	0.019574
VINCI (EX.SGE)	0.055948	0.0706	0.000422	0.376909	0.147256	0.005221	0.016464	1.21E-07	0.091912	0.014218
VIVENDI UNIVERSAL	0.074614	0.06696	0.002236	0.270463	0.22277	0.001851	0.002939	9.75E-07	0.013901	0.009113

Table 24: Summary of out-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, dynamical PCA-ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.1124	0.1254	0.0002	0.5893	0.3713	0.0096	0.0214	2.04E-08	0.1220	0.0471
AGF-ASS.GEN.FRANCE	0.1315	0.1508	0.0019	0.7841	0.4689	0.0173	0.0421	1.63E-06	0.2605	0.0966
AIR LIQUIDE	0.0774	0.0713	0.0031	0.2963	0.2691	0.0144	0.0258	1.28E-05	0.1104	0.0971
ALCATEL	0.1050	0.0996	0.0005	0.4260	0.3157	0.0023	0.0040	3.28E-08	0.0205	0.0109
ARCELOR	0.1084	0.1136	0.0002	0.6382	0.2736	0.0030	0.0074	5.32E-09	0.0499	0.0099
AXA	0.0860	0.1147	0.0001	0.5745	0.3761	0.0033	0.0091	1.41E-09	0.0535	0.0232
BNP PARIBAS	0.0746	0.0591	0.0030	0.2228	0.2123	0.0041	0.0059	4.13E-06	0.0226	0.0203
BOUYGUES	0.1784	0.0998	0.0111	0.4831	0.3712	0.0104	0.0113	2.85E-05	0.0563	0.0362
CAP GEMINI	0.1542	0.1534	0.0004	0.7424	0.3671	0.0191	0.0369	5.89E-08	0.2112	0.0559
CARREFOUR	0.0658	0.0541	0.0034	0.1954	0.1877	0.0032	0.0048	5.23E-06	0.0174	0.0154
CASINO GUICHARD	0.1175	0.1085	0.0007	0.5122	0.3049	0.0197	0.0349	3.25E-07	0.2070	0.0708
CREDIT AGRICOLE	0.1361	0.1877	0.0022	0.9664	0.4518	0.0096	0.0283	9.05E-07	0.1662	0.0371
DANONE	0.0461	0.0401	0.0020	0.1680	0.1155	0.0048	0.0078	5.14E-06	0.0377	0.0178
DEXIA	0.0808	0.1070	0.0024	0.5401	0.1958	0.0024	0.0077	7.84E-07	0.0393	0.0051
EADS	0.1821	0.1915	0.0093	0.8733	0.5076	0.0123	0.0267	1.33E-05	0.1380	0.0466
FRANCE TELECOM	0.1120	0.1305	0.0036	0.7202	0.3245	0.0062	0.0169	2.76E-06	0.1104	0.0219
L'OREAL	0.0841	0.0922	0.0014	0.4330	0.2550	0.0096	0.0212	1.19E-06	0.1158	0.0410
LAFARGE	0.1003	0.1272	0.0030	0.5902	0.4237	0.0161	0.0408	5.15E-06	0.2049	0.1199
LAGARDERE S.C.A.	0.1197	0.1074	0.0032	0.5779	0.3270	0.0112	0.0232	4.51E-06	0.1476	0.0475
LVMH	0.1011	0.1017	0.0026	0.5089	0.3093	0.0118	0.0269	3.91E-06	0.1484	0.0545
MICHELIN	0.1473	0.1557	0.0029	0.8255	0.4773	0.0155	0.0373	2.96E-06	0.2382	0.0760
PERNOD-RICARD	0.0801	0.0762	0.0027	0.2958	0.2333	0.0103	0.0168	6.21E-06	0.0747	0.0441

Table 25: (Continued)Summary of out-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, dynamical PCA-ARMA model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.0803	0.1156	0.0003	0.5650	0.3176	0.0074	0.0213	4.49E-08	0.1193	0.0400
PIN.-PRINT.REDOUTE	0.1178	0.1230	0.0007	0.4889	0.3461	0.0238	0.0433	4.07E-07	0.2096	0.1007
RENAULT	0.1324	0.1300	0.0097	0.5451	0.4259	0.0189	0.0355	5.15E-05	0.1690	0.0957
SAINT GOBAIN	0.0986	0.0901	0.0034	0.4038	0.3256	0.0064	0.0114	4.43E-06	0.0582	0.0381
SANOFI-AVENTIS	0.0964	0.0920	0.0019	0.4874	0.2485	0.0097	0.0208	2.03E-06	0.1339	0.0326
SCHNEIDER ELECTRIC	0.1020	0.1386	0.0003	0.8831	0.3116	0.0150	0.0574	5.83E-08	0.4026	0.0496
SOCIETE GENERALE	0.0600	0.0565	0.0057	0.3377	0.1262	0.0044	0.0114	2.08E-05	0.0733	0.0106
SODEXHO ALLIANCE	0.1245	0.1277	0.0017	0.6786	0.4180	0.0073	0.0169	6.43E-07	0.1047	0.0417
STMICROELECTRONICS	0.0765	0.0861	0.0001	0.3577	0.2893	0.0030	0.0064	3.85E-09	0.0296	0.0193
SUEZ	0.0948	0.1034	0.0002	0.5106	0.2787	0.0028	0.0066	4.85E-09	0.0402	0.0109
TF1	0.1187	0.1022	0.0006	0.3841	0.3648	0.0065	0.0100	8.44E-08	0.0396	0.0367
THALES	0.0991	0.1170	0.0022	0.5874	0.4063	0.0058	0.0150	1.3E-06	0.0853	0.0402
THOMSON (EX:TMM)	0.1677	0.2161	0.0098	0.9597	0.7623	0.0129	0.0335	1.62E-05	0.1667	0.1032
TOTAL	0.0498	0.0528	0.0002	0.2104	0.1627	0.0071	0.0131	5.47E-08	0.0598	0.0375
VEOLIA ENVIRON.	0.1353	0.1575	0.0058	0.7708	0.5241	0.0083	0.0210	6.43E-06	0.1214	0.0507
VINCI (EX.SGE)	0.0787	0.1067	0.0011	0.5503	0.2740	0.0112	0.0324	8.45E-07	0.1959	0.0470
VIVENDI UNIVERSAL	0.1066	0.1223	0.0006	0.6139	0.3406	0.0048	0.0119	6.95E-08	0.0733	0.0195

Table 26: Summary of out-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, dynamical PCA-SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
ACCOR	0.1121	0.1244	0.0021	0.6061	0.3671	0.0095	0.0211	1.6E-06	0.1291	0.0455
AGF-ASS.GEN.FRANCE	0.1209	0.1413	0.0015	0.7887	0.3503	0.0149	0.0411	1.03E-06	0.2636	0.0539
AIR LIQUIDE	0.0818	0.0757	0.0005	0.3143	0.2707	0.0161	0.0285	2.66E-07	0.1325	0.0969
ALCATEL	0.1079	0.0955	0.0005	0.3944	0.3420	0.0023	0.0038	2.75E-08	0.0176	0.0130
ARCELOR	0.1062	0.1146	0.0007	0.4960	0.3214	0.0030	0.0058	6.91E-08	0.0302	0.0136
AXA	0.0889	0.1234	0.0032	0.6210	0.4045	0.0037	0.0105	1.61E-06	0.0625	0.0258
BNP PARIBAS	0.0742	0.0590	0.0006	0.2568	0.2068	0.0041	0.0062	1.49E-07	0.0301	0.0193
BOUYGUES	0.1773	0.0978	0.0099	0.5087	0.3608	0.0102	0.0114	2.27E-05	0.0624	0.0342
CAP GEMINI	0.1491	0.1322	0.0024	0.4774	0.3913	0.0161	0.0224	2.54E-06	0.0873	0.0627
CARREFOUR	0.0638	0.0562	0.0019	0.2193	0.2154	0.0032	0.0054	1.61E-06	0.0219	0.0207
CASINO GUICHARD	0.1129	0.1076	0.0021	0.5265	0.3595	0.0187	0.0377	3.46E-06	0.2186	0.0979
CREDIT AGRICOLE	0.1102	0.1375	0.0001	0.6769	0.4637	0.0056	0.0143	2.44E-09	0.0848	0.0371
DANONE	0.0531	0.0441	0.0023	0.1751	0.1611	0.0062	0.0095	7.06E-06	0.0393	0.0347
DEXIA	0.0779	0.1018	0.0002	0.5367	0.1759	0.0022	0.0070	6.57E-09	0.0388	0.0041
EADS	0.1404	0.1359	0.0070	0.6248	0.4196	0.0070	0.0138	7.54E-06	0.0762	0.0314
FRANCE TELECOM	0.1080	0.1257	0.0028	0.7210	0.3492	0.0058	0.0168	1.73E-06	0.1106	0.0257
L'OREAL	0.0832	0.0888	0.0004	0.4281	0.2448	0.0091	0.0203	1.08E-07	0.1132	0.0355
LAFARGE	0.1075	0.1358	0.0068	0.6370	0.4483	0.0184	0.0480	3.15E-05	0.2492	0.1343
LAGARDERE S.C.A.	0.1141	0.1003	0.0048	0.4333	0.3482	0.0099	0.0169	1.06E-05	0.0769	0.0538
LVMH	0.0959	0.1001	0.0007	0.5160	0.2879	0.0110	0.0255	2.92E-07	0.1526	0.0472
MICHELIN	0.1513	0.1653	0.0021	0.8349	0.5016	0.0170	0.0398	1.48E-06	0.2436	0.0838
PERNOD-RICARD	0.0745	0.0706	0.0010	0.2963	0.2182	0.0089	0.0154	9.04E-07	0.0711	0.0417

Table 27: (Continued)Summary of out-sample estimated costs of execution of VWAP order for period from September 2, 2003 to December 16, 2003, dynamical PCA-SETAR model.

	MAPE					MSPE				
Company	Mean	Std	Min	Max	Q95	Mean	Std	Min	Max	Q95
PEUGEOT	0.0801	0.0960	0.0000	0.4719	0.3046	0.0059	0.0144	6.12E-12	0.0832	0.0367
PIN.-PRINT.REDOUTE	0.0998	0.1119	0.0013	0.4484	0.3359	0.0184	0.0352	1.33E-06	0.1762	0.0893
RENAULT	0.1287	0.1138	0.0001	0.5084	0.3845	0.0163	0.0279	3.7E-09	0.1470	0.0859
SAINT GOBAIN	0.0952	0.0775	0.0027	0.3280	0.2713	0.0054	0.0082	2.71E-06	0.0389	0.0265
SANOFI-AVENTIS	0.0897	0.0944	0.0027	0.4746	0.2861	0.0092	0.0211	4.12E-06	0.1270	0.0433
SCHNEIDER ELECTRIC	0.1027	0.1417	0.0023	0.8921	0.3239	0.0155	0.0588	2.89E-06	0.4108	0.0536
SOCIETE GENERALE	0.0617	0.0600	0.0009	0.3533	0.1601	0.0048	0.0124	4.59E-07	0.0803	0.0178
SODEXHO ALLIANCE	0.1182	0.1280	0.0030	0.6053	0.3861	0.0070	0.0163	2E-06	0.0833	0.0345
STMICROELECTRONICS	0.0768	0.0867	0.0025	0.3791	0.2882	0.0031	0.0065	1.5E-06	0.0333	0.0192
SUEZ	0.0908	0.0970	0.0011	0.4763	0.3022	0.0025	0.0057	1.76E-07	0.0350	0.0128
TF1	0.1118	0.1040	0.0009	0.4264	0.3402	0.0062	0.0103	1.9E-07	0.0488	0.0306
THALES	0.1027	0.1270	0.0042	0.6337	0.3967	0.0066	0.0178	4.71E-06	0.0992	0.0401
THOMSON (EX:TMM)	0.1398	0.1780	0.0014	0.8393	0.4116	0.0087	0.0232	3.2E-07	0.1275	0.0273
TOTAL	0.0508	0.0515	0.0017	0.2184	0.1594	0.0070	0.0138	4.09E-06	0.0644	0.0343
VEOLIA ENVIRON.	0.1286	0.1511	0.0005	0.7291	0.4065	0.0076	0.0188	4.99E-08	0.1087	0.0323
VINCI (EX.SGE)	0.0755	0.0969	0.0009	0.4896	0.2544	0.0096	0.0267	4.68E-07	0.1551	0.0405
VIVENDI UNIVERSAL	0.1020	0.1012	0.0017	0.4977	0.2591	0.0038	0.0080	4.6E-07	0.0481	0.0128

Table 28: Comparison of VWAP predictions, based on mean absolute percentage error (MAPE), for period from September 2 to December 16, 2003.

Models	*Mean*	*STD*	*Min*	*Max*	*Q95*
Result of in-sample estimation					
PC-SETAR	0.0706	0.0825	0.0017	0.4526	0.2030
PC-ARMA	0.0772	0.0877	0.0019	0.4813	0.2173
Classical approach	0.1140	0.1358	0.0017	0.7054	0.3702
Result of out-sample estimation					
PC-SETAR theoretical	0.0770	0.0942	0.0020	0.5070	0.2432
PC-ARMA theoretical	0.0833	0.0956	0.0017	0.5009	0.2498
PC-SETAR with dynamical adjustment of forecast	0.0898	0.0954	0.0020	0.4560	0.2854
PC-ARMA with dynamical adjustment of forecast	0.0922	0.0994	0.0018	0.4866	0.2854
Classical approach	0.1006	0.1171	0.0025	0.5787	0.3427

Note:The cost is expressed in as a percentage of the end of day volume weighted price. The classical approach is based on calculating averages from historical volume data.

Table 29: Robustness check: Comparison of VWAP predictions, based on mean absolute percentage error (MAPE), for period from January 2 to April 20, 2004.

Models	*Mean*	*STD*	*Min*	*Max*	*Q95*
Result of in-sample estimation					
PC-SETAR	0.0679	0.0681	0.0010	0.3792	0.1908
PC-ARMA	0.0742	0.0786	0.0011	0.4560	0.2207
Classical approach	0.1099	0.1290	0.0010	0.7363	0.3442
Result of out-sample estimation					
PC-SETAR theoretical	0.0978	0.1047	0.0018	0.5303	0.2997
PC-ARMA theoretical	0.1043	0.1110	0.0034	0.5462	0.3145
PC-SETAR with dynamical adjustment of forecast	0.1116	0.1177	0.0027	0.5430	0.3495
PC-ARMA with dynamical adjustment of forecast	0.1142	0.1209	0.0026	0.5681	0.3505
Classical approach	0.1200	0.1345	0.0021	0.6523	0.3780

Note:The cost is expressed in as a percentage of the end of day volume weighted price. The classical approach is based on calculating averages from historical volume data.

Table 30: Summary of estimated costs of execution of the VWAP order for different intraday volume models. The panels present summary in cases when estimated volume weighed prices are smaller or higher from observed ones, upper and lower panel respectively.

Models	*Mean*	*Frequency*	*STD*	*Min*	*Max*	*Q95*
SETAR	0.0751	49.2	0.0924	0.0016	0.5681	0.2032
ARMA	0.0824	49.9	0.0915	0.0016	0.5291	0.2300
Classical approach	0.1122	52.0	0.1358	0.00158	0.7661	0.3527
SETAR	0.0795	50.8	0.0881	0.0013	0.5023	0.2340
ARMA	0.0856	50.1	0.0910	0.0020	0.5040	0.2471
Classical approach	0.1147	48.0	0.1310	0.0019	0.7231	0.3390

Note:The cost is expressed in as a percentage of the end of day volume weighted price. The classical approach is based on calculating averages from historical volume data.

Table 31: Comparison of execution risk exposure.

Companies	*Classical approach (in %)*	*Theoretical PCA-SETAR (in %)*	*Dynamical PCA-SETAR (in %)*	*Difference*		
				Theo. SETAR Class. approach	*Dyn. SETAR Class. approach*	*Theo. SETAR Dyn. SETAR*
ACCOR	0.1047	0.0906	0.1121	-0.0141	0.0074	-0.0215
AGF-ASS.GEN.FRANCE	0.1316	0.1023	0.1209	-0.0293	-0.0107	-0.0186
AIR LIQUIDE	0.0801	0.0726	0.0818	-0.0075	0.0017	-0.0092
ALCATEL	0.1336	0.0845	0.1079	-0.0491	-0.0257	-0.0234
ARCELOR	0.1171	0.0665	0.1062	-0.0506	-0.0109	-0.0397
AXA	0.0930	0.0720	0.0889	-0.0210	-0.0041	-0.0169
BNP PARIBAS	0.0782	0.0710	0.0742	-0.0072	-0.0040	-0.0032
BOUYGUES	0.1715	0.1623	0.1773	-0.0092	0.0058	-0.0150
CAP GEMINI	0.2323	0.1448	0.1491	-0.0875	-0.0832	-0.0043
CARREFOUR	0.0628	0.0537	0.0638	-0.0091	0.0010	-0.0101
CASINO GUICHARD	0.1465	0.1054	0.1129	-0.0411	-0.0336	-0.0075
CREDIT AGRICOLE	0.1389	0.0902	0.1102	-0.0487	-0.0287	-0.0200
DANONE	0.0548	0.0459	0.0531	-0.0089	-0.0017	-0.0072
DEXIA	0.1099	0.0848	0.0779	-0.0251	-0.0320	0.0069
EADS	0.1947	0.1434	0.1404	-0.0513	-0.0543	0.0030
FRANCE TELECOM	0.1398	0.1006	0.108	-0.0392	-0.0318	-0.0074
L'OREAL	0.0866	0.0698	0.0832	-0.0168	-0.0034	-0.0134
LAFARGE	0.1076	0.0964	0.1075	-0.0112	-0.0001	-0.0111
LAGARDERE S.C.A.	0.1003	0.0816	0.1141	-0.0187	0.0138	-0.0325
LVMH	0.1131	0.0913	0.0959	-0.0218	-0.0172	-0.0046
MICHELIN	0.1541	0.138	0.1513	-0.0161	-0.0028	-0.0133
PERNOD-RICARD	0.0775	0.0532	0.0745	-0.0243	-0.0030	-0.0213

Means of MAPE and drops in the execution risk measured by the difference of means of MAPE. The first column, named *Difference* , is the difference between the theoretical implementation PCA-SETAR model and the classical approach. A negative value means that the theoretical implementation PCA-SETAR model out-performs the classical approach since it reduces the execution risk to use the first approach instead of the latter one. The second column, is the difference between the dynamic implementation PCA-SETAR and the classical and the last one is the difference between theoretical and dynamic implementation.

Table 32: (Continued) Comparison of execution risk exposure.

Companies	*Classical approach (in %)*	*Theoretical PCA-SETAR (in %)*	*Dynamical PCA-SETAR (in %)*	*Difference*		
				Theo. SETAR Class. approach	*Dyn. SETAR Class. approach*	*Theo. SETAR Dyn. SETAR*
PEUGEOT	0.0762	0.059	0.0801	-0.0172	0.0039	-0.0211
PIN.-PRINT.REDOUTE	0.1389	0.0778	0.0998	-0.0611	-0.0391	-0.0220
RENAULT	0.1406	0.1076	0.1287	-0.0330	-0.0119	-0.0211
SAINT GOBAIN	0.0979	0.0895	0.0952	-0.0084	-0.0027	-0.0057
SANOFI-AVENTIS	0.0999	0.0707	0.0897	-0.0292	-0.0102	-0.0190
SCHNEIDER ELECTRIC	0.0865	0.0788	0.1027	-0.0077	0.0162	-0.0239
SOCIETE GENERALE	0.0699	0.0653	0.0617	-0.0046	-0.0082	0.0036
SODEXHO ALLIANCE	0.1233	0.0806	0.1182	-0.0427	-0.0051	-0.0376
STMICROELECTRONICS	0.0906	0.0802	0.0768	-0.0104	-0.0138	0.0034
SUEZ	0.0968	0.0725	0.0908	-0.0243	-0.0060	-0.0183
TF1	0.1103	0.0899	0.1118	-0.0204	0.0015	-0.0219
THALES	0.0959	0.0782	0.1027	-0.0177	0.0068	-0.0245
THOMSON (EX:TMM)	0.1460	0.0784	0.1398	-0.0676	-0.0062	-0.0614
TOTAL	0.0528	0.0496	0.0508	-0.0032	-0.0020	-0.0012
VEOLIA ENVIRON.	0.1300	0.0899	0.1286	-0.0401	-0.0014	-0.0387
VINCI (EX.SGE)	0.0774	0.0559	0.0755	-0.0215	-0.0019	-0.0196
VIVENDI UNIVERSAL	0.1095	0.0746	0.102	-0.0349	-0.0075	-0.0274

Note:Means of MAPE and drops in the execution risk measured by the difference of means of MAPE. The first column, named *Difference* , is the difference between the theoretical implementation PCA-SETAR model and the classical approach. A negative value means that the theoretical implementation PCA-SETAR model out-performs the classical approach since it reduces the execution risk to use the first approach instead of the latter one. The second column, is the difference between the dynamic implementation PCA-SETAR and the classical and the last one is the difference between theoretical and dynamic implementation.

Figure 1: Autocorrelation and partial autocorrelation functions of the two components, TOTAL stock.

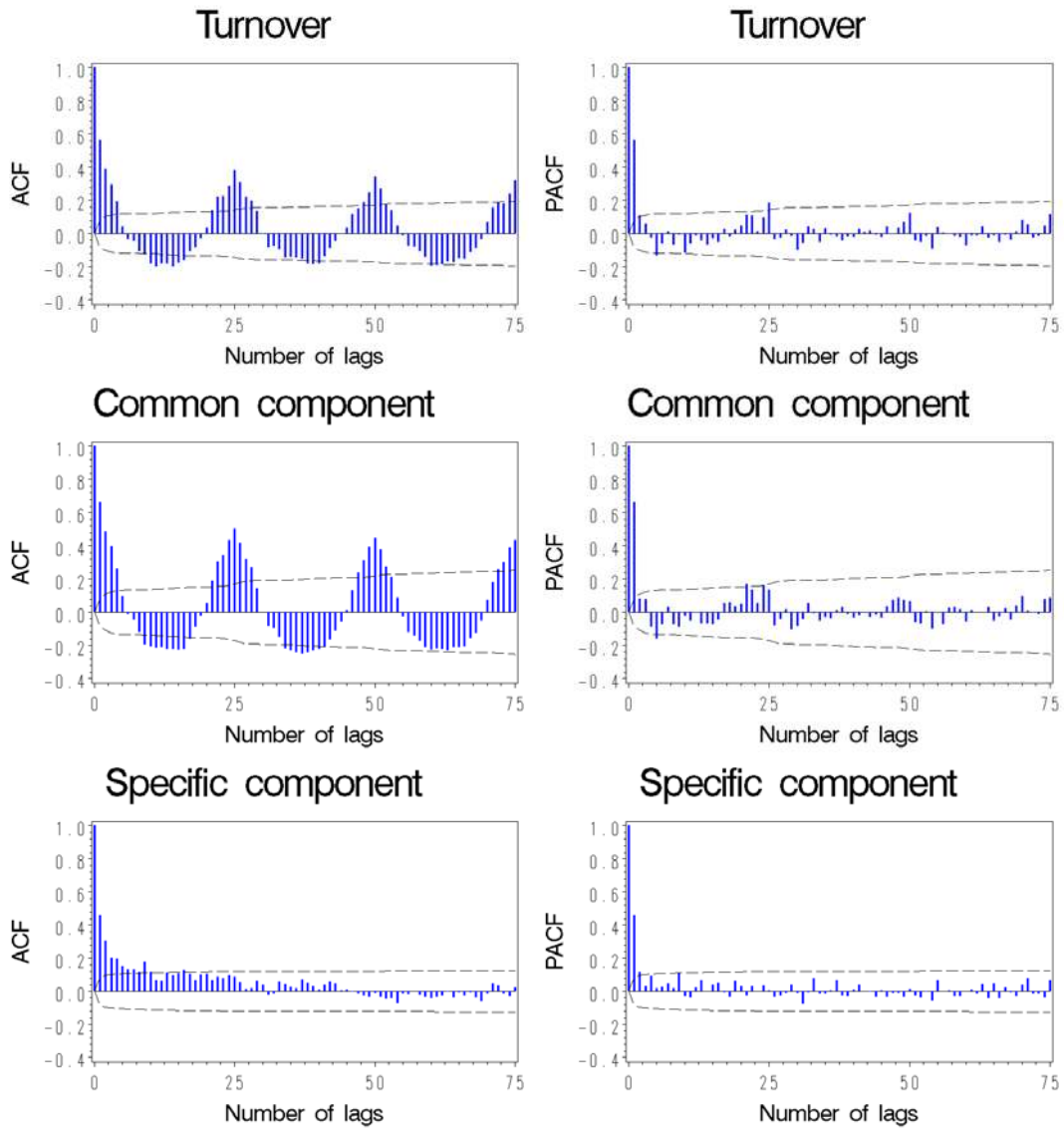

Figure 2: Autocorrelation functions of ARMA (left graph) and SETAR (right graph) residuals for specific component of EADS, SANOFI-AVENTIS and TOTAL stock.

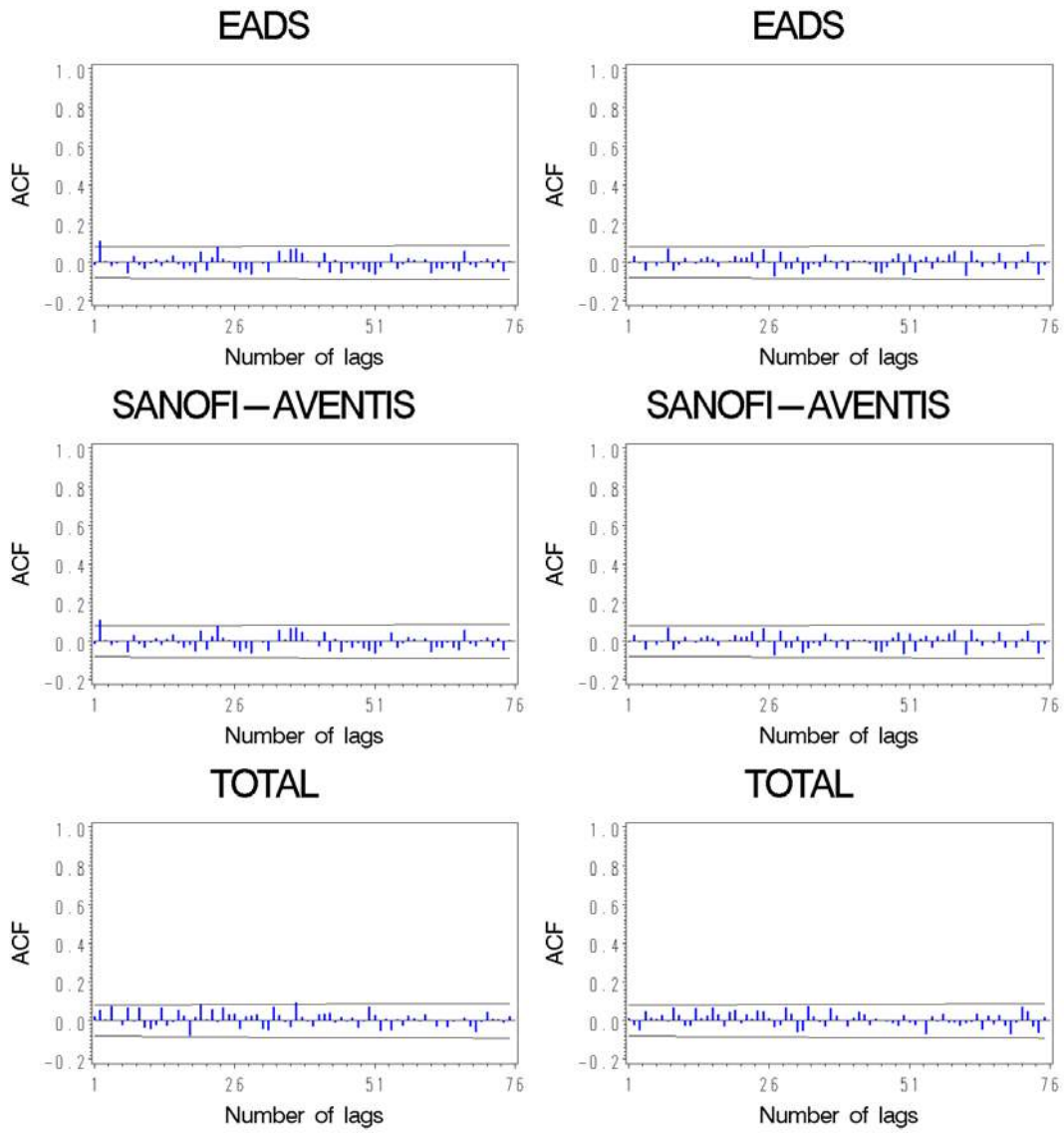

Figure 3: TOTAL stock daily volume patterns on September 9 and 10, 2003, left and right respectively. The first two graphs represent the intraday turnover evolution. The next two give the common component evolution and the final two, the specific component evolution.

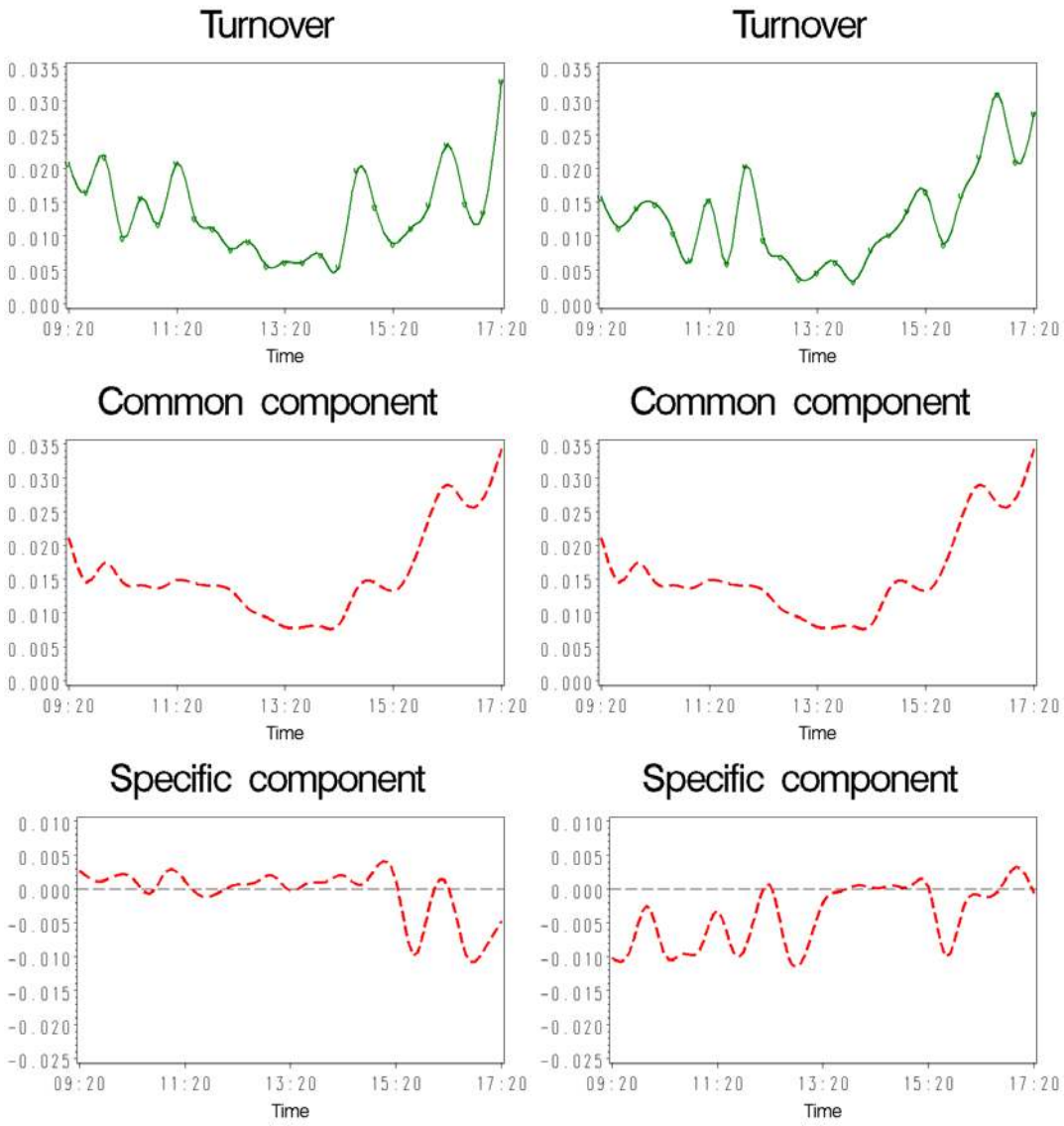

Figure 4: The dependence between classical approach tracking error and gain and loss for dynamical strategy.