

## In-Plane Magnetic Field Effect on the Transport Properties in a Quasi-3D Quantum Well Structure

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### Abstract

The transport properties of a quasi-three-dimensional, 200 layer quantum well structure are investigated at integer filling in the quantum Hall state. We find that the transverse magnetoresistance  $R_{xx}$ , the Hall resistance  $R_{xy}$ , and the vertical resistance  $R_z$  all follow a similar behavior with both temperature and in-plane magnetic field. A general feature of the influence of increasing in-plane field  $B_{in}$  is that the Hall conductance quantization first improves, but above a characteristic value  $B_{in}^C$ , the quantization is systematically removed. We consider the interplay of the chiral edge state transport and the bulk (quantum Hall) transport properties. This mechanism may arise from the competition of the cyclotron energy with the superlattice band structure energies. A comparison of the results with existing theories of the chiral edge state transport with in-plane field is also discussed.

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## INTRODUCTION

The integer quantum Hall effect has been observed in many quasi-three dimensional ( $Q3D$ ) structures<sup>1-5</sup>, where the interlayer tunneling band-width is much smaller than the two dimensional ( $2D$ ) quantum Hall gap  $E_g = \hbar\omega_c$ . In such materials, in a quantum Hall (QH) state, the edge of the sample is enveloped by a sheath of current-carrying Chiral edge states which is the  $2D$  extension of the  $1D$  states at the edge of a single layer QH fluid. These chiral edge states have been predicted theoretically<sup>6, 7</sup>, and confirmed experimentally<sup>2</sup>. A curious property of these  $Q3D$  systems at integer filling is the observation of a very similar temperature dependence of the in-plane resistance at the center of the QH state ( $R_{xx}^{\min}$ ) and the inverse of the vertical transport resistance ( $G_{zz} \sim 1/R_{zz}$ ). In particular, with magnetic field perpendicular to the layers, both follow an activated behavior (with a gap much smaller than the Landau gap  $\hbar\omega_c$ ) at higher temperatures, typically above 0.3 K, and a Coulomb gap-like behavior at lower temperatures where both  $R_{xx}^{\min}$  and  $G_{zz}$  approach some asymptotic (residual) value<sup>8</sup>. This is the case for the system discussed here, and also appears to be the case for  $R_{xx}^{\min}$  and  $G_{zz}$  in an independent study<sup>2</sup>. The motivation for the present work has been to try to understand the origin of this universal temperature dependence, and to consider the influence of finite in-plane field on the properties of the transport tensor. This second point is particularly important since there are theoretical predictions<sup>9</sup> for the in-plane field dependence of the chiral surface transport.

## BRIEF REVIEW OF OUR RESULTS.

The main results of the present work are the observation that in tilted magnetic fields, the quantization first improves at integer filling (as seen by the reduction in dissipation in  $R_{xx}$ , the enhancement of  $R_{zz}$ , and the broadening of the  $R_{xy}$  Hall plateau), followed by the gradual disappearance of the quantum Hall state above a characteristic in-plane magnetic field  $B_{in}^c$ . We further note that the temperature dependence of all measured components of the transport tensor also follow the same behavior. The results

suggest an interplay between the bulk quantum Hall state and the chiral edge state systems, and provide a test of the theoretical models for the transport properties of the chiral edge state.

## BRIEF DESCRIPTION OF OUR EXPERIMENT

To explore both the bulk quantum Hall and chiral edge state behaviors in a quasi-three-dimensional system, we employed a 200 layer *GaAs/AlGaAs* quantum well structure. The samples used were prepared by MBE, and involved 200 periods of 19nm *GaAs* quantum wells separated by *Si*  $\delta$ -center-doped ( $n_d = 4.0 \times 10^{11} \text{ cm}^{-2}$ ) *Al<sub>0.1</sub>Ga<sub>0.9</sub>As* barriers of thickness 4nm. To prevent layer depletion and to offset the surface pinning potential, a cap layer and a sequence of *Al<sub>0.1</sub>Ga<sub>0.9</sub>As* doped layers were grown<sup>10</sup>. The relevant parameters for this structure are: the interlayer spacing  $c = a + b = 23 \text{ nm}$ , where  $a = 19 \text{ nm}$  and  $b = 4 \text{ nm}$  are the widths of the well and barrier respectively, barrier height  $V_0 = 77 \text{ meV}$  calculated using the Shrodinger equation with doping density of Poission distribution; and low-field mobility  $\mu_H = 6562 \text{ cm}^2 / \text{Vs}$ . The Kronig-Penny model miniband structure for the superlattice potential (in the  $z$  direction) is shown in Fig. 1.

A conventional Hall bar configuration was used in measuring the in-plane magnetoresistance  $R_{xx}$  and  $R_{xy}$ , as shown in Figs. 2 and 3. To measure the vertical  $R_{zz}$  transport, sections from the same sample were processed by a vertical etching process<sup>10</sup> to provide a mesa-like structure as shown in Fig. 4. We note that  $R_{zz}$  was a four-terminal measurement, and that independent two-terminal measurements<sup>11, 12</sup> did not show any mixing of  $R_{xx}$  and  $R_{xy}$  into the  $R_{zz}$  signal. Measurements were carried out with standard ac lock-in methods with a current of 50 nA/layer. No evidence for heating or hot-electron gas effects were observed. For all measurements shown here, a rotating platform immersed in the mixture of a dilution refrigerator, associated with a superconducting magnet was employed.

The evidence that the Q3D sample exhibited complete quantization is shown in Fig. 3. At 8.7 tesla, concomitant with  $R_{xx}$  minima ( $R_{xx}^{\text{min}}$ ) in Fig. 2, and the  $R_{zz}$  maximum ( $R_{zz}^{\text{max}}$ ) in Fig. 4 there is a well developed Hall plateau in  $R_{xy}$ , which saturates at

below, the crossover may be a result of the modification of the mini-band structure with increasing in-plane magnetic field. We note that in our four terminal measurements, we find no systematic evidence for universal conductance fluctuations, although extensive efforts were made to measure these effects, as reported in Ref. 2.

## DISCUSSION

The results discussed above indicate an unusual coupling of the bulk quantum well transport ( $R_{xx}$  and  $R_{xy}$ ) at integer filling with the corresponding behavior of the chiral edge state transport ( $1/R_z$ ). This is true both in terms of the temperature dependence, and in the angular (in-plane field) dependence. At zero angle, both  $R_{xx}^{\min}$  and  $1/R_z^{\max}$  remain finite in the low temperature limit, indicating the presence of dissipation in both the bulk quantum Hall state and in the chiral edge state. And, in finite in-plane field, the angular dependence of these two tensor components consistently track each other, first as the quantization improves, and then as it is removed. Indeed, the two states appear to be interconnected. The crossover from activated-to-power law behavior above the optimum angle also appears in both parameters. The finite dissipation at low temperatures in  $1/R_z^{\max}$ , which is also observed in the measurements of Druist et al.<sup>2</sup>, is at odds with theoretical expectations for a dissipationless 2D chiral metal as predicted by Balents and Fisher<sup>7</sup>. Although the theoretically predicted geometrical relationship<sup>6</sup> of  $G_z^{\max} \approx C/L$  (where  $C$  and  $L$  are the height and circumference of a mesa-type structure respectively) has been demonstrated in experiment<sup>2</sup>, the anomalous low temperature dissipation is observed in both reported measurements of  $G_z^{\max}$ . Given the apparent coupling of the bulk and edge state transport properties, it is not clear that the surface transport is truly decoupled from the bulk.

We next turn to a discussion of the effects of the in-plane field. Chalker and Sondhi have treated the chiral edge state conductivity  $\sigma(B_{in})$  with in-plane field<sup>9</sup>  $B_{in}$ . Their results show that  $R_{zz}$  should exhibit positive magnetoresistance, following a

Fig. 5 a more detailed study of these trends is presented in the form  $R_{xx}^{\min}$  and  $1/R_{xx}^{\max}$ . For the case of no in-plane field, there is a clear relationship between the measured transverse resistance  $R_{xx}$  and the conductivity,  $G_{xx} \sim 1/R_{xx}$ . In this paper, we take the view point that a variable tilted field at integer filling is simply the case where the transverse field is constant, and the in-plane field varies. Hence we retain the definition that  $G_{xx} \sim 1/R_{xx}$  (at  $\nu=2$  filling in this case) with increasing in-plane field. Since the total field  $\mathbf{B}$  must be increased to maintain the  $\nu=2$  filling with increasing angle, the optimum angle (more accurately  $27^\circ$ ) corresponds to a perpendicular field of  $B_{\perp} = 8.7$  T and an in-plane field  $B_{\parallel} = 4.4$  T. We will return to the influence of the in-plane magnetic field in the discussion.

Another striking feature of the transport properties is their temperature dependence, as is shown in Fig. 6. Here we show  $R_{xx}^{\min}$  and  $1/R_{xx}^{\max}$  ( $\sim G_{xx}$ ) vs. temperature for  $\theta=0^\circ$ , and also for  $\theta=36^\circ$ , which is well above the optimum angle where the quantization is reduced. There are two temperature ranges of interest. First, at high temperatures (above 0.3 K) and  $\theta=0^\circ$ ,  $R_{xx}^{\min}$  and  $1/R_{xx}^{\max}$  both show activated behavior ( $R = R_0 e^{-(\Delta/2KT)}$  and  $G = G_0 e^{-\Delta/2KT}$  respectively), and at low temperatures (below 0.3 K) both exhibit Coulomb gap-like behavior ( $R = R_{res} + R_0 e^{-\sqrt{T_c}/T}$  and  $G = G_{res} + G_0 e^{-\sqrt{T_c}/T}$  respectively). In contrast, in tilted magnetic field, and above the optimum angle, both exhibit power law (non activated) behavior at high temperatures ( $R = R_0 + R_1 T^\alpha$  and  $G = G_0 + G_1 T^\alpha$  respectively). However, the low temperature behavior remains Coulomb gap-like. (The parameter values are defined in the caption of Fig. 6.) The nearly identical temperature dependence of  $R_{xx}^{\min}$  and  $1/R_{xx}^{\max}$  is further shown in Fig. 7a where they are plotted against each other for the different temperature ranges and angles. The crossover from activated to power law dependence with angle is more clearly demonstrated in Fig. 7b, where, for each tensor component, the zero angle and tilted values are plotted against each other. This is to emphasize the different functional dependence for the two orientations. The crossover from activated to power law behavior is an additional indication of the degradation of the quantum Hall state. As discussed

64.45±0.010Ω. This value corresponds to a quantization of  $R_{xy} = (1/200)(h/2e^2)$  where all 200 layers participate, and demonstrates a spin-unpolarized integer *QHE* at the filling factor  $\nu=2$ . This demonstrates that for the *Q3D* sample studied here, with the field perpendicular to the layers, the Fermi energy lies in a gap, and that none of the layers are depleted. This last point was carefully checked by demonstrating that we could sequentially deplete individual layers with a top gate structure<sup>8</sup>. Such a procedure also reduces or eliminates parallel conductance channels. However, in the presence of a gated sample with several integer layers depleted, the asymptotic behavior of  $R_{xx}^{\min}$  was still evident, as was the full quantization in  $R_{xy}$ . Hence it is unlikely that the finite, residual dissipation observed in  $R_{xx}^{\min}$  in the low temperature limit is due to non-integer depletion or a parallel conductance channel.

## RESULTS

In Figs. 2-4 we present the three transport tensor components  $R_{xx}$ ,  $R_{xy}$ , and  $R_{zz}$  at low temperatures (30 mK) vs. total magnetic field  $B$  for the *Q3D* system studied. The full curves represent the cases where the magnetic field was perpendicular to the layers ( $\theta=0$ ). The inset curves in Figs. 2-4 represent the details of the angular dependence of the transport components in the vicinity of integer filling  $\nu=2$ . Here the angle is measured from the normal to the conducting layers to the direction of the magnetic field. We define the field normal to the conducting layers as  $B_{\perp} = B \cos(\theta)$ , and the field parallel to the conducting layers as the in-plane field  $B_{\parallel} = B \sin(\theta)$ . Starting with Fig. 2 we see that at  $\nu=2$  there is a minimum in the dissipation near  $30^\circ$ , *i.e.*  $R_{xx}^{\min}$  approaches the lowest value at this angle, and at higher angles the dissipation increases. In Fig. 3 the width of the  $\nu=2$  Hall plateau is largest at  $30^\circ$  then rapidly decreases with increasing angle (note the inset is plotted vs.  $B_{\perp}$ , not the total field  $B$ ). And finally, in Fig. 4 the maximum of the vertical resistance  $R_{zz}^{\max}$  has a slight extremum at  $30^\circ$ , followed by a reduction at higher angles. In all three cases the effect of increasing angle corresponds to an initial improvement of the quantization, followed by a rapid removal of quantization above an optimum angle. In

Drude formula, with a field scaling field  $B_0 = \Phi_0 / a\ell_{cl}$ , where  $\Phi_0$  is the flux quanta,  $a$  is the lattice spacing and  $\ell_{cl}$  is the in-plane elastic length according to the relation  $\sigma(B_{in}) = \frac{\sigma(B_{in} = 0)}{1 + (B_{in} / B_0)^2}$ . To test this prediction, we have treated the data below the optimum angle shown in Fig. 5. We find that this description only fits a short range of in-plane field  $< 4$  T, with the fitting parameters:  $\sigma(B_{in} = 0) = 5.15 \times 10^{-3} (\Omega^{-1})$ ,  $B_0 = 7.15 T$ ,  $\ell_{cl} = 252 \text{ \AA}$ . These parameters indicate, assuming that the theoretical relationship is valid in this range, that the elastic scattering length is comparable to the inter-layer spacing of  $230 \text{ \AA}$ . These parameters indicate that the elastic scattering length is comparable to the inter-layer spacing of  $a = 230 \text{ \AA}$ , and also the magnetic length  $l_B = 220 \text{ \AA}$ . The value of these parameters are clearly outside the limits of applicability of the theory<sup>9</sup>, which requires  $\ell_{cl} \gg l_B^2 / a$ . The absence of macroscopic conductance fluctuation in the present case may be an additional indication that our system is outside the limits of the theory. This would also be consistent with our lack of observation of conductance fluctuations, due to the small length scales involved.

We may further consider the behavior of the transport tensor components above the optimum angle, where the quantization is removed with increasing in-plane field. There are several treatments of multiple well structures in tilted magnetic fields. Marlow and co-workers have studied cyclotron resonance in a coupled two layer quantum well in tilted magnetic fields<sup>13</sup>. They find wavefunction hybridization and subband energy splitting which result from the in-plane magnetic field. Although a detailed description of the removal of the quantization must be worked out theoretically, a general argument for the mechanism may be made. In reference to Fig. 1, we note that the condition for quantization, with the Fermi level in a gap, will change with increasing in-plane field due to fact that the eigen values of the Hamiltonian will change when the in-plane field is added. It would appear then, from our experiments, that the band structure as given in Fig. 1 starts to change significantly above an in-plane field of 4 T, and it is no longer possible to maintain the quantization condition with the Fermi level in



a gap. Hence the results may be viewed a crossover from a quasi-two-dimensional to a quasi-three-dimensional electronic structure (i.e. bands closing with respect to the Fermi level ) with increasing in-plane field.

## SUMMARY

In summary, we have studied the transport properties associated with the integer quantum Hall effect in a 200 layer quantum well superlattice in the Hall-bar configuration, and also the vertical transport associated with the chiral edge state in the mesa configuration, both on the same material. Furthermore, the transport properties have also been studied in tilted magnetic fields while maintaining the position of the  $\nu = 2$  filling, for increasing in-plane field. We find that there is a direct correlation between the in-plane magnetoresistance  $R_{xx}$  and the vertical conductivity  $1/R_{zz}$ , both in terms of the temperature dependence, and in the angular dependence. For in-plane fields up to 4T, the quantization improves, but for larger in-plane fields, the quantization is removed. These results suggest a strong correlation between the bulk quantum Hall states and the chiral edge states. This correlation persists even as the system crosses over from a quasi-two-dimensional to a quasi-three-dimensional electronic structure in tilted field, an effect which is most likely due to the modification of the subband structure at high in-plane magnetic fields. If we assume that the chiral edge state is present, then its properties are substantially different from theoretical expectations, since there appears to be considerable dissipation in the low temperature limit, and, since the chiral behavior is correlated with the bulk behavior. Interaction effects may play an important role in the modification of the mini-band structure with in-plane field, and further theoretical work is needed to fully understand the behavior in tilted magnetic fields.

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## FIGURE CAPTIONS

Figure 1. Mini-band structure and Fermi level for the 200 layer quantum well superlattice from the Kronig-Penny model.

Figure 2. Field dependence of magnetoresistance  $R_{xx}$  at 30mK with zero titled angle. The inset shows the  $R_{xx}$   $\nu=2$  minimum at different angles vs. the total field.

Figure 3. Field dependence of Hall resistance  $R_{xy}$  at 30mK with zero titled angle. The inset shows the  $R_{xy}$   $\nu=2$  plateaus at different angles vs. the in-plane field.

Figure 4. Field dependence of vertical magnetoresistance  $R_{zz}$  at 30mK with zero titled angle. The inset figure illustrates the  $R_{zz}$   $\nu=2$  peaks at different angles vs. the total field. The mesa configuration is also indicated.

Figure 5a. In-plane magnetic field dependence of  $1/R_{zz}^{\max}$  and  $R_{xx}^{\min}$  at 30mK. There is an optimal value of the in-plane field  $B_{\parallel}$ . Here,  $1/R_{zz}^{\max}$  and  $R_{xx}^{\min}$  are plotted on different scales. With the in-plane field increasing from zero to 8 tesla,  $R_{xx}^{\min}$  changes by almost an order of magnitude, but the change in  $1/R_{zz}^{\max}$  is very small by comparison.

Figure 5b. In-plane magnetic field dependence of  $1/R_{zz}^{\max}$  and  $R_{xx}^{\min}$  at 548mK.

Figure 6a. Temperature dependence of  $R_{xx}^{\min}$  at different titled angles. ( $\blacktriangle$ :  $\theta=0^{\circ}$ , left hand scale;  $\circ$ :  $\theta=36^{\circ}$ , right hand scale). The solid lines are low temperature (below 0.3 K) fits to the coulomb gap form. Above 0.3 K, the short dashed line for  $\theta=0^{\circ}$  is the fit to the activated form, and the long dashed line for  $\theta=36^{\circ}$  is the fit to the power law form.

The parameters of the fits are: Coulomb gap-  $T_c=1.25\text{K}$ ,  $R_0=29.1\Omega$  and  $R_{res}=2.28\Omega$  at  $\theta=0^{\circ}$  and  $T_c=4.82\text{K}$ ,  $R_0=62.2\Omega$ ,  $R_{res}=12.5\Omega$  for  $\theta=36^{\circ}$ ; Activation -  $\Delta=0.73\text{K}$ ,  $R_0=15.44\Omega$  at  $\theta=0^{\circ}$ ; Power law -  $R_0=12.3\Omega$ ,  $R_1=6.07\Omega$ ,  $\alpha=1.23$  at  $\theta=36^{\circ}$ .

Figure 6b. Temperature dependence of  $1/R_{zz}^{\max}$  at different titled angles. Symbols and lines have the same representation as in Fig. 6a. The parameters of the fits are: Coulomb gap-  $T_c = 16K$ ,  $\sigma_0=0.004\Omega^{-1}$  and  $\sigma_{\text{res}}=3.7\times 10^{-5}\Omega^{-1}$  at  $\theta=0^0$  and  $T_c=42.5K$ ,  $\sigma_0=0.085\Omega^{-1}$  and  $\sigma_{\text{res}}=0.0043\Omega^{-1}$  for  $\theta=36^0$ ; Activation -  $\Delta/K=2.21$ ,  $\sigma_0=0.004\Omega^{-1}$  at  $\theta=0^0$ ; Power law -  $\sigma_0=0.0039\Omega^{-1}$ ,  $\sigma_1=6.0\times 10^{-5}\Omega^{-1}$ ,  $\alpha=0.64$  at  $\theta=36^0$ .

Figure 7a. Demonstration of the direct proportionality between  $R_{xx}^{\min}$  and  $1/R_{zz}^{\max}$  in both the high and low temperature regimes. Here  $R_{xx}^{\min}$  and  $1/R_{zz}^{\max}$  are plotted against each other for the two temperature limits, and for the two field orientations. The linear slopes indicate direct proportionality.

Figure 7b. Demonstration of difference in functional dependence between  $\theta=0^0$  activated and  $\theta=36^0$  power law behavior at higher temperatures. Here  $R_{xx}^{\min}(\theta=0^0)$  vs.  $R_{xx}^{\min}(\theta=36^0)$  and  $1/R_{zz}^{\max}(\theta=0^0)$  vs.  $1/R_{zz}^{\max}(\theta=36^0)$  are plotted to show that the functional dependence with temperature has changed for each between the two field orientations. ( $\theta=36^0$  is above the optimal angle for quantization.) The non-linear slopes indicate the functional difference between activated and power-law dependence between  $\theta=0^0$  and  $\theta=36^0$ .

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