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Properties of g^*s^* Closure,

g^*s^* Interior and g^*s^* Derived Sets

in Topological Spaces

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Abstract

The aim of this paper is to introduce the g^*s^* closure, g^*s^* interior, g^*s^* derived set and discuss some basic properties of the g^*s^* -closure, g^*s^* interior, g^*s^* derived set in a topological space.

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1. Introduction

In 1963, N. Levine [3] introduced semi-open sets in topological space. In 1970, N. Levine [4] initiated the study of generalized closed sets. In 1987, Bhattacharya

and Lahiri [2] defined and studied the concepts of semi generalized closed sets. In 1990, Arya and Nour [1] introduced the concept of generalized semi closed sets. In 2000, M.K.R.S.Veerakumar [6] studied the notion of g^* closed sets which is properly placed between closed sets and generalized closed sets. In 2014, R.Rajendiran[5] introduced and discussed the notion of g^*s^* closed sets. In this paper we analyze the various properties of g^*s^* -closure, g^*s^* interior, g^*s^* derived set in a topological space.

2. **PRELIMINARIES**

Definition 2.1. A subset of a topological space (X, τ) is called

- (i) generalized closed (briefly g closed) [4] if $cl(A) \subset U$ whenever $A \subset U$ and U is open in X.
- (ii) generalized semi closed (briefly gs closed) [1] $scl(A) \subset U$ whenever $A \subset U$ and U is open in X.
- (iii) Semi generalized closed(briefly sg closed)[2] $scl(A) \subset U$ whenever $A \subset U$ and U is semi open in X.
- (iv) Generalized star closed (briefly g^* closed)[6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g open.
- (v) g^*s^* closed [5] if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* open.

3. g^*s^* Interior and their properties

Definition 3.1. Let (X, τ) be a topological space and $x \in X$. A subset *N* of *X* is said to be g^*s^* neighborhood of *x* if there exists a g^*s^* open set *U* in *X* such that $x \in U \subset N$.

Definition: 3.2. Let A be a subset of a topological space (X, τ) , a point $x \in X$ is said to be g^*s^* interior point of A, if there exists a g^*s^* open set U such that $x \in U \subset A$. The set of all g^*s^* interior points of A is called g^*s^*

interior of A and is denoted by $g^*s^*Int(A)$.

Theorem 3.3. Let A be a subset of a space X, then $g^*s^*Int(A)$ is the union of all g^*s^* open sets which are contained in A.

Proof: Let $x \in g^*s^*Int(A)$. Then x is a g^*s^* interior point of A. Hence there exists a g^*s^* open set G such that $x \in G \subset A$. Therefore $x \in \bigcup \{G/G \text{ is } g^*s^*$ open and $G \subset A\}$. So $g^*s^*Int(A) \subseteq \bigcup \{G/G \text{ is } g^*s^*$ open and $G \subset A\}$. Conversely, let $x \in \bigcup \{G/G \text{ is } g^*s^*$ open and $G \subset A\}$. Then x belongs to a g^*s^* open set G contained in A. Hence x is an g^*s^* interior point of A.That is $x \in g^*s^*Int(A)$. So $\bigcup \{G/G \text{ is } g^*s^*$ open and $G \subset A\} \subseteq g^*s^*Int(A)$. Therefore $g^*s^*Int(A) = \bigcup \{G/G \text{ is } g^*s^*$ open and $G \subset A\}$.

Theorem. 3.4. Let A be a subset of a space X, then $g^*s^*Int(A) \subset A$.

Proof: Let $x \in g^*s^*Int(A)$. Then there exists a g^*s^* open set G such that $x \in G \subset A$. Hence $x \in A$. Therefore $g^*s^*Int(A) \subset A$.

Theorem 3.5. If A is g^*s^* open in a topological space (X, τ) , then $g^*s^*Int(A) = A$.

Proof: For any subset $A \text{ of } X, g^*s^*Int(A) \subset A$. Let $x \in A$. Then $x \in A \subset A$ and A is g^*s^* open which implies x is a g^*s^* interior of A. Hence $x \in g^*s^*Int(A)$. Therefore $A \subset g^*s^*Int(A)$. Hence $g^*s^*Int(A) = A$.

Theorem 3.6. Let A and B be subsets of a space X. If B is any g^*s^* -open set contained in A, then $B \subset g^*s^*Int(A)$.

Proof: Let $x \in B$. Since B is g^*s^* -open set contained in A, x is a g^*s^* interior point of A. $x \in g^*s^*Int(A)$. Hence $B \subset g^*s^*Int(A)$.

Theorem 3.7. If A is g^*s^* open in a topological space (X, τ) , then $g^*s^*Int(A)$ is also a g^*s^* open set in X.

Proof: From the theorem 3.5, if A is g^*s^* open in a topological space (X, τ) , then $g^*s^*Int(A) = A$. Therefore $g^*s^*Int(A)$ is a g^*s^* open set in X.

Theorem 3.8. Let A be a subset of a topological space (X, τ) , then

 $g^*s^*int(g^*s^*Int(A)) \subset g^*s^*Int(A).$

Proof: Let $x \in g^*s^*int(g^*s^*Int(A))$. Then x is g^*s^* interior point of $g^*s^*Int(A)$. Hence there exists an g^*s^* open set U such that $x \in U \subset g^*s^*Int(A) \subset A$. So that there exists a g^*s^* open set U such that $x \in U \subset A$. Therefore x is a g^*s^* interior point of A That is $x \in g^*s^*Int(A)$. Therefore $g^*s^*int(g^*s^*Int(A)) \subset g^*s^*Int(A)$.

Theorem. 3.9. If (X, τ) is a topological space, then $g^*s^*Int(\emptyset) = \emptyset$ and $g^*s^*Int(X) = X$.

Proof: $g^*s^*Int(\emptyset) = \bigcup \{G/G \text{ is } g^*s^* \text{ open and } G \subset \emptyset\}$. Since \emptyset is the only g^*s^* open set contained in \emptyset , $g^*s^*Int(\emptyset) = \emptyset$.

 $g^*s^*Int(X) = \bigcup \{G/G \text{ is } g^*s^* \text{ open and } G \subset X\}$. Since X is g^*s^* open set contained in X, $g^*s^*Int(X) = X \cup \{G/G \text{ is } g^*s^* \text{ open and } G \subset X\} = X$.

Theorem 3.10. If A and B are any two subsets of a topological space (X, τ) and $A \subset B$ then $g^*s^*Int(A) \subset g^*s^*Int(B)$.

Proof: Let $x \in X$ and $x \in g^*s^*Int(A)$. Then by the definition of g^*s^* interior, there exists a g^*s^* open set U such that $x \in U \subset A$. Since $A \subset B$, then $x \in U \subset A \subset B$. Hence $x \in g^*s^*Int(B)$. Therefore $g^*s^*Int(A) \subset g^*s^*Int(B)$.

Theorem 3.11. If A and B are any two subsets of a topological space (X, τ) and $A \cap B = \emptyset$ then $g^*s^*Int(A) \cap g^*s^*Int(B) = \emptyset$

Proof: Given $A \cap B = \emptyset$. To prove that $g^*s^*Int(A) \cap g^*s^*Int(B) = \emptyset$. We prove this by method of contradiction. Assume that $x \neq \emptyset \in g^*s^*Int(A) \cap g^*s^*Int(B)$. Therefore $x \in g^*s^*Int(A)$ and $x \in g^*s^*Int(B)$. Hence there exists an g^*s^* open sets U and V such that $x \in U \subset A, x \in V \subset B$. So $x \in U \cap V \subset U \subset A$ and $x \in U \cap V \subset V \subset B$. For this reason $x \in A \cap B$. This is contradiction to $A \cap B = \emptyset$. Consequently, $g^*s^*Int(A) \cap g^*s^*Int(B) = \emptyset$.

Theorem 3.12. If A and B are any two subsets of a topological space (X, τ) then $g^*s^*Int(A) \cup g^*s^*Int(B) \subset g^*s^*Int(A \cup B)$.

Proof: Let $x \in g^*s^*Int(A) \cup g^*s^*Int(B)$. Then $x \in g^*s^*Int(A)$ or $x \in g^*s^*Int(B)$. $x \in g^*s^*Int(A)$ implies that there exists a g^*s^* open set U such

that $x \in U \subset A \subset A \cup B$. So $\in g^*s^*Int(A \cup B)$. $x \in g^*s^*Int(B)$ implies that there exists a g^*s^* open set V such that $x \in V \subset B \subset A \cup B$. So $x \in$ $g^*s^*Int(A \cup B)$. Consequently $g^*s^*Int(A) \cup g^*s^*Int(B) \subset g^*s^*Int(A \cup B)$. B).

Theorem 3.13. If *A* and *B* are two subsets of a topological space (X, τ) then $g^*s^*Int(A \cap B) \subset g^*s^*Int(A) \cap g^*s^*Int(B)$

Proof: Let $x \in g^*s^*Int(A \cap B)$. Then there exists a g^*s^* open set U such that $x \in U \subset A \cap B \subset A$. This implies that $x \in g^*s^*Int(A)$. Also $x \in U \subset A \cap B \subset B$. So $x \in g^*s^*Int(B)$. Hence $x \in g^*s^*Int(A) \cap g^*s^*Int(B)$. Therefore $g^*s^*Int(A \cap B) \subset g^*s^*Int(A) \cap g^*s^*Int(B)$.

4. g^*s^* closure and their properties

Definition 4.1. For any subset A in the space X, the g^*s^* closure of A, denoted by $g^*s^*Cl(A)$, is defined by the intersection of all g^*s^* closed sets containing A.

Theorem 4.2. For a subset A of a space X, then $A \subset g^*s^*Cl(A)$

Proof: Let $x \in A$. By the definition of g^*s^* closure of , $x \in g^*s^*Cl(A)$. So $A \subset g^*s^*Cl(A)$.

Theorem 4.3. The g^*s^* closure of A is the intersection of all g^*s^* closed sets containing A.

Proof: By the definition of g^*s^* closure of *A*,

 $g^*s^*Cl(A) = \cap \{F/F \text{ is } g^*s^* \text{ closed and } A \subset F\}.$

Theorem 4.4. If B is any g^*s^* closed set and $A \subset B$ then $g^*s^*Cl(A) \subset B$.

Proof: By the definition of g^*s^* closure, $g^*s^*Cl(A) = \cap \{F/F \text{ is } g^*s^* \text{ closed} \text{ and } A \subset F\}$. Therefore $g^*s^*Cl(A)$ is contained in every g^*s^* closed set containing A. Since B is g^*s^* closed set and $A \subset B$, $g^*s^*Cl(A) \subset B$. **Theorem 4.5.** If A is g^*s^* closed set in (X, τ) then $A = g^*s^*Cl(A)$.

Proof: By the definition of g^*s^* closure, $A \subset g^*s^*Cl(A)$. Also $A \subset A$ and A is g^*s^* closed set, by theorem 4.4, $g^*s^*Cl(A) \subset A$. Hence $A = g^*s^*Cl(A)$.

Theorem 4.6. In a topological space (X, τ) , $g^*s^*Cl(\emptyset) = \emptyset$ and $g^*s^*Cl(X) = X$.

Proof: By the definition of g^*s^* closure, $g^*s^*cl(\emptyset) =$ Intersection of all g^*s^* closed sets containing \emptyset . Hence $g^*s^*cl(\emptyset) = \emptyset$. Also $g^*s^*cl(X) =$ Intersection of all g^*s^* closed sets containing X. Hence $g^*s^*cl(X) = X$.

Theorem 4.7. If A and B are any subsets of a space (X, τ) and $A \subset B$ then $g^*s^*Cl(A) \subset g^*s^*Cl(B)$.

Proof: Let $x \in g^*s^*Cl(A)$. By the definition, $g^*s^*cl(B) = \bigcap \{F/B \subset F \in g^*s^*C(X)\}$. If $B \subset F \in g^*s^*C(X)$, then $g^*s^*cl(B) \subset F$. Since $A \subset B$, $A \subset B \subset F \in g^*s^*C(X)$, we have $g^*s^*cl(A) \subset F$. Therefore $g^*s^*cl(A) \subset F = \bigcap \{F/B \subset F \in g^*s^*C(X)\} = g^*s^*cl(B)$. That is $g^*s^*Cl(A) \subset g^*s^*Cl(B)$.

Theorem 4.8. If *A* and *B* are any subsets of a space (X, τ) then $g^*s^*Cl(A) \cup g^*s^*Cl(B) \subset g^*s^*Cl(A \cup B)$.

Proof: Let A and B be subsets of X. Clearly $A \subset A \cup B$ and $B \subset A \cup B$. By theorem 4.7, we have $g^*s^*Cl(A) \subset g^*s^*Cl(A \cup B)$ and $g^*s^*Cl(B) \subset g^*s^*Cl(A \cup B)$. Hence $g^*s^*Cl(A) \cup g^*s^*Cl(B) \subset g^*s^*Cl(A \cup B)$.

Theorem 4.9. If A and B are any subsets of a space (X, τ) and $g^*s^*Cl(A) \cap g^*s^*Cl(B) = \emptyset$ then $A \cap B = \emptyset$.

Proof: Let $g^*s^*Cl(A) \cap g^*s^*Cl(B) = \emptyset$. To prove that $A \cap B = \emptyset$. We prove this by the method of contradiction. Assume that $x \in A \cap B$. Then $x \in$ $A \& x \in B$. Therefore $x \in g^*s^*Cl(A) \& x \in g^*s^*Cl(B)$. So $x \in g^*s^*Cl(A) \cap$ $g^*s^*Cl(B)$ which is contradiction. Hence $A \cap B = \emptyset$.

Theorem 4.10. If *A* and *B* are any subsets of a space (X, τ) then $g^*s^*Cl(A \cap B) \subset g^*s^*Cl(A) \cap g^*s^*Cl(B)$.

Proof: Let A and B be subsets of X. Also $A \cap B \subset A \& A \cap B \subset B$. Therefore by Theorem.4.7, we have $g^*s^*Cl(A \cap B) \subset g^*s^*Cl(A)$ and $g^*s^*Cl(A \cap B) \subset g^*s^*Cl(B)$. Therefore $g^*s^*Cl(A \cap B) \subset g^*s^*Cl(A) \cap g^*s^*Cl(B)$.

Theorem 4.11. For an $x \in X$, $x \in g^*s^*Cl(A)$ if and only if $V \cap A \neq \emptyset$ for every g^*s^* open set V containing x.

Proof: Let $x \in X, x \in g^*s^*Cl(A)$. To prove $V \cap A \neq \emptyset$ for every g^*s^* open set V containing x. We prove by contradiction. Suppose that there exists a g^*s^* open set V containing x such that $V \cap A = \emptyset$. Then $A \subset X \setminus V$ and $X \setminus V$ is g^*s^* closed set . Hence $g^*s^*cl(A) \subset X \setminus V$. Therefore $g^*s^*cl(A) \cap V = \emptyset$. This implies that $x \notin g^*s^*Cl(A)$ which is contradiction. So $V \cap A \neq \emptyset$ for every g^*s^* open set V containing x.

Conversely, let $V \cap A \neq \emptyset$ for every g^*s^* open set V containing x. To prove that $x \in g^*s^*Cl(A)$. We prove this by contradiction. Assume that $x \notin g^*s^*Cl(A)$. Then there exists a g^*s^* open set V containing x such that $V \cap A = \emptyset$ which is contradiction. Hence $x \in g^*s^*Cl(A)$.

Theorem 4.12. If A is g^*s^* closed set in X then $g^*s^*Cl(A)$ is also g^*s^* closed set in X.

Proof: If A is g^*s^* closed set in X then $g^*s^*Cl(A) = A$. Therefore $g^*s^*Cl(A)$ is also a g^*s^* closed set in X.

Theorem 4.13. If A is a subset of (X, τ) then $g^*s^*Cl(A) \subset g^*s^*Cl\{g^*s^*Cl(A)\}$

Proof: If A is a subset of (X, τ) then $g^*s^*Cl(A)$ is also a a subset of (X, τ) . By definition of g^*s^* closure, $g^*s^*Cl(A) \subset g^*s^*Cl\{g^*s^*Cl(A)\}$.

Theorem. 4.14. For any subset A of a space X, $X \setminus g^*s^*Cl(A) = g^*s^*Int(X \setminus A)$. **Proof:** For any point $x \in X, x \in X \setminus g^*s^*Cl(A)$ implies $x \notin g^*s^*Cl(A)$. Then there exists g^*s^* open set U containing $x, A \cap U = \emptyset$. So $x \in U \subset X \setminus A$. Thus $x \in g^*s^*Int(X \setminus A)$. Conversely, let $x \in g^*s^*Int(X \setminus A)$. There exists a g^*s^* open set U such that $x \in U \subset X \setminus A$. So $x \notin g^*s^*Cl(A)$. This implies that $\in X \setminus g^*s^*Cl(A)$.

Theorem 4.15. For any subset A of a space X, $X \setminus g^*s^*Int(A) = g^*s^*Cl(X \setminus A)$. **Proof:** Let $x \in X \setminus g^*s^*Int(A)$. Then $x \notin g^*s^*Int(A)$. That is every g^*s^* open set U containing x is such that $U \not\subset A$. That is every g^*s^* open set U containing x is such that $U \cap A^C \neq \emptyset$. Then by theorem 4.11, $x \in g^*s^*Cl(X \setminus A)$. A). Therefore $X \setminus g^*s^*Int(A) \subset g^*s^*Cl(X \setminus A)$. Conversely, $x \in g^*s^*Cl(X \setminus A)$. Then by theorem 4.11, every g^*s^* open set U containing x is such that $U \cap A^C \neq \emptyset$. That is every g^*s^* open set U containing x is such that $U \not\subset A$. This implies that by the definition of g^*s^* interior, $x \notin g^*s^*Int(A)$. That is $x \in X \setminus g^*s^*Int(A)$. So $g^*s^*Cl(X \setminus A) \subset X \setminus g^*s^*Int(A)$. Consequently, $g^*s^*Int(A) = g^*s^*Cl(X \setminus A)$.

Theorem 4.16. For any subset A of a space X, $g^*s^*Cl(A) = X \setminus g^*s^*Int(X \setminus A)$ **Proof:** $x \in g^*s^*Cl(A)$. Then by theorem 4.11, every g^*s^* open set U containing x is such that $U \cap A \neq \emptyset$. Hence every g^*s^* open set U containing x is such that $U \not\subset X \setminus A$. Therefore $x \not\in g^*s^*Int(X \setminus A)$. Hence $x \in X \setminus g^*s^*Int(X \setminus A)$. Therefore $g^*s^*Cl(A) \subset X \setminus g^*s^*Int(X \setminus A)$. Conversely, let $x \in X \setminus g^*s^*Int(X \setminus A)$. Then $x \notin g^*s^*Int(X \setminus A)$. Hence every g^*s^* open set U containing x is such that $U \not\subset X \setminus A$. Then every g^*s^* open set U containing x is such that $U \not\subset X \setminus A$. Then every g^*s^* open set U containing x is such that $U \not\subset X \setminus A$. Then every g^*s^* open set U containing x is such that $U \not\subset X \setminus A$. Then every g^*s^* open set U containing x is such that $U \cap A \neq \emptyset$. So $x \in g^*s^*Cl(A)$. That is $X \setminus g^*s^*Int(X \setminus A) \subset g^*s^*Cl(A)$. Thus $g^*s^*Cl(A) = X \setminus g^*s^*Int(X \setminus A)$.

Theorem 4.17. For any subset A of a space , $g^*s^*Int(A) = X \setminus g^*s^*Cl(X \setminus A)$. **Proof:** Let $x \in g^*s^*Int(A)$. Then there exists a g^*s^* open set U such that $x \in U \subset A$. Hence $x \notin g^*s^*Cl(X \setminus A)$. Therefore $x \in X \setminus g^*s^*Cl(X \setminus A)$. Hence $g^*s^*Int(A) \subset X \setminus g^*s^*Cl(X \setminus A)$. Conversely, let $x \in X \setminus g^*s^*Cl(X \setminus A)$. This implies that $x \notin g^*s^*Cl(X \setminus A)$. Then there exists a g^*s^* open set U such that $x \in U \cap X \setminus A = \emptyset$. That is there exists a g^*s^* open set U such that $x \in U \cap X \setminus A = \emptyset$. Therefore $X \setminus g^*s^*Cl(X \setminus A) \subset g^*s^*Int(A)$. Hence $g^*s^*Int(A) = X \setminus g^*s^*Cl(X \setminus A)$.

5. g^*s^* derived set and their properties

Definition 5.1. Let A be a subset of a topological space (X, τ) . A point $x \in X$ is said to be g^*s^* limit point of A if for each g^*s^* open set U containing x, $U \cap \{A \setminus \{x\}\} \neq \emptyset$. The set of all g^*s^* limit points of A is called a

 g^*s^* derived set of A and is denoted by $g^*s^*D(A)$.

Theorem 5.2. In a topological space (X, τ) , $g^*s^*D(\emptyset) = \emptyset$.

Proof: For all g^*s^* open set U and for all $x \in X, U \cap \{\emptyset \setminus x\} = \emptyset$. Hence $g^*s^*D(\emptyset) = \emptyset$.

Theorem 5.3. Let A be a subset of a space (X, τ) . If $x \in g^*s^*D(A)$, then $x \in g^*s^*D(A \setminus \{x\})$

Proof: Let $x \in g^*s^*D(A)$. Then for each g^*s^* open set U, $U \cap \{A \setminus \{x\}\} \neq \emptyset$. This implies that $U \cap \{(A \setminus \{x\})\{x\}\} \neq \emptyset$. Hence $x \in g^*s^*D(A \setminus \{x\})$.

Theorem 5.4. Let A and B be subsets of a space X. If $A \subset B$ then $g^*s^*D(A) \subset g^*s^*D(B)$.

Proof: Let $x \in g^*s^*D(A)$. Then for each g^*s^* open set U containing x, $U \cap \{A \setminus \{x\}\} \neq \emptyset$. Since $A \subset B$, $U \cap \{B \setminus \{x\}\} \neq \emptyset$. Therefore $x \in g^*s^*D(B)$. Hence $g^*s^*D(A) \subset g^*s^*D(B)$

Theorem 5.5. Let A and B be subsets of a space (X, τ) . Then $g^*s^*D(A) \cup g^*s^*D(B) \subset g^*s^*D(A \cup B)$

Proof: Let $x \in g^*s^*D(A) \cup g^*s^*D(B)$. This implies that $x \in g^*s^*D(A)$ or $\in g^*s^*D(B)$. If $x \in g^*s^*D(A)$ then for each g^*s^* open set U containing x, $U \cap \{A \setminus \{x\}\} \neq \emptyset$. Since $A \subset A \cup B$, $U \cap \{A \cup B \setminus \{x\}\} \neq \emptyset$. This shows that $x \in g^*s^*D(A \cup B)$. Otherwise if $x \in g^*s^*D(B)$ then for each g^*s^* open set U containing x, $U \cap \{B \setminus \{x\}\} \neq \emptyset$. Since $B \subset A \cup B$, $U \cap \{A \cup B \setminus \{x\}\} \neq \emptyset$. This shows that $x \in g^*s^*D(A \cup B)$. Otherwise if $x \in g^*s^*D(B)$ then for each g^*s^* open set U containing x, $U \cap \{B \setminus \{x\}\} \neq \emptyset$. Since $B \subset A \cup B$, $U \cap \{A \cup B \setminus \{x\}\} \neq \emptyset$. This shows that $x \in g^*s^*D(A \cup B)$. So $g^*s^*D(A) \cup g^*s^*D(B) \subset g^*s^*D(A \cup B)$. **Theorem 5.6.** Let A and B be subsets of a space X. Then $g^*s^*D(A \cap B) \subset g^*s^*D(A) \cap g^*s^*D(B)$

Proof: Let $x \in g^*s^*D(A \cap B)$. Then for each g^*s^* open set U containing x, $U \cap \{(A \cap B \setminus \{x\}\} \neq \emptyset$. Since $A \cap B \subset A, U \cap \{(A \setminus \{x\}\} \neq \emptyset$. This implies that $x \in g^*s^*D(A)$. Also $A \cap B \subset B, U \cap \{(B \setminus \{x\}\} \neq \emptyset$. This implies that $x \in$ $g^*s^*D(B)$. Therefore $x \in g^*s^*D(A) \cap g^*s^*D(B)$. Hence $g^*s^*D(A \cap B) \subset$ $g^*s^*D(A) \cap g^*s^*D(B)$.

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