In Transit to Constant Time Shortest-Path Queries in Road Networks

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- for general graphs with non-negative edge weights, exact solution given by Dijkstra's algorithm in $O(m + n \log n)$ where $n = \#$ nodes, $m = \#$ edges

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- so far, best solutions allow for a query time in the order of *milliseconds* (with preprocessing)

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- yields shortest path queries in the order of *milliseconds* on the US roadmap (after preprocessing)
- any sensible reason to aim for faster query times ?
  ⇒YES! Web services, traffic simulations, logistics . . .

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– let's say more than 50 miles –

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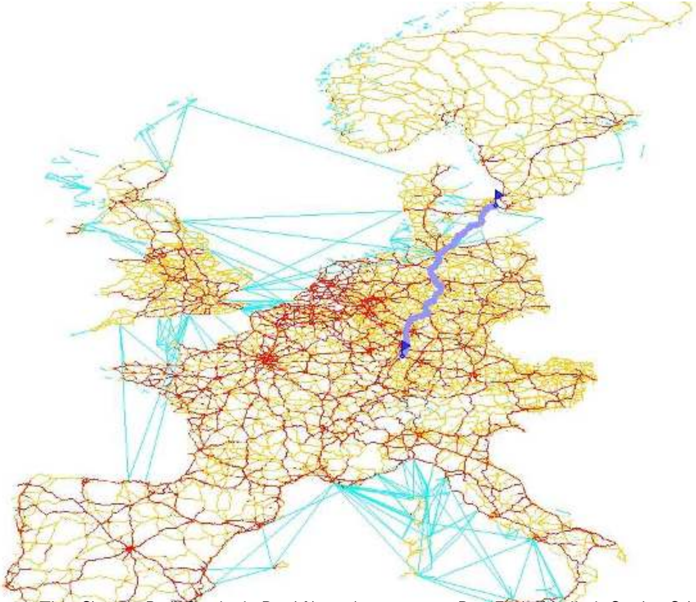
Only VERY few!

Example: Karlsruhe → Copenhagen



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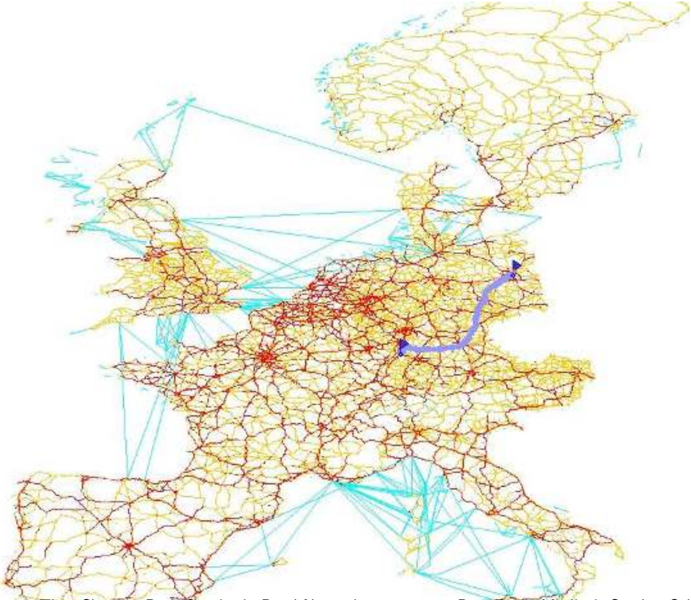

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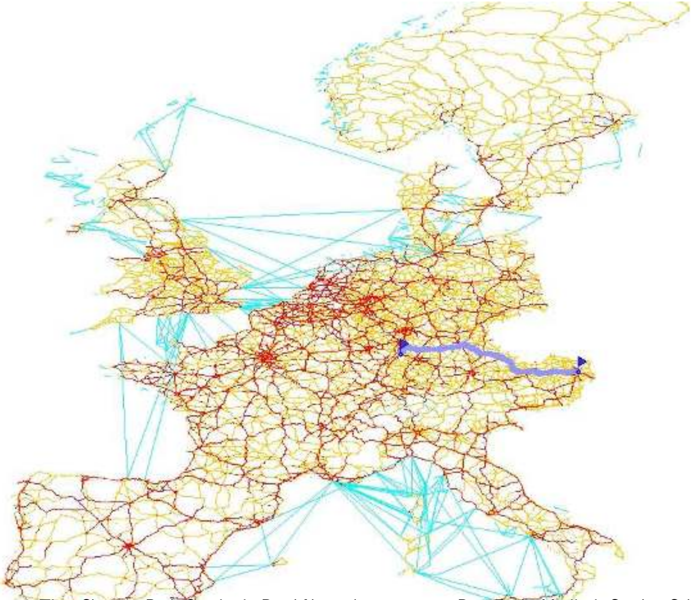

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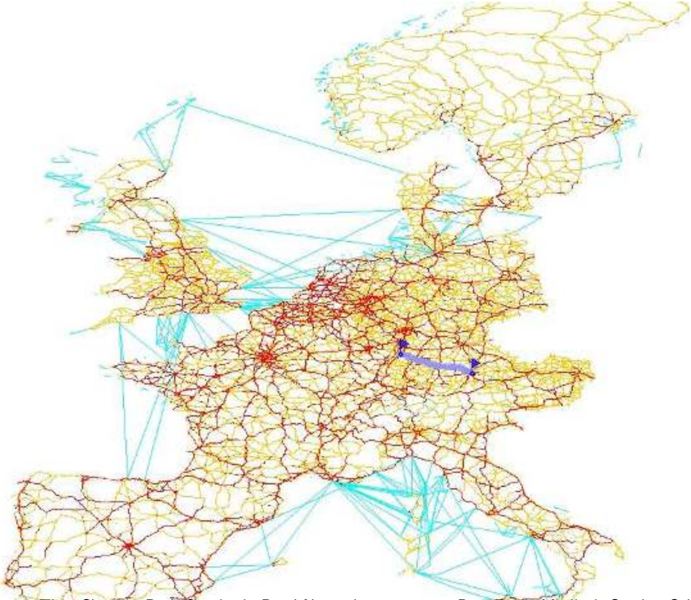

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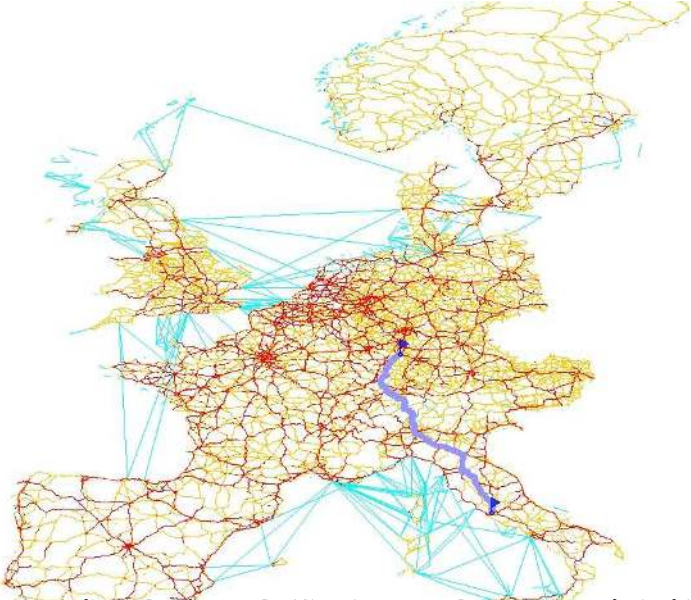

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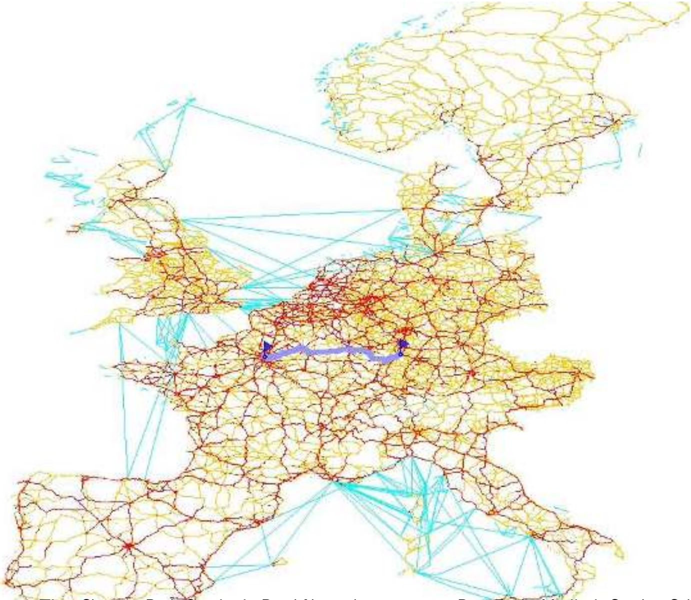

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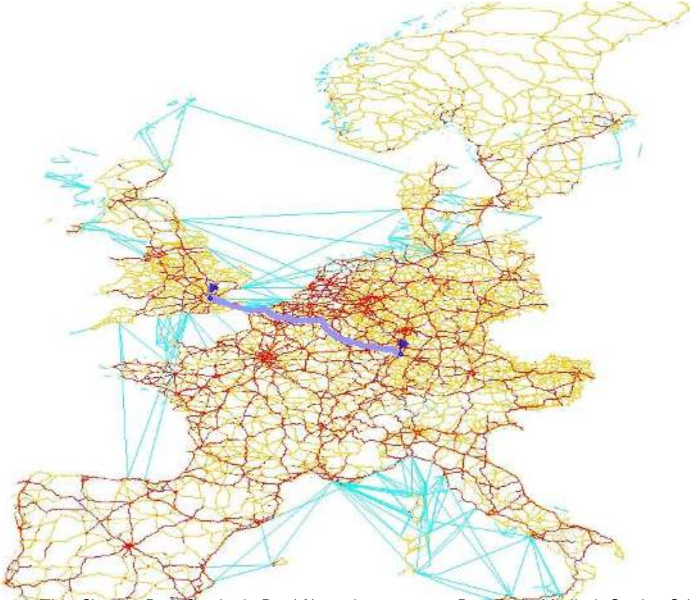

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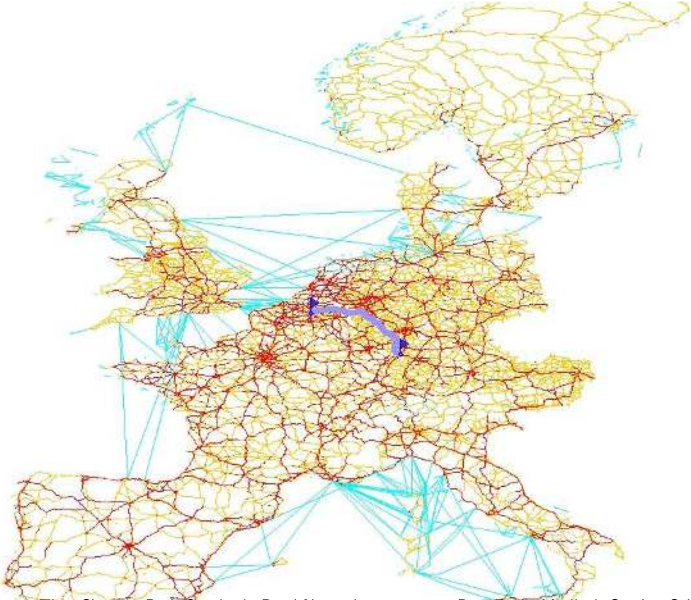

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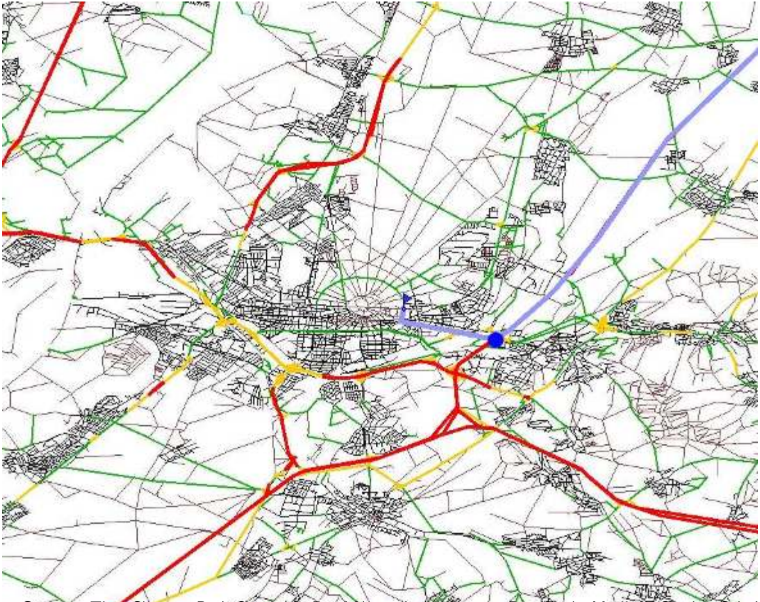

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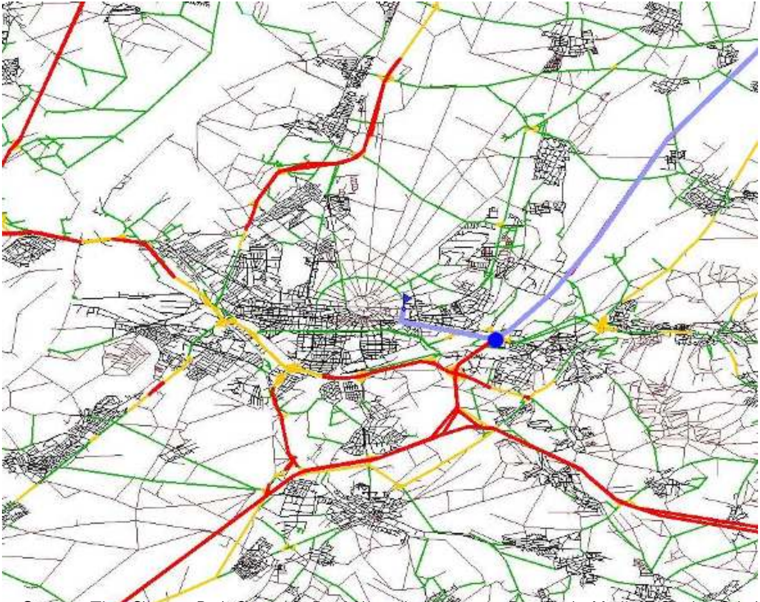

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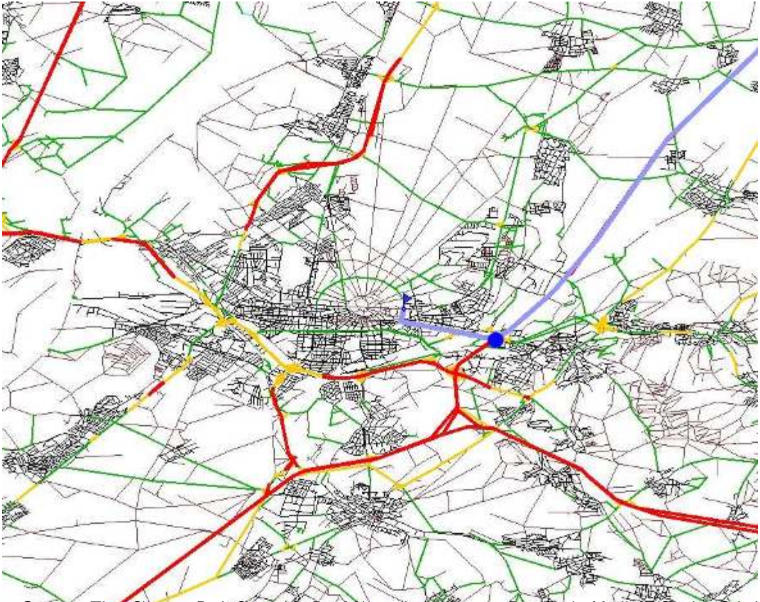

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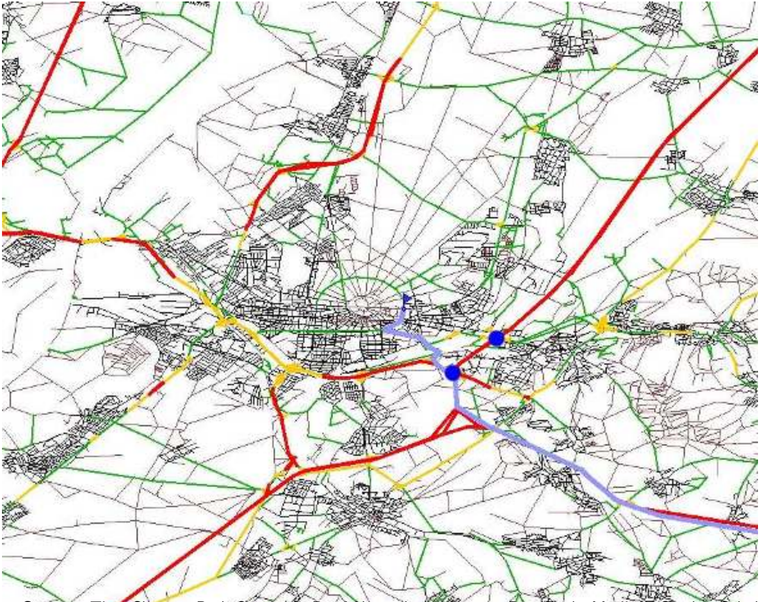

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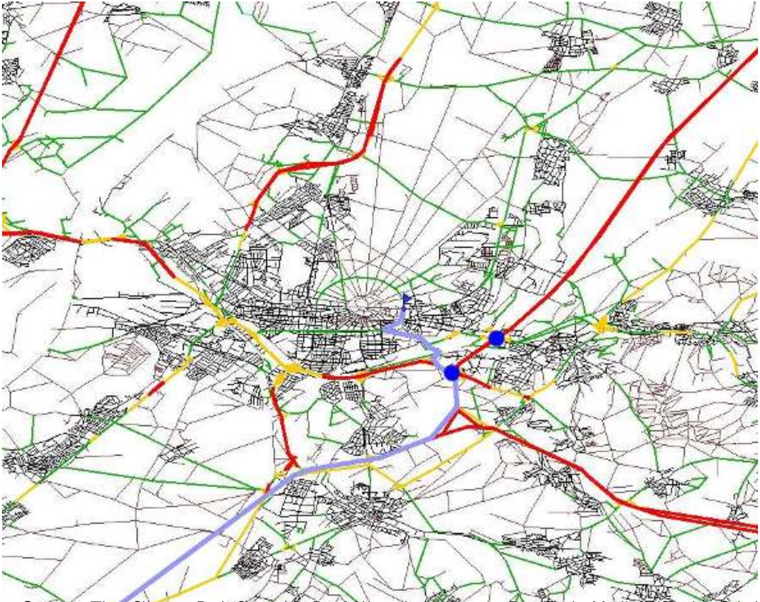

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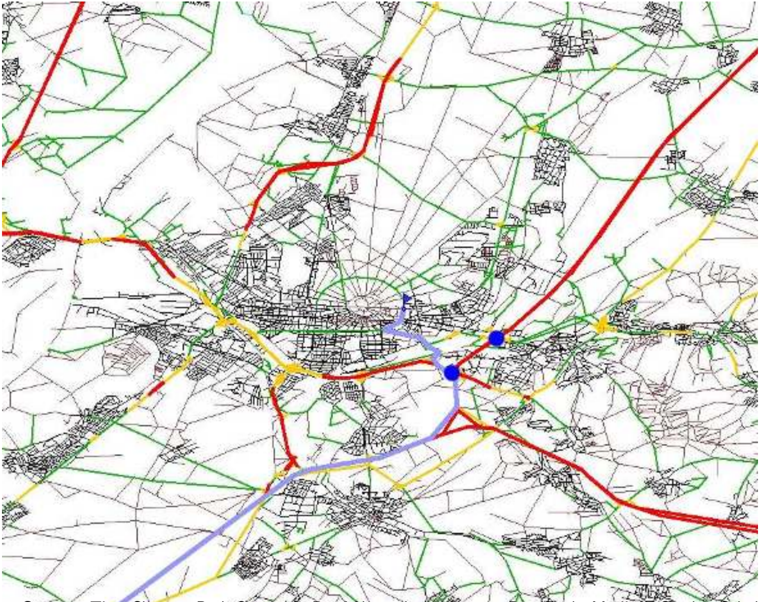

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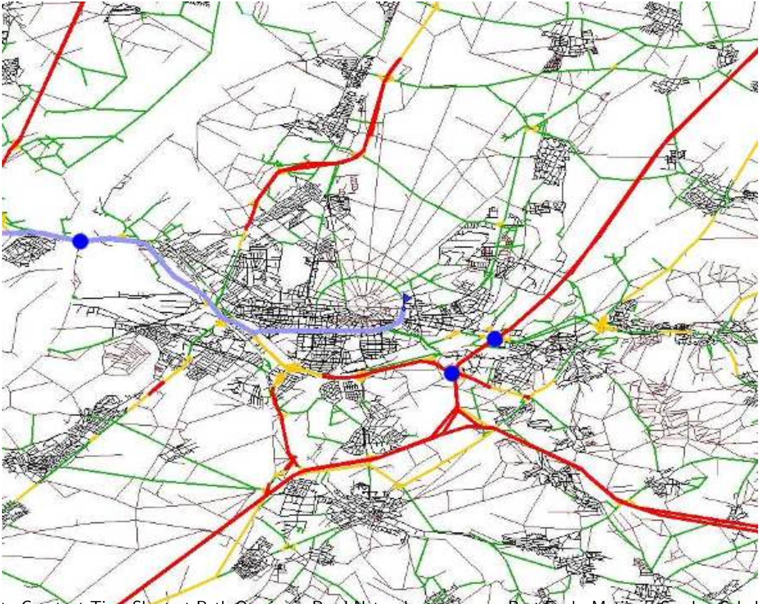

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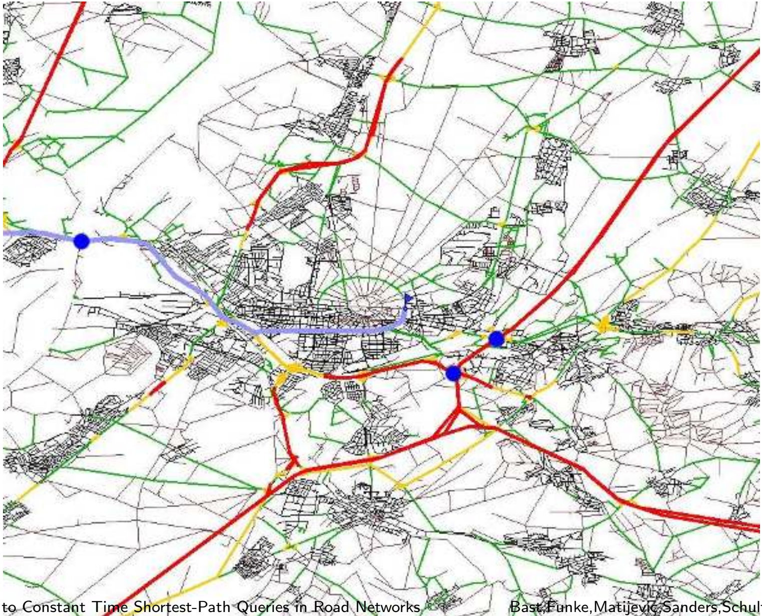

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- reduction of shortest path distance queries to a constant number of table-lookups
- various trade-offs between space and query-/preprocessing times:
  - avg. query times between $5\mu s$ and $63\mu s$ (on the US road map)
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  - a per node space overhead of 21 to 244 bytes

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- $\Rightarrow$ Query times *orders of magnitudes* better than previously reported results

Milliseconds (10^{-3}) vs. Microseconds (10^{-6})!

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What to do with the transit/access nodes ?

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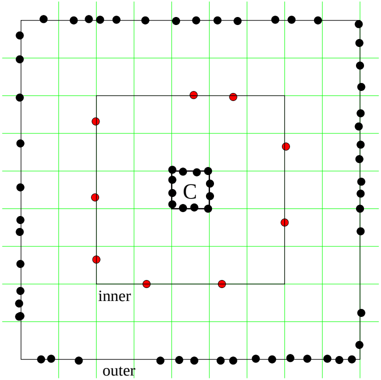
- NO $\rightarrow$ use favourite SP data structure – HH, edge reach ...

Grid-based Implementation

First attempt – adhoc realization of the core idea.

- impose a grid, e.g. 128×128 over the network



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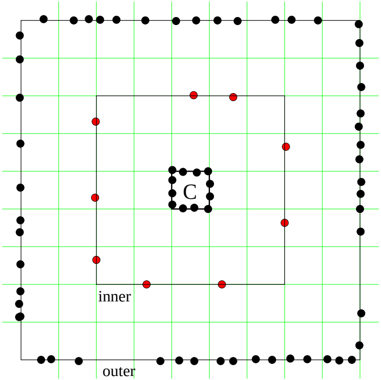

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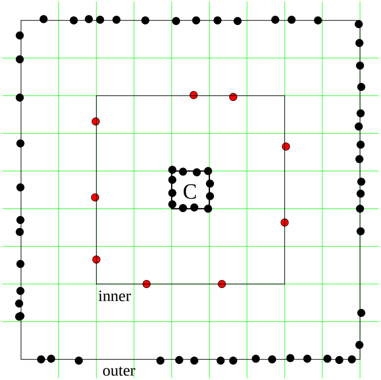

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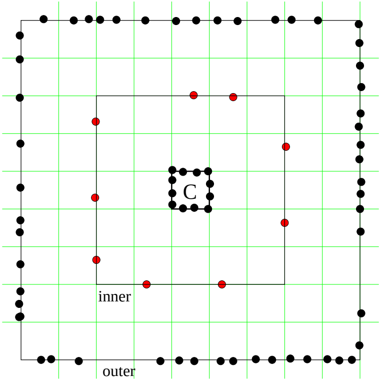

Grid-based Implementation: What are 'long' paths?

Path/Query between source s and target t 'long'/non-local

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s and t at least 4 grid cells (horiz./vert.) apart

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- multi-layered implementation also possible
- can be made very space-efficient

Experiments: US roadmap ($n = 24$ Mio, $m = 58$ Mio)

Grid	$\|\mathcal{T}\|$	$\frac{\|\mathcal{T}\|\times\|\mathcal{T}\|}{\text{node}}$	avg. $\|\mathcal{A}\|$	% 'long' queries	construction of transit nodes
64 × 64	2 042	0.1	11.4	91.7%	498 min
128× 128	**7 426**	**1.1**	**11.4**	**97.4%**	**525 min**
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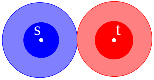
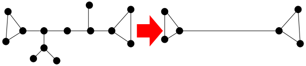
non-local (99%)	local (1%)	all queries	preproc.	space/node
12μs	5112μs	63μs	20h	**21 bytes**

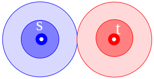
Results for 2-layer data structure.

Highway Hierarchies [Sanders/Schultes ESA'05/'06]



- Gutman's insight with 2nd metric = Dijkstra rank
- complete search within a local area
- identify *highway network* = minimal subgraph that preserves all 'long' shortest paths
- contract network
- iterate ⇒ *highway hierarchy*





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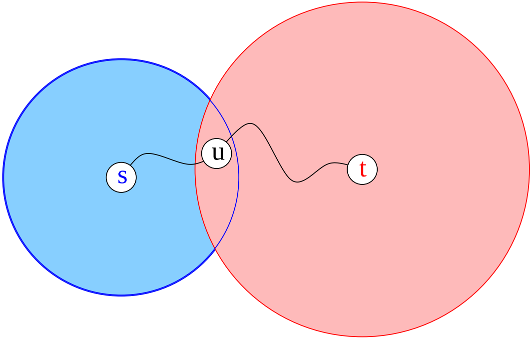
  $\Rightarrow$ high level in HH canonical choice for transit nodes $\mathcal{T}$
- Using several levels from HH induce multi-layer solution in a very natural way
- Local queries can be handled very efficiently by HH ($\Leftrightarrow$ grid-based approach)

HH-based Transit Node Routing: What are 'long' paths?

Compute for each node v a radius $r(v)$ such that a query (s,t) is non-local/the path is considered long, if respective disks with radii $r(s)$ and $r(t)$ do **not** overlap.



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- A more economical version in terms of storage space and preprocessing times uses levels 5 and 3 in a 2-layer structure
- Implemented and benchmarked also for directed graphs, e.g. roadmap of Europe, and *distances* instead of travel times

Experiments: US roadmap ($n = 24$ Mio, $m = 58$ Mio)

Preprocessing:

	variant	layer 1 $\lvert\mathcal{T}\rvert$	layer 1 $\lvert\mathcal{A}\rvert$	layer 2 $\lvert\mathcal{T}_2\rvert$	layer 2 $\lvert\mathcal{A}_2\rvert$	space [B/node]	time [h]
	eco	12 111	6.1	184 379	4.9	111	*0:59*
USA	gen	10 674	5.7	485 410	4.2	244	3:25

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Query:

	variant	layer 1 [%] wrong	layer 1 [%] cont'd	layer 2 [%] wrong	layer 2 [%] cont'd	time
	eco	0.14	1.13	0.0064	0.2780	11.5 μs
USA	gen	0.11	0.80	0.0014	0.0138	*4.9 μs*

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- we presented two ways to exploit this insight:
  - simple, grid-based implementation tuned for space efficiency (63μs avg.query; 20h preprocessing; 21 bytes/node)
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- One of the main challenges: deal with dynamics of real-world networks

Thank you for your attention!