# Incentive Effects of Terminations: Applications to the Credit and Labor Markets

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Firms often respond to low output of a worker not by lowering that worker's wage, but by firing the worker; landlords at times react to poor crops by evicting the sharecropper rather than lowering his share of output; similarly, banks often deny future loans to defaulters rather than raising the interest rate that a defaulter would have to pay. These are all examples of contingency contracts in which bad outcomes lead to a firm-employee, landlord-tenant, or bank-borrower relationship being terminated. This paper addresses three questions: When is it desirable to employ contingency contracts? What does the structure of the equilibrium contingency contract look like? Under what conditions does the equilibrium contingency contract entail the termination of the relationship? We argue that contingency contracts have certain desirable incentive properties. In each of the cases cited where termination occurs, the threat of termination encourages behavior that the principal (employer, landlord, bank) finds desirable while avoiding some of the negative incentive and sorting effects that could result from penalizing agents (employee, tenant, borrower) through changes in the terms of the contract. For example, in the labor market, a lower wage could result either in a decrease in effort by workers, or an increase in quits (particularly by better workers). Similarly,

banks might not penalize defaulters by raising the interest rate they are charged because if they did, their profits might decline.

In our 1981 article, we showed two reasons for the inverse relationship between the interest rate charged borrowers and bank profits. First, raising the interest rate may increase the average riskiness of those applying for loans (the *adverse selection effect*). Bankruptcy bounds the downside risk of a borrower to his collateral: the nature of the loan contract bounds the upside return of a lender to the interest rate. Consequently, among projects with the same expected gross return, riskier projects (in the sense of mean preserving spreads) have higher returns to borrowers and lower returns to lenders. Among the set of borrowers whose projects had the same expected gross return, the borrowers deterred by higher interest rates will be those whose projects were safest; that is, the most desirable customers for the bank. The second reason for bank profits to fall with increases in interest rates is that higher interest rates induce borrowers to switch from safe to risky projects (the *incentive effect*). This switch occurs because the probability of that interest being paid is lower for projects that are more likely to fail. Thus the cost of an increase in the interest rate is lower the higher is the probability of default.

Since a model that was sufficiently general to explain all instances in which a principal responds to bad outcomes by terminating the principal-agent relationship would obscure the unique feature of each of these markets, we focus our attention on understanding why banks refuse to lend to defaulters. In Section VI, we apply the same method of analysis to explain why workers are fired. In both cases, the agents that are terminated are "superior" (in a sense defined below) to the new agents the principal is engaging.

These results hold even if the following restrictions are imposed on the terms of contracts. First, it must be in the interest of one of the parties to enforce every term of the contract: if a contract calls for defaulters to be denied credit, the bank must find denying credit to be profitable; if borrowers who repay loans are assured additional credit, either the bank or the borrower must find those loans desirable. Second, the incentive effects of contracts must take into account the responses of other agents; for example, if other banks lend to borrowers who have been denied credit by their current borrower, the incentive effect of "cutting-off credit" is the fall in the borrowers' welfare from borrowing elsewhere, not the fall in welfare from being excluded from the credit market.

## I. The Model

We initially assume that both borrowers and lenders are risk neutral, that all borrowers are identical,[1] and that borrowers only live for two periods. (As we proceed, it should be clear that our results can be extended to cases in which borrowers live for many periods. In Section IV, we treat the case of risk-averse borrowers.) In each period, borrowers choose a project to invest in. The set of feasible projects may differ in the two periods. We further assume that if a borrower *can* repay his loan, he *must* repay it.[2] The bank offers loan contracts to new customers consisting of:[3,4]

$r_1$ = a first-period interest rate;

$r_2$ = the second-period interest rate the borrower is charged if the first-period loan is repaid;

$r_2^f$ = the second-period interest rate the borrower is charged if he defaults on the first-period loan;

$\gamma^s$ = the probability that the bank offers a second-period loan to a borrower who repays his first-period loan;

$\gamma^f$ = the probability that the bank offers a second-period loan to a borrower who defaulted on his first-period loan.

The vector $C \equiv \{r_1, r_2, r_2^f, \gamma^s, \gamma^f\}$ completely describes a loan contract.

It is easy to show that under a wide range of circumstances, banks will lend to borrowers who repaid their previous loans; thus, to ease the notation, we assume those conditions hold and set $\gamma^s = 1$.

There is one further important set of controls, the omission of which requires some comment: the bank can affect the riskiness of the projects undertaken by altering collateral requirements. (Similarly, it can require a higher equity-debt ratio.)

Since we are concerned with defaults which are costly to the lender, we assume that the size of projects and the wealth of borrowers are such that the amount of the loan always exceeds the collateral. Obviously, if the value of the collateral were equal to or greater than the magnitude of the loan, there would be no risk associated with the loan and the rate of interest associated with all loans would be the same as the rate of U.S. Treasury bills. The assumptions that there is some probabil-

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[1] This enables us to focus exclusively on incentive issues; in our 1981 article, we treated the adverse selection problem for the credit market.

[2] Other models of the credit market such as Daniel Benjamin (1978) have assumed that borrowers will "take the money and run" whenever the size of the loan exceeds the value (to the borrower) of the collateral. In our model we assume that there is an enforcement system precluding that sort of behavior. However, since our principal results hold when there is an excess demand for loanable funds so that borrowers make positive profits on their investment, a strategy of defaulting whenever the value of the loan exceeds the value of collateral is not, in general, profit maximizing for borrowers (except in the last period). See Franklin Allen (1981).

[3] There are three kinds of new customers: first-period borrowers, second-period borrowers who borrowed from some other bank in the previous period, and second-period borrowers who were denied credit in the previous period. We will initially deal with the loan contract offered new borrowers: in Section III we show that in equilibrium, banks will not lend to second-period borrowers who did not borrow from them in the previous period.

[4] We assume that "success" or "failure" of a project is observed, but not the magnitude of the return. See the discussion below.

ity of default, and that banks lose something when borrowers default, seem plausible. For simplicity, we ignore collateral requirements.[5]

The bank cannot directly observe the project chosen by the borrower,[6] but knows how the terms of the contract affect the borrower's behavior (the bank knows the age and experience of every potential borrower).

In each period, potential borrowers apply for credit at the bank offering the contract which maximizes the expected return for that borrower and choose the investment yielding the maximum return for that contract. Banks know this and choose the terms of the contract to maximize their expected profit. (This is a standard indirect control problem.) Our objective is to determine the characteristics of the equilibrium loan contract. In particular, we show that there may exist a Nash equilibrium in which banks do not lend to borrowers who do not repay their first-period loans.

For simplicity, we focus on the case when projects have only two possible outcomes: success with a return $R$ and failure with return zero. The probability of success for a project with return $R$ when it is undertaken in period $i$ is denoted $p_i(R)$. Clearly $p_i'(R) < 0$ since a project with a lower return and lower probability of success than an alternative will never be chosen.[7] Moreover, we assume that $\forall R$, $p_1(R) \le p_2(R)$: older individuals learn from their experiences.[8]

First we *assume* that no banks lend to the defaulters of their competitors and show that there are conditions under which all borrowers who default the first period are denied credit in the second period. We then show that in "equilibrium," contracts may be designed so that defaulters who are denied loans by their current bank would not be able to obtain credit elsewhere. In this section, we focus on the case where, in equilibrium, there is excess demand for credit. In the following section, we consider the case where banks compete for both borrowers and depositors.

*Borrowers*: Borrowers (given the putative strategies of banks, in particular not lending to defaulters of other banks) choose a value of $R$ the first period ($R_1$), a value of $R$ the second period if they are successful the first period ($R_2$), and a value of $R$ the second period if they are unsuccessful the first period, but manage to get a loan ($R_2^f$). These investments are chosen to maximize (the pres-

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[5]This simplification can be justified in several ways. If, for instance, borrowers consume all their wealth in each period, no collateral, other than the assets acquired through the loan, can be provided. Alternatively, under a variety of conditions, increases in collateral requirements may lead to a lowering of the banks' rate of return. In our 1981 article, we showed, for example, that if borrowers had different wealth holdings, but all had the same utility function that was characterized by decreasing absolute risk aversion, then increases in collateral requirements would discourage the most risk-averse borrowers and hence could decrease the expected return of lenders. Hildegard Wette (1983) has shown that increasing collateral requirements may cause adverse selection even if borrowers are risk neutral. The modifications required in our formulae by the introduction of a fixed collateral level are straightforward. In Section V, we analyze the determination of the optimal debt-equity ratio.

[6]If project choices are observed, the loan contract would simply specify the project to be undertaken; there would be no incentive problem. Of course, banks often have some control over the borrower's choice of a project—we are focusing on the residual choices available to borrowers. If a bank had complete control, there would be no distinction between the bank and the borrower; according to present banking laws, the bank would forfeit its claims on the assets of the borrower in case of default.

[7]The analysis would be essentially unchanged were the return in the case of failure positive. Similarly, the analysis may easily be extended to the case where the return is a random variable. Then all that is relevant, for our risk-neutral borrowers, is the mean return if the project does not default (corresponding to $R$ in our model here), and the probability of default. For the lender, all that is relevant is the probability of default and the mean return in the event of a default. In this case, observing a particular level of return may not convey much information (in the case discussed in this text, observing the level of output conveys perfect information about the project chosen by the borrower). The calculations for this case are, however, complicated, because the probability of bankruptcy will in general be a function of the interest rate charged by the bank.

[8]This assumption is made to make our results more interesting. It would be trivial to show that defaulters would be denied credit if their projects were less attractive than those of new applicants. We have given ourselves the more difficult task of proving that result when *defaulters have better projects*.

ent discounted value of) expected profits $\pi$:

$$(1) \quad \max_{\{R_1, R_2, R_2^f\}} \pi = p_1(R_1)[R_1 - (1 + r_1)]$$
$$+ \delta p_1(R_1) p_2(R_2)[R_2 - (1 + r_2)]$$
$$+ \delta \gamma^f [1 - p_1(R_1)] p_2(R_2^f)[R_2^f - (1 + r_2^f)],$$

where $\delta$ is the rate at which future returns are discounted (assumed the same for borrowers and lenders). We now examine how the choice of projects responds to the costs of borrowing. Define

$$(2) \quad \pi_2(r) \equiv \max_R p_2(R)(R - (1 + r)),$$

the expected profits from a second-period loan; and

$$(3) \quad x \equiv (1 + r_1) - \delta(\pi(r_2) - \gamma^f \pi(r_2^f)),$$

where $x$ represents the total cost imposed by the bank on a borrower who repays his first-period debt. The value $1 + r_1$ is the interest rate that is repaid, and $\delta(\pi(r_2) - \gamma^f \pi(r_2^f))$ is the present value of the better borrowing position afforded borrowers who repay. This cost $x$ is the key first-period control variable of the bank and completely determines the borrower's first-period investment, $R_1$.

We can rewrite (1) as

$$(1') \quad \pi = p_1(R_1)[R_1 - x] + \delta \gamma^f \pi_2(r_2^f).$$

Notice that $R_1$ depends on $x$ while $R_2$ and $R_2^f$ depend only on $r_2$ and $r_2^f$, respectively:

$$(4) \quad \partial \pi / \partial R_1 = p_1'(R_1 - x) + p_1 = 0;$$

$$\partial \pi / \partial R_2 = \delta p_1 [p_2'(R_2 - (1 + r_2)) + p_2] = 0;$$

$$\partial \pi / \partial R_2^f$$
$$= \delta \gamma^f [1 - p_1][p_2'(R_2^f - (1 + r_2^f) + p_2^f] = 0;$$

where $p_1$ denotes $p_1(R_1)$, $p_2$ denotes $p_2(R_2)$ and $p_2^f$ denotes $p_2(R_2^f)$; and we are assuming $R_1$, $R_2$, and $R_2^f$ are unbounded from above. In each case, increasing the cost of a loan increases the riskiness of the investment.

Implicitly differentiating (4),

$$(5) \quad dR_1/dx = p_1'/[p_1''(R_1 - x) + 2p_1'] > 0;$$

$$dR_2/dr_2$$
$$= p_2'/[p_2''(R_2 - (1 + r_2)) + 2p_2'] > 0;$$

$$dR_2^f/dr_2^f$$
$$= p_2^{f'}/[p_2^{f''}(R_2^f - (1 + r_2^f)) + 2p_2^{f'}] > 0.$$

The signs of the derivatives are implied by second-order conditions and the fact that $p_1'$, $p_2'$, and $p_2^{f'}$ are negative.

*Lenders*: The bank chooses the terms of the contract to maximize its profit, given the known responses to its terms by borrowers that we have just described. Let the bank's expected profit be denoted by $Y = \nu L$ where $L$ is the number of loans it makes and $\nu$ is its expected profit per loan.

$$(6) \quad \nu = p_1(R_1)(1 + r_1) - \rho^*$$
$$+ \delta p_1(R_1)[p_2(R_2)(1 + r_2) - \rho^*]$$
$$+ \delta \gamma^f(1 - p_1(R_1))[p_2(R_2^f)(1 + r_2^f) - \rho^*],$$

where $\rho^*$ is the bank's cost per dollar loaned. Competition for depositors forces $\rho^*$ to the value where $\nu = 0$. At that value of $\rho^*$, the contract that maximizes the bank's profits also maximizes its profits per borrower.[9] Thus $\rho^*$ is the risk-free rate of interest; our model enables us to treat $\rho^*$ as an endogenous variable.

This result follows immediately from substituting $\nu = 0$ into $\partial Y / \partial z = L \partial \nu / \partial z + \nu \partial L / \partial z$, where $z$ represents any of the choice variables of the bank. Intuitively, if all banks are maximizing their profits per loan, and yet are earning zero profits per loan, any bank

[9]It is important to observe that similar results obtain with alternative formulations of the economic environment of the bank. For instance, if the bank is a monopoly bank in some locality with a fixed number of customers, but can obtain funds from the international market at a cost $\rho^*$, then profit maximization by the bank entails maximizing $\nu$, and in such markets $\nu$ may be positive.

that does not maximize its profits per loan will incur losses.

For the rest of this section we examine the case where each bank offers contract $\hat{C}$ which maximizes $v$, and the resulting interest rate paid depositors is such that there is an excess demand for loanable funds. In other words, borrowers are offered $\hat{C}$; depositors are offered interest rate $\rho(\hat{C}|v=0)$, where $\rho(\hat{C}|v=0)$ represents the interest rate offered depositors when contract $\hat{C}$ is offered and competition for depositors bids up the interest rate paid depositors to the level where $v=0$; and $L^D(\hat{C}) > L^S(\rho(\hat{C}|v=0))$, where $L^S$ and $L^D$ are loan supply and demand, respectively.[10] (To ease the notation we will represent $\rho(C|v=0)$ by $\rho(C)$ when no confusion would result.) We now characterize the contract offered to risk-neutral borrowers. Recalling that $R_2$ is determined solely by $r_2$, define

$$(7a) \qquad S_2(r_2) \equiv p_2 R_2 - \rho^*,$$

the expected social return from a second-period loan, and

$$(7b) \qquad v_2(r_2) = p_2(1+r_2) - \rho^*,$$

the bank's expected return from a second-period loan; let $\{r_1^*, r_2^*, r_2^{f*}, \gamma^{f*}\}$, denote the values that maximize $v$, and let $v^*$ be the corresponding value of $v$.

Let $\hat{r}$ be the value of $r$ that maximizes $v_2(r)$, and let $\tilde{r}$ be the value of $r$ that maximizes $S_2(r)$. It is immediate from the definition of $S_2(r)$ and $\pi_2(r)$ that $1+\tilde{r}_2 = 0$. When $r_2 = -1$, $\partial S_2(r)/\partial R_2 \equiv \partial \pi_2(r)/\partial R_2$ so that a borrower maximizing $\pi_2(r)$ would also be maximizing $S_2(r)$. Therefore second-period borrowers who are charged any positive interest rate will undertake riskier projects than would be socially optimal.[11] Because loans

are only repaid when the project succeeds, investment decisions are distorted in favor of riskier projects which succeed less often, but have larger returns when successful.

We can rewrite (6) using equations (7a), (7b), (2), and (3), as

$$(8) \quad v = p_1 x - \delta\gamma^f p_1 \pi_2(r_2^f) + \delta p_1 S_2(r_2)$$
$$+ \delta\gamma^f(1-p_1) v_2(r_2^f) - \rho^*.$$

Equation (8) expresses bank profits per loan as a function of $x$, $r_2$, $r_2^f$ and $\gamma^f$. The rates $r_2$ and $r_2^f$ must satisfy the enforceability constraints, that is, it must not be in the interests of both bank and borrower to have the rate lowered. Thus $r_2^* \leq \hat{r}_2$ and $r_2^{f*} \leq \hat{r}_2$.

Because $x$ is linear in $r_1$, and $r_1$ is unbounded, the bank's choice of $r_2$, $r_2^f$, and $\gamma^f$ does not restrict its choice of $x$. Recalling that $R_1$ is a function solely of $x$, we can write the first-order conditions for the banks optimization problem as[12]

$$(9a) \qquad p_1 + p_1'(dR_1/dx)$$
$$\left[ x + \delta S_2(r_2) - \delta\gamma^f \pi_2(r_2^f) - \delta\gamma^f v_2(r_2^f) \right] = 0;$$

$$(9b) \qquad \partial v/\partial r_2^f$$
$$= \delta\gamma^f \left[ -p_1 \pi_2'(r_2^f) + (1-p_1) v_2'(r_2^f) \right] \geq 0;$$

$$(9c) \qquad \partial v/\partial r_2 = \delta p_1 S_2'(r_2) \geq 0.$$

The inequalities in (9b) and (9c) hold only if the enforceability constraint is binding on the relevant interest rate. We now see what (9a)–(9c) imply about the structure of the equilibrium contract when borrowers are risk neutral.

### A. Second-Period Interest Rates

Equation (9c) implies $r_2 = \tilde{r}_2$, so that the second-period interest rate charged successful borrowers is the socially optimal one; that is, $1 + r_2^* = 0$, and the enforceability constraint is not binding. (As we show in Section IV, if borrowers are risk averse, then $1 + r_2^* > 0$.)

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[10] The reader may have observed that we have not defined what we mean by equilibrium in this market, or even argued that a steady-state equilibrium exists; nor have we discussed the contract offered by banks if $L^D(\hat{C}) < L^S(\rho(\hat{C}|v=0))$. We keep the presentation of the bank's optimization problem as simple as possible by deferring our discussion of these issues until Section III.

[11] We use the term somewhat loosely to refer to that project which maximizes expected net national product.

[12] In deriving (9a), we made use of the fact that $\pi_2 + v_2 = S_2$.

On the other hand, it is clear from (9b) that, absent the enforceability constraint, borrowers who default in the first period would be charged the interest rate which maximizes $\delta\gamma^f[(1-p_1)v_2(r_2^f)-p_1\pi_2(r_2^f)]$. However, since $\pi_2'(r_2^f) < 0$, the enforceability constraint is always binding. Thus defaulters are charged interest rate $\hat{r}_2$.[13] The enforceability constraint ensures that they cannot be charged a higher interest rate. Up to that point, an increase in the interest rate increases the returns to second-period loans *and* enables the firm to charge a higher first-period rate of interest while, at the same time, ensuring that the firm does not alter the riskiness of the project undertaken the first period. These results have one further implication: defaulters who get loans undertake riskier projects than do successful borrowers. They have higher failure rates, not because they are less competent (by assumption all individuals are identical), but because they have been "punished" by being charged a higher interest rate. It is this higher interest rate which induces them to invest in riskier projects.

## B. *First-Period Interest Rate*

From equations (3), (7a), (7b), (9a), and $r_2^f = \hat{r}_2$, we obtain

$$p_1'(1+r_1)(dR_1/dx)+p_1$$
$$+ \delta(dR_1/dx)\, p_1'\big[v_2(r_2)-\gamma^f v_2(\hat{r}_2)\big]=0.$$

As we have shown, $v_2(r_2) < v_2(r_2^f)$ and $v_2(r_2) < 0$ (provided $\rho^* > 0$, which we will assume). Define $\hat{r}_1$ analogously to $\hat{r}_2$ as the solution to

$$p_1'(1+r_1)(dR_1/dr_1)+p_1=0,$$

and call $1+\hat{r}_1$ the (myopic) first-period bank optimal interest rate. Since the bracketed term is negative for $\gamma^f = 0$ and $\gamma^f = 1$, it is negative for all $\gamma^f$, and $1+r_1^* \ge 1+\hat{r}_1$. The

ability to make the terms of the second-period loan contingent on the outcome of the first-period loan causes the bank to increase the interest rate it charges the first period.

In the one-period model formulated in our 1981 article, the only instrument available to the bank to affect behavior was the interest rate charged in that period. These incentive effects led the bank to charge a lower interest rate than it would if the bank could control the actions of the borrower. Contingency contracts give the bank more instruments through which to control borrowers, thus leading the bank to raise its first-period interest rate.

### C. *Credit Restrictions, the Value of $\gamma^f$*

Note that $v$ is linear in $\gamma^f$, and the range of $x$ is not restricted by $\gamma^f$.[14] Hence

(10)

$$\gamma^{f*}= \begin{cases} 0 \\ 1 \end{cases} \text{ as } p_1\pi_2(r_2^f)-(1-p_1)v_2(r_2^f)\gtrless 0,$$

where $p_1, \pi_2(r_2^f), v_2(r_2^f)$ are evaluated at the values they would take if $r_1 r_2, r_2^f$ were chosen to maximize $v$ subject to $\gamma^f = 1$.[15]

Thus if the enforceability condition is imposed $\gamma^{f*} = 0$ if and only if[16]

(11)     $$p_1 p_2^f\big(R_2^f-(1+\hat{r}_2)\big)$$
$$> (1-p_1)\big[p_2^f(1+\hat{r}_2)-\rho^*\big].$$

We can see the intuition for (10) by looking at the case where $\gamma^f = 1$. Then from equation (3) we can see that eliminating loans to defaulters (setting $\gamma^f = 0$) allows the bank to raise $1+r_1$ by $\delta\pi(r_2^f)$, holding $R_1$ fixed. Since interest rates are only repaid when the project succeeds, that higher interest rate has a value to the bank of $p_1\delta\pi_2(r_2^f)$. This term is

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[13] If we had not imposed the enforceability constraint, the interest rate charged defaulters would equal $\arg\max\{(1-p_1)v_2(r_2)-p_1\pi_2(r_2)\}$. One can substitute that expression everywhere for $r_2^{f*}$ and show that our qualitative results are unchanged.

[14] The result that $v$ is linear in $\gamma^f$, so that $\gamma^{f*}$ is either 0 or 1, depends on the borrowers being risk neutral. In Section IV we show that if borrowers are risk averse, $\gamma^{f*}$ may have an interior solution.

[15] Because $\gamma^f$ is either zero or one, we need only to evaluate $dv/d\gamma^f$ at $\gamma^f = 1$ ($r_2^f$ can take any value when $\gamma^f = 0$, thus preventing (10) from being evaluated at $\gamma^f = 0$).

[16] One can show that (10) will only be an equality by happenstance.

the cost to the bank of lending to defaulters because of the effect those loans have on the first-period interest rate the bank can charge and still elicit the desired actions by the borrower. On the other hand, $(1 - p_1)\delta\nu_2(r_2^f)$ is the expected present value of those loans. Loans are only made to defaulters if the profits they generate outweigh the adverse incentive effects.

Since we are only concerned with showing that the contract that maximizes $\nu$ *may* be characterized by $\gamma^{f*} = 0$, let us look at the case where $\forall R$, $p_2(R) \equiv p_1(R)$. It is somewhat easier to treat the bank's problem as maximizing $\rho^*$, the interest rate paid depositors, rather than $\nu$, its return per loan. Competition for depositors causes the solutions to these maximization problems to be identical.

Let us consider the suboptimal contract $\tilde{C} = \{ r_1 = r_2^f = \hat{r}_2, 1 + r_2 = 0, \gamma^f = 1 \};$[17] then

$$(12) \qquad \rho^* > \rho(\tilde{C}) = \left[ p_2^f(1 + \hat{r}_2) \right.$$

$$\left. + \delta p_2^f(1 + \hat{r}_2) \right] / (1 + \delta).$$

Consequently,

$$(13) \qquad p_2^f(1 + \hat{r}_2) - \rho^* < 0.$$

Since the left-hand side of (11) is positive, we see that if $p_1(R) = p_2(R)$, the bank maximizes its return per borrower by denying loans to defaulters. The intuition is that, if second-period projects are no more desirable than are first-period ones, banks can costlessly increase $p_1$ by denying credit to defaulters. Since this strictly increases bank profits, the policy of restricting credit may still be profit maximizing even if $p_2(R)$ is greater than $p_1(R_1)$ for all $R$, and $\hat{r}_2 > \hat{r}_1$.

### D. *Summary*

*The contract which maximizes the bank's return per dollar loaned takes one of two forms*:

(a) *The bank makes the availability of credit the second period contingent on the individual not defaulting the first period ($\gamma^f = 0$)*;

(b) *The bank makes loans available the second period regardless of performance in the first period. Defaulters are charged a higher interest rate than are borrowers who repaid their first-period loans.*

In both cases, the interest rate the bank charges the first period is higher than the rate it would have charged had it not been able to link the availability and terms of credit the second period to performance the first (i.e. $r_1^* > \hat{r}_1$). Banks deny credit to defaulters if the leverage they gain over first-period projects from cutting off credit enables them to raise the first-period interest rate and increase first-period profits by an amount greater than the present discounted value to the bank of second period loans to defaulters.

## II.  Competition Among Banks

In the preceding section, we assumed that whenever a bank refused to lend to a firm, the firm would be unable to find an alternate source of funds. In this section, we examine whether this is a valid assumption.

There is a simple mechanism by which banks ensure that defaulters won't obtain credit elsewhere. By placing a provision in its first-period loan contract that the first-period loan has seniority over later loans, the bank can lower the expected return to a second-period lender. In particular, even if a bank could profitably lend to unencumbered experienced (second-period) borrowers, it may not choose to make those loans when the borrower has outstanding obligations which must be repaid before the new loan is repaid.[18]

Let us now allow borrowers to declare bankruptcy, eliminating previous debts. There still may exist an equilibrium in which a borrower who was denied a second-period loan is turned down by all other banks. The

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[17]As we have shown, the optimal contract is characterized by $r_1^* > \hat{r}_1$.

[18]This result does not depend on the assumption that borrowers only live for two periods. Even if agents were infinitely lived, subordinated debt, requiring borrowers to repay their debts in the order in which they were incurred could make it unprofitable for any bank, or potential entrant, to lend to borrowers who defaulted on loans to a competing bank.

maximum return on a second-period loan is $v_2(\hat{r}_2)$. In the case where $\gamma^f = 0$, a bank lending to a first-period borrower will be able to pay depositors a return valued at

$$(14) \quad \rho^* = \frac{p_1[1 + r_1 + \delta p_2(1 + r_2)]}{1 + \delta p_1}.$$

If $\rho^* > p_2(1 + \hat{r}_2)$, then no bank lending to defaulters would be able to attract depositors.[19] To show that this inequality can hold without violating the conditions under which $\gamma^f = 0$ is optimal for the bank, assume $r_2 = \hat{r}_2$ (this restriction can only decrease $\rho^*$ relative to the optimal $r_2$). It is sufficient to show conditions exist under which $\gamma^{f*} = 0$ and $v^* > v(\hat{r}_2)$. Clearly this inequality is satisfied if $\forall R$, $p_1(R) = p_2(R)$, since then $p_1(1 + r_1^*) > p_2(1 + \hat{r}_2)$.

Suppose a *myopic* bank would find second-period loans *more* profitable than first-period loans. Even then contingency contracts may be more profitable to banks than are second-period loans. The profitability of first-period loans is due to the additional leverage available from making the contract terms for second-period borrowers a function of whether the first-period loan was repaid. Under those circumstances, competition for deposits eliminates loans to defaulters.[20]

### III. Equilibrium in the Credit Market

Thus far we have shown that the contract that maximizes the bank's steady-state profits may be characterized by defaulters being denied credit. We have, however, avoided discussing whether that contract is a market equilibrium or whether that result holds if banks are competing for both borrowers and depositors. Suppose we now consider the case of $L^D(\hat{C}) < L^S(\rho(\hat{C}|v = 0))$, where it will be recalled that $\hat{C}$ is the contract where maximizers $v$, and $\rho(\hat{C}|v = 0)$ is the return to depositors when contract $\hat{C}$ is offered borrowers and banks earn zero profits. In this section, we show that even if banks are competing for borrowers, the market equilibrium may be characterized by defaulters being denied credit.

To simplify the exposition, we only analyze symmetric equilibria. The bank's equilibrium strategy is denoted by $S^*$ and consists of a loan contract, $C^*$, an interest rate paid depositors, $\rho^*$, and a commitment period for deposits (deposits may be precluded from being withdrawn after one period). We define a strategy $S^*$ to be a symmetric equilibrium if it is offered by all banks and there does not exist a strategy $S \neq S^*$ which attracts both depositors (inequality (15) below) and borrowers (inequality (16) below).[21] That is, there cannot exist a strategy $\tilde{S}$ satisfying:

$$(15) \quad \rho(\tilde{S}) \geq \rho(S^*);$$

and either

$$(16a) \quad \pi(\tilde{C}) \geq \pi(C^*)$$

for borrowers who are getting loans, or

$$(16b) \quad \pi(\tilde{C}) \geq 0$$

for borrowers who are being denied loans, where $\pi(C)$ is the expected profit for borrowers offered contract $C$,[22] and either (15), (16a), or (16b) is a strict inequality.

---

[19] The condition that $\rho^* > p_2(1 + \hat{r}_2)$ is also necessary and sufficient for denying loans to defaulters to be an enforceable provision of the contract. If that inequality holds, banks prefer to lend to inexperienced borrowers; experienced borrowers who repaid debts will enforce the terms of their contract which dictate their loan's renewal; the bank will enforce the terms of the contract which mandates denying credit to defaulters.

[20] The reader should bear in mind that this argument assumes that at the profit-maximizing contract for the bank, there is an excess demand for loanable funds.

[21] This equilibrium notion does not allow an equilibrium to be broken by a contract which attracts all the depositors, and then gains borrowers because no other bank has funds to lend. This equilibrium concept is closely related to the core of a game without side payments. That is, there is no coalition of borrowers and lenders which can form and make every member of the new coalition better off when payoffs are completely determined by the terms of the contracts.

[22] This formulation assumes that depositors are not rationed. This can easily be shown to be always true. Intuitively, rationing depositors does not make sense since rationed depositors would drive down the return $\rho$ to the point where either there is no excess supply or $\rho = 0$ and depositors are indifferent between holding their capital in a bank or at home.

It is immediate that the equilibrium contract earns zero profits for the bank; any non-zero-profit equilibrium could be broken by a bank giving a portion of its profits to depositors (or borrowers).

We restrict ourselves to considering contracts that are both *enforceable* and *credible*. The concept of enforceability was discussed earlier (Section I). For a strategy to be credible, it must have the property that if the contingency contract implies making a certain quantity of future second-period loans to first-period borrowers, the strategy must be such that the required funds will *always* be available. Similarly, a strategy that offers depositors an interest rate $\tilde{\rho}$ over two periods in anticipation of making a certain percentage of new loans to inexperienced borrowers is only credible if those new borrowers are *always* forthcoming. That is, the strategy must not only attract new borrowers given the strategies of the other banks, but must attract new borrowers when confronting any profitable (but not necessarily credible) strategy that could be offered by another bank. These credibility conditions are only satisfied if all borrowers who sign two-period contingency contracts are matched with depositors who have invested a fraction $p_2$ of their funds for two periods.[23,24] By imposing

these credibility requirements, we prevent steady-state equilibria from being broken by a bank that expands, increasing its proportion of loans to inexperienced borrowers and using the higher profits during the expansion to increase the interest rate it pays depositors.

The structure of the equilibrium depends on whether $L^D(\hat{C}) \gtrless L^S(\rho(\hat{C}|\nu = 0))$. If the supply and demand for loanable funds are such that when $\hat{C}$, the contract that maximizes $\rho^*$, is offered by all banks there is an excess demand for loans, then, as we showed, maximizing $\nu$ is equivalent to maximizing $\rho^*$. The results developed in Sections I and II are applicable here: under reasonable conditions, market equilibrium is characterized by $\gamma^f = 0$. However, because

$$L^D(\hat{C}) > L^S(\rho(\hat{C}|\nu = 0)),$$

not only are all defaulters denied credit but some first-period borrowers are also denied credit.[25]

The second case of interest is when banks compete for borrowers: that is, $L^S(\rho(\hat{C}|\nu = 0)) > L^D(\hat{C})$. This equilibrium may also be characterized by defaulters being denied credit. Let $\pi^0$ denote the equilibrium profit level of borrowers; banks compete for borrowers, so the equilibrium contract is the solution to

$$\max \rho(C), \text{ subject to } \pi(C) \geq \pi^0.$$

We form the Lagrangian $H = \rho(C) - \lambda(\pi^0 - \pi(C))$. In equilibrium

$$\partial H / \partial r_1 = \partial \rho(C) / \partial r_1 + \lambda \, d\pi / dr_1 = 0;$$

$\lim_{C \to \hat{C}} \partial \rho(C) / \partial r_1 = 0$, and $\lim_{C \to \hat{C}} d\pi / dr_1 < 0$. Suppose markets clear for contracts near

[23] Notice that long-term borrowers who are not linked to long-term depositors will not obtain second-period loans unless the bank is able to attract new depositors in the second period (its total number of loans is increasing). Those depositors will not be forthcoming if some other bank chooses to pursue a similar expansionary policy beginning one period later. On the other hand, a depositor would not invest for two periods if the "promised" interest rate were predicated on the bank lending to new inexperienced borrowers in the second period. There is no reason for the depositor to believe that those new borrowers will be forthcoming. Consequently, the notion of credible strategies precludes strategies which increase the returns to borrowers or depositors by constructing a Ponzi-type game in which the bank exploits the profitability of loans to new borrowers and constantly covers its losses on the experienced borrowers through new loans. The credibility requirement prevents banks from being forced to rely on new loans (or deposits) in order to honor its commitments to either borrowers or depositors.

[24] Notice that we do not need to impose any restrictions on the lives of depositors, since even if a depositor is only going to live for one period, he can make

multiperiod commitments to a bank; these commitments have the effect of either forcing his heirs to wait before selling those assets or to sell the assets in some secondary market.

[25] For a fuller discussion of first-period rationing, see our 1981 article. Generally there exists a market-clearing interest rate where demand for loans equals supply. But it is not, in general, a market equilibrium. A bank could increase its return by lowering one or more of the interest rates it charges.

$\hat{C}$. Then, since at the equilibrium contract, $\partial H/\partial r_1 = 0$, $\lambda$ will be small. The equilibrium contract $C^*$ is characterized by $\gamma^f = 0$ if

$$(17) \quad \left.\frac{\partial H}{\partial \gamma^f}\right|_{\gamma^f = 0} = \frac{\partial \rho(C)}{\partial \gamma^f} + \frac{\lambda \, d\pi}{d\gamma^f} \le 0.$$

Consider a set of parameter values for which $\partial \rho / \partial \gamma^f < 0$ at $C = \hat{C}$. Then for sufficiently small values of $\lambda$, $\lim_{C \to \hat{C}} \partial H / \partial \gamma^f |_{\gamma^f=0} < 0$. Consequently, if markets clear for contracts near $\hat{C}$, then $\lambda$ is small and the equilibrium contract is characterized by $\gamma^f = 0$.

The intuition is clear. Suppose at $\hat{C}$ there is an excess supply of funds. Banks compete for borrowers by changing the contract to increase $\pi(C)$, the borrower's expected return. They try to do this at minimal cost to depositors. Banks change their contracts in ways such that the increase in $\pi(C)$ is large relative to the fall in $\rho(C)$. However, at $C = \hat{C}$, a decrease in $r_1$ has only a second-order effect on $\rho(C)$ ($r_1$ has an interior solution). Thus $\pi(C)$ could be costlessly increased by decreasing $r_1$. On the other hand, because $\gamma^f$ is at a boundary point, the increase in $\gamma^f$ required to increase $\pi(C)$ would have a first-order effect, decreasing $\rho(C)$.

If $L^S(\rho(\hat{C})) > L^D(\hat{C})$, that is, if there is excess supply of loanable funds at $\hat{C}$, the equilibrium contract may be characterized by $\gamma^f$ taking any value in the interval $[0,1]$: some defaulters may be denied credit even though all first-period borrowers get loans. There may be contingency rationing without arbitrary rationing of first-period borrowers.[26]

FIGURE 1

From the first-order conditions of maximizing $H$, we know that for $\gamma^{f*} > 0$, equilibrium is characterized by $\partial H / \partial \gamma^f = 0$, $\lambda > 0$, and

$$(18) \quad \frac{d\pi/dr_1}{\partial \rho(C)/\partial r_1} = \frac{d\pi/d\gamma^f}{\partial \rho(C)/\partial \gamma^f}.$$

Equation (18) states that if the market equilibrium is at an interior solution, the marginal cost per dollar of marginal benefit (in terms of increased profit by borrowers) will be the same for each instrument.

Some equilibria are illustrated in Figure 1. We have drawn the isoprofit lines of borrowers and banks in $r_1, \gamma^f$ space holding

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[26]Thus far, for the case where $L^S(\rho(\hat{C})) < L^D(\hat{C})$, we have only looked at contracts which generate the maximum values of $\rho^*$. However, in somewhat "perverse" cases, there may also be market-clearing equilibria even if $L^S(\rho(\hat{C}))i < L^D(\hat{C})$. Suppose there exists some contract $\tilde{C} \ne \hat{C}$, such that $L^S(\rho(\tilde{C})) = L^D(\tilde{C})$ and for any $C \ne \tilde{C}$ having the property that $\rho(C) > \rho(\tilde{C})$, it is the case that $\pi(C) < \pi(\tilde{C})$, and if $\pi(C) > \pi(\tilde{C})$, then $\rho(C) < \rho(\tilde{C})$. Then $\tilde{C}$ is an equilibrium since any alternative contract makes either borrowers or depositors worse off. It may be the case that $\tilde{C}$ is characterized by $\gamma^f = 1$, so that defaulters get loans, even when $\hat{C}$ is characterized by $\gamma^f = 0$. In that case,
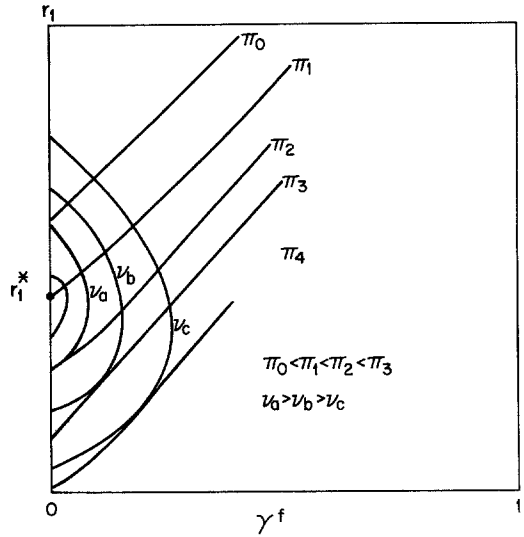
there is both a market-clearing and a non-market-clearing symmetric equilibrium. This situation occurs either because the supply of funds is increased by measures which decrease interest rates paid depositors, or the demand for loans is decreased through policies which increase the profits of borrowers. These somewhat perverse price effects must be large enough to outweigh the direct effect by which increasing $\gamma^f$ increases loan demand. The existence of multiple equilibria when $L^S(\rho(\hat{C})) < L^D(\hat{C})$ is sensitive to our definition of equilibrium. If we required only that the competing contracts attract depositors, the unique symmetric equilibrium when $L^S(\rho(\hat{C})) < L^D(\rho(\hat{C}))$ would be contract $\hat{C}$.

$1 + r_2 = 0$ and $r_2^f = \hat{r}_2$. The upward-sloping curves are isoprofit lines for borrowers, movements down and to the right increase borrowers' profits. In the case we are illustrating, $\nu$ is maximized at $\gamma^f = 0$, and $r_1 = r_1^*$. If the equilibrium return to borrowers corresponds to an isoprofit line that intercepts $r_1$ at $r_1^*$, such as $\pi_1$, then in equilibrium $r_1 = r_1^*$, and $\gamma^f = 0$. This equilibrium could be characterized by an excess demand for loans; however, contracts that lower the profits of borrowers also lower $\nu$, the return to lenders.

If the equilibrium return to borrowers is along an isoprofit line that is greater than $\pi_1$, but not tangent to any of the isorevenue curves of the bank, for example, $\pi_2$, then the equilibrium contract is characterized by $\gamma^f = 0$ and $r_1 < r_1^*$. Banks compete for borrowers by lowering their interest rates.

When the equilibrium isoprofit curves such as $\pi_3$ or $\pi_4$ have interior tangency points with the corresponding bank isoreturn curves $\nu_c$, the equilibrium contracts are characterized by equation (18).

## IV. Risk-Averse Borrowers

The analysis thus far has assumed that both borrowers and lenders are risk neutral. This assumption reasonably approximates the preferences of banks with respect to a single loan, the unit we are concerned with. However, the preferences of borrowers may be significantly distorted by assuming that they are maximizing the present discounted value of their lifetime income. Had we assumed a more general, additively separable, von Neumann-Morgenstern utility function for borrowers, some of the results obtained above would no longer hold.

The equilibrium is again characterized by banks choosing a contingency contract to maximize $\rho(C)$ subject to borrowers choosing $R_1$, $R_2$ and $R_2^f$ to maximize their expected utility $\hat{U}$.

$$(19) \quad \hat{U} = p_1\big[U(R_1 - (1 + r_1)) + \delta p_2 U(R_2$$
$$- (1 + r_2)) - \delta\gamma^f p_2^f U\big(R_2^f - (1 + r_2^f)\big)\big]$$
$$+ \delta\gamma^f p_2^f U\big(R_2^f - (1 + r_2^f)\big),$$

where $U$ is the borrower's utility function in each period. Let $U_1$ be the utility to the borrower of the first-period contract, and let $U_2^S$ and $U_2^f$ be the utility of the second-period contracts offered to borrowers whose first-period contracts succeeded or failed, respectively.

Because borrowers choose $R_1$ to maximize $\hat{U}$,

$$(20) \quad d\hat{U}/dR_1 = p_1'\big[U_1 + \delta p_2 U_2^S$$
$$- \delta\gamma^f p_2^f U_2^f\big] + p_1 U_1' = 0.$$

A particular technique, $R_1$, will be chosen for any pair $\{\gamma^f, r_1\}$ satisfying (20); hence

$$\left.\frac{d\gamma^f}{dr_1}\right|_{R_1} = -\frac{p_1' U_1' + p_1 U_1''}{\delta p_1' U_2^f p_2^f} < 0,$$

$$\left.\frac{d^2\gamma^f}{dr_1^2}\right|_{R_1} = \frac{2 p_1' U_1'' + p_1 U'''}{\delta p_1' p_2^f U_2^f}.$$

If $U''' > 0$, as would be implied by decreasing absolute risk aversion, then, holding $R_1$ fixed, $r_1$ is a concave decreasing function of $\gamma^f$. We now show that $\nu$ may be maximized at a value of $\gamma^f$ between 0 and 1. In Figure 2, we have plotted the values of $\{\gamma^f, r_1\}$ consistent with the firm choosing the (optimal) value of $R_1 = R_1^*$. It is concave from below. Holding $R_1^*$ fixed, it is immediate from equation (8) that the isoprofit lines of the banks in $r_1, \gamma^f$ space are linear. Consequently the maximum value of $\nu$ may be achieved at a value of $\gamma^f$ between 0 and 1.

On the other hand, if borrowers are risk neutral so that $U_1'' = U_1''' = 0$, then the graph of values of $\gamma^f$ and $r_1$ which induce a particular investment decision will be linear. The isoprofit lines of the bank corresponding to that investment are again linear, and unless the slopes of the two lines happen to be identical, $\nu$ is maximized at a boundary point: $\gamma^f = 0$ or $\gamma^f = 1$.

As is apparent from the discussion thus far, the market equilibrium may be characterized by banks randomly rationing credit to borrowers who defaulted in the previous period even if they are simultaneously competing for new borrowers. If the contract $\hat{C}$
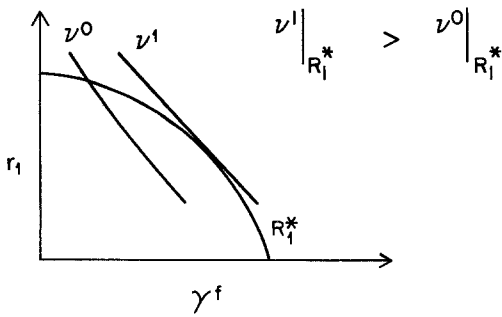
FIGURE 2

which maximizes $\rho(C)$ is characterized by $0 < \gamma^f < 1$ and $L^S > L^D$, banks will lower $r_1$ and raise $\gamma^f$ as they compete for borrowers.

Just as the result that $\gamma^f = 0$ or 1 was sensitive to the assumption of risk-neutral borrowers, so also is the result that $1 + r_2 = 0$. If borrowers are risk averse, small increases in $r_2$ in the neighborhood of $1 + r_2 = 0$ will have relatively small effects on the utility of borrowers. Banks can increase $r_2$ and simultaneously decrease $r_1$, so as to hold $R_1$ fixed. Because borrowers are risk averse, the increase in $r_2$ is large relative to the decrease in $r_1$. Therefore $\nu$ increases. Hence $1 + r_2^* > 0$.

## V. The Equilibrium Debt-Equity Ratio

This paper has been concerned with the problem of incentives. Banks cannot perfectly monitor the actions of the borrowers; they can only tell whether, at the end of each period, projects were successful. Thus, to induce borrowers to undertake safe projects, banks have to rely on two critical terms of the contract: the availability of credit in future periods and the interest rates charged.

Although we have explored many of the consequences of these fixed-fee contingency contracts, we have not explained why, given the obvious difficulties (inefficiencies) they cause, loan contracts are used, rather than profit-sharing contracts. In the basic model presented earlier, since success or failure but not output ($R$) was observed, the contracts we examined were the only feasible ones. Even if output is observable, it may still be desirable to use loan contracts. There is a substantial literature (dating at least back to

Alfred Marshall) showing the incentive problems of revenue-sharing contracts (such as sharecropping or equity finance) which are the alternatives to the contracts we discussed. Since the borrower doesn't gain all the (marginal) returns from additional expenditures of effort (or other inputs), those arrangements elicit too little effort from agents.

In any model where effort plays no role, the equilibrium contract would be a pure equity contract. On the other hand, if the probability of success is not under the control of the agent (he has no choice of technique) but the return, if successful, is a function of his effort, and agents are risk neutral, the equilibrium contract is a pure loan contract. If firm managers face both a choice of effort and a choice of technique which affects the probability of bankruptcy, there is an equilibrium debt-equity ratio representing a balancing of the (marginal) deadweight losses from these two incentive problems.[27] (See our 1980 paper for a derivation of the equilibrium debt-equity ratio in a particular context.) The importance of the absence of moral hazard consideration (both with respect to effort and choice of projects) for the Modigliani-Miller theorem should thus be apparent.

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[27]It is sometimes suggested that the incentive problems would be eliminated by requiring the "agent" to post a bond, or in the case of borrowing, providing collateral. In our 1981 article, we showed why these schemes often do not suffice. Many individuals do not have the collateral (or the funds to purchase the bond) and restricting the provision of funds to those with collateral may not be profitable (or socially efficient): first, they may be ineffectual managers of funds; second, individuals with a large amount of liquid funds may be those who in previous periods undertook risky projects and are likely to undertake risky projects in the future; third, if wealthier borrowers are less risk averse, then increasing collateral requirements increases the average risk preference of the bank's borrowers, and may consequently decrease the bank's profits. For all these reasons, collateral requirements may be set at a sufficiently low level that the bank's losses upon default exceed the collateral, and the bank may ration first-period borrowers. At the same time, it should be noted that since second-period borrowers whose first-period projects failed have less wealth, introducing collateral requirements effectively excludes defaulters from the credit market.

## VI. Firing Workers

In our introduction, we asserted that the method of analysis we would develop could be used to explain the form taken by contingency contracts in a wide variety of principal-agent relationships. In particular, our model can be generalized to provide an understanding not just of why banks cut off credit, but also of why firms fire workers, landlords evict tenant-sharecroppers, and insurance companies refuse to renew policies.

To show how the methodology developed above can be applied to other markets, let us consider a stylized model of the labor market. Assume workers are risk neutral, and work for only two periods; in period $i$ they contribute an amount of effort $e_i$, which affects the probability $p_i(e_i)$ that the project they are working at succeeds. Successful projects have a value of $vQ_i$ where $Q_i$ is the output of the worker and $v$ is the price of each unit of output. We assume that $vQ_i$ is independent of the individual worker or his effort; competition among firms forces $v$ to adjust to the point where each firm earns zero profits. Both workers and firms are risk neutral. As in the credit market example, to make the problem interesting we assume $vQ_2 \geq vQ_1$. The contingency contract firms offer workers consists of:

$\omega_1$ = the wage a newly hired worker receives if his project succeeds;

$\omega_2^s$ = the second-period wage a worker receives if his first- and second-period projects both succeed;

$\omega_2^f$ = the second-period wage if a worker's first-period project failed and his second-period succeeds;

$\gamma$ = the probability of being rehired in the second period if the worker's first-period project failed. (As in the credit market model, it is trivial to show that agents whose projects succeeded in the previous period will not be terminated.)

The firm cannot observe the effort of individuals, only whether the project succeeded. We also assume that firms cannot penalize failure by fining workers (paying a negative wage). Thus, since workers are risk neutral, it is trivial to show that they are paid a zero wage if their project fails. We further simplify the

exposition by assuming a zero discount rate for firms and workers, constant returns to scale in production for the firm, and a single period utility function for individuals of the form $\omega_i - e_i$.

The firm chooses $\{\omega_1, \omega_2^s, \omega_2^f, \gamma\}$ to maximize

$$\pi = p_1(e_1)[vQ_1 - \omega_1]$$
$$+ p_1(e_1)p_2(e_2^s)[vQ_2 - \omega_2^s]$$
$$+ \gamma(1 - p_1(e_1))p_2(e_2^f)[vQ_2 - \omega_2^f],$$

where $e_2^s$ and $e_2^f$, respectively, denote the effort expended in period 2 by a worker whose first-period project succeeded or failed.

A firm maximizes $\pi$ subject to workers choosing $e_1, e_2^s, e_2^f$ to maximize their lifetime expected utility[28]

$$U \equiv p_1(e_1)\omega_1 - e_1 + p_1[p_2(e_2^s)\omega_2^s - e_2^s]$$
$$+ (1 - p_1)\gamma[p_2(e_2^f)\omega_2^f - e_2^f].$$

To recruit workers, it is necessary that $\max U \geq \hat{U}_1 + \hat{U}_2$, where $\hat{U}_i$ is the utility of the wage a worker could receive elsewhere in period $i$.

To retain workers the second period,

$$p(e_2^s)\omega_2^s - e_2^s \geq \hat{U}_2; \quad p(e_2^f)\omega_2^f - e_2^f \geq \hat{U}_2,$$

To ease the exposition we assume that the utility constraints are not binding. Let

$$U_2^s = p_2(e_2^s)\omega_2^s - e_2^s; \quad U_2^f = p_2(e_2^f)\omega_2^f - e_2^f.$$

Then

$$\partial U/\partial e_1 = p_1'(e_1)[\omega_1 + U_2^s - \gamma U_2^f] - 1 = 0.$$

Let $y = \omega_1 + U_2^s - \gamma U_2^f$, and note that $e_1$ is a function solely of $y$. Rewriting $\pi$ we find

$$\pi = p_1(e_1)[-y - e_2^s + \gamma e_2^f + vQ_1 + p_2^s vQ_2]$$
$$+ \gamma p_2(e_2^f)[(1 - p_1(e_1))vQ_2 - \omega_2^f]$$
$$\delta\pi/\delta\gamma = p_1(e_1)e_2^f$$
$$+ [1 - p_1(e_1)]p_2^f(e_2^f)vQ_2 - p_2^f(e_2^f)\omega_2^f;$$

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[28]We have implicitly normalized effort so that one unit changes in income or in effort have the same consequences for utility. None of our results depend on this normalization.

since the range of $y$ is not restricted by $\gamma$, and $\pi$ is linear in $\gamma$, we see that when the utility constraints are not binding $\gamma$ is either zero or one.

$$\gamma = \begin{cases} 0 \\ 1 \end{cases} \text{ as}$$

$$(1 - p_1(e_1))p_2^f(e_2^f)\left[vQ_2^f - \omega_2^f\right] \lessgtr p_1(e_1)U_2^f,$$

where $e_1, e_2^f, \omega_2^f, U_2^f$ are evaluated at the values they would take if $\omega_1, \omega_2^s, \omega_2^f$ were chosen to maximize $\pi$ subject to $\gamma = 1$.[29] The necessary and sufficient condition for $\gamma = 1$ is that the expected profits the firm anticipated from an employee whose previous project failed must exceed the savings in first-period wages (holding first-period effort constant), the firm would gain by firing workers whose projects failed rather than penalizing them through low-wage contracts. The leverage the firm gains is proportional to $U_2^f$, the value workers place on being rehired in the second period, multiplied by $p_1(e_1)$, the probability that the first-period wage is paid. Note that this result is the labor market equivalent of the conditions in the credit market for defaulters to have their loans renewed.

Differentiating $U$ with respect to $e_2^s$ and $e_2^f$, we see that $e_2^s$ is a function solely of $\omega_2^s$ and $e_2^f$ is a function solely of $\omega_2^f$. Let $S_i(e_i) = p_i(e_i)vQ_i - e_i$, the social return to effort $e_i$ in period $i$, and let $\pi_i = p_i(e_i)[vQ_i - \omega_i]$ the expected profit to the firm in period $i$ from paying wage $\omega_i$ and inducing effort $e_i$. Then we can see that when the utility constraints are not binding the firm chooses $\omega_2^s$ and $\omega_2^f$ to satisfy

$$\frac{d\pi}{d\omega_2^s} = p_1(e_1^s)S_2'(e_2^s)\frac{de_2^s}{d\omega_2^s} = 0, \text{ or}$$

$$S_2'(e_2^s) = 0,$$

$$\frac{d\pi}{d\omega_2^f} = \gamma\frac{d\pi_2}{d\omega_2^f} - p_1(e_1)vQ_2\frac{\partial p_2^f(e_2^f)}{\partial e_2^f}\frac{de_2^f}{d\omega_2^f} = 0$$

These first-order conditions imply that if the first-period project succeeds, the second-

[29] $\partial\pi/\partial\gamma$ cannot be evaluated at $\gamma = 0$; any value of $\omega_2^f$ is consistent with profit maximization when $\gamma = 0$.

period contract will be chosen to induce the socially optimal level of effort. Note that this result is analogous to the result in the credit market where the socially optimal investment is chosen by borrowers whose first-period project succeeded. If the first-period project failed and $\pi_2$ is concave in $\omega_2^f$, the wage offered will be below $\hat{\omega}_2$, the wage which maximizes the firm's second-period profits. Of course, if we impose an enforceability condition, the wage would be equal to $\hat{\omega}_2$. The intuition is that the firm gains additional profits by punishing poor performance through its effect on effort in the previous period. Even if the enforceability condition is imposed, $\omega_2^s > \omega_2^f$: a favorable history is rewarded with a better contract.

By extending the analysis to a complete market equilibrium, all the results derived from the credit market in Section III continue to hold in the appropriately modified forms. The market equilibrium may be characterized by $\gamma$ taking any value between zero and one, so equilibrium may be characterized by some, all, or none of the poor performers being fired, and $\omega_2^s > \omega_2^f$ so that poor performers who are retained receive a worse contract than do good performers who are retained.

## VII. Conclusion

We have established that in markets with important moral hazard problems, competitive behavior, in general, features intertemporal linkages.[30] Contracts in those markets may include contingencies under which a principal-agent relationship is terminated, even when principals are competing for agents, and the return functions for all terminated agents stochastically dominates

[30] We use the term competitive in the sense that firms are small and have no monopoly power; it should be obvious that we are not using it to refer to equilibria where no agent can charge a price other than the market clearing price. In that case, it is trivial that if an equilibrium exists, it is market clearing. In our model, there are market-clearing prices (interest rates), but we have shown that if those rates are too high, any bank would increase its profits per borrower by lowering them. Since any bank would thus have an incentive to deviate from the market-clearing prices (interest rates) it seems unreasonable to describe those prices as being determined by competitive forces.

the return function of newly engaged agents. That is, experienced borrowers are denied credit, workers are fired, and sharecroppers are evicted, even though in each case the expected performance of the terminated borrower, worker, or sharecropper exceeds that of his replacement. In addition, each term of the contingency contract would voluntarily be enforced by either the principal or the agent. Moreover, even though terminated agents are now worse off than those who are retained, no other principal will offer them a contract.

Although this paper has focused on the case where the intertemporal linkage of contracts takes place by denying loans to defaulters or firing workers, we also observed that the linkage may take the form of making prices (rents, wages) contingent on previous behavior. Linkage takes the form of terminating the principal-agent relationship only when either:

(a) When contract terms which make the agent worse off also make the principal worse off. For example, if at lower wages workers work less or if the better workers are more likely to quit if the wages are lowered, then cutting the wages of workers whose output is low may be less profitable than firing those workers.

(b) Fees are constrained by legislation such as minimum wage laws or usury laws, or

(c) Agents whose returns are low (who fail) in early periods are more likely to have low returns (fail) in later periods.

If, as in our model, experienced, but terminated, agents are more capable than newly contracted agents, it appears likely that a government policy which forbids terminations could make everyone better off. In our 1980 paper, we constructed a sequence of government interventions with that property. We show that although the market equilibrium is Pareto inefficient, no bank or coalition of banks can effect a Pareto improvement.[31]

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[31]The sequence of interventions in our 1980 paper are first usury laws, then a transitory restriction on the proportion of loans to new borrowers, and finally a prohibition on banks denying credit to defaulters. Each

Our results on the incentive effects of terminations were obtained in a model in which all agents are identical. The arguments would be strengthened if we took into account differences among agents. Then having one's credit cut off (being fired) may convey information about one's desirability as a borrower (worker). As Bruce Greenwald has shown, one can extend George Akerlof's argument showing that used car markets may be thin or nonexistent, to "used intertemporal labor markets"—or, equivalently, to "used borrowers."

The interlinkage of contracts suggests a critical distinction between *ex ante* and *ex post* competition: before contracts are signed, banks (employers) compete actively for customers (workers); after the contract has been signed, borrowers (workers) who feel dissatisfied cannot costlessly move to another bank (employer). There is only limited *ex post* competition. This limited competition is essential for the intertemporal linkage of contracts and serves to enforce long-term commitments. The full implication of limited competition in a richer model of markets with heterogeneous principals and agents who have incomplete knowledge of the characteristics of other participants remains an important question for future research.

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of these interventions is accompanied by taxes and transfer which ensure that neither borrowers nor depositors are being made worse off. It is necessary to order the interventions to ensure that no cohort of borrowers or depositors is made worse off during the transition to a new steady-state equilibrium.

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