JOURNAL OF THE EXPERIMENTAL ANALYSIS OF BEHAVIOR 1982, 38, 217-232 NUMBER 2 (SEPTEMBER)

INCENTIVE THEORY: II. MODELS FOR CHOICE

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Incentive theory is extended to account for concurrent chained schedules of reinforcement. The basic model consists of additive contributions from the primary and secondary effects of reinforcers, which serve to direct the behavior activated by reinforcement. The activation is proportional to the rate of reinforcement and interacts multiplicatively with the directive effects. The two free parameters are q, the slope of the delay of reinforcement gradient, whose value is constant across many experiments, and b, a bias parameter. The model is shown to provide an excellent description of all results from studies that have varied the terminal-link schedules, and of many of the results from studies that have varied initial-link schedules. The model is extended to diverse modifications of the terminal links, such as varied amount of reinforcement, varied signaling of the terminal-link schedules, and segmentation of the terminal-link schedules. It is demonstrated that incentive theory provides an accurate and integrated account of many of the phenomena of choice.

Key words: concurrent chained schedules, mathematical models of choice

In an earlier article (Killeen, 1982), I outlined a general framework for analyzing motivational phenomena based on the arousing or instigating function of incentives. The present treatment of concurrent schedule performance was developed in that context but may be appreciated independenty of it. My concern here is concurrent schedules that terminate not in reward, but in schedules for reward—that is, concurrent chained schedules.

In the later 1950's, work at the Harvard laboratories, stimulated in part by Wyckoff's (1952) article on observing behavior, focused on procedures for measuring the strength of conditioned reinforcers as a function of the rate of reinforcement they signaled. Herrnstein (Note 1) suggested the concurrent-chains technique as a tool for such measurement and this led to the 1960 dissertation of Autor (see 1969) and articles by Reynolds (1963) and Herrnstein (1964) employing concurrent chained schedules. The popularity of this paradigm now approaches that of older techniques. The *Handbook of Operant Behavior* (Honig & Staddon, 1977) divided the subject of conditioned reinforcement into a section on relative measures of strength generated by concurrent chained and observing schedules (Fantino, 1977) and a section on absolute measures of strength generated by single chained schedules and schedules of brief stimulus presentation (Gollub, 1977). In his chapter Gollub warned that "[in concurrent schedules] the organism is enmeshed in a complex set of contingencies . . . It is naive to consider concurrent chained schedules and related procedures as a simple technique for preference scaling of the terminal components" (p. 298). The terms "choice" and "preference" do indeed belong to another language game, one originating in the description of complex human behavior. Whereas strong verbal control of such behavior in humans may emancipate it from complete control by immediate contingencies, we expect no such overshadowing by covert stimuli in pigeons and rats. But even if the convenient terms "choice" and "preference" paint an oversimplified picture of what is measured in the concurrent chained paradigm, we need not infer that the laws of such behavior must be complicated. The data from such procedures have been quite orderly, and I propose in this article simple mathematical models that seem to account for much of the variance in them.

Most analyses of concurrent chained schedules treat the conditioned reinforcers (the ter-

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Some of the early work on these models was conducted under the aegis of grant BMS74-23566 from the National Science Foundation. I thank Rick Shull, Tony Nevin, and Dick Herrnstein for their comments on the manuscript. Reprints are available from Peter R. Killeen at the Department of Psychology, Arizona State University, Tempe, Arizona 85287.

minal-link stimuli) as conveying the total impact of primary reinforcement on behavior. Herrnstein (1964) concluded that "the relative rate at which each pigeon pecked to obtain a secondary reinforcer equalled the relative rate of primary reinforcement in its presence" (p. 27). Fantino (1977) called the implication of such reports the "reinforcement density hypothesis" but had earlier argued that "the relative rate of reinforcement should not generally be found by dividing the number of reinforcements by the time spent in the presence of the S[timulus]" (Fantino, 1967, p. 42). Research over the next decade sought other ways of calculating density that would yield general accounts of choice behavior. None succeeded generally.

The present account holds that behavior in the initial links of concurrent chained schedules is reinforced both by the direct (although delayed) effects of the primary reinforcer and by the immediate (although diluted) effects of the conditioned reinforcers (the terminal-link cues). The conditioned reinforcers do not usurp the power of the primary reinforcer; they abet it. This assumption may seem to violate intuitions about the "conservation of energy" of reinforcers: From where do the conditioned reinforcers derive their strength, if not from the primary reinforcer? But if the primary reinforcer gives up some of its impact to the conditioned reinforcers, is it not itself thereby diminished? These intuitions are reinforced by associative theories in which the total associative strength is assumed constant (Deluty, 1977; Donahoe, 1977; Rescorla & Wagner, 1972). However, it may be argued that conditioned reinforcing effects are always present; it is not that they are something added in the concurrent-chains paradigm, but that they are ignored in other paradigms. In situations where the experimenter does not schedule conditioned reinforcers, background cues may become conditioned and support diffuse activities that are not usually measured (Gibbon & Balsam, 1981; Tomie, Rohr-Stafford, & Schwam, 1981). I view conditioned reinforcers as stimuli that may be used to focus some of the impact of a primary reinforcer on a target response. If such stimuli are not explicitly scheduled to enhance measured responses, they will affect unmeasured ones; the term for conditioned reinforcers might then disappear from our models, but the control they exerted will not necessarily revert to the primary effects of reinforcers. Thus the dual effects of reinforcers asserted by these models is assumed to be ubiquitous and not a special effect of the paradigms studied.

THE MODEL

The Delay of Primary Reinforcement

Hull (1943) interpreted the rapid decay of efficacy that occurs when reinforcement is delayed as a result of the fading of a neural trace. Performance was said to be maintained over delays by the mediation of fractional antedating goal responses, which "backchained" (Denny, 1971) from the eventual reward to the response that had actually precipitated it. But such chains quickly ramify and dilute the impact of any one event. Let us call the probability that a reward will become associated with a response "p." After a delay of one second, there is some chance of another response intervening and being conditioned with a probability q; the probability that the "correct" response will retain its association decreases to $p(1-q)$. After two seconds it is $p(1-q)(1-q)$, and after t seconds it is $p(1-q)^t$. As implied by this model, the opportunity to engage in alternative behavior during the delay greatly steepens the gradient (Salmon & D'Amato, 1981; Spence, 1956; Wilkie, Summers, & Spetch, 1981). Special effects, such as long-delay taste aversion learning, may be accommodated by selection of values for p and q appropriate to those contexts.

If we choose as our units of time not seconds, but some smaller unit (perhaps—to honor Hull's speculation—connected with the refractory period for neural transmission), this geometric series may be approximated by the exponential:

$$P = p\,exp(-qt), \qquad (1)$$

which has the primary effect (P) of a reinforcer decrease exponentially with its delay. Note that this logic does not limit the application of Equation 1 to primary reinforcers: Punishers or conditioned reinforcers whose onset is delayed from a target response will suffer a similar fate. Another way of verbalizing Equation 1 is to say that reinforcement of a target response may be blocked by intervening responses (Williams, 1978) and that this occurs with constant probability during the delay.

The Dilution of Conditioned Reinforcement

Terminal-link cues typically occur immediately after a response, so that their impact on the response should be simply p. However, conditioned reinforcers are not as effective as primary reinforcers. Let us take the strength of the conditioned reinforcer to be equal to the immediacy of the primary reinforcer that it signals and to specify that as simply $1/t$. (If the onset of the conditioned reinforcer is itself delayed, then the fraction is multiplied by the appropriate exponential term, as in Equation 14). The equation for conditioned reinforcement is then $C = p/t$, and, assuming that the primary and conditioned reinforcers are equally associable with the response, the combined directive effect of the schedule (S_d) equals

$$S_d = P+C=p[exp(-qt)+1/t]. \qquad (2)$$

Although the combined effects of primary and conditioned reinforcement focus the target response at the moment of reinforcement, that response is motivated (or "aroused"; Killeen, Hanson, & Osborn, 1978) by the overall rate of reinforcement provided on that key (at least in the case of a prepared, or "terminal," response such as key pecking). We designate this rate as

$$R = 1/(I + T), \qquad (3)$$

where I is the duration of the initial link and T the duration of the terminal link. Where there are large differences in the number of entries into the terminal links, I must be calculated by dividing the total obtained time in the initial links by the number of terminal-link entries for that schedule. For most simple schedules, $T = t$, but when there are multiple reinforcers in the terminal link or when its offset is extended beyond the end of reinforcement, their values will differ. Equation 3 is an approximation to a more exact expression

$$\mathrm{R} = \lambda/\{1-exp[-\lambda(I+T)]\}, \qquad (4)$$

derived from an exponentially-weighted moving average of the reinforcement rate (Killeen, 1982). The parameter lambda is the rate constant for the update of the average (or, equivalently, for the decay of arousal). Equation 3 is accurate only for small and moderate values of $I + T$, say, less than 3 or 4 minutes, for typical values of lambda (~.002/sec). Although it is not likely that lambda remains invariant over all scheduling conditions, it was possible to assume that it remained small so that Equation 3 could be used in the current analysis. We take the "strength" of a schedule to be the combination of these three factors:

$$S = R\,(P+C) \qquad (5)$$

or, explicitly

$$S = p[exp(-qt)+1/t]/(I+T). \qquad (6)$$

Here the notion of reinforcement strength is extended to schedules, to indicate their effectiveness in eliciting responses and directing them to an operandum.

The multiplicative relation between motivation and direction is required by the data, but it is also consistent with the logic of those constructs. Even highly motivated organisms will not emit many measured responses if those are not focused on the operandum by the primary and conditioned reinforcers. Thus we are led to a model in which the directive effects of reinforcement, including "sign-tracking" effects, summate, and this sum (S_d) is modulated multiplicatively by the motivational factors (here simply R, but in a more general model including satiation, biorhythms, and other factors). Note also the separate accounting of R for each of the concurrent schedules, suggesting that R represents not a diffuse organismic activation but one that is itself under stimulus control.

The relative rate of responding in the initial link is equal to the relative strength of the chained schedules:

$$B_1/(B_1 + B_2) = S_1/(S_1 + S_2). \qquad (7)$$

This model has one free parameter, q, which measures the steepness of the delay gradient. But such parsimony requires equipotent association to the two responses. Where there is substantial bias, values of p will not cancel, and their ratio must be retained as a second parameter multiplying S_1 and hereafter labeled *"b."*

THE PREDICTIONS

Terminal-Link Schedules

I applied Equations 6 and 7 to all of the published studies that yielded more than three

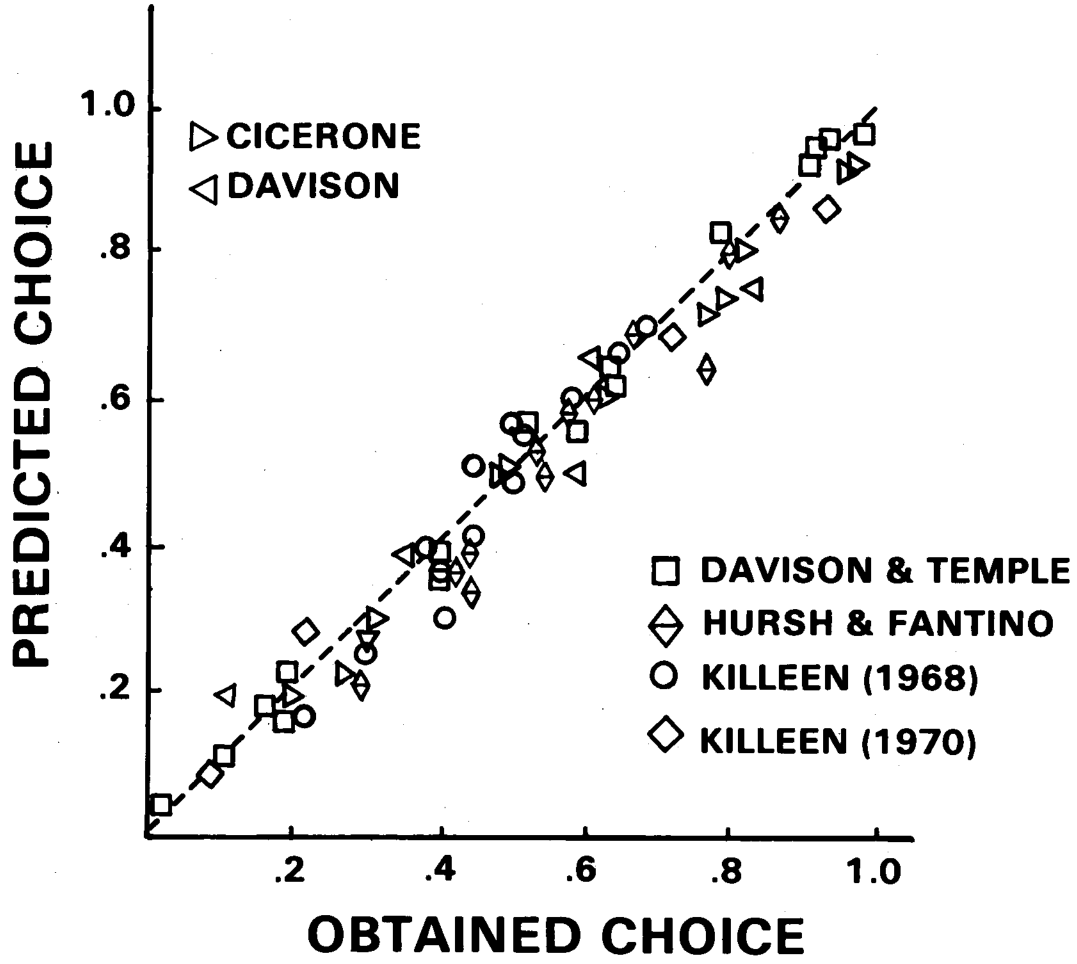

Fig. 1. Predicted versus obtained relative rates of responding in the initial links of the concurrent chained schedules from the studies listed. The parameter q was set at .125 for all of the studies; there was little key-bias, so that the parameter b was fixed at 1. (After Killeen, 1982; reprinted with permission of the University of Nebraska Press.)

data points, averaged over subjects. Figure 1 shows the results. A single value sufficed for q in these studies, $q = .125/\text{sec}$. Note that the data come from studies comparing fixed-interval (FI) schedules in the terminal links (Davison, 1969; Killeen, 1970); fixed- and variable-interval (VI) schedules (Cicerone, 1976; Davison & Temple, 1973; Hursh & Fantino, 1973; Killeen, 1968), and variable-interval schedules (Killeen, 1968). In the past, investigators coped with results from these studies by linear, logarithmic, square, and cubic transformations, but here a single formulation suffices.

In predicting the strength of variable-interval schedules, S_d is evaluated for each of the intervals, averaged, and then divided by the average duration of that chain $(I+T)$. Simulation has shown that Equation 8 provides a good approximation to the obtained values of S_d:

$$S_d = k/(vq+1), \tag{8}$$

where v is the mean of the VI schedule, q is the decay constant, and k depends on the nature of the distribution, ranging around 1.0 for rectangular distributions and 1.5 for constant-probability distributions. (Because this parameter will cancel out of a relative measure, its precise value is unimportant).

The subjects in several studies evinced a bias for one of the keys, and the bias parameter b was introduced for them. The fit of the model to those data using the two parameters is demonstrated in Figure 2. Wherever one of

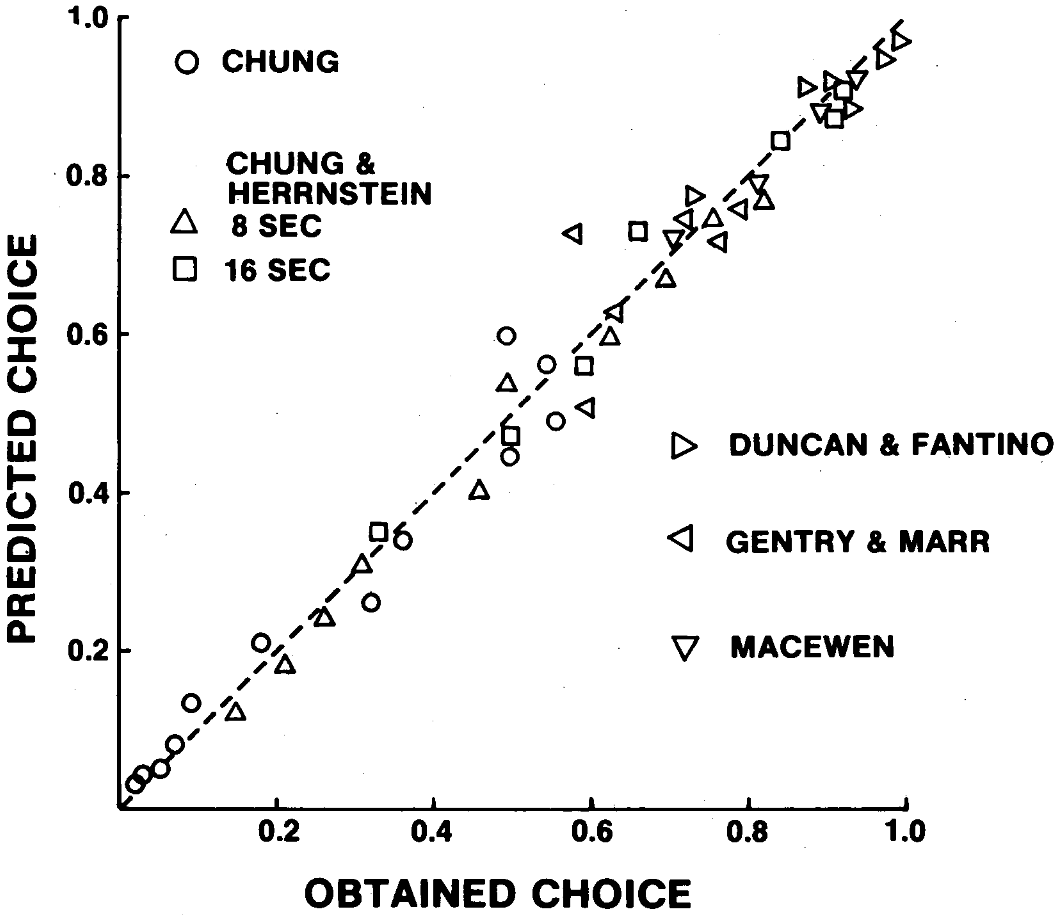

Fig. 2. Predicted versus obtained relative rates of responding in the initial links of the concurrent chained schedules from the studies listed; the parameters *q* and *b* are: Chung, 1965 (.15,1.3); Chung and Herrnstein, 1967 (8"Std: .10,1.2; 16"Std: .10,1.8); Duncan and Fantino, 1970 (.125,2.4); Gentry and Marr, 1980 (.25,.33); MacEwen, 1972 (.08, 1.5).

the terminal links was nominally a zero-second delay, as in Chung (1965), a value of one-half second was assumed as the actual delay. As in the previous figure, the fit of predictions to data is excellent.

Some of the error variance and parameter variability in these figures is due to hysteresis and bias, to which concurrent chained schedules seem especially susceptible (Marcatillio & Richards, 1981; Williams & Fantino, 1978). There is no simple solution to this problem; randomization across conditions and subjects will remove systematic bias, but when such data are averaged, they will regress toward indifference; this regression is maximum when the conditions are strictly alternated. Simple models of learning may be fit to the data to "covary out" hysteresis (e.g., Davison & Hunter, 1979), but this adds complexity and makes it more difficult to test the asymptotic models. Between-groups designs may be employed, but this substitutes sampling error for sequential biases. Accuracy of data may be increased by several tactics but only at the cost of resources that may be better spent testing the extension of the models in new contexts. For the current models, variation of the parameters will accommodate some of these types of bias but at the cost of decreasing the cross-experiment constancy of their values. Where the preferred options are alternated from one key to the other, as is often the case in research by Davison, Fantino, Gentry, and Marr, values for *q* tend to be higher than .125; where the preferred options are kept on the same key, as is the case for MacEwen, values for *q* tend to be lower than .125. In general, I have kept *q* constant at .125 and *b* at 1.0, unless other values would increase the percentage of variance the model accounts for by more than 2%.

The above figures, incorporating the results of some dozen experiments, constitute the major application of the model. In the remainder of the paper, I interpret the rendering of the parameters and attempt to apply the model and its offspring to diverse other experiments. It is such extensions that force clarification of the mappings among experimental paradigms, models, and data, and which thus provide the richest source of theory development. Although the models are often spoken of as "predicting" the data, the shape of the models was in fact determined by the nature of the data to which they were applied.

Separation of Primary and Conditioned Reinforcement

Figure 2 includes data from Chung and Herrnstein (1967), where the terminal links were both signaled by blackout, not by distinctive stimuli. At first glance, it seems that we should set $C = 0$ in applying the model. This tactic may not be appropriate, however, because the blackouts must have had some status as conditioned reinforcers; they signaled that the animals had entered the terminal links and had thus moved closer to primary reinforcement. In fact, in conjunction with the position of the animals in front of the left or right key at the moment of transition, they gave complete information. And the model works best for Chung and Herrnstein's data if we presume complete conditioned reinforcement.

If we confuse the animals by signaling entry into the terminal link with keylights that are randomly associated with each of the terminal schedules (Williams & Fantino, 1978), preferences decrease toward indifference. In Chung's (1965) experiment, blackouts that did not terminate in reinforcement were randomly imposed on the nondelay key. For these data, the model with $C = 0$ works just as well as the original model. Killeen and Johnson-Haight (Note 2) used a change-over key technique (Findley, 1958) to minimize postural information and gave pigeons a choice between an 8-sec and a 16-sec delay-of-reinforcement. The three pigeons showed an average preference of 76% for the shorter delay when the terminal links were distinctively signaled and 78% when the chamber was blacked-out during both terminal links. The model predicts a preference of 74% in the first case and 75% in the second; the predictions lack power because the conditioned-reinforcement function is quite similar to the primary-reinforcement function, and for these short intervals will not make a distinctive contribution to relative measures of strength. In the last condition of the experiment, Killeen and Johnson-Haight added the cue for the longer schedule only and found a decrease in preference for the shorter, unsignaled schedule of 7%, exactly the amount predicted by the model. Qualitatively similar results were found by Nevin and Mandell (1978). However, the blackout has become (at least potentially) a distinctive cue for the 8-sec schedule, and we should expect preferences to eventually return to the level found when both or neither schedules were distinctively signaled. The experiment was not run long enough to test that implication.

Another technique for separating the effects of primary and conditioned reinforcement involves additional operanda that provide conditioned reinforcers, while primary reinforcers are available via a common operandum. Dinsmoor, Mulvaney, and Jwaideh (1981) employed this observing-response type of paradigm and varied the duration (D) of the conditioned reinforcers available on each of two side keys, while the center key was either a variable-interval or extinction schedule. Since the conditioned reinforcers could signal any number of primary reinforcers occurring during their presence, we predict their strength to be proportional to the sum of the immediacies of primary reinforcers they signal, or approximately $S = \int 1/t = ln(D)$—constant. This prediction accounts for 95% of the data variance, with a value of .93 for the additive constant (this value stipulates that integration begins .4 sec [the antilog of −.93] after the onset of the keylights).

In summary, direct evidence for the separate and additive effect of primary and conditioned reinforcement is positive but meager. Most support comes from the success of this assumption in accounting for the results of concurrent chained experiments, as evidenced in Figures 1 and 2, and for the relative invariance of q across experiments that the assumption permits.

Initial-Link Schedules

Fantino (1969a,b) noted that preference between two terminal-link schedules is not independent of the value of the initial-link schedules but rather moves towards indifference as the initial link is lengthened. He found that preference for a VI 30-sec vs. a VI 90-sec schedule decreased from around 95% when the initial links were VI 40-sec schedules, to around 60% when the initial links were VI 90-sec schedules. The present account also predicts a decrease in preference, due to the factor *R* in Equation 5. However, the predicted decrease is not quite as great as that found by Fantino; even when key biases are accounted for, the average deviation of predictions from data is 7%.

Incentive theory does better with the data of Squires and Fantino (1971) and Davison (1976). Squires and Fantino modified Fantino's model to accommodate initial-link schedules that were of different lengths for each key, causing unequal numbers of entries into the terminal links. The first condition employed equal variable-interval schedules in the terminal links. Because of this our predictions are simple: The values of S_d (Equation 2) cancel, and we may predict performance simply by taking ratios of *R*s (Equation 3). The predictions are shown in Figure 3, alongside the predictions of the Squires and Fantino model. The two are equivalent.

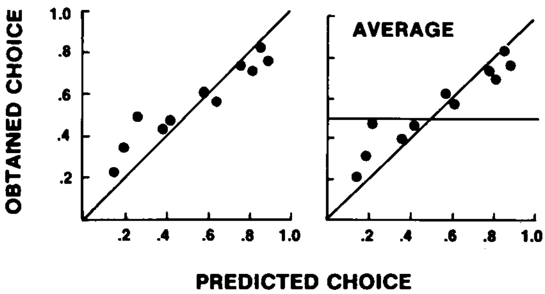

Fig. 3. Obtained versus predicted relative rates of responding in an experiment where the duration of the initial links was varied. Left panel: predictions from incentice theory; right panel: predictions from the delay reduction model of Squires and Fantino (1971). The data and right panel are from Squires and Fantino; reproduced with permission from the Society for the Experimental Analysis of Behavior.

Incentive theory also predicts the major trends in data collected by Davison (1976), who employed a concurrent-chains procedure in which the terminal links were fixed-interval schedules of 5 sec or 15 sec, one initial link was always VI 27-sec and the other was varied over a range of VI 38-sec to VI 181-sec. There was substantial key bias (extrapolation of the 5/5 condition to an initial-link duration of 27 sec for both keys projects a preference of 68%, where indifference should have obtained), and some nonmonotonicities. In all, deviation of the model from the data is on the same order as the variability intrinsic to the data (see Figure 4).

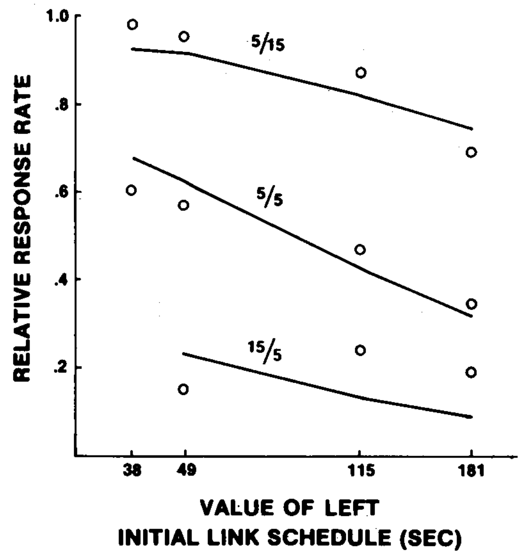

Fig. 4. Relative rate of responding in an experiment where the duration of the right initial link was kept constant at 27 sec and that of the left initial link was varied over the range indicated on the x-axis. The parameters are values of the left and right terminal link FI schedules. The data are from Davison, 1976; values of *q* and *b* were .18 and 2.8.

Wardlaw and Davison (1974) arranged equal VI schedules in the initial links and different FI schedules in the terminal links. Again, they found that preference was more extreme with short initial links. Incentive theory accounts for 90% of the data variance (with $q = .2$ and $b = 1.35$); however, there was some systematic deviation, with predictions being less extreme than they should be for short initial links and more extreme than they should be for long initial links.

Finally, Hursh and Fantino (1974) studied preference between equal mixed and multiple schedules of reinforcement using initial-link schedules of different length. Incentive theory correctly predicts a preference for the multiple schedule (see below). But, because the values of R for the concurrent schedules were always equal, it predicts no difference in the degree of preference as a function of the initial-link duration. A difference was found.

In summary, as currently developed, incentive theory accounts for some, but not all, of the data on initial-link duration.

Duration of Terminal Links

If more than one reinforcer is delivered in the terminal link, each successive reinforcer will strengthen initial-link responding to a lesser extent than the earlier ones: Equation 5 is extended to

$$S = R\Sigma(P+C), \quad (9)$$

where the values of t are calculated from the time of the last initial-link response until the delivery of each reinforcer, and include the time for delivery of intervening reinforcers. Unless the terminal-link schedules are very dense, the marginally increasing strength will be offset by the decreasing overall value of R. This effect is demonstrated most impressively in the first condition of a study by Moore (1979).

Moore's first experiment varied the duration of terminal links in which reinforcement was scheduled according to various VI schedules. In the first condition the schedules were VI 20-sec, and the terminal-link durations were 20 sec and 180 sec. Despite a ninefold advantage in the number of reinforcers in the long-duration component, incentive theory predicts a preference of 54% for the shorter duration; the obtained preference was 49% (cf. Fantino's delay reduction hypothesis, which predicts a preference of 18%). Because the multiple reinforcers were delivered according to variable-interval schedules, it was necessary to simulate the experiment and apply Equations 9 and 7 to each of the outcomes to obtain the prediction, using values of .125 for q and 1.0 for b.

Moore's second experiment permits analytic solutions; the terminal links consisted of one FI 30 schedule, and one FI 30 followed by four FI x schedules. The preference for the multiple feedings is graphed in Figure 5, along with the predictions from incentive theory. The parameter q was set at a value of .125 and b at 1.0. It can be seen that as x increases, preference for the multiple reinforcers decreases—to the point where once again a preference for fewer reinforcers begins to emerge.

These data force a particular construction of the parameter C. It must be calculated not as the sum of the reinforcers in a period divided by the time in that period, but as the sum of the reciprocals of the times to each of the reinforcers in the period. Thus, the strength of a conditioned reinforcer is not a homogenous function of the density of reinforcers that it signals but rather is a unique function of the point in time at which it is initiated (i.e., makes contact with a response) in relation to each of the forthcoming reinforcers. Rearrangement of the timing of the reinforcers within the period will affect the strength of a stimulus even though it leaves the average density of reinforcement that it signals unchanged.

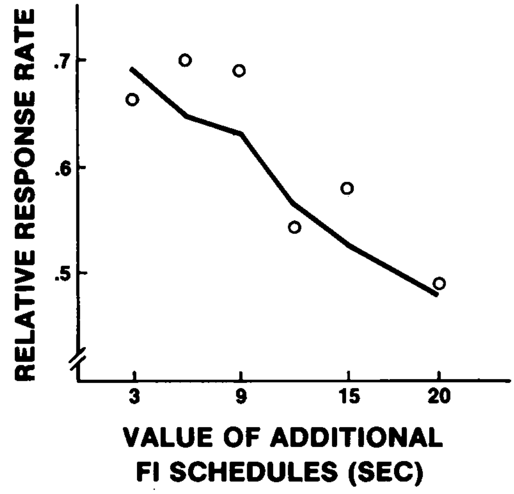

Fig. 5. The effect of delaying additional reinforcers in a terminal link upon the preference for that schedule. After an FI 30 schedule, four additional reinforcers were delivered according to FI schedules whose values are given as the abscissae. The data are from Moore, 1979; values of q and b were .125 and 1.

Squires and Fantino (1971) tested their model by scheduling 2, 5, or 10 additional reinforcers in the terminal links to compensate for the unequal initial links. Because their model is additive and linear, they predict that a terminal link having, say, 10 reinforcers should just balance an initial link 10 times as long as the comparison and yield indifference between the schedules. Equation 9 predicts that additional reinforcers will have marginally decreasing utility that will not offset the overall decreased rate of reinforcement: It predicts preferences of .39, .24, and .15 for the schedule with the additional reinforcers. The obtained preferences were approximately .43, .42, and .30; preference decreased, and although not nearly as steeply as predicted, certainly steeper than the horizontal line at .5 predicted by Squires and Fantino. (I note that the scheduling of conditions in this study was a strict alternation of all subjects from a condition that generated a preference for the right to one that generated a preference for the left, and so on, so that all hysteresis effects would be regressive and would force data toward indifference.)

Shull, Spear, and Bryson (1981) let pigeons switch from a variable-interval schedule into a schedule with a fixed duration of 240 sec, with a varied number of reinforcers available throughout the period. The rank-order correlation between the calculated strength of the schedule using incentive theory and the average rate of switching into it is .96. Shull and his associates found that the sum of the reciprocals of the delays to each reinforcement was also correlated .96 with average switching rate. Their model works well because the reciprocal of the delay is a reasonable approximation to S_d and because the duration of the period was fixed and therefore did not differentially impact R.

This study forces further clarification of the parameters. I take R to be simply 1/240, the duration of the fixed period, with no initial link included in the calculation. I do this because the alternate condition was a "terminal link" in its own right, providing primary reinforcers and competing with the fixed period, rather than constituting part of it and under the control of its reinforcing strength. Furthermore, in another part of this study the fixed period was terminated after varying durations without a reinforcer. The pigeons switched into this condition less frequently if it involved a long period after reinforcement before reverting to the other link. This indicates that such periods were not discounted by the pigeons and should be included in calculating R.

Probabilistic Reinforcement/ Observing Behavior

The previous applications of incentive theory have assumed that every terminal link contained a reinforcer. When this is not the case, the strength of the terminal links will be reduced proportionately. Taking pr as the probability that a terminal link will end in reinforcement:

$$S = pr(P+C)/(I+T). \tag{10}$$

Note, however, that this model is most appropriate when the extinction components are signaled by distinctive cues; when they are not, the conditioned reinforcing strength of the terminal-link cues is further debased. Let us rewrite the above equation for the case of "mixed" schedules in the terminal links.

$$S = pr[P+(1-U)C]/(I+T). \tag{11}$$

The function $(1-U)$ denotes the impact of a reduction in informativeness of the terminal-link cues in these situations. Since the existing data only weakly constrain the form of U, I employ the traditional information metric (Garner, 1962):

$$U = -\Sigma p \log p, \tag{12}$$

where p is the relative frequency with which a stimulus is associated with a particular outcome, and log p is the logarithm to the base 2 of p. The expression measures the uncertainty of an event of probability p, and $(1-U)$ measures the information provided by a signal of that reliability. Equation 12 predicts zero strength for stimuli that signal food or extinction with equal likelihood (for $p = .5$, $[1 - U] = 0$) and has strength grow to maximum as the informativeness approaches certainty (as $p \rightarrow 0$ or 1, $[1-U] \rightarrow 1$). Of course, even a completely ambiguous terminal-link cue tells the animal that it is in the terminal rather than the initial link, and may therefore be somewhat reinforcing. This potential source of strength is not captured in the model and may be a source of some of the error variance, although the effect should be operative for both schedules and therefore have reduced impact on a relative measure of behavior.

I have applied Equations 10 and 11 to several sets of data. The first is that of Kendall (1975), who gave pigeons a choice between two terminal-link schedules of equal durations, with $pr = .50$. The independent variable was the duration of the terminal links, being either 1, 3, 7, or 15 sec. The dependent variable was the relative response rate for the terminal link that had distinctive stimuli signaling the reinforced components (i.e., the terminal link with the multiple schedule for which p=1, as opposed to the mixed schedule for which p=.5). The predicted (and obtained) preferences for the informative schedule as a function of increasing terminal-link duration were: .64 (.60), .66 (.71), .87 (.85), 1.0 (.93). However, q took on the unusually high value of .6, suggesting either an exceptionally rapid decay of the primary effects of reinforcement or the inappropriateness of the model as construed for these data.

The second experiment is that of Green and Rachlin (1977) who kept the duration of the terminal links constant at 30 sec and varied the value of pr. Their data are displayed in Figure 6. Note that for pr=p=1.0 and in other control conditions the pigeons were not indifferent but displayed a preference of .62, requiring a value of 1.67 for b. Equation 11 with q=.125 and with p=1 for the multiple schedule and p=pr for the mixed schedule generated the solid line in that figure.

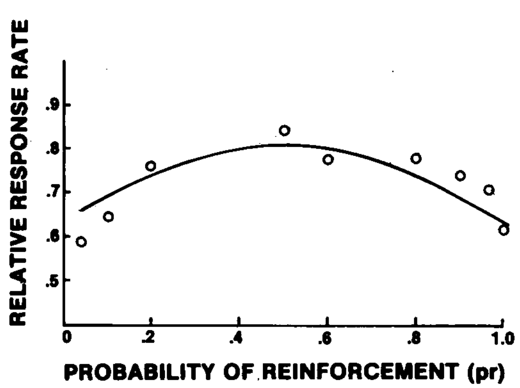

Fig. 6. Pigeons' preference for the more informative terminal link as a function of the probability of reinforcement. The data are from Green and Rachlin, 1977; values of q and b were .125 and 1.67.

The third data set comes from Green (1980), who kept the terminal links constant at 20 sec, with pr=.5, and tested the preference of pigeons for multiple schedules whose cues varied in accuracy. In the first study he compared various values of p on one key with p=.5 on the other. A p of .5 means that half the time a cue was associated with food and half the time with extinction—it is noninformative; a p of .85 means that 85% of the time the cue will be associated with one of the outcomes in particular. In the second study he compared a p=.85 with a range of values. A value of .19 for q generated the predictions shown in Table 1, which account for 90% of the data variance.

Table 1

Data from Green (1980), who varied the reliability (p) of cues on two response keys. The predictions come from Equation 11 with parameter values of q = .19 and b = 1.0.

p		*Preference*	
Left	*Right*	*Obtained*	*Predicted*
.50	.50	.50	.50
.65	.50	.59	.54
.75	.50	.64	.59
.85	.50	.67	.66
1.00	.50	.76	.77
.85	.65	.60	.63
.85	.75	.53	.57
.85	.85	.47	.50
.85	.95	.42	.42

Equation 11 suggests that multiple schedules will generally be preferred over mixed schedules, and we may infer that animals involved in mixed schedules will respond to transfer themselves into an equivalent multiple schedule. Experiments that permit such transfers are called "observing response" experiments, and they generally demonstrate the expected results. Fantino (1977) has reviewed the research and discussed alternative theoretical interpretations. Incentive theory belongs in his category of conditioned reinforcement interpretations. Although we can predict a general preference for multiple schedules, more exact predictions are difficult because the contingencies in this paradigm encourage all-or-none preferences (e.g., Killeen, Wald & Cheney, 1980). Variants of the paradigm that provide continuous measures of preference (e.g., McMillan, 1974) involve procedures that do not permit us to specify the values of the independent variable for our equations. Equation 12 does entail a symmetric change in conditioned reinforcement strength with changes in p, in accord with the data recorded in the above figure and table; this symmetry has not generally been found in the observing paradigm nor was it found in a concurrent study by Fantino and Moore (1980). Whether the disparity is due to the inappropriateness of Equation 12, or to changes in R, or to interactions of the schedule parameters with the observing behavior is unknown.

Varied Amount of Reinforcer

Incentive theory implies that varied amounts of the reinforcers will affect Equation 5 in a multiplicative fashion (Killeen, 1982). But this is not what happens. Two studies make clear that the amount of the reinforcer interacts with the delay value. The first is an unpublished dissertation by D. Green (1969), in which the hopper time was set at 3 sec and 9 sec for the terminal links of a concurrent chained schedule, which were equal fixed-interval schedules ranging from 1 to 60 sec. As the value of the terminal-link delay was increased, choice of the larger amount increased from 55% to 92% (see Figure 7). Ito and Asaki (1982) and Navarick and Fantino (1976) have published similar results. The second study is that of Green and Snyderman (1980), whose parametric data provide welcome exercise for any model of choice. These authors kept the amounts of the reinforcers fixed at 2 and 6 sec and varied the values of the terminal-link FI schedules over a range of values, keeping their ratio fixed at either 6/1, 3/1, or 3/2. Their results are shown in Figure 8. Control conditions employing equal amounts and delays, and equal amounts with different delays, are shown to the left in each figure.

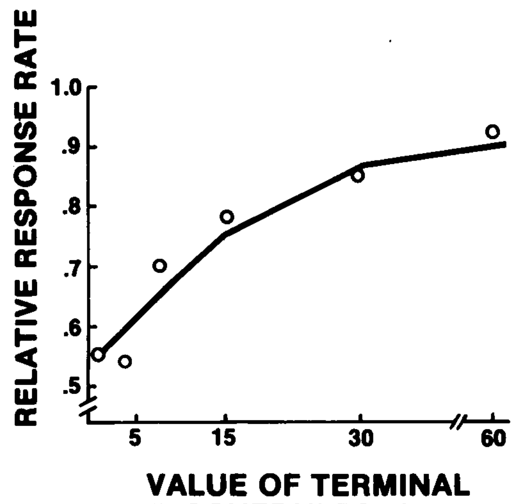

Fig. 7. Preference for the larger of two reinforcers as a function of their delay. The data are from D. Green, 1969; values of q and b were .11 and 1.2 (taking A to be 1 and 3 for the 3 and 9 sec amounts, for consistency with the earlier specifications of q).

A modification of incentive theory that enables it to cope with these data assumes that increasing the amount of the reinforcer decreases the interference from other behavior that might block primary reinforcement; this may be effected simply by dividing q by the amount of the reinforcer (A) received:

$$P = e^{-(q/A)t}. \tag{13}$$

This new model is consistent with all of the previous analyses (although the absolute values of q will be increased) and generates the solid lines through the data in the above figures.

The fit of the model to data is generally encouraging but is the assumption required to get it a reasonable one? Should increases in the amount of a reinforcer decrease competition? If so, how does the system decide which activities are "competitive," and which are "instrumental"? Staddon and Simmelhag (1971) have postulated such a differential effect of reinforcement, with the class of behavior that is incited (or, in their schema, not diminished) by reward being called "terminal behavior." Killeen, Hanson, and Osborne (1978) noted that the function relating arousal to the rate of incitement varied for different responses, being steepest for "prepared" responses such as key pecking and shallower for "unprepared" responses such as lever pressing by rats. We have then two theories that posit differential impact of incentives on different classes of behavior. Data are supplied by Reid and Dale (in press), who found that increases in the amount of food given rats increased their terminal behavior and decreased their interim behavior, as implied by the theories and by Equation 13. As an untested corollary of this assumption, we might expect the data from different organisms or involving different response topographies (i.e., interim versus terminal) to show different patterns than those displayed in Figures 7 and 8 and to be inconsistent with Equation 13. Recent results by Ito and Asaki (1982) with rats and lever pressing replicated the interaction found by Navarick and Fantino (1976), suggesting the appropriateness of Equation 13 for that response.

Our model also makes the prediction that differential amounts of the reinforcer will have a greater differential effect in short schedules.

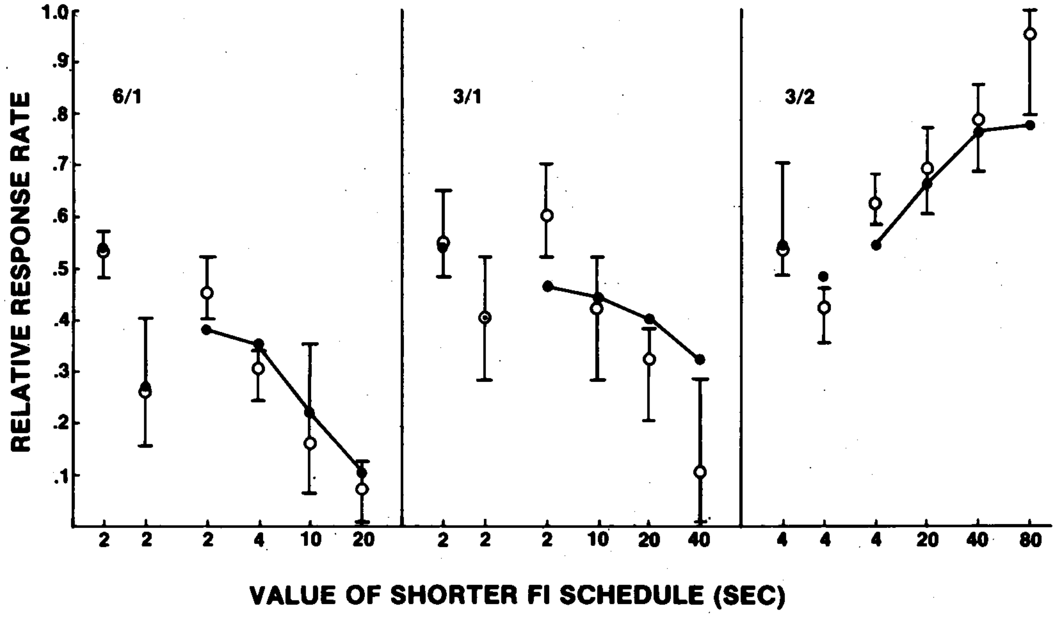

Fig. 8. Preference for the larger of two reinforcers as a function of their delay, with the delays of the shorter interval shown as the abscissae. For the first two conditions in each panel equal amounts were scheduled on both keys; in the first of those conditions the delays to food were equal. In subsequent conditions two seconds of food was scheduled on one key and six seconds on the other and the ratio of delays was fixed at the parameter shown in the panel. The data are from Green and Snyderman, 1980; the value for q was .23 (same convention as in Figure 7) and for b, 1.2.

Data to test this prediction come from Lendenmann, Myers, and Fantino (1982), who employed equal-valued interval schedules in the initial and terminal links. In the two cases that permit analytic predictions they gave 2, 3, or 5 seconds of access to grain to pigeons on FI 30 schedules or FI 90 schedules. The prediction was validated, although the small number of subjects and possible floor effects on response rates limit the strength of the demonstration.

Self-Control

Equation 13, in concert with Equations 5 and 7, permits us to extend incentive theory to the "commitment," or "self-control" paradigm (Ainslie, 1975; Rachlin & Green, 1972). In this paradigm animals are given a choice between the options of a small immediate reward and a large delayed reward. As the choicepoint is moved away from temporal contiguity with the options, animals show increasing preference for the larger delayed rewards. These paradigms require a fixed number of responses to complete the initial-link choice, and such scheduling generates a large bias in the number of entries into each terminal link. This bias favors the preferred link and is absent only at the point of indifference (furthermore, we expect that point to vary as a function of the direction from which it is approached). Therefore, we do not attempt to predict degree of preference throughout the range of delays, but we should be able to predict the points of indifference. As Navarick and Fantino (1976) have noted, the interaction of amount with delay rules out the early multiplicative models for self-control postulated by Ainslie and Rachlin. To proceed we must reconsider our measure of conditioned reinforcement. In these paradigms, the options ("terminal links") are remote from the point of choice, and so their strength will be weakened as an exponential function of the distance from the choicepoint to their onset:

$$C = exp(-qt_D)/t_0, \tag{14}$$

where t_D is the delay from the choice to the onset of the option, and t_0 is the delay from onset of the option to reinforcement. Of course, as t_D approaches zero, C approaches $1/t_0 = 1/t$, which is consistent with our first specification of C. The obtained duration of the options is used for t_0, with a minimum of .5 sec. When hopper duration is not timed with a photocell, .5 sec is subtracted from all values of A to allow for hopper approach time. The primary reinforcement function (Equation 13) operates on the obtained delay between the choice response and reinforcement; R is a function of $t_D + t_0 + A$, plus the obtained initial-link choice time. With these details specified, we may apply the model to some of the existing data, choosing the characteristic value of .125 for q and rescaling A as in Figures 7 and 8. Rachlin and Green (1972) found animals to be indifferent at $t_D = 4$-sec; our model predicts indifference at $t_D = 5$-sec. Green, Fisher, Perlow, & Sherman (1981) found indifference at $t_D = 10$-sec; our model predicts 9 sec. Ainslie and Herrnstein (1981) found indifference at $t_D = 5$-sec; our model predicts 8 sec. (These authors, like Green et al. [1981], kept rate of reinforcement constant by varying the duration of blackout after the reinforcers. If we assume the pigeons included this blackout period in "calculating" R, our prediction improves to 5 sec—but for Green et al. it worsens to 6 sec. Because of the large range of indifference points found for different subjects, these studies are not the ones to use in deciding whether or not blackout periods should generally contribute to our measure of R.) A different test of Equation 14 is found in the next section.

Segmented Terminal Links

If a terminal link is itself composed of a chained schedule, in which one or more different stimuli are interposed between the onset of the terminal link and the delivery of the reinforcer, we might expect the strength of that schedule to be weakened. This is what Duncan and Fantino (1972) found. In Part A of their experiments, they compared a Chain FI x-sec FI x-sec with a simple FI t-sec, where $t=2x$. Their results are shown in Panel A of Figure 9. In Part B they compared Chain FI 5-sec FI 5-sec with FI t-sec. In Part C they compared Chain FI t-sec FI t-sec with Chain FI x-sec FI x-sec FI x-sec, where $3x=2t$. There is a clear preference for the unsegmented or less segmented interval, one that increases with increases in the length of the terminal links. How shall we go about capturing these results within our framework?

The major problem concerns the assignment of a strength to the first segment of the terminal link. It is not contiguous with reinforcement, and we expect a large decrement because of that (Kamin 1965; Kaplan & Hearst, 1982; Rescorla, 1982). For simplicity, we shall assume the decrement to be complete: Behavior is maintained by the delayed primary reinforcer and by the less delayed conditioned reinforcer—the segment that is contiguous with reinforcement—and by nothing else. This assumption lets us get away without any new equations: We invoke 5, 7, and 14 and a value of .24 for q to generate the solid lines in Figure 9. Although there is systematic deviation, perhaps it is not so bad given the simplifying assumptions that we made. Here, as in all previous figures, the attempt at a quantitatively accurate model exposes the limits of our assumptions (and often the fallibility of our data) to an extent not possible with qualitative models.

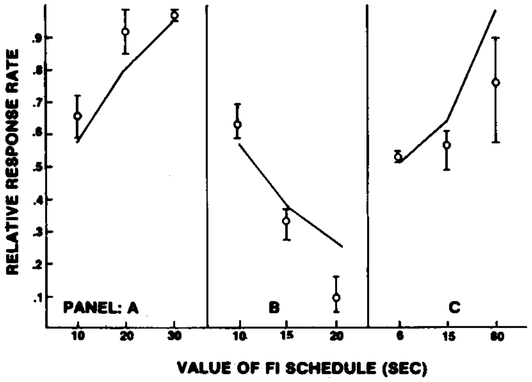

Fig. 9. Preference for the unsegmented or less segmented terminal link. In the first panel the abscissae are the values of fixed-interval schedules that are contrasted with a chain whose two links sum to that value; in the second panel the abscissae are the values of fixed-interval schedules that are contrasted with a chain FI 5 FI 5 schedule; in the third panel the abscissae are the lengths of one member of a chain FI t FI t schedule that is contrasted with a triple chain schedule whose components sum to the same value. The data are from Duncan and Fantino, 1972; values for q and b were .24 and 1.

DISCUSSION

Because most of the laws of physics are couched in mathematical terms, many people have come to see a mathematical description as the end of a research program, rather than as a means toward a greater understanding of phenomena. The formalism of incentive theory should be viewed as a means of understanding choice behavior. It is a description that may be useful and informative without displacing other descriptions. The precision that it permits in developing predictions should encourage modifications of the theory when the structure of the data is clearly incongruent with that of the specific model.

The need for validation of the current models is apparent. The identification of *P* with primary reinforcing effects and *C* with conditioned reinforcing effects is reasonable but far from assured. The presumption that increased amounts of the reinforcer will affect *P* but not *C* was congenial to the data, but may be less than congenial to our intuitions; that they affect it by decreasing competition from nonterminal behavior is problematic but easily tested. The impact of discontiguities with primary reinforcement on the value of *C* for early segments of a chain is a feature that has yet to be quantified due to the paucity of relevant data and the absence of an obvious model.

An example of the heuristic utility of incentive theory may be found in its application to the provocative data of Catania (1975, 1980a,b; Catania & Sagvolden, 1980), who demonstrated that pigeons prefer terminal links that provide two operanda on which to earn food over links that provide only one operandum—even though the schedules on the alternative operanda are identical and only one of them will eventuate in reinforcement. Since the operanda—pigeon keys—also contain the terminal-link stimuli, we may hypothesize that the animals are receiving twice as much conditioned reinforcement in these terminal links and evaluate the predictions our models would make if that were the case. (A more direct test of the hypothesis would be to separate the stimuli from the operanda and vary the number of each independently. Those data are not available. Catania did vary the size of the keys without any effect on behavior, but number may be much more potent than size in controlling choice.) If we double *C* in Equation 5 and apply that equation with $q=.125$ to an experiment in which the keys signal FI schedules of 10, 20, and 30 sec, we predict preferences for the doubled terminal link of .56, .58, and .61. Catania (1980a) obtained values of .52, .54, and .63, with the range easily including the predicted values.

But other predictions of this approach are wrong. If we treble or quadruple the number of options available, we must expect a linear increase in preference for that terminal link. Catania (1980a) found no reliable increase in preference beyond that for the two options. Although we might introduce notions such as decreasing marginal utility for number of conditioned reinforcers, the available data will not support that level of modeling.

My purpose has been twofold: to extend incentive theory to concurrent chained schedules and to demonstrate the utility of using such a formal representation as an exploratory tool. The different paradigms addressed with the theory required the clarification and extension of the original model. The problems still faced, such as failure to predict some of the results when the initial-link schedules are varied, provide a challenge for further modification of the models. It must be remembered, however, that data as well as theories are corrigible; some of the current models may need to be discarded because they are incorrect in general and merely approximate the data on which they were based, or they may need to be discarded because those data themselves are incorrect—biased for any one of a number of reasons. The utility of a strong formal theory is that, when well grounded in the basic phenomena of a paradigm, it provides a nexus within which both new models and new data may be generated, tested for accuracy, and either incorporated into the schemapiric nexus or rejected as beyond the boundaries of that system.

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