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Inclined Rectangular Weir-Flow Modeling

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Abstract

Sharp-crested weirs are the most commonly used devices in channels for relatively accurate flow measurement. Attempts have been made to study flow over different shapes of normal weirs, side weirs and oblique weirs. The sharp-crested weir is more commonly used in laboratory rather in the field channels due to its major drawback in development of afflux. This paper is concerned with the development of discharge–head-inclination model to measure flow over inclined Rectangular weir. The considerable increase in the discharging capacity of the weir, with increase in inclination along the vertical plane of weir is ascertained. A new index called inclined-weir-discharging index is defined which is a measure of the discharging capacity of the weir. The established discharge-head-inclination equation through the model can provide an accurate method to predict precise flow over the weir. The higher Inclined-weir discharging index associated with angle α will help to improve discharge capacity and reduce afflux.

Key words: Weirs; Inclined Notches, Rectangular Notch, Discharge characteristics, Flow measurement, Afflux, Inclined-weir discharging index

Introduction

A large number of weir profiles have been developed by many investigators for flow measurements with the main intention of reducing the afflux, increasing the discharging capacity and practically usable and acceptable weir profile. The arrangements of weirs have been standardized to be normal to the flow axis and lot of experimental work has been done. However, sometimes, these weirs are also arranged oblique to flow axis known as oblique weir and side weirs to gain some benefits. Further, any weir is associated with weir constant, called discharge coefficient which is a measure of its discharging capacity.

De Marchi (1934), Frazer (1957), Subramanya and Awasthy (1972), Nadasamoorthy and Thomson (1972), Ranga Raju *et al.* (1972), Hager (1987), Cheong (1991), Singh *et al.* (1994) have analyzed flow over rectangular side weir. Aichel (1953) gave table relating the discharge coefficient of a round crested skew weir to that of a normal weir. Ganapathy *et al.* (1964) conducted experiments on broad crested skew weir and gave curves for discharge coefficient and head with skew angle ‘θ’ as the third parameter. Mahapatra (1965) followed approach of Aichel for rectangular sharp-crested weirs. Muralidhar (1965) conducted experiments on weirs of finite crest width. Swamee *et al.* (1994) developed equation for elementary discharge coefficient for rectangular side weir.

Jain and Fischer (1982) have studied weirs with crest oblique to the flow axis in the channel. Multi fold skew weirs called Labyrinth weirs were developed and tested by Hay and Taylor (1970). They presented results in the form of curves between discharge coefficient and weir head to weir crest ratio and compared the results with normal weir.

Tullis *et al.* (1995) studied Labyrinth weir having trapezoidal plan forms and presented results in the form of curves between discharge coefficient and E/w with angle 'θ' as the third parameter.

Ramamurthy and Vo Ngoc-Diep (1994) have shown that with the increase in downstream slope for circular crested weir discharge coefficient can be improved. Talib Mansoor (2001) has developed equation for elementary discharge coefficient for rectangular skew weirs. Shesha Prakash and Shivapur (2002) have studied the variation of discharge coefficient with the angle of inclination of rectangular weir plane with respect to the normal (standard) position of plane of weir for a sharp-crested triangular weir. Most of the researchers have reported their study on establishing discharge coefficient in terms of weir head to weir (height/length) and skew angle. The majority of these investigators have used other type of sharp-crested weirs to calibrate the weir which is under investigation, limiting the accuracy of the results obtained.

In the present investigation, sharp-crested rectangular weir of 150 mm crest length is fixed inclined with respect to the bed of the channel. The conventional method of volumetric measurement is used to find actual discharge over the weir. As can be seen from Fig.1, the flow length along the plane of the weir increases with increase in inclination with the normal plane which increases the discharging capacity of the weir.

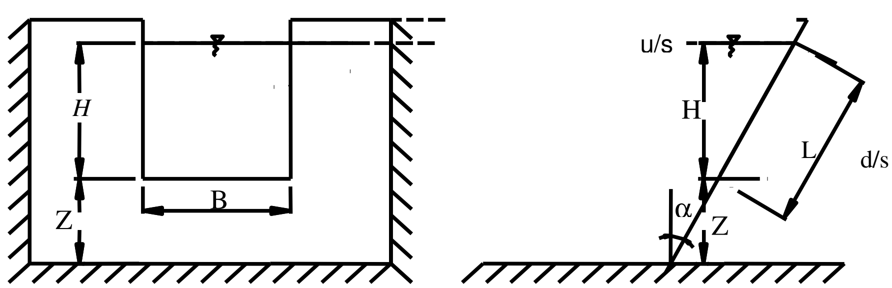

Fig.1: Arrangement of inclined rectangular weir.

Experiments

Experiments were carried on inclined rectangular notch fixed normal to the flow direction (0^0), 15^0, 30^0, 45^0, and 60^0 inclinations with respect to the normal plane (Vertical) along the flow axis. The experimental channel is rectangular in section and having dimensions 0.3m wide, 0.45m deep and of 11.5m length. The channel is constructed of Flexy glass and has smooth walls and bed with nearly horizontal bed to reduce the boundary frictional force. It is connected to a Head tank of dimensions 1m x 1.5m x 1.5m. The inclined rectangular notch weir is made of 8mm Flexy glass with a crest thickness of 1 mm and a 45^0 chamfer given on downstream side to get a springing nappe. The experimental set up is shown in Fig. 2. Water is supplied to the channel by an inlet valve provided on supply pipe. Over head tank is provided with overflow arrangement to maintain constant head. Smooth, undisturbed, steady-uniform flow was obtained by making the water to flow through graded aggregates and the surface waves were dampened by tying gunny bags at the surface near the tank. The head over the weir is measured using a electronic point gauge placed in piezometer located at a distance of about 1.40m on upstream of inclined rectangular notch. A collecting tank of size 1.465m length, 1.495m breadth, and of 1.5m depth is provided with a piezometer. Water after running through the experimental setup is collected in an underground sump from which it is re-circulated by pump by lifting it back to the overhead tank.

In the present study, the conventional method of volumetric discharge measurement is used, which increases the accuracy of the work. The measurements are done through electronic point gauge which automatically detects the water level and records the gauge reading. The volumetric measurement is done through self regulated timer for a fixed rise of water level automated through sensors.

Fig.2: Experimental setup.

Procedure

The step by step procedure followed for experimentation is as follows:

1. A sharp-crested inclined rectangular notch weir was installed at a desired inclination ‘α’ and height ‘z’.
2. The crest reading was taken by electronic point gauge when the water was in verge of flowing past the weir crest.

3. The discharge in the channel was controlled by the valve provided on supply pipe.
4. When flow attained a steady-uniform condition, the head over the weir crest 'h' was measured through the electronic point gauge.
5. For 100 mm, 200 mm, 300 mm and 400 mm rise of water level in collecting tank time in seconds was recorded (to eliminate the human error in piezometric readings in the collecting tank, time was auto-recorded by an electronic timer, for the above interval of water rise in the tank and averaged, by considering the cumulative volume and the accumulated time).
6. After having varied the discharge uniformly with a near constant increments in channel using a control gate of supply pipe, the steps 4 to 5 was repeated.
7. Steps from 1 to 6 were repeated for new desired angle of inclination of notch plane'α'.

The present investigation was carried out on the range of variables shown in Table-1.

Table-1: Range of variables studied.

Position of the weir	**Normal**	**Angle of inclination in degrees**			
	α=0^o	15^o	30^o	45^o	60^o
Actual discharge (m^3/s)	0.0019 to 0.019	0.0012 to 0.02	0.0016 to 0.021	0.0013 to 0.02	0.0017 to 0.02
Head over the crest (m)	0.0352 to 0.162	0.0229 to 0.16	0.028 to 0.153	0.0213 to 0.126	0.00169 to 0.098
No. of runs	40	35	38	33	24

Analysis of results

Initially the obtained and computed values of Head and discharge were non-dimensionalised as below so that obtained equation will be more generic in nature.

The discharge for flow through rectangular weir, (as $\frac{V^2}{2g}$, the velocity of approach is very small it is neglected), is given by

$$q = \frac{2}{3}\sqrt{2g}Lh^{\frac{3}{2}}$$

Where q is the discharge through the weir in m^3/s, L is the crest width in m and h is the head over the crest in m.

Non-dimensionalising the above equation, we get

$$Q = H^{\frac{3}{2}}$$

Where $Q = \dfrac{q}{\frac{2}{3}\sqrt{2g}L^{\frac{5}{2}}}$; $\qquad H = \dfrac{h}{L}$

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A plot of Non-dimensional discharge verses non-dimensionalised head for various positions of plane of rectangular notch weir have been shown in Fig.3. It shows that the discharge increases with increase in inclination angle α.

Hence, sharp-crested rectangular weir can be installed at a suitable inclination angle to the bed of the channel without any alteration to the conventional simple geometry of the weir so that the discharging capacity of the weir can be much higher, corresponding to the same head, as compared to conventional normal weir, which is evident from Fig. 3. This will help in reducing free board requirement on upstream of weir position. Further, it can also be used in the existing channels with least effect of afflux.

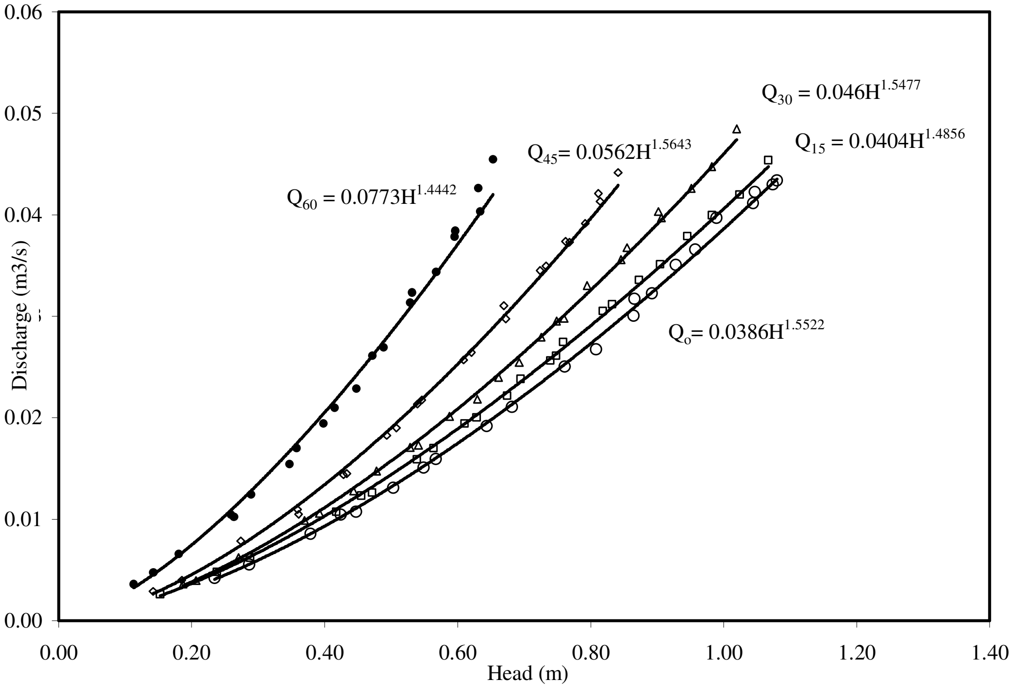

Fig.3: Non-dimensional Head-discharge plot for various values of inclination α.

Mathematical Modeling:

This paper proposes a programmable iterative algorithm to intermittently improve the efficiency of the interpolation by the well-known simple and popular ***Newton-Gregory Forward Interpolation Formula***, using statistical perspective of ***Reduced-Bias***. The impugned formula uses the values of the simple forward differences using values of the unknown function $f(x)$ at equidistant-points/ knots in the Interpolation-Interval say $[x_0, x_n]$. The basic perspective motivating this iterative algorithm is the fuller use of the information available in terms of these values of the unknown function $f(x)$ at the $(n+1)$ equidistant-points/knots. This information is used to reduce the Interpolation Error, which is statistically equivalent to the well-known concept of bias. The potential of the improvement of the interpolation is tried to be brought forth per an empirical study for which the function is assumed to be known in the sense of simulation. The numerical metric of the improvement uses the sum of absolute errors *i.e.* the differences between the actual (assumed to be known in the sense of the simulating nature of the empirical study) and the interpolated values at the mid-points of the equidistant-points/knots in Interpolation-Interval, (say $[0,1]$). This leads to the calibrations of the respective Percentage Relative (Relative to actual value of the function at that point) Errors and hence that of the respective Percentage Relative Gains in terms of the reduced values of the Percentage Relative Error, compared to that with the use of the Newton-Gregory original forward difference formula. Newton`s forward difference formula is significant in many contexts. It is a finite difference identity capable of giving an interpolated value between the tabulated points $\{f_k\}$ in terms of the first value f_0 and powers of the forward difference "Δ".

For $k \in [0,1]$; the formula states

$$f_k = f_0 + k\Delta_1 + \frac{k(k-1)}{2!}\Delta_2 + \frac{k(k-1)(k-2)}{3!}\Delta_3 + ... + \frac{k(k-1)(k-2)....(k-n+1)}{n!}\Delta_n .$$

Where, k is any real number; $\Delta_1, \Delta_2, \Delta_3, ...\Delta_n$ are respectively the first, second, third,………. nth forward differences.

This formula looks apparently like an analog of a Taylor Series expansion. It would be important to note that we have taken the Interpolation-Interval as $[0,1]$ rather than say $[a,b]$ without any loss of generality. In fact, $C[0,1]$ and $C[a,b]$ are essentially identical, for all practical purposes, in as much as they are linearly isometric as normed spaces, order isomorphic as lattices, and isomorphic as algebras (rings). The following section details the algorithm enabling the fuller use of the "information" available in terms of these values of the unknown function $f(x)$ at the $(n+1)$ equidistant-points/knots. As the same perspective (of Reduced Error/Bias) is available to be used at the end of a particular iteration, beginning from the first iteration, the proposed algorithm is an iterative one.

The discharge-head-inclination equation can be expressed as

$$Q = f(\alpha) H^{\phi(\alpha)} \qquad (01)$$

The modeling part is subdivided into two stages.

In the first stage the actual head-discharge data values are statistically fit with the exponential equation

$$Q = KH^n$$

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Table-2: Calibrated Head-discharge equations for various angles α.

S.No	α	$Q=KH^n$
1	0	$Q = 0.039\ H^{1.552}$
2	15	$Q = 0.040\ H^{1.486}$
3	30	$Q = 0.046\ H^{1.548}$
4	45	$Q = 0.056\ H^{1.564}$
5	60	$Q = 0.077\ H^{1.444}$

In the second stage adopting the ***Newton-Gregory Forward Interpolation Formula*** to develop the model for the present problem, we see that there are 5 values and hence we can adopt only value up to n-1, i.e. 4th order polynomial curve can be fit to the data and simplifying the equations, we get the final general head-discharge-angle expression for any given rectangle, of and inclination as under:

$$Q=\left(4.68\times10^{-9}\theta^{4}-3.54\times10^{-7}\theta^{3}+1.62\times10^{-5}\theta^{2}-4.76\times10^{-5}\theta+3.86\times10^{-2}\right)H^{\left(5\times10^{-8}\theta^{4}-1\times10^{-5}\theta^{3}+7\times10^{-4}\theta^{2}-0.011\ \theta+1.552\right)}$$

Where $Q=\dfrac{q}{\frac{2}{3}\sqrt{2g}L^{\frac{5}{2}}}$; $\qquad H=\dfrac{h}{L}$

The same is obtained as an Excel curve fit as shown in Fig. 4.

It can be seen that the equation developed by the model agrees with the one obtained by Excel and further, the regression coefficient in both the cases is exactly unity. This improves the credibility of the analysis and practical usage of the notch. Even though obtained discharge-head-inclination equation is complicated, it reduces to simple equation once the α values are substituted and simplified.

Error analysis:

Error analysis is carried out by computing the percentage deviation of the Computed discharge from the actual discharge for various inclinations as shown in Table-3.

Table-3: Maximum percentage deviation of Computed to Actual discharge.

Angle of inclination	0	15	30	45	60
Max Error	4.3723	4.7973	4.8166	8.9669	9.6916

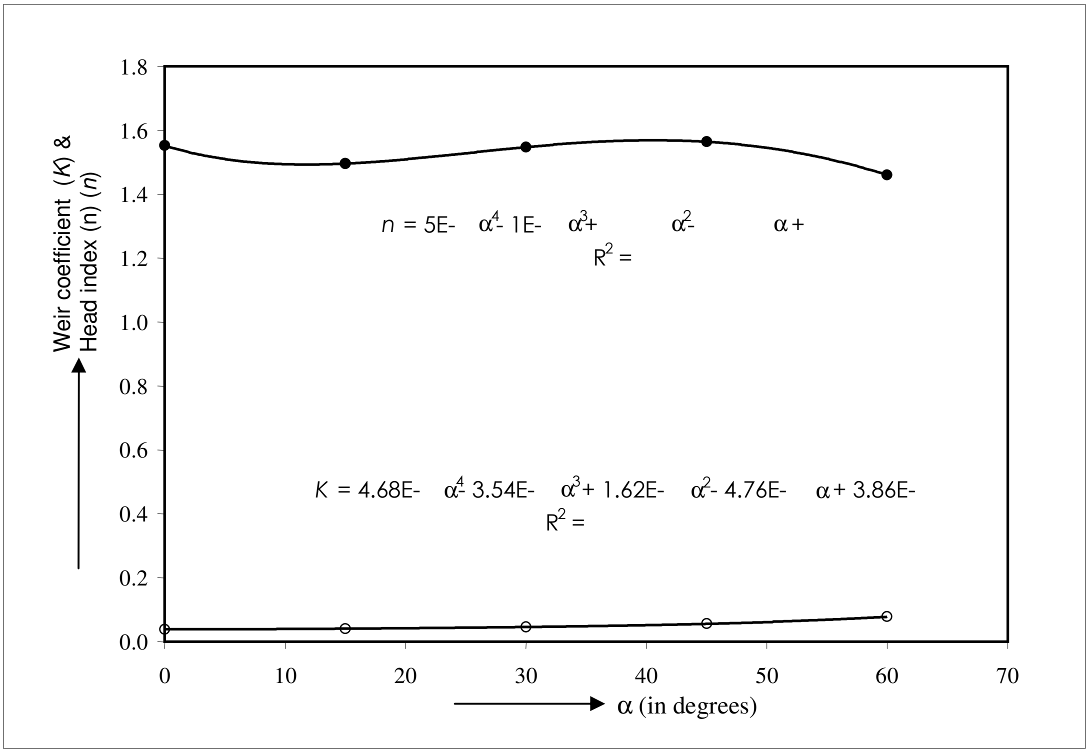

Fig.4: Variation of weir coefficient (*K*) and Head indices (*n*) with weir inclination (*α*).

Discharging capacity:

The discharging capacity of the inclined rectangular weir relative to normal position is shown in Fig.5. ***Inclined-weir discharging index*** is the ratio of the discharging capacity of an inclined weir to that of normal weir. *Inclined-weir discharging index* for inclined rectangular weir for various inclinations are as plotted in Fig. 5.

$$C_{di} = 0.0005\alpha^2 - 0.0111\alpha + 1.0478$$

If only the last three points are considered *viz*. 30°, 45° and 60° and the increase is found to be exponential as shown below.

$$C_{di} = 0.609e^{0.0208\alpha}$$

Analysis of afflux:

The Non-Dimensional Head is computed for the Maximum Non-dimensional Discharge for various inclinations of the rectangular notch as shown in Table-4. It can be seen that the maximum reduction is with 60° inclination.

Table-4: Reduction in afflux for various inclinations of rectangular notch relative to its normal position.

Q_{max}	4.85E-02		$H = \left[\frac{Q}{K}\right]^{\frac{1}{n}}$	% age Reduction in Afflux
α	K	n		
0	0.03861	1.55	1.157773	0%
15	0.040586	1.50	1.125953	3%
30	0.046001	1.55	1.034325	11%
45	0.056203	1.56	0.909687	21%
60	0.07822	1.46	0.72054	38%

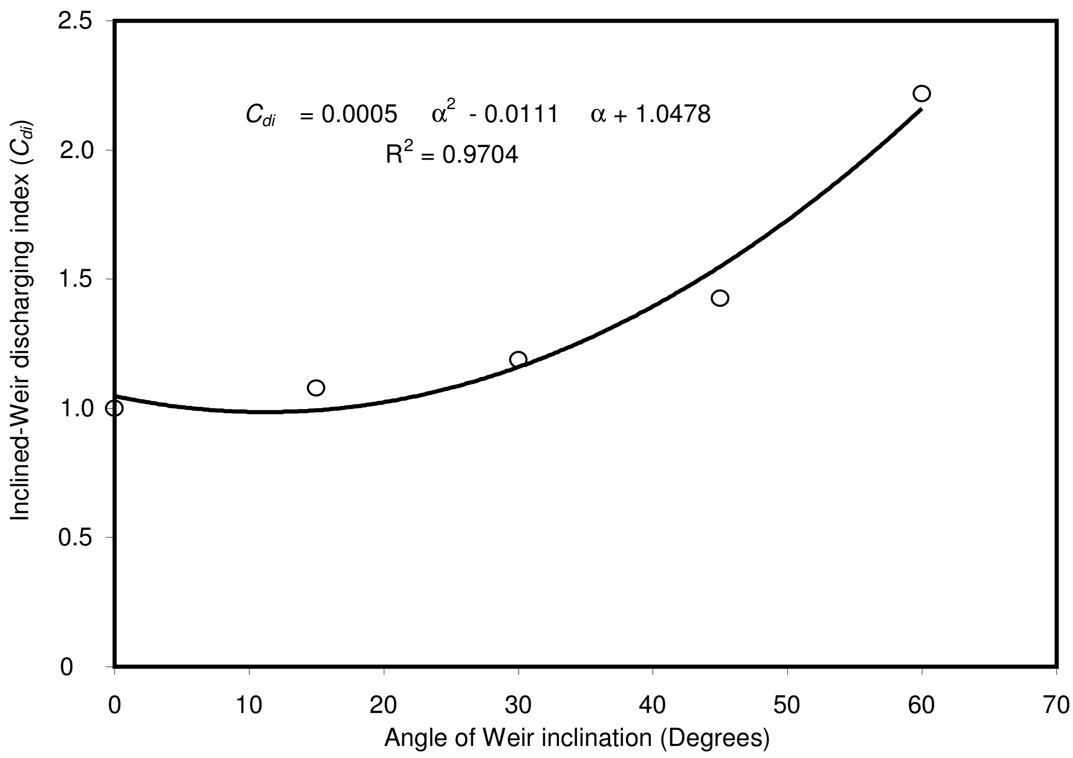

Fig.5: Inclined-weir discharging index of Inclined Rectangular Weir for various inclination.

Conclusions

Following conclusions were drawn based on the experimental investigation and the subsequent analysis by the authors.

- The discharging capacity of the weir increases with the increase in inclination of the plane of weir. In particular it is found to increase exponentially from 30° to 60°.
- It is observed from Table-3 that the percentage deviation of error in computation of discharge increases with increase in angle of inclination of the notch.
- Larger area of flow is possible in the inclined notches relative to the conventional normally positioned notches. From Table-4, it is seen that afflux is found to be about 38% with 60° inclination of rectangular notch relative to its normal position. The property of increase in ***Inclined-weir discharging index*** with increase in inclination of weir plane can be used to discharge more water quickly without increasing afflux on upstream side in pre-designed canal structure during flood season, without changing the pre-installed weir (which is practically very difficult).
- Due to the simple geometry and ease of construction, inclined rectangular notch-weirs find its applications as a simple measuring devices in irrigation, chemical and sanitary engineering for flow measurement and flow control.
- The Mathematical modeling results in a single head-discharge-inclination equation which can be used for any rectangular weir of any desired inclination.

Limitation:

The experiment can be done with larger discharge in larger channels and the Head-Discharge-Inclination equation can be improved by using the model.

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