# Incomplete Markets and the Output-Inflation Tradeoff[*]

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## Abstract

This paper analyses the effects of money shocks on macroeconomic aggregates within a flexible price, incomplete-markets environment that generates persistent wealth inequalities amongst agents. In this framework, unexpected money shocks redistribute wealth from the cash-rich employed to the cash-poor unemployed and induce the former to increase labour supply in order to maintain their desired levels of consumption and precautionary savings. The reduced-form dynamics of the model is a textbook 'output-inflation tradeoff' equation, whereby inflation shocks raise current output. The limiting impact of mean inflation and money growth persistence on this non neutrality mechanism are also examined.

*Keywords*: Incomplete markets; borrowing constraints; short-run non-neutrality.
*JEL codes*: E24; E31; E32.

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# 1 Introduction

This paper analyses the effects of money shocks on macroeconomic variables within a flexible price, incomplete-markets environment that generates persistent wealth inequalities amongst agents. More specifically, we explore the aggregate and welfare effects of unexpected monetary injections within a Bewley-type model where money serves as a short-run store of value allowing agents to self-insure against idiosyncratic income fluctuations. As was first shown by Bewley (1983), and analysed further in a number of contributions including Kehoe *et al.* (1990), Imrohoroglu (1992), and Akyol (2004), this role for money arises naturally in environments where insurance markets are missing and agents cannot borrow against future income. We draw on this literature by emphasising the role of money as a buffer stock against labour income fluctuations, where money partly substitutes for the lack of insurance and credit markets. Unlike this existing work, however, we study the short-run non neutrality of monetary shocks, rather than focusing on the potential long-run, non superneutral effects of steady state inflation.

The central result of this paper lies in the derivation of a textbook 'output-inflation tradeoff' equation, whereby inflation shocks contemporaneously raise labour supply and total output. In our economy, inflation shocks redistribute wealth from the cash-rich employed (i.e., those who pay the inflation tax) to the cash-poor unemployed (i.e., those who benefit from the inflation subsidy), thereby forcing the former to increase their labour supply to replete their income and maintain their desired levels of consumption and money wealth. The implied increase in hours then raises current output, with the underlying tradeoff mechanism differing from traditional ones like those based on sticky prices (e.g., Ball *et al.*, 1988) or imperfect information (see Lucas, 1973).

What does the size of this monetary non-neutrality depend on? In our model, the redistributive effect of an inflation shock is positively related to the gap between the real balances of cash-rich and cash-poor agents, i.e., the degree of inequality in the distribution of money holdings. High mean inflation, in as much as it lowers the desirability of real balances as a means of self insurance, tends to deter employed households from accumulating them and thus lowers both money wealth inequalities and the implied impact of inflation shocks. Highly persistent money growth shocks, to the extent that they forecast high inflation taxes on future real balances, transitorily lowers real money demand and induce a *negative* effect of

money growth shocks on labour supply and output. Thus, the output effects of inflationary money shocks are all the more likely to be large that both the mean and the persistence of money growth are low. In the extreme opposite situation where both are very large, the intertemporal effects on future inflation taxes may come to dominate the intratemporal wealth redistribution effect and even lead to a reversal in the slope of the tradeoff.

Our model follows the route opened by Bewley (1983), Scheinkman and Weiss (1986) and Kehoe *et al.* (1990). Bewley and Kehoe *et al.* focused on the optimal long-run inflation rate and did not analyse the short-run non-neutrality of money under incomplete markets. Scheinkman and Weiss were the first to identify the non-neutrality of money shocks when borrowing constraints make cash holdings heterogenous; however, the infinite-dimensional wealth distribution of their model did not allow them to derive the output-inflation tradeoff, let alone relate its size to the underlying deep parameters of the model such as, unemployment risk mean inflation, or the persitence of shocks. Given the lack of tractability of heterogenous-agent models with infinite-state wealth distributions, an alternative approach to ours is to solve them computationally. However, computational limitations have thus far limited the applicability of these models to the study of optimal steady-state inflation, again leaving aside the analysis of the short-run effects of inflation shocks (e.g., Imrohoroglu, 1992, and Akyol, 2004). We circumvent this difficulty by deriving a closed form solution to the model with a finite-state wealth distribution and a finite number of agent types. Finally, our work is related to that of Doepke and Shneider (2006, Section 4), who look at the aggregate effects of wealth redistribution through inflation within an overlapping generations model. In their framework, inflation episodes transfer wealth from old retirees to young workers, thereby inducing a decrease in labour supply and output in the short and medium run. In contrast, our model features infinitely-lived agents occasionally hit by borrowing constraints and is able to generate a positive short run relation between inflation and ouput.

Section 2 introduces the model and spells out the optimality and market clearing conditions in the general case. Section 3 derives a specific closed-form equilibrium with four types of agents and two possible levels of real money balances. Section 4 analyses the properties of the short-run output-inflation tradeoff generated by the model, with particular attention being paid to the role of mean inflation and the persistence of shocks in affecting the slope of the tradeoff.

# 2 The model

The economy is populated by a large number of firms, as well as a unit mass of infinitely-lived households $i \in [0, 1]$, all interacting in perfectly competitive labour and goods markets. Firms produce output, $y_t$, from labour input, $l_t$, using the CRS technology $y_t = l_t$; they thus adjust labour demand up to the point where the real wage is equal to 1. Households' behaviour, on the other hand is potentially affected by both the (uninsurable) idiosyncratic income uncertainty that they are facing and the aggregate shock.

## 2.1 Uncertainty

*Individual states.* In every period, each household can be either employed or unemployed. We denote by $\chi_t^i$ the status of household $i$ at date $t$, where $\chi_t^i = 1$ if the household is employed and $\chi_t^i = 0$ if the household is unemployed. Households switch randomly between these two states, with $\alpha = \text{prob}(\chi_{t+1}^i = 1 | \chi_t^i = 1)$ and $\rho = \text{prob}(\chi_{t+1}^i = 0 | \chi_t^i = 0)$, $(\alpha, \rho) \in (0, 1)^2$, being the probabilities of staying employed and unemployed, respectively. Given this Markov chain for individual states, the asymptotic unemployment rate is:

$$U = (1 - \alpha) / (2 - \alpha - \rho) \tag{1}$$

The history of individual shocks up to date $t$ is denoted $e_i^t$, where $e_i^t = \{\chi_0^i, .., \chi_t^i\}$. $E^t = \{0, 1\} \times ..\{0, 1\}$ is the set of all possible histories up to date $t$, and $\mu_t^i : E^t \to [0, 1]$, $t = 0, 1, ...$ denotes the probability measures of individual histories (for example, $\mu_t^i(e_t^i)$ is the probability of individual history $e_t^i$ for agent $i$ at date $t$). Following convention, we use the notation $e_{t+1}^i \succeq e_t^i$ to indicate that $e_{t+1}^i$ is a possible continuation of $e_t^i$. Finally, we limit the ability of households to diversify this idiosyncratic unemployment risk away by assuming that it is *uninsurable* and that agents *cannot borrow* against future labour income.

*Aggregate states.* Money growth shocks are the only source of economy-wide uncertainty that we consider. The history of these shocks up to date $t$ is denoted $h_t$, while $H^t$ is the set of all possible histories for these shocks up to date $t$. Let $\nu$ denote the probability measure over histories up to date $t$: $\nu_t : H^t \to [0, 1], t = 0, 1, ...$ As before, $\nu_t(h_t)$ is the probability of history $h_t$ and $h_{t+1} \succeq h_t$ indicates that $h_{t+1}$ is a possible continuation of $h_t$.

In every period, a real amount $\gamma_t(h_t) > 0$ of newly issued money is given symmetrically to all households (we show below how the latter is related to money growth, $\tau_t(h_t)$). Moreover,

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in equilibrium the price level and the inflation rate are functions of the history of aggregate states. These are denoted $P_t(h_t)$ and $\pi_t(h_t) \equiv P_t(h_t)/P_{t-1}(h_{t-1}) - 1$, respectively.

## 2.2 Households' behaviour

The household's instantaneous utility function is $u(c) - \phi l$, where $c$ is consumption, $l$ is labour supply, $\phi > 0$ a scale parameter, and where $u$ is a $C^2$ function satisfying $u' > 0$, $u'' < 0$ and $\sigma(c) \equiv -u''(c)c/u'(c) \leq 1 \ \forall c \geq 0$ (i.e., consumption and leisure are not gross complements). Fiat money is the only asset that households can use to smooth consumption. Employed households (i.e., those for whom $\chi_t^i = 1$) choose their labour supply, $l_t^i$, at the current wage rate $(= 1)$, while unemployed households (i.e., for whom $\chi_t^i = 0$) earn no labour income but a fixed amount of 'home production', $\delta > 0$.[1] Let $M_t^i$ denote the nominal money holdings of household $i$ at the end of date $t$, and $m_t^i \equiv M_t^i/P_t$ the corresponding real money holdings (by convention, let us denote $M_{-1}^i$ the nominal money holdings of household $i$ at the beginning of date 0). Household $i$'s problem is to choose the sequences of functions

$$
\left.
\begin{array}{l}
c_t^i : H^t \times E^t \to R_+ \\[4pt]
l_t^i : H^t \times E^t \to R_+ \\[4pt]
m_t^i : H^t \times E^t \to R_+
\end{array}
\right\} t = 0, 1, ...,
$$

that maximise

$$
\sum_{t=0}^{\infty} \beta^t \sum_{h_t \in H^t} \nu_t(h_t) \sum_{e_t^i \in E^t} \mu_t^i(e_t^i) \left( u\left( c_t^i(h_t, e_t^i) \right) - \phi l_t^i(h_t, e_t^i) \right),
$$

where $\beta \in (0,1)$ is the discount factor, subject to

$$
P_t c_t^i(h_t, e_t^i) + M_t^i(h_t, e_t^i) = M_{t-1}^i(h_{t-1}, e_{t-1}^i) + P_t\left( \chi_t^i l_t^i(h_t, e_t^i) + (1 - \chi_t^i)\delta + \gamma_t(h_t) \right), \quad (2)
$$

$$
c_t^i(h_t, e_t^i),\ l_t^i(h_t, e_t^i),\ M_t^i(h_t, e_t^i) \geq 0. \quad (3)
$$

Eq. (2) is the nominal budget constraint of household $i$ at date $t$, while the last inequality in (3) indicates that agents cannot have negative asset holdings. The Lagrangian function

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[1] Alternatively, $\delta$ can be interpreted as an unemployment subsidy financed through a compulsory lump sum contribution $e = (1 - \alpha)\delta/(1 - \rho)$ paid by all employed households and ensuring the balance of the unemployment insurance scheme. In this case, steady state labour supply and output are higher than under the home production interpretation (as working households attempt to offset the wealth effect of the unemployment contribution), but the behaviour of the stochastic economy is unchanged.

associated with household $i$'s problem, formulated in real terms, is:[2]

$$L = \sum_{t=0}^{\infty} \beta^t \sum_{h_t \in H^t} \nu_t (h_t) \sum_{e_t^i \in E^t} \mu_t (e_t^i) \times$$

$$\left[ \begin{array}{c} u \left( c_t^i \left( h_t, e_t^i \right) \right) - \phi l_t^i \left( h_t, e_t^i \right) + \varphi_t^i \left( h_t, e_t^i \right) m_t^i \left( h_t, e_t^i \right) \\ + \eta_t^i \left( h_t, e_t^i \right) \left( \frac{m_{t-1}^i \left( h_{t-1}, e_{t-1}^i \right)}{1 + \pi_t (h_t)} + \chi_t^i l_t^i \left( h_t, e_t^i \right) + \left( 1 - \chi_t^i \right) \delta + \gamma_t (h_t) - c_t^i \left( h_t, e_t^i \right) - m_t^i \left( h_t, e_t^i \right) \right) \end{array} \right],$$

where the Lagrange multipliers $\eta_t^i$ and $\varphi_t^i$ are positive functions defined over $H^t \times E^t$ (we check below that the non-negativity constraints on $c_t^i$ and $l_t^i$ are always satisfied in the equilibrium under consideration). From the Kuhn-Tucker theorem, the optimality conditions are, for $t = 0, 1, \ldots$ and for all $(h_t, e_t^i) \in H^t \times E^t$,

$$u' \left( c_t^i \left( h_t, e_t^i \right) \right) = \eta_t^i \left( h_t, e_t^i \right), \tag{4}$$

$$\eta_t^i \left( h_t, e_t^i \right) = \phi \text{ if } \chi_t^i = 1 \text{ and } l_t^i \left( h_t, e_t^i \right) = 0 \text{ if } \chi_t^i = 0, \tag{5}$$

$$\eta_t^i \left( h_t, e_t^i \right) - \varphi_t^i \left( h_t, e_t^i \right) = \beta \sum_{h_{t+1} \succeq h_t} \nu_{t+1} (h_{t+1}) \sum_{e_{t+1}^i \succeq e_t^i} \mu_{t+1}^i \left( e_{t+1}^i \right) \frac{\eta_{t+1}^i \left( h_{t+1}, e_{t+1}^i \right)}{1 + \pi_{t+1} (h_{t+1})}, \tag{6}$$

$$\varphi_t \left( h_t, e_t^i \right) m_t^i \left( h_t, e_t^i \right) = 0, \tag{7}$$

$$\lim_{t \to \infty} \beta^t u' \left( c_t \left( h_t, e_t^i \right) \right) m_t^i \left( h_t, e_t^i \right) = 0. \tag{8}$$

Eq. (4) defines household $i$'s marginal utility, while Eqs. (5) and (6) are the intratemporal and intertemporal optimality conditions, respectively. Eq. (7) states that either the borrowing constraint is binding for household $i$ ($\varphi_t^i > 0$), implying that cash holdings are zero ($m_t^i = 0$), or the constraint is slack ($\varphi_t^i = 0$) and the household uses real balances to smooth consumption over time ($m_t^i \geq 0$). The transversality condition (8) always hold along the equilibria that we will consider. Note that Eq. (6) can be written more compactly as:

$$u' \left( c_t^i \left( h_t, e_t^i \right) \right) = \beta E_t \left( \frac{u' \left( c_{t+1}^i \left( h_{t+1}, e_{t+1}^i \right) \right)}{1 + \pi_{t+1} (h_{t+1})} \right) + \varphi_t^i \left( h_t, e_t^i \right). \tag{9}$$

## 2.3 Market clearing

*Goods market.* Equilibrium in the market for goods requires that, at each date and for all histories of aggregate states $h_t \in H^t$, the sum of each type of agent's consumption be equal

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[2] As will become clear below, our choice of using the Lagrangian function, rather than the Bellman equation, allows a more transparent derivation of the equilibrium on which our analysis focuses.

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to total production. Given the production function assumed, total production is simply the sum of individual labour supplies and home production, so that we have:

$$\int_0^1 \left( \chi_t^i l_t^i \left( h_t, e_t^i \right) + \left( 1 - \chi_t^i \right) \delta \right) di = \int_0^1 c_t^i \left( h_t, e_t^i \right) di,$$

where the summation operator $\int$ is over individual households.

*Money market.* Let $M_t \left( h_t \right)$ denote the nominal quantity of money at date $t$, then money-market clearing may equally be written as:

$$M_t \left( h_t \right) = \int_0^1 M_t^i \left( h_t, e_t^i \right) di.$$

Denote real money supply by $m_t \left( h_t \right) \equiv M_t \left( h_t \right) / P_t \left( h_t \right)$ and the (gross) rate of money growth by $\tau_t \left( h_t \right) \equiv M_t \left( h_t \right) / M_{t-1} \left( h_{t-1} \right)$. Then, symmetric real money injections can be written as:

$$\gamma_t \left( h_t \right) = \frac{M_t \left( h_t \right) - M_{t-1} \left( h_{t-1} \right)}{P_t \left( h_t \right)} = \frac{m_{t-1} \left( h_{t-1} \right) \times \left( \tau_t \left( h_t \right) - 1 \right)}{1 + \pi_t \left( h_t \right)}, \tag{10}$$

while the law of motion for the real quantity of money is:

$$m_t \left( h_t \right) = \frac{m_{t-1} \left( h_{t-1} \right) \times \tau_t \left( h_t \right)}{1 + \pi_t \left( h_t \right)}. \tag{11}$$

An equilibrium is defined by a set of individual consumption sequences, $\{ c_t^i \left( h_t, e_t^i \right) \}_{t=0}^\infty$, individual real money holdings sequences, $\{ m_t^i \left( h_t, e_t^i \right) \}_{t=0}^\infty$, individual labour supply sequences, $\{ l_t^i \left( h_t, e_t^i \right) \}_{t=0}^\infty$, $i \in [0, 1]$, and aggregate variables, $\{ y_t \left( h_t \right), m_t \left( h_t \right), \pi_t \left( h_t \right) \}_{t=0}^\infty$, such that the optimality conditions (4)-(8) hold for every household $i$ and the goods and money markets clear, given the forcing sequence $\{ \tau_t \left( h_t \right) \}_{t=0}^\infty$.

# 3    A closed-form solution

In general, heterogenous-agent models such as that described above generate an infinite-state distribution of agent types, as all individual characteristics (i.e., agents' wealth and implied optimal choices) depend on the personal history of every single agent. In this paper, we derive a closed-form solution of the model with a finite number of household types by considering an equilibrium where the cross-sectional distribution of money wealth is two-state. The derivation involves three steps. First, we conjecture the general shape of the

solution; second, we identify the conditions under which the hypothesised solution results; and third, we set the relevant parameters (the productivity of home production, here) in such a way that these conditions are always fulfilled along the equilibrium under consideration.

## 3.1 Conjectured equilibrium

We conjecture the existence of an equilibrium along which

$$\varphi_t\left(h_t, e_t^i\right) = 0 \text{ if } \chi_t^i = 1 \text{ and } m_t^i\left(h_t, e_t^i\right) = 0 \text{ if } \chi_t^i = 0, \tag{12}$$

that is, one where no employed household is borrowing-constrained (so that all of them store cash to smooth consumption), while all unemployed households are borrowing-constrained (and thus hold no cash). From here on, we simplify notation by simply using the $i$-index for variables that depend on individual histories and the $t$-index for those that depend on aggregate history.

Consider first the consumption level of an unemployed household. If this household was employed in the previous period, then from (2) and (12) their current consumption is:

$$c_t^i = m_{t-1}^i/\left(1 + \pi_t\right) + \delta + \gamma_t \ (> 0), \tag{13}$$

On the other hand, from (2) the consumption level of unemployed households who were already unemployed in the previous period is identical across such households and given by:

$$c_t^{uu} = \gamma_t + \delta \ (> 0). \tag{14}$$

We now turn to employed households. From Eqs. (4) and (5), their consumption level is identical across employed households and independent of aggregate history, i.e.,

$$c^e = u'^{-1}\left(\phi\right) \ (> 0). \tag{15}$$

From Eqs. (9) and (12), the intertemporal optimality condition for an employed household is $\phi = \beta E_t\left(\eta_{t+1}^i/\left(1 + \pi_{t+1}\right)\right)$. If this household is employed in the following period, which occurs with probability $\alpha$, then $\eta_{t+1}^i = \phi$ (see Eq. (5)). If the household moves into unemployment in the next period, then from (4) $\eta_{t+1}^i = u'\left(c_{t+1}^i\right)$, where by construction $c_{t+1}^i$ is given by Eq. (13). The Euler equation for employed households is thus:

$$\phi = \alpha\beta E_t\left(\frac{\phi}{1 + \pi_{t+1}}\right) + (1 - \alpha)\beta E_t\left(u'\left(\frac{m_t^i}{1 + \pi_{t+1}} + \delta + \gamma_{t+1}\right) \times \frac{1}{1 + \pi_{t+1}}\right), \tag{16}$$

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which in turns implies that all employed households wish to hold the same quantity of real balances, denoted $m_t^e$ (i.e., $\forall i \in [0,1]$, $\chi_t^i = 1 \Rightarrow m_t^i = m_t^e$). We may thus rewrite Eq. (13), giving the consumption level of unemployed households which were previously employed, as follows:

$$c_t^{eu} = m_{t-1}^e / (1 + \pi_t) + \delta + \gamma_t. \tag{17}$$

The labour supplies of employed households depend on whether they were employed or not in the previous period. Using Eqs. (2) and (15) these are given by, respectively,

$$l_t^{ee} = u'^{-1}(\phi) + m_t^e - m_{t-1}^e / (1 + \pi_t) - \gamma_t, \tag{18}$$

$$l_t^{ue} = u'^{-1}(\phi) + m_t^e - \gamma_t, \tag{19}$$

with Eqs. (28)–(29) below establishing that both $l_t^{ee}$ and $l_t^{ue}$ are positive in equilibrium.

In other words, when all unemployed households are borrowing-constrained and no employed household is, households can be of four different types, depending only on their current and past employment status, with their personal history before $t - 1$ being irrelevant. This distributional simplification is essentially the outcome of the joint assumption that all unemployed households liquidate their asset holdings (i.e., $\chi_t^i = 0 \Rightarrow m_t^i = 0$), while all employed households choose the same levels of consumption and asset holdings thanks to linear labour disutility (i.e., $\chi_t^i = 1 \Rightarrow m_t^i = m_t^e$). We denote these four households types $ee$, $eu$, $ue$ and $uu$, where the first and second letters refer to date $t - 1$ and date $t$ employment states, respectively. Since our focus is on the way idiosyncratic unemployment risk affects self-insurance by the employed, we consider the effect of variations in $\alpha$ taking $U$ in (1) as given (the implied probability of leaving unemployment is thus $\rho = 1 - (1 - \alpha)(1 - U)/U$). We then write the asymptotic shares of households as:

$$\omega^{ee} = \alpha(1 - U), \ \omega^{eu} = \omega^{ue} = (1 - \alpha)(1 - U), \ \omega^{uu} = U - (1 - \alpha)(1 - U), \tag{20}$$

and we abstract from transitional issues regarding the distribution of household types by assuming that the economy starts at this invariant distribution. Given the consumption and labour supply levels of each type of household, goods-market clearing now implies that:

$$\omega^{ee} l_t^{ee} + \omega^{ue} l_t^{ue} + U\delta = (1 - U)c^e + \omega^{eu} c_t^{eu} + \omega^{uu} c_t^{uu}. \tag{21}$$

In the equilibrium under study, which we assume to prevail from date 0 onwards, unemployed households hold no money while all employed households hold the real quantity $m_t^e$.

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Money-market clearing thus requires that:

$$(1 - U) \, m_t^e = m_t. \tag{22}$$

## 3.2 Conditions for the closed-form equilibrium to exist

The condition for the distribution just derived to be an equilibrium is that the borrowing constraint never be binding for $ee$ and $ue$ households but always be binding for both $uu$ and $eu$ households. The constraint is not binding for employed households if the latter never wish to borrow. Thus, interior solutions to (16) must always be such that:

$$m_t^e \geq 0. \tag{23}$$

On the other hand, the Lagrange multiplier $\varphi_t^i$ must be positive when households are unemployed, so that from (4)-(6) we must have $\eta_t^i > \beta E_t \eta_{t+1}^i / (1 + \pi_{t+1})$. First consider $uu$ households, whose current consumption is just $\delta + \gamma_t$ (see (14)), and thus for whom $\eta_t^i = u'(\delta + \gamma_t)$. These households remain unemployed with probability $\rho$, in which case they will also consume $\delta + \gamma_t$ in the following period and thus $\eta_{t+1}^i = u'(\delta + \gamma_{t+1})$. They leave unemployment with probability $1 - \rho$ and will then consume $u'^{-1}(\phi)$ in the following period, so that $\eta_{t+1}^i = u'(u'^{-1}(\phi)) = \phi$. Thus, $uu$ households are borrowing-constrained whenever:

$$u'(\delta + \gamma_t) > \rho \beta E_t \left( \frac{u'(\delta + \gamma_{t+1})}{1 + \pi_{t+1}} \right) + (1 - \rho) \beta E_t \left( \frac{\phi}{1 + \pi_{t+1}} \right). \tag{24}$$

We now turn to $eu$ households. Their current consumption is given by Eq. (17), so that $\eta_t^i = u'(m_{t-1}^e / (1 + \pi_t) + \delta + \gamma_t)$, while, just like $uu$ households, they will be either $uu$ or $ue$ households in the following period. Thus, $eu$ households are borrowing-constrained whenever:

$$u'\left( \frac{m_{t-1}^e}{1 + \pi_t} + \delta + \gamma_t \right) > \rho \beta E_t \left( \frac{u'(\delta + \gamma_{t+1})}{1 + \pi_{t+1}} \right) + (1 - \rho) \beta E_t \left( \frac{\phi}{1 + \pi_{t+1}} \right). \tag{25}$$

If (23) holds then (25) is more stringent than (24), so (25) is a sufficient condition for both $uu$ and $eu$ households to be borrowing-constrained. We show in the Appendix that when mean inflation is non-negative and $\delta$ lies inside a range $(\delta_-, \delta_+)$, where $0 \leq \delta_- < \delta_+$, then both (23) and (25) hold for all $t \geq 0$, provided that aggregate shocks have sufficiently

small support. Intuitively, for our equilibrium to exist home production must be sufficiently productive to deter unemployed households from saving, whilst at the same time being sufficiently unproductive to induce positive precautionary savings by employed households.

# 4    Incomplete markets and short-run nonneutrality

## 4.1    An output-inflation tradeoff equation

We can now derive the solution dynamics of the closed-form equilibrium. Using Eqs. (10), (11), (16) and (22), we can summarise the dynamic behavior of the economy by a single forward-looking equation, i.e.,

$$
m_t^e = \alpha\beta E_t\left(\frac{m_{t+1}^e}{\tau_{t+1}}\right) + \frac{(1-\alpha)\,\beta}{\phi}E_t\left(\frac{m_{t+1}^e}{\tau_{t+1}}u'\left(\delta + \frac{m_{t+1}^e\left(U + (1-U)\,\tau_{t+1}\right)}{\tau_{t+1}}\right)\right). \tag{26}
$$

Eq. (26) determines the equilibrium dynamics of real money balances held by employed households, $\{m_t^e\}$, as a function of the (exogenous) money growth sequence $\{\tau_t\}$.

In order to examine the redistributive effect of inflation shocks in isolation, it is convenient to start by focusing on the effect of i.i.d. money growth shocks on aggregates (auto-correlated shocks are introduced in the next Section). From now on, we further assume that mean money growth is positive (i.e., $\tau > 0$), and that shocks have small bounded support (so that $\tau_t \geq 0\ \forall t$). Then, in first approximation the solution to (26) is the following constant path for $m_t^e$:[3]

$$
m_t^e = m^e = \frac{1+\pi}{1+(1-U)\,\pi}\left(u'^{-1}\left(\frac{(1+\pi-\alpha\beta)\,\phi}{(1-\alpha)\,\beta}\right) - \delta\right), \tag{27}
$$

where   unindexed variables denote steady state values (all of these are summarised in the Appendix). Two properties of $m^e$ are worth mentioning at this stage. First, $m^e$ falls with $\alpha$, as lower idiosyncratic unemployment risk reduces employed households' incentives to self-insure against this risk. Second, under our maintained assumption that $\sigma\,(c) \leq 1$, $m_t^e$ falls with $\pi$: as inflation increases, the return to holding real balances decreases and money becomes less valuable as a self-insurance device against idiosyncratic unemployment shocks.[4]

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[3]This can be checked by linearising (26) around steady state real balances and money growth, $(m^e, \tau)$, and solving the equation obtained forwards. That $\sigma\,(c) \leq 1$ implies that the equilibrium is unique and non-cyclical, while i.i.d shocks preclude time-variations in real balances.

[4]A sufficient condition for $\partial m^e/\partial\,(1+\pi) < 0$ is that $\partial\,(1+\pi)\,u'^{-1}\left((1+\pi-\alpha\beta)\,\phi/\,(1-\alpha)\,\beta\right)/\partial\,(1+\pi) <$

We can now turn to the effects of inflation shocks on the labour supplies of $ee$ and $eu$ agents and on market output. Substituting (10), (22) and (27) into (18)–(19), we obtain:

$$l_t^{ee} = u'^{-1}(\phi) + m^e U \pi_t / (1 + \pi_t) \ (> 0), \tag{28}$$

$$l_t^{ue} = u'^{-1}(\phi) + m^e (1 + U \pi_t) / (1 + \pi_t) \ (> 0), \tag{29}$$

Note that $l_t^{ee}$ rises while $l_t^{ue}$ falls as $\pi_t$ increases. After an inflation shock, the households who pay the inflation tax in period $t$ are those who hold money at the beginning of period $t$ ($ee$ and $eu$ households), while the households who benefit from the corresponding inflation subsidy are those who do not hold money at the beginning of period $t$ ($ue$ and $uu$ households). Consequently, $ee$ households are hurt by the shock and increase their labour supply to maintain their desired levels of consumption and money wealth, while $ue$ households can afford to work less than they would have had the shock not occurred. Now, substituting (28)–(29) into market output, $y_t = \omega^{ee} l_t^{ee} + \omega^{ue} l_t^{ue}$, we may rewrite the latter as:

$$y_t = (1 - U) u'^{-1}(\phi) + (1 - U) m^e \times \frac{U \pi_t + 1 - \alpha}{1 + \pi_t}. \tag{30}$$

Market output increases with current inflation (i.e., greater labour supply by $ee$ households dominates the lower supply of $ue$ households) provided that $\alpha + U > 1$, or, equivalently from (1), that $\alpha + \rho > 1$ (that is, the 'average' persistence in employment status must be sufficiently high) For small shocks, the latter equation can be approximated by the following linear 'ouput-inflation tradeoff' relation:

$$y_t = y + \mu (\pi_t - \pi), \tag{31}$$

where
$$\mu = \frac{U + \alpha - 1}{(1 + \pi)(1 + (1 - U)\pi)} \left( u'^{-1} \left( \frac{(1 + \pi - \alpha\beta)\phi}{(1 - \alpha)\beta} \right) - \delta \right). \tag{32}$$

This tradeoff equation is reminiscent of those derived by Lucas (1973) or Ball *et al.* (1988); the underlying mechanism that we emphasise here works very differently, however. In Lucas, agents raise production after an inflationary money shock because they cannot

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0. Since $u'^{-1}((1 + \pi - \alpha\beta)\phi / (1 - \alpha)\beta) = c^{eu}$ (see the Appendix), this condition may be written as:

$$c^{eu} + (1 + \pi)\phi / (1 - \alpha)\beta u''(c^{eu}) < 0,$$

or, after rearranging, $\sigma(c^{eu}) < (1 + \pi) / (1 + \pi - \alpha\beta)$. This is always true since $\sigma(c) \leq 1 \ \forall c$ by assumption

fully disentangle changes in relative prices from variations in the general price level; in Ball *et al.*, the output-inflation tradeoff naturally arises from nominal rigidities. In contrast, our model features perfect information, fully flexible prices, but heterogenous cash balances. Consequently, lump-sum monetary injections redistribute wealth from cash-rich households to cash-poor ones, thereby inducing employed households to alter their labour supplies in order to offset the implied wealth effects. Interestingly, the model predicts that higher trend inflation lowers the impact of inflation shocks on output (i.e., $\partial \mu / \partial \pi < 0$), because it lowers money holdings by employed households and thus mitigates the redistributive effects of these shocks. We may thus conclude that this negative relation is perfectly compatible with price flexibility, contrary to the claim by Ball *et al.* (1988) that it supports the hypothesis of nominal rigidities. We summarise the results obtained so far in the following proposition:

**Proposition 1**. Steady state real money holdings by employed households, $m^e$, increase with idiosyncratic unemployment risk, $1 - \alpha$, and decrease with mean inflation, $\pi$. With i.i.d. money growth shocks and $U + \alpha > 1$ (or, equivalently, $\alpha + \rho > 1$), inflationary shocks raise current output, $y_t$, the effect being stronger the lower is mean inflation.

## 4.2   Persistent money growth shocks

Central to the transmission of monetary shocks here is the rôle, and determinants, of real money holdings held by the employed as a buffer against idiosyncratic unemployment risk. Under i.i.d money growth shocks, these holdings are constant over time as they are immediately and entirely repleted by employed households (through variations in labour supply) following a shock that redistributes current wealth. Obviously, this simple adjustment to exogenous disturbances is complicated if real money demand is itself affected by the current shock. In our framework, this precisely occurs when money growth shocks display persistence; in this case, a relatively high current inflation tax on employed households *future* high future inflation taxes, thereby lowering the desirability of money as a means of self-insurance and reducing households' incentives to supply labour to acquire it. This intertemporal effect induced by the current shock thus runs counter the effect induced by intratemporal wealth redistribution, presumably limiting, or even reverting, the effect of money shocks on total labour supply and market output. To illustrate this point, let us now assume that money

growth obeys the following AR(1) process:

$$\tau_t = (1 - \chi)\tau + \chi\tau_{t-1} + \epsilon_t, \tag{33}$$

where $\chi \in (0, 1)$ and $\{\epsilon_t\}_{t=0}^{\infty}$ is a white noise process with mean zero and small bounded support. Linearising (26) around the steady state, we obtain:

$$\hat{m}_t^e = AE_t\left(\hat{m}_{t+1}^e\right) - BE_t\left(\hat{\tau}_{t+1}\right), \tag{34}$$

where hated values denote proportional deviations from steady state (e.g., $\hat{m}_t^e = (m_t^e - m^e)/m^e$) and $A$, $B$ are the following constants:

$$A = 1 - \frac{(1 + \pi - \alpha\beta)(1 - \delta/c^{eu})\sigma(c^{eu})}{1 + \pi} \in (0, 1),$$

$$B = 1 - \frac{(1 + \pi - \alpha\beta)(1 - \delta/c^{eu})\sigma(c^{eu})}{1 + \pi}\left(\frac{U}{1 + (1 - U)\pi}\right) \in (A, 1).$$

Then, iterating (34) forwards under the transversality condition (8), and using (33), gives:

$$\hat{m}_t^e = -\left(\frac{B\chi}{1 - A\chi}\right)\hat{\tau}_t, \tag{35}$$

where $B\chi/(1 - A\chi) > 0$. Equation (35) summarises the effect of current money growth on current real balances working through changes in *expected* money growth, both relative to steady state. To the extent that higher-than-steady state money growth forecasts high future money growth (that is, whenever $\chi > 0$), then it also forecasts high future inflation taxes that discourage current real money accumulation. How do such adjustments in the demand for real balances modify the labour supplies of employed households and implied market output? First, use equations (10), (22) and (18)–(19) again to write the market output equation (30) in the following slightly more general form:

$$y_t = (1 - U)u'^{-1}(\phi) + (1 - U)m_t^e\left(\frac{U(\tau_t - 1) + 1 - \alpha}{\tau_t}\right). \tag{36}$$

Persistent money growth shocks lower $m_t^e$ (because of the future inflation taxes on real money), but raise $(U(\tau_t - 1) + 1 - \alpha)\tau_t^{-1}$ provided that $U + \alpha > 1$ (through contemporaneous wealth redistribution). The actual slope of the tradeoff thus ultimately depends on the relative strengths of these two effects. Linearising equation (36) and using (35), we obtain the following results.

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**Proposition 2**. Assume that money growth, $\tau_t$, follows an AR(1) process with autocorrelation parameter $\chi \in (0,1)$. Then, the higher $\chi$, the lowers is the impact of monetary shocks on output, while a necessary and sufficient condition for these shocks to raise output is:

$$\frac{U + \alpha - 1}{U\pi + 1 - \alpha} > \frac{B\chi}{1 - A\chi}$$

Whether the latter condition holds or not ultimately depends on the deep parameters that enter both sides of the inequality. When $\chi \to 0$, the analysis of the previous Section applies and monetary shocks raise current output (provided that $U + \alpha - 1 > 0$). As $\chi$ is increased, the intertemporal effect gains importance and lowers the impact of shocks, possibly (but not necessarily) leading to a reversal in the slope of the tradeoff for large values of $\chi$. Finally, it is straightfoward to show that sufficiently high values of mean inflation always lead to the violation of this condition, as they tend to mitigate the intratemporal redistributive effects of shocks.

For sake of illustration, Figure 1 displays the dynamic effects of a persistent money growth shock on monetary and aggregate supply variables (first and second row, respectively). We set $\beta = 0.99$, $\chi = 0.6$, and $\tau = 1.005$ (the time perid is to be interpreted as a quarter). There are two ways of interpreting $U$ in the context of our model. Strictly speaking, it refers to the unemployment rate. However, the central mechanism underlying the nonneutrality of money here is the redistribution of wealth from cash-rich asset holders to cash-poor, borrowing-constrained agents. Since many real world employed are borrowing-constrained due to low labour income, we interpret $U$ as the share of borrowing-constrained households in the economy and set it to 20%, following Jappelli (1990). We set $\alpha = 0.95$ (so that the share of cash-holding households who will meet the borrowing constraint in the next quarter is 5%) and $\delta = 0.9c^{eu}$. (These parameters ensure that $U + \alpha > 1$, which is necessary for money shocks to raise output, and that the existence conditions in Section 3.2 are satisfied.)

# 5    Some welfare considerations

Since the nonneutrality mechanism described in this paper relies on wealth redistribution (both at the time of the shock and in the future), it directly affects the welfare of every single agent. Obviously, there potential are loosers and winners resulting from wealth redistribution, meaning that we should not expect monetary shocks to unambiguously lead to better
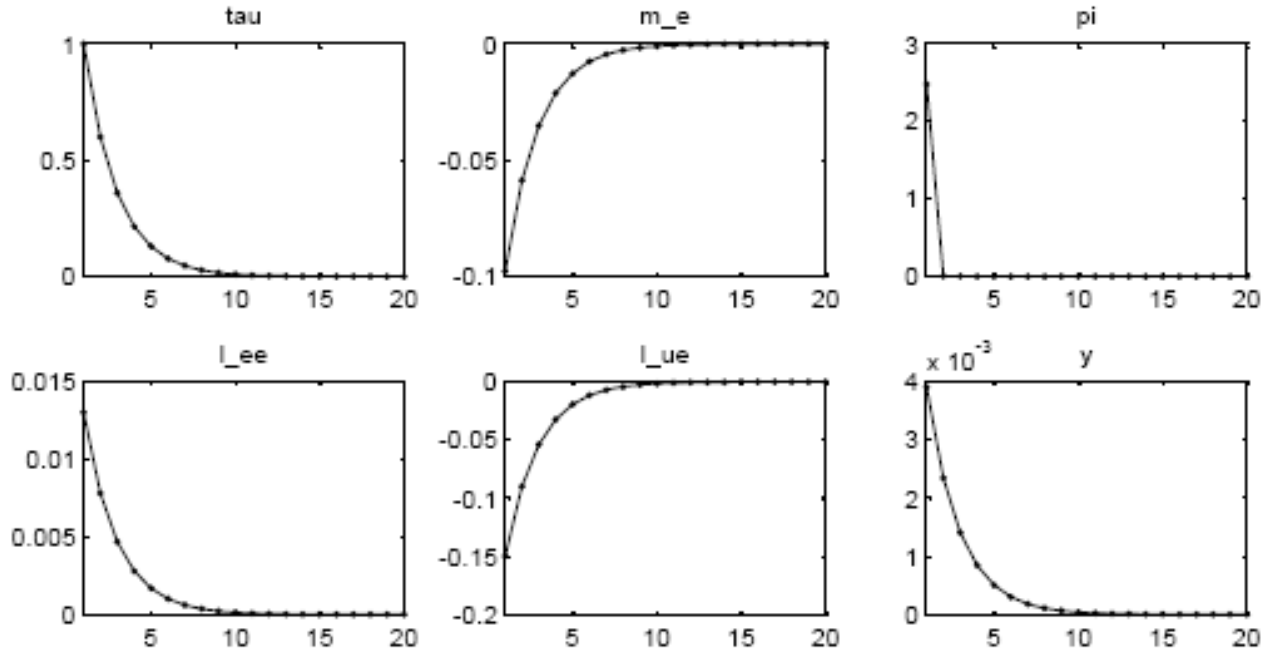
Figure 1: Dynamic responses to a money growth shock

or worse dynamic equilibria in the Pareto sense. Who gains and who looses following money growth shocks? It may seem at first sight that households who benefit from the inflation tax at the time of the shock ($uu$ and $ue-$households, i.e., those who hold no cash at the beginning of the period) always see their utility increase, while those who pay for this tax ($ee$ and $eu-$households, who are cash-rich at the beginning of the period) necessarily experence a welfare loss. But this reasonning is only valid when money growth is i.i.d. but no longer holds when they display auto-correlation. In the latter case indeed, the ongoing transitions of households accross employment status implies that todays' winners may be tomorrow's loosers, so that the effect of the current shock on the total expected utility of a particular household is of ambiguous sign.

To understand this point further, note first that all time-$t$ variables can be expressed as a function of the only state variable of the model, current money growth $\tau_t$, and that $\tau_{t+1}$ only depends on $\tau_t$. Call $W^i(\tau_t)$ the value function of agent $i$ when current money growth is $\tau_t$ and $W^i_\tau = \partial W^i(\tau_t)/\partial \tau_t$ its (time invariant) first derivative. Then, given the transition probabilities across employment status and the fact that $\partial \tau_{t+1}/\partial \tau_t = \chi$, taking the first

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derivative of the Bellman equations associated with each agent type gives:

$$W_\tau^{ee} = -\phi \frac{\partial l_t^{ee}}{\partial \tau_t} + \alpha\beta\chi W_\tau^{ee} + (1-\alpha)\beta\chi W_\tau^{eu},$$

$$W_\tau^{ue} = -\phi \frac{\partial l_t^{ue}}{\partial \tau_t} + \alpha\beta\chi W_\tau^{ee} + (1-\alpha)\beta\chi W_\tau^{eu},$$

$$W_\tau^{eu} = \frac{\partial u\left(c_t^{eu}\right)}{\partial \tau_t} + \rho\beta\chi W_\tau^{uu} + (1-\rho)\beta\chi W_\tau^{ue},$$

$$W_\tau^{uu} = \frac{\partial u\left(c_t^{uu}\right)}{\partial \tau_t} + \rho\beta\chi W_\tau^{uu} + (1-\rho)\beta\chi W_\tau^{ue}.$$

The solution to this system expresses the four value function's first derivatives as (cumbersome) functions of $\partial l_t^{ee}/\partial \tau_t$, $\partial l_t^{ee}/\partial \tau_t$, $\partial u\left(c_t^{eu}\right)/\partial \tau_t$, $\partial u\left(c_t^{uu}\right)/\partial \tau_t$, which themselves depend on the deep parameters of the model. When $\chi = 0$ (the limiting i.i.d. case), the welfare responses to the current shock are just given by the responses of households' current labour supplies and consumption demands, and it is then easy to show that $W_\tau^{ee} < 0$, $W_\tau^{ue} > 0$, $W_\tau^{eu} < 0$ and $W_\tau^{uu} > 0$. However, when $\chi > 0$, one may construct examples where some of these signs are reverted. For sake of illustration, compute the limit of $W_\tau^{ee}$ as $\alpha = 0.5$ and $\rho \to 1$ and $\chi \to 1$ (that is, a situation where currently employed households who pay for the inflation tax at the time of the shock are likely to benefit from it in the future). From the above system, we get:

$$\lim_{(\rho,\chi)\to(1,1)} W_\tau^{ee} = -\left(\frac{2\phi}{2-\beta}\right)\frac{\partial l_t^{ee}}{\partial \tau_t} + \left(\frac{1}{2-\beta}\right)\frac{\partial u\left(c_t^{eu}\right)}{\partial \tau_t} + \left(\frac{\beta^2}{1-\beta}\right)\frac{\partial u\left(c_t^{uu}\right)}{\partial \tau_t}.$$

It is easy to show that $\partial l_t^{ee}/\partial \tau_t$ is positive but bounded above, that $\partial u\left(c_t^{eu}\right)/\partial \tau_t$ is negative but bounded below, and that $\partial u\left(c_t^{uu}\right)/\partial \tau_t$ is positive and bounded above. Thus, if $ee$-households are sufficiently patient (i.e., $\beta^2/(1-\beta)$ is sufficiently large), $W_\tau^{ee}$ reverts sign as $(\rho, \chi) \to (1, 1)$ and these households actually benefit, rather than suffer, from a positive money growth shock.

Another potentially relevant welfare criterion is the one that would be followed by a benevolent social planner who would give equal weight to every household's utility. Can this latter one overcome the limitations of the Pareto criterion and yield unambiguous results as to the effects of money growth shocks? To answer this question, first use equations (10), (14), (17), (18)–(19) and (22) above to compute the discounted weighted sum of households'

utilities as follows:

$$W^{sp}(\tau_t) = \sum_{k=0}^{\infty} \beta^{t+k} \sum_{j,l=e,u} \omega^{jl} \left( u(c_{t+k}^{jl}) - \phi l_{t+k}^{jl} \right)$$

$$= \Omega + \sum_{k=0}^{\infty} \beta^{t+k} \left( \omega^{eu} u \left( c_{t+k}^{eu} \right) + \omega^{uu} u \left( c_{t+k}^{uu} \right) - \phi y_{t+k} \right),$$

where $\Omega$ is a constant and where the $y_{t+k}$s terms summarise the welfare losses incurred by employed agents through higher work effort. Writting the Bellman equation associated with this welfare criterion and taking its derivative with respect to current money growth, we get:

$$\frac{\partial W_t^{sp}}{\partial \tau_t} = \left( \frac{1}{1 - \beta \chi} \right) \left( \omega^{eu} \frac{\partial u \left( c_t^{eu} \right)}{\partial \tau_t} + \omega^{uu} \frac{\partial u \left( c_t^{uu} \right)}{\partial \tau_t} - \phi \frac{\partial y_t}{\partial \tau_t} \right).$$

Here again, we find that the sign of $\partial W_t^{sp} / \partial \tau_t$ depends on all deep parameters of the model. Thus, under both criteria the welfare effects of money growth shocks are ambiguous.

# 6  Conclusion

This papers has uncovered the short-run implications of a simple Bewley-type monetary model with idiosyncratic labour income risk as to the dynamic and welfare effects of monetary shocks. A prerequisite to the derivation of our results was the construction of, and then the focus on, a closed-form equilibrium with limited heterogeneity (both in terms of wealth and agents types) and which may be of independent interest.[5] We have shown that money growth shocks that contemporary redistribute real money wealth across agents tend to raise output, unless this direct effect is conterbalanced by the (indirect) effect of future redistribution on the real demand for cash. Since monetary innovation are inflationary (at least at the time of the shock), our model thus tends to generate the positive output-inflation relation that has repeatedly been observed in the data. Finally, the fact wealth is redistributed both at the time of the shock and in the future (provided that money growth variations are persistent), combined with the fact that households alternate employment status and thus cash holding levels, implies that the welfare effects of monetary shocks are in general ambiguous.

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[5]Kehoe and Levine (2001) have emphasised the inherent difficulty of analysing 'liquidity constrained' Bewley models with both indiosyncratic and aggregate uncertainty, due to the very large number of agent types that they typically generate. Our closed-form equilibrium aims to provide a partial answer to this concern.

# Appendix A: Steady state of the model

We use variables without time indexation to indicate steady state values. From Eqs. (10)-(11), steady state inflation and real transfers are $1 + \pi = \tau \; (> 1)$ and $\gamma = m\pi/(1 + \pi)$, respectively. Substituting these values into (16) and using (22), we find that steady-state real money holdings by employed households, $m^e$, are:

$$m^e = \frac{1 + \pi}{1 + (1 - U)\,\pi} \left( u'^{-1} \left( \frac{(1 + \pi - \alpha\beta)\,\phi}{(1 - \alpha)\,\beta} \right) - \delta \right).$$

The values of $c^{uu}$, $c^{eu}$, $l^{ee}$, $l^{ue}$ and $y$ can then be derived straightforwardly. For example,

$$c^{eu} = u'^{-1} \left( \frac{(1 + \pi - \alpha\beta)\,\phi}{(1 - \alpha)\,\beta} \right). \tag{A1}$$

Since we are considering fluctuations occurring arbitrarily close to the steady state, a sufficient condition for our closed-form solution to be an equilibrium is that both (23) and (25) hold with strict inequalities in steady state. From (27), the first condition is simply:

$$\delta < u'^{-1} \left( \frac{(1 + \pi - \alpha\beta)\,\phi}{(1 - \alpha)\,\beta} \right) \equiv \delta_+.$$

In steady state, the left hand side of (25) is $c^{eu}$. Using (A1), inequality (25) becomes:

$$\frac{(1 + \pi)\,(1 + \pi - \alpha\beta)}{(1 - \alpha)\,\beta} - (1 - \rho)\,\beta > \frac{\rho\beta}{\phi} u'\,(\delta + \gamma). \tag{A2}$$

In steady state, $\gamma = (1 - U)\,m^e\pi/(1 + \pi)$ (see Eqs. (10) and (22)). Substituting $\gamma$ into (A2), using Eq. (27) and rearranging, we may rewrite the latter inequality as:

$$\frac{(1 + \pi)\,(1 + \pi - \alpha\beta)}{(1 - \alpha)\,\beta} - (1 - \rho)\,\beta >$$
$$\frac{\rho\beta}{\phi} u' \left( \frac{\delta}{1 + (1 - U)\,\pi} + \frac{(1 - U)\,\pi}{1 + (1 - U)\,\pi} u'^{-1} \left( \frac{(1 + \pi - \alpha\beta)\,\phi}{(1 - \alpha)\,\beta} \right) \right). \tag{A3}$$

The left-hand side of (A3) is positive at $\pi = 0$ and thus for all $\pi > 0$. The right hand side of (A3) is decreasing and continuous in $\delta$ over $[0, \infty)$. Thus, if (A3) holds when evaluated at $\delta = \delta_+$, then by continuity there exists $\delta_- < \delta_+$ such that (A3) holds for all $\delta > \delta_-$. Setting $\delta = \delta_+$ in (A3) and rearranging, we find:

$$(1 + \pi - \rho\beta)\,(1 + \pi - \alpha\beta) - (1 - \rho)\,(1 - \alpha)\,\beta^2 > 0,$$

which is always true when $\pi > 0$ because the left-hand side increases with $\pi$ and is positive at $\pi = 0$.

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