Inconsistencies, Negations and Changes in Ontologies

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Abstract

Ontology management and maintenance are considered cornerstone issues in current Semantic Web applications in which semantic integration and ontological reasoning play a fundamental role. The ability to deal with inconsistency and to accommodate change is of utmost importance in real-world applications of ontological reasoning and management, wherein the need for expressing negated assertions also arises naturally. For this purpose, precise, formal definitions of the the different types of inconsistency and negation in ontologies are required. Unfortunately, ontology languages based on Description Logics (DLs) do not provide enough expressive power to represent axiom negations. Furthermore, there is no single, well-accepted notion of inconsistency and negation in the Semantic Web community, due to the lack of a common and solid foundational framework. In this paper, we propose a general framework accounting for inconsistency, negation and change in ontologies. Different levels of negation and inconsistency in DL-based ontologies are distinguished. We demonstrate how this framework can provide a foundation for reasoning with and management of dynamic ontologies.

Keywords: Negation, Inconsistency, Ontology Change, Semantic Web

Introduction

The ability to deal with inconsistency and to accommodate change is of utmost importance in real-world applications of Description Logic based ontological reasoning and management (Baader *et al.* 2003; Horrocks, Sattler, & Tobies 2000). For example, one of the typical scenarios in deployed Semantic Web applications is ontology reuse, where users build their own ontologies from existing ones, rather than starting from scratch. After adding new axioms into an existing ontology, users may find that revised ontologies become inconsistent. A remedy for such a situation would require the removal of a minimal part of the ontology in order to make the resulting ontology consistent (Haase *et al.* 2005). This type of change is usually required to meet some rationality postulates, similar to those in the AGM theory in the belief revision (Alchourrón, Gärdenfors, & Makinson 1985). Another example is reasoning with inconsistent ontologies (Huang, van Harmelen, & ten Teije 2005), where querying systems should return meaningful answers to queries on inconsistent ontologies. The latter suffers from "entailment explosion" as any formula is a consequence of an inconsistent logical theory.

Addressing effectively the issues raised in these examples requires precise, formal definitions of inconsistency and negation. Unfortunately, DL-based ontology languages, such as OWL DL (Patel-Schneider, Hayes, & Horrocks 2004), do not provide enough expressive power to represent axiom negations. Furthermore, there is no single, well-accepted notion of inconsistency and negation in the Semantic Web community, due to the lack of a common and solid foundational framework. (Schlobach & Cornet 2003) proposed an approach to debug inconsistent ontologies, in which inconsistency is identified with the existence of unsatisfiable concepts. (Huang, van Harmelen, & ten Teije 2005) developed a framework of reasoning with inconsistent ontologies, in which inconsistency is given a classical first-order logic interpretation. In (Haase *et al.* 2005), the definition of axiom negation is merely mentioned in an example at a footnote, without proper discussion in the paper.

In this paper, we propose a general framework accounting for inconsistency, negation and change by which we aim at providing a unique foundation of inconsistency and change processing for DL-based ontologies. We distinguish different levels of inconsistency and negation in DL-based ontologies, and investigate the relationship among the different notions. Accordingly, we lay the foundations of a formal theory of ontology change, based on a set of rationality postulates inspired by the AGM theory of belief change. Furthermore, we discuss how this proposed framework can provide a foundation for the tasks of ontology management and reasoning. Specifically, we show how a bridge connecting two main ontology change operations - revision and contraction - can be built under the proposed framework.

The rest of this paper is organized as follows: DL-based ontologies and the AGM theory are briefly introduced in the next section. Section **Inconsistencies and Negations** distinguishes different notions of negation and inconsistency in DLs. Section **Postulates for Ontology Change** investigates their application to ontology reasoning and management for accommodating ontology changes. The final section summarizes the paper and discusses further research directions.

Preliminaries

Ontologies An *ontology* (Uschold & Gruninger 1996) typically consists of a hierarchical description of important concepts in a domain, along with descriptions of the properties of each concept, and constraints on these concepts and properties. In this paper, following the W3C Web Ontology language OWL (Patel-Schneider, Hayes, & Horrocks 2004), we consider Description Logics (DLs) based ontologies. Description Logics are a family of class-based (concept-based) knowledge representation formalisms, equipped with well-defined model-theoretic semantics (Baader *et al.* 2003). The $\mathcal{SHOIN}(\mathbf{D}^+)$ DL underpins OWL DL, the key sub-language of OWL.

Let $\mathcal{K}$ be a Description Logic, C, D $\mathcal{K}$-concepts, R, S $\mathcal{K}$-roles, and $\mathtt{a}, \mathtt{b}$ individuals. An *interpretation* (written as $\mathcal{I}$) of an ontology consists of a *domain* $\Delta^\mathcal{I}$ (a nonempty set), and an *interpretation function* (written as $\cdot^\mathcal{I}$), which maps each individual name $\mathtt{a}$ to an element $\mathtt{a}^\mathcal{I} \in \Delta^\mathcal{I}$, each concept name CN to a subset $CN^\mathcal{I} \subseteq \Delta^\mathcal{I}$ of the domain and each role name RN to a binary relation $RN^\mathcal{I} \subseteq \Delta^\mathcal{I} \times \Delta^\mathcal{I}$. The interpretation function can be extended to give semantics to $\mathcal{K}$-concepts and $\mathcal{K}$-roles, which are concepts and role descriptions built by $\mathcal{K}$-constructors. Example concept constructors of $\mathcal{SHOIN}(\mathbf{D}^+)$ are $\neg C, C \sqcap D, C \sqcup D, \exists R.C, \forall R.C, \geqslant nR, \leqslant nR$ and $\{\mathtt{a}\}$ (where n is a natural number). A $\mathcal{K}$-ontology (or simply ontology) O is a finite set of axioms of the following forms:[1] concept inclusion axioms $C \sqsubseteq D$, transitivity (abstract) role axioms $\mathsf{Trans}(R)$, role inclusion axioms $R \sqsubseteq S$, concept assertions $C(\mathtt{a})$, role assertions $R(\mathtt{a}, \mathtt{b})$ and individual (in)equalities $\mathtt{a} \approx \mathtt{b}$ ($\mathtt{a} \not\approx \mathtt{b}$, respectively). In an ontology, we use *TBox* (*RBox*, *ABox*) to refer to the set of concept (role, individual, respectively) axioms. An interpretation $\mathcal{I}$ satisfies the concept inclusion axiom $C \sqsubseteq D$ if $C^\mathcal{I} \subseteq D^\mathcal{I}$. Due to the limitation of space, the reader is referred to (Baader *et al.* 2003) for more details of the semantics of DL constructors and axioms. An interpretation $\mathcal{I}$ satisfies an ontology O iff $\mathcal{I}$ satisfies all its axioms. An ontology O is *consistent* iff it has an interpretation. A concept C is *satisfiable* w.r.t. O iff there exists an interpretation $\mathcal{I}$ of O s.t. $C^\mathcal{I} \neq \emptyset$. A concept C is subsumed by a concept D w.r.t. O iff, for every interpretation $\mathcal{I}$ of O, we have $C^\mathcal{I} \subseteq D^\mathcal{I}$. Given an axiom φ, an ontology O *entails* φ, written as $O \models \varphi$, iff, for all interpretations $\mathcal{I}$ of O, we have $\mathcal{I}$ satisfies φ. An ontology O_1 entails an ontology O_2, written as $O_1 \models O_2$, iff, for all interpretations $\mathcal{I}$ of O_1, we have $\mathcal{I}$ satisfies O_2.

Given a (monotonic) Description Logic $\mathcal{K}$, we consider a pair $\langle L, Cn\rangle$, where L is the set of possible $\mathcal{K}$-axioms and Cn is a consequence operator such that, given a $\mathcal{K}$-ontology O, $Cn(O) = \{\varphi \mid O \models \varphi\}$. In the rest of the paper, we will use $\langle L, Cn\rangle$ (or $\langle L^\mathcal{K}, Cn\rangle$ when necessary) to refer to the Description Logic $\mathcal{K}$. $\langle L, Cn\rangle$ is a very general model introduced by Tarski in 1928; to guarantee rationality, Tarski required that Cn satisfies *iteration*, *inclusion* and *monotony*; see (Fuhrmann 1991).

[1]The kinds of role axioms that can appear in O depend on the expressiveness of $\mathcal{K}$.

AGM Theory and its Variations The theory of Alchourrón, Gärdenfors and Makinson 1985 — the *AGM theory* — is probably the most influential work in the area of belief change. This theory sets the foundations for future research on belief change, by defining a set of widely accepted properties that any rational operators should satisfy.

More specifically, AGM studied 3 different operators, namely *expansion*, *revision* and *contraction*. Expansion is the addition of a sentence to a knowledge base (KB), without taking any special provisions for maintaining consistency; revision is similar, with the important difference that the result should be a consistent set of beliefs; contraction is required when one wishes to consistently remove a sentence from their beliefs instead of adding one. AGM introduced a set of postulates for revision and contraction that formally describe the properties that such an operator should satisfy (expansion was skipped, as it is trivial).

The AGM theory is based on the *coherence model*. In practice, this model states that both the explicitly represented knowledge and the implied knowledge are of equal value and should be considered when deciding the changes to be made upon the KB. In the context of ontologies, however, it seems more natural to use the *foundational model*, under which there is a clear distinction between the explicitly represented knowledge (i.e., the one contained in the KB) and the implicit one (i.e., knowledge implied by the explicitly represented one). Under this model, changes can be made in the explicit knowledge only; implicit knowledge can only be indirectly affected through changes in the explicit knowledge.

The foundational model greatly restricts our options for a "proper" modification of knowledge. This fact was verified in (Fuhrmann 1991), in which an attempt to define a foundational version of the AGM theory was made. There it was shown that, in the logics originally considered by AGM, no contraction operator can be defined that satisfies the foundational version of the AGM postulates.

A second problem related to the application of the AGM theory in the DL context is caused by the assumptions made by AGM in the formulation of their theory: even though the intuition behind the AGM postulates is independent of the logic used for the representation of the KB, the formulation of the postulates themselves is based on certain assumptions, disallowing their direct use in logics such as DLs (Flouris, Plexousakis, & Antoniou 2004). For example, well known DLs do not provide enough expressive power to represent negations of all the axioms. This fact is both a curse and a blessing. On the one hand, it implies that the AGM theory cannot be directly applied to DLs; on the other hand, if we could reformulate the AGM theory in a more general context, then the result of (Fuhrmann 1991) might not be applicable in DLs, as they do not satisfy the AGM assumptions.

This problem was originally addressed in (Flouris, Plexousakis, & Antoniou 2004), where the AGM theory (and postulates) were recast so as to be applicable in a wider class of logics, which includes DLs. That work studied the AGM theory under both the coherence and the foundational model, but was restricted to the operation of contraction only. It was shown that there are certain conditions under which a logic admits a contraction operator satisfying the AGM postulates in each of the two paradigms (coherence, founda-

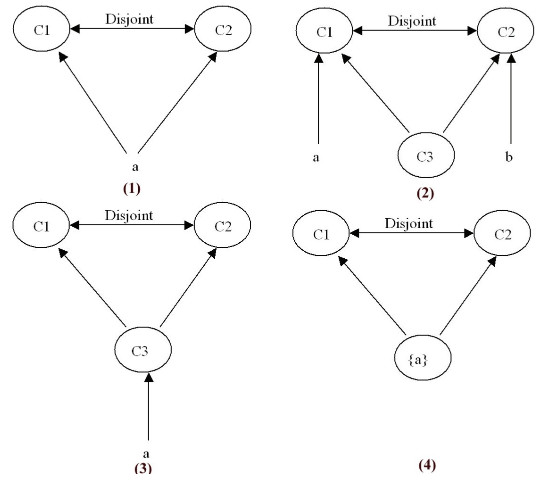

Figure 1: Examples of variant inconsistency and incoherence.

tional). Such logics were termed *AGM-compliant* and *base-AGM-compliant*, respectively.

In this paper, we focus on the foundational model; we will show that the conditions introduced in (Flouris, Plexousakis, & Antoniou 2004) for a base-AGM-compliant logic are too restrictive, overruling practically all interesting DLs. Following this observation, we propose a weakening of the AGM postulates which is applicable in our context (DLs under the foundational model) and present some ideas on the operation (and postulates) of revision and its interrelationship with contraction.

Inconsistency and Negation

Different notions of inconsistency in DLs have been used in the Semantic Web community, as we have discussed them in the **Introduction**. In this section we define different notions of inconsistency and examine their relations. We start from the most primitive inconsistency, i.e., the unsatisfiability of a single concept.

Definition 1 (Unsatisfiable Concept) *A named concept C in the ontology O is unsatisfiable iff, for each interpretation $\mathcal{I}$ of O, $C^{\mathcal{I}} = \emptyset$.*

That would lead us to consider the kinds of ontologies with unsatisfiable concepts.

Definition 2 (Incoherent Ontology) *An ontology O is incoherent iff there exists an unsatisfiable named concept in O.*

The incoherence can be considered as a kind of the inconsistency in the TBox, i.e. the terminology part, of an ontology. An incoherent ontology has an incoherent TBox. However, the incoherence does not provide the classical sense of the inconsistency because there might exist a model for an incoherent ontology. Thus, we need the classical inconsistency for ontologies.

Definition 3 (Inconsistent Ontology) *An ontology is inconsistent iff it has no interpretation.*

We now briefly discuss the relationships of the two kinds of inconsistencies of ontologies. Firstly, an ontology is inconsistent does not necessarily imply that it is incoherent, and vice versa. There exist different combinations of the inconsistency and the incoherence. Figures 1 presents several examples to show the variants of inconsistency and incoherence. Figure1(1) is an example of inconsistent but coherent ontology, in which the two disjoint concepts $C1$ and $C2$ share an instance a. Figure1(2) is an example of consistent but incoherent ontology, in which the two disjoint concepts $C1$ and $C2$ share a sub-concept $C3$. Figure1(3) is an example of an inconsistent and incoherent ontology, in which the two disjoint concepts $C1$ and $C2$ share a sub-concept $C3$, which has an instance a. Figure1(4) is an example of inconsistent but coherent TBox, in which the two disjoint concepts $C1$ and $C2$ share a sub-concept which is a nominal $\{a\}$.

Secondly, coherence and consistency are somehow related. We can introduce a fresh individual i_C for each named concept C in an ontology O. Accordingly, an enhanced ontology $O^+ = O \cup \{C(i_C) \mid \text{for all named concepts } C \text{ in } O\}$ can be constructed by adding these individual axioms about these fresh individuals into the ontology. It is easy to see that the following propositions hold:

Proposition 1 *(a) Given an ontology O, if its enhanced ontology O^+ is consistent, then O is coherent.*
(b) Given a consistent ontology O, if O is coherent, then its enhanced ontology O^+ is consistent.

Axiom Negation in Ontologies

Negated axioms are closely related to inconsistencies and changes in ontologies. They are one of the main sources of ontology inconsistencies. For example, an ontology containing the mutually negated axioms $C(a), \neg C(a)$ is inconsistent. Furthermore, negated axioms are one of the key stones connecting the contraction and revision operators in the AGM theory, although unfortunately well known DL-based ontology languages do not provide enough expressive power to represent negations of all the axioms. Similar to the notion of inconsistency, the definition of the negation is different from an approach to another approach in the Semantic Web community (Haase *et al.* 2005; Huang, van Harmelen, & ten Teije 2005), as we have briefly discussed in the **Introduction**.

Based on the distinction between ontology consistency and coherence, in the following we propose two corresponding axiom negations.

Definition 4 (Consistency-Negation) *An axiom ψ is said to be a consistency-negation of an axiom ϕ, written $\psi = \neg\phi$, iff*
(i)(Inconsistency) $\{\phi, \psi\}$ is inconsistent,
(ii) (Minimality) There exist no other ψ' such that ψ' satisfies the condition (i) and $Cn(\{\psi'\}) \subset Cn(\{\psi\})$.

The inconsistency condition states the relationship between axiom negation and ontology inconsistency, which is based on the classical notion of negation. We introduce the minimality condition to make the negation minimal so that it would not include any unnecessary additional part. Note that this does not enforce a unique consistency-negation though. Similarly we have the following axiom negation which corresponds with incoherence.

Definition 5 (Coherence-Negation) *An axiom ψ is said to be a coherence-negation of an axiom ϕ, written $\psi = \sim\phi$, iff*
(i)(Incoherence) $\{\phi, \psi\}$ is incoherent,
(ii) (Minimality) There exist no other ψ' such that ψ' satisfies the condition (i), and $Cn(\{\psi'\}) \subset Cn(\{\psi\})$.

Note that it is possible to extend our notion of negated axioms from a single axiom to a set of axioms, where a set of axioms represent the negation of another set of axioms. This extension goes beyond the scope of this paper.

Example 1 *Let us consider the consistency negation and the coherence negation of an axiom $C \sqsubseteq D$, where C and D are named concepts.*

$$\neg(C \sqsubseteq D) = \exists(C \sqcap \neg D), \ \sim(C \sqsubseteq D) = C \sqsubseteq \neg D$$

where $\exists(C \sqcap \neg D)$ is an existence axiom (Horrocks & Patel-Schneider 2003), which states there exists some instance of the concept $C \sqcap \neg D$. Note that, in any ontologies containing $C \sqsubseteq D$ and $C \sqsubseteq \neg D$, the concept C is unsatisfiable.

It should be noted that the minimality condition of the consistency-negation prevents the counter-intuitive property that any axiom ψ is qualified to be a consistent-negation of an inconsistent axiom ϕ (such as $C \sqsubseteq \neg C$). It is easy to see that its consistency-negation must be the tautology T because the tautology T is implied by any axiom ψ, i.e. $Cn(\emptyset) = Cn(\{T\}) \subseteq Cn(\{\psi\})$. Thus no other axioms can meet the minimality condition. For example, we have $\neg(\{a\} \sqsubseteq \bot) = T$; similarly, we have $\sim (D \sqsubseteq \bot) = T$.

In the following we will briefly discuss whether the proposed negations satisfy the following important properties:

1. *Existence*: It should exist in (almost) every DL.
2. *Classicality*: If the definition of negation is applied in a classical logic, it should coincide with the classical negation.
3. *Decidability*: The problem of checking whether or not an axiom is the negation of another axiom should be decidable.

Existence Definitions 4 and 5 improve the Existence property by giving up the restriction on double negations; i.e., an axiom ψ should be logically equivalent to the negation of the negation of ψ. Due to the limitation of space, here we only illustrate our point with some examples. For a DL $\langle L, Cn\rangle$ that does not provide concept existence axioms, we cannot use $\exists(C \sqcap \neg D)$ as a negation of $C \sqsubseteq D$; however, Definition 4 allows $C \sqcap \neg D(a)$ (where a is a fresh individual) as a consistency-negation of $C \sqsubseteq D$. For a DL $\langle L, Cn\rangle$ that does not provide any role constructors, we cannot use $\exists(R \sqcap \neg S)$ as a negation of the role inclusion $R \sqsubseteq S$; however, Definition 5 allows $C \sqsubseteq \exists R.\top \sqcap \forall S.\bot$ (where C is a fresh named concept) as a coherent-negation of $R \sqsubseteq S$.

Classicality The classical negation has the following intuitive properties:
(i) $Cn(\{\phi\}) \cap Cn(\{\neg\phi\}) \subseteq Cn(\emptyset)$ (only the tautology appears in both the consequences set of an axiom and its negation);
(ii) $Cn(\{\phi\} \cup \{\neg\phi\}) = L$ (the consequence set of the negation is the complement set of the consequence set of the axiom).

It can be shown that, under standard assumptions (Flouris, Plexousakis, & Antoniou 2004), the properties are guaranteed by the consistency-negation.

Decidability Given a DL $\langle L, Cn\rangle$, let us first consider the consistency-negation. The checking of the inconsistency condition is indeed a knowledge base satisfiability problem of $\langle L, Cn\rangle$. The minimality condition can be checked by trying to replace some sub-concepts (or sub-roles) with more general ones.

Proposition 2 *Given a DL $\langle L, Cn\rangle$, if the knowledge base satisfiability problem of $\langle L, Cn\rangle$ is decidable, then the consistency-negation checking in $\langle L, Cn\rangle$ is decidable.*

Proposition 3 *Given a DL $\langle L, Cn\rangle$, if the problem of concept satisfiability w.r.t. to a TBox in $\langle L, Cn\rangle$ is decidable, then the coherent-negation checking in $\langle L, Cn\rangle$ is decidable.*

Postulates for Ontology Change

In this section we propose certain postulates which describe two rational change operators for DL-based ontologies and even the use of the different kind of negation for revision. Our approach will be based on the AGM theory presented in Section **Preliminaries**.

Postulates for Contraction

The main result that motivates our quest for a new set of contraction postulates is summarized in the next lemma and its corollary. Note that, as mentioned above, we use $\langle L, Cn\rangle$ (or $\langle L^{\mathcal{K}}, Cn\rangle$ when necessary) to refer to the Description Logic $\mathcal{K}$.

Lemma 1 *For a DL $\langle L, Cn\rangle$, if there is an axiom $x \in L$ and a set of axioms $Y \subseteq L$ such that $Cn(\emptyset) \subset Cn(Y) \subset Cn(\{x\})$, then $\langle L, Cn\rangle$ is not base-AGM-compliant.*

Corollary 1 *Any DL that is at least as expressive as $\mathcal{FL}_0$ and whose alphabet allows at least two concept names and one role name is non-base-AGM-compliant.*

Corollary 1 practically overrules the use of the postulates that appeared in (Flouris, Plexousakis, & Antoniou 2004) in the DL context. The reason for this failure is related to the so-called base recovery postulate (B-6). Here, we will propose a different set of postulates that satisfy the following:

1. *Existence*: For every monotonic DL $\langle L, Cn\rangle$, there is a contraction operator satisfying the proposed postulates.
2. *AGM Rationality*: Whenever possible (i.e., for base-AGM-compliant DLs), the proposed postulates allow exactly the same contraction operators as the AGM postulates do.

It turns out that the following set of postulates satisfies both goals:

(O-1) $O - X \subseteq O$.

(O-2) If $O \not\models X$, then $O - X = O$.

(O-3) If $\emptyset \not\models X$, then $O - X \not\models X$.

(O-4) If $X \cong Y$, then $O - X = O - Y$.

(O-5) If $Cn((O-X) \cup X) \subset Cn(Y \cup X)$ for some $Y \subseteq O$, then $Y \models X$ and $\emptyset \not\models X$.

The postulates (O-1)-(O-4) are equivalent reformulations of the postulates discussed in (Flouris, Plexousakis, & Antoniou 2004), i.e. (B-2)-(B-5), respectively; postulate (B-1) from (Flouris, Plexousakis, & Antoniou 2004) was ignored because it is trivial. These postulates follow the AGM intuition: contraction is an operation that is used to remove knowledge from an ontology, so the result should not contain any new, previously unknown, information (O-1); if the contracted axiom is not part of our original knowledge, nothing should be removed (O-2); but if it is, then contraction is supposed to return a new ontology such that the contracted expression is no longer explicitly asserted or entailed (O-3); finally, the result should be syntax-independent (O-4).

Postulates (O-1)-(O-4) fail to capture the *Principle of Minimal Change* (Gärdenfors 1992) which states that a contraction operator should remove as little information from the ontology as possible. This principle was originally captured by postulate (B-6) in (Flouris, Plexousakis, & Antoniou 2004), while in our case it weakened to form (O-5). (B-6) states that a contraction operation should only remove axioms which are relevant to the contracted axiom; this is guaranteed by restricting the union of the result of the contraction ($O - X$) and the contracted axiom (X) to entail (or be equivalent to) the original ontology (O):

(B-6) $O \subseteq Cn((O - X) \cup X)$

(O-5) comes very close to that by restricting $Cn((O - X) \cup X)$ to be maximal out of all the possible selections for $O-X$ that satisfy the other postulates: if there is any $Y \subseteq O$ giving a "larger" set $Cn(Y \cup X)$, then Y will necessarily entail X (so Y would not be a possible subset of $O - X$, by (O-3)). The latter implication ($\emptyset \not\models X$) was included in (O-5) in order to capture a certain limit case.

Now let us see why this set of postulates satisfies the required properties. The Existence property is easy to show. As a DL based ontology, O contains a finite number of axioms. Thus, there is only a finite number of subsets of O, so one can find at least one $Y \subseteq O$ for which $Cn(Y \cup X)$ is maximal. Once some technical details and limit cases are taken care of, the following proposition can be shown:

Proposition 4 *For any logic $\langle L, Cn \rangle$, there is a contraction operator '$-$' such that the operation $O - X$ satisfies (O-1)-(O-5) for all finite $O \subseteq L$ and all $X \subseteq L$.*

The second property, AGM Rationality, is more difficult to show, so we will break its proof in two parts. Firstly, we will show that if (B-1)-(B-6) are satisfied by a contraction operator, then (O-1)-(O-5) are also satisfied. This is trivial for (O-1)-(O-4), as these are equivalent reformulations of (B-2)-(B-5) respectively. To show that (O-5) is satisfied as well, notice that (B-6) requires that O is entailed by $Cn((O - X) \cup X)$. If $O \models X$, then $O \models Cn(Y \cup X)$, for all $Y \subseteq O$. This fact, combined with the requirement imposed by (B-6), shows that the "if part" of (O-5) cannot be true for any $Y \subseteq O$, so (O-5) trivially holds. If, on the other hand, $O \not\models X$ then (B-3) (equivalently, (O-2)) indicates $O - X = O$; thus, again, the "if part" of (O-5) cannot be true for any $Y \subseteq O$, so (O-5) holds. This gives the following result:

Proposition 5 *If a contraction operator satisfies* (B-1)-(B-6)*, then it satisfies* (O-1)-(O-5)*.*

This result implies that the original set of postulates (B-1)-(B-6) is stronger than (O-1)-(O-5); this should be expected, by Proposition 4, as the result of this proposition does not hold for (B-1)-(B-6) (see Lemma 1 and (Flouris, Plexousakis, & Antoniou 2004)).

To show AGM Rationality, we should also show that the two sets of postulates are actually equivalent whenever possible (i.e., in base-AGM-compliant DLs). The proof follows similar steps as the proof of Proposition 5. The only non-trivial task is to show that whenever (O-1)-(O-5) are satisfied, (B-6) is also satisfied. This is shown by the fact that, in base-AGM-compliant logics, there is always a $Y \subseteq O$ which does not entail X, such that $Cn(Y \cup X)$ entails O; thus, (O-5) guarantees that $O - X$ will be selected in such a way that $Cn((O - X) \cup X)$ will imply O, thus satisfying (B-6). Once some limit cases are taken care of (one of which justifies the use of the implication $\emptyset \not\models X$ in (O-5)), the following can be shown:

Proposition 6 *For a base-AGM-compliant logic, if a contraction operator satisfies* (O-1)-(O-5)*, then it satisfies* (B-1)-(B-6)*.*

Finally, it is important to note that the proposed postulates, as well as Propositions 4-6, are applicable not only to DLs, but also to all logics that comply with the $\langle L, Cn \rangle$ model.

Postulates for Revision

To the our knowledge, there has been no attempt to recast the AGM postulates for revision in the context of the foundational model; furthermore, there has been no attempt to generalize these postulates in the sense of (Flouris, Plexousakis, & Antoniou 2004). The main reason for the latter shortcoming are postulates which requires the definition of a negation. The definitions of negation presented in the previous section allow us to overcome this problem and present some initial thoughts on these issues for DLs.

The original AGM postulates for revision (K+1)-(K+6) can be found in (Alchourrón, Gärdenfors, & Makinson 1985) and are omitted due to space limitations. Postulate (K+1) requires that the result of revision is a theory; in our context, this should be dropped, as we are working on the foundational model. Postulates (K+2)-(K+5) can be reformulated as follows:

(O+1) $X \subseteq O + X$.

(O+2) If $Cn(O \cup X) \neq L$, then $O + X = O \cup X$.

(O+3) If $Cn(X) \neq L$, then $Cn(O + X) \neq L$.

(O+4) If $X \cong Y$, then $O + X \cong O + Y$.

It can be easily shown that each of (O+1)-(O+4) is equivalent to (K+2)-(K+5) in the standard case. Postulate (K+6) poses some extra problems, because it requires the definition of negations of DL axioms. A straightforward (and equivalent) reformulation of (K+6) follows:

(O+5) $(O + X) \cap O \cong O - \neg X$.

In (O+5), the '$\neg$' symbol may be replaced by the standard negation, consistency negation or coherence negation, depending on our needs and on which type(s) of negation exist in the underlying logic.

In the AGM theory, there is a close connection between revision and contraction, as this is expressed by the *Harper Identity* (which is equivalent to (O+5)) and *Levi Identity*; here we present a generalized version of these identities:

Harper: $O - X \cong Cn(O + \neg X) \cap Cn(O)$.
Levi: $O + X \cong Cn(O - \neg X) \cup Cn(X)$.

Again, in place of the symbol '$\neg$', any of the negations that we proposed could be used. In the AGM setting, it has been shown that for any given revision operator that satisfies the AGM postulates for revision, the contraction operator defined by the Harper identity satisfies the AGM postulates for contraction; moreover, for any given contraction operator that satisfies the AGM postulates for contraction, the revision operator defined by the Levi identity satisfies the AGM postulates for revision. One of our most important goals for future work is the proof that these facts hold for the generalized versions of the postulates, the Levi and the Harper identities.

If the coherence negation is used for (O+5), the Levi and the Harper identities, then it is more appropriate to replace the postulates (O+2) and (O+3) with following coherence-based postulates:

(O+2*) If $O \cup X$ is coherent, then $O + X = O \cup X$.

(O+3*) If X is coherent, then $O + X$ is coherent.

The coherence postulates are useful for the revision on the ontologies which have only T-boxes, because their incoherence appears much more often than their inconsistency. It is more meaningful to avoid their hidden inconsistency, i.e. their incoherence.

Conclusions and Outlook

As has been mentioned in the **Introduction**, inconsistencies, as well as negations in ontologies are closely related to ontology change. In this paper we have proposed a general framework accounting for negation, inconsistency and change for DL-based ontologies, which aims at providing a foundation for reasoning and management of dynamic ontologies. Such a foundation is of utmost importance for the deployment of real-world applications in the context of the Semantic Web. In our framework, we have shown how to use the proposed negations to achieve the Harper identity and Levi identity for ontology change, by which we can make a close connection between the ontology revision and contraction operations. The distinction between incoherence and inconsistency provides us two different approaches for devising rationality postulates for ontology revision, which cover different needs in different application scenarios.

Encouraged by our results so far, we plan to further generalise our postulates w.r.t. the Levi and the Harper identities. Furthermore, it would be useful to investigate detailed impacts of this proposed framework in some specific inconsistency processing in ontology reasoning and management, such as the diagnosis and debugging of inconsistent ontology, reasoning with inconsistent ontologies, multiple-version ontology reasoning, and ontology evolution and change.

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