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Alessandro Tavoni\*

## Incorporating fairness motives into the Impulse Balance Equilibrium and Quantal Response Equilibrium concepts: an application to 2x2 games

**Abstract.** Substantial evidence has accumulated in recent empirical works on the limited ability of the Nash equilibrium to rationalize observed behavior in many classes of games played by experimental subjects. This realization has led to several attempts aimed at finding tractable equilibrium concepts which perform better empirically; one such example is the impulse balance equilibrium (Selten, Chmura, 2008), which introduces a psychological reference point to which players compare the available payoff allocations. This paper is concerned with advancing two new, empirically sound, concepts: equity-driven impulse balance equilibrium (EIBE) and equity-driven quantal response equilibrium (EQRE): both introduce a distributive reference point to the corresponding established stationary concepts known as impulse balance equilibrium (IBE) and quantal response equilibrium (QRE). The explanatory power of the considered models leads to the following ranking, starting with the most successful in terms of fit to the experimental data: EQRE, IBE, EIBE, QRE and Nash equilibrium.

*JEL classification:* C72, C91, D01, D63

*Keywords:* Fairness, Inequity aversion, Aspiration level, Impulse balance, Quantal Response, Behavioral economics, Experimental economics

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## 1. From efficiency to equality: the “distributive” reference point

In recent years experimental economists have accumulated considerable evidence that steadily contradicts the self-interest hypothesis embedded in equilibrium concepts traditionally studied in game theory, such as Nash's. The evidence suggests that restricting the focus of analysis to the strategic interactions among perfectly rational players (exhibiting equilibrium behavior) can be limiting, and that considerations about fairness and reciprocity should be accounted for<sup>1</sup>.

In fact, while models based on the assumption that people are exclusively motivated by their material self-interest perform well for competitive markets with standardized goods, misleading predictions arise when applied to non-competitive environments, for example those characterized by a small number of players (cf. Fehr and Schmidt, 2001) or other frictions. For example, Kahneman, Knetsch and Thaler (1986) find empirical results indicating that customers are extremely sensitive to the fairness of firms' short-run pricing decisions, which might explain the fact that some firms do not fully exploit their monopoly power.

One prolific strand of literature on equity issues focuses on relative measures, in the sense that subjects are concerned not only with the absolute amount of money they receive but also about their relative standing compared to others. Bolton (1991), formalized the relative income hypothesis in the context of an experimental bargaining game between two players.

Kirchsteiger (1994) followed a similar approach by postulating envious behavior. Both specify the utility function in such a way that agent  $i$  suffers if she gets less than player  $j$ , but she's indifferent with respect to  $j$ 's payoff if she is better off herself. The downside of the latter specifications is that, while consistent with the behavior in bargaining games, they fall short of explaining observed behavior such as voluntary contributions in public good games<sup>2</sup>.

A more general approach has been followed by Fehr & Schmidt (1999), who instead of assuming that utility is either monotonically increasing or decreasing in the well being of the other players,

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<sup>1</sup> For supporting arguments see, among the many available literature reviews, the updated one provided in Gowdy (2008).

<sup>2</sup> A substantial departure from the models considered here, which are solely based on subjective considerations to differences in payoffs, is represented by models where agents' responses are also driven by the motivations behind the actions of the other player. This is the case for Falk et al. (2006), as well as Levine (1997). While without doubt one can argue that our social interactions are to some extent influenced by judgments we hold on others, these efforts inevitably run into the questionable assumption of perfect (or high degree of) knowledge of the preferences. For this reason, we restrict attention here to more parsimonious models that nevertheless account for reference dependence in several dimensions, as will be explained below.

model fairness as self-centered inequality aversion. Based on this interpretation, subjects resist inequitable outcomes, that is they are willing to give up some payoff in order to move in the direction of more equitable outcomes. More specifically, a player is altruistic towards other players if their material payoffs are below an equitable benchmark, but feels envy when the material payoffs of the other players exceed this level. To capture this idea, the authors consider a utility function which is linear in both inequality aversion and in the payoffs. Formally, for the two-player case ( $i \neq j$ ):

$$u_i = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\} \quad (1)$$

Where  $x_i, x_j$  are player 1 and player 2's payoffs respectively and  $\beta_i, \alpha_i$  are player  $i$ 's inequality parameters satisfying the following conditions:  $\beta_i \leq \alpha_i$  and  $0 \leq \beta_i \leq 1$ .

The second term in the equation is the utility loss from disadvantageous inequality, while the third term is the utility loss from advantageous inequality. Due to the above restrictions imposed on the parameters, for a given payoff  $x_i$ , player  $i$ 's utility function is maximized at  $x_i = x_j$ , and the utility loss from disadvantageous inequality ( $x_i < x_j$ ) is larger than the utility loss incurred if player  $i$  is better off than player  $j$  ( $x_i > x_j$ ).

Fehr and Schmidt (1999) show that the interaction of the distribution of types with the strategic environment explains why in some situations very unequal outcomes are obtained while in other situations very egalitarian outcomes prevail. In referring to the social aspects introduced by this utility function, one could think of inequality aversion in terms of an interactive framing effect (reference point dependence)<sup>3</sup>.

This payoff modification has proved successful in many applications, mainly in combination with the Nash equilibrium concept, and will therefore be employed in this study, although in conjunction with different equilibrium concepts. In the next section, the main features of the impulse balance equilibrium will be introduced, while the remainder of the paper is concerned with advancing two equity-driven concepts: sections 3 and 4 deal with the proposed modification of IBE and its ability to match observed behavior by individuals playing experimental games, while sections 5 and

<sup>3</sup> See Kahneman and Tversky (1979) for the pioneering work that introduced the standard reference dependence concept.

6 with equity-driven quantal response equilibrium and its fit to the experimental data. Section 7 provides a discussion of the results.

## **2. The “psychological” reference point**

The predictive weakness of the Nash equilibrium is effectively pointed out by Erev and Roth (1998), who study the robustness and predictive power of learning models in experiments involving at least 100 periods of games with a unique equilibrium in mixed strategies. They conclude that the Nash equilibrium prediction is, in many contexts, a poor predictor of behavior, while claiming that a simple learning model can be used to predict, as well as explain, observed behavior on a broad range of games, without fitting parameters to each game. A similar approach, based on within-sample and out-of-sample comparisons of the mean square deviations, will also be employed in this paper to assess to what extent is the proposed model able to fit and predict the frequencies of play recorded by subjects of an experiment involving several games with widely varying equilibrium predictions.

Based on the observation of the shortcomings of mixed Nash equilibrium in confronting observed behavior in many classes of games played by experimental subjects, an alternative tractable equilibrium has been suggested by Selten and Chmura (2008). IBE is based on learning direction theory (Selten and Buchta, 1999), which is applicable to the repeated choice of the same parameter in learning situations where the decision maker receives feedback not only about the payoff for the choice taken, but also for the payoffs connected to alternative actions. If a higher parameter would have brought a higher payoff, the player receives an upward impulse, while if a lower parameter would have yielded a higher payoff, a downward impulse is received. The decision maker is assumed to have a tendency to move in the direction of the impulse. IBE, a stationary concept which is based on transformed payoff matrices as explained in Section 3, applies this mechanism to 2x2 games. The probability of choosing one of two strategies (for example Up) in the considered games is treated as the parameter, which can be adjusted upward or downward<sup>4</sup>. It is assumed that the second lowest payoff in the matrix is an aspiration level determining what is perceived as profit or loss (with losses weighing twice as much as gains). In impulse balance equilibrium expected upward and downward impulses are equal for each of both players simultaneously.

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<sup>4</sup> Section 3 and Appendix 4 provide more detail on the experimental setup utilized here.

The main result of the paper by Selten and Chmura (2008) is that, for the games they consider, impulse balance theory has a greater predictive success than the other stationary concepts they compare it to: Nash equilibrium, action-sampling equilibrium, payoff-sampling equilibrium and quantal response equilibrium. While having the desirable feature of being a parsimonious parameter-free concept as the Nash equilibrium, and of outperforming the latter, the aspiration level framework (to be described) has the less appealing featuring of requiring the use of transformed payoffs in place of the original ones for the computation of the equilibrium<sup>5</sup>.

The aspiration level can be thought of as a psychological reference point, as opposed to the social one considered when modeling inequality aversion: the idea behind the concept proposed in section 4 is that of utilizing the equilibration between upward and downward impulses which is inherent to the IBE, but replacing the aspiration level associated to own-payoff considerations only with equity considerations related to the distance between own and opponent's payoff. The motivation follows from the realization that in non-constant sum games (considered here) subjects' behavior also reflects considerations of equity. In fact, while finite repetition alone has been shown to have limited effectiveness in enlarging the scope for cooperation or retaliation, non-constant sum games offer some cooperation opportunities, and it seems plausible that fairness motives would play an important role in repeated play of this class of games. A suitable consequence of replacing the aspiration level framework with the inequality aversion one is that the original payoffs can be utilized (and should, in order to avoid mixing social and psychological reference points).

### 3. Experimental setup: IBE

The table in Appendix A shows the 12 games, 6 constant sum games and 6 non-constant sum games on which Selten and Chmura (2008) have run experiments, which have taken place with 12 independent subject groups for each constant sum game and with 6 independent subject groups for each non-constant sum game. Each independent subject group consists of four players 1 and four players 2 interacting anonymously in fixed roles over 200 periods with random matching. In summary:

Players:  $I=\{1,2\}$

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<sup>5</sup> When the IBE is applied to the payoffs belonging to the games truly played by the participants, the gains in fit of the concept over the Nash equilibrium appear to be greatly reduced, indicating that its explanatory superiority depends to a large extent on the payoff transformation, which is itself dependent on the choice of the aspiration level (the pure strategy maximin payoff) and the double weight assigned to losses relative to gains.

Action space:  $\{U,D\} \times \{L,R\}$

Estimated choice probabilities in mixed strategy:  $\{P_u, 1-P_u\}$  and  $\{Q_l, 1-Q_l\}$

Sample size: (54 sessions) x (16 subjects) = 864

Time periods: T=200

In Appendix A, a non-constant sum game next to a constant sum game has the same best reply structure (characterized by the Nash equilibrium choice probabilities  $P_u, Q_l$ ) and is derived from the paired constant sum game by adding the same constant to player 1's payoff in the column for  $R$  and to player 2's payoff in the row for  $U$ . Games identified by a smaller number have more extreme parameter values than games identified by a higher number; for example, Game 1 and its paired non-constant sum Game 7 are near the border of the parameter space ( $P_u \cong 0.1$  and  $Q_l \cong 0.9$ ), while Game 6 and its paired non-constant sum Game 12 are near the middle of the parameter space ( $P_u \cong 0.5$  and  $Q_l \cong 0.6$ ).

As pointed out in Section 2, IBE involves a transition from the original game to the transformed game, in which losses with respect to the aspiration level get twice the weight as gains above this level. The impulse balance equilibrium depends on the best reply structure of this modified game, which is generally different from that of the original game, resulting therefore in different predictions for the games in a pair.

The present paper utilizes the data on the experiments involving 6 independent subject groups for each of the 6 non-constant sum games (games 7 through 12 in Appendix A). As anticipated above, this class of games is particularly conceptually suitable to the application of the inequality aversion framework. Further, in completely mixed 2x2 games, mixed equilibrium is the unambiguous game theoretic prediction when they are played as non-cooperative one-shot games. Since non-constant sum games provide incentives for cooperation, such attempts to cooperation may have influenced the observed relative frequencies in the experiment by Selten and Chmura (2008). Along these lines, it is particularly relevant to see whether inequality aversion payoff modifications can help improve the fit with respect to these frequencies.

The application of inequality aversion parameters to the impulse balance equilibrium provides an opportunity for testing the fairness model by Fehr & Schmidt (1999) in conjunction with the latter, which is itself a simple yet powerful concept which has proven to be empirically successful in fitting the data in different categories of games while nevertheless being parsimonious (see footnote 5 for remarks on the not fully parameter-free nature of IBE). By including a fairness dimension to

it, the hope is to supply favorable empirical evidence and provide further stimulus to expand the types of games empirically tested.

Formally, this involves first modifying the payoff matrices of each game in order to account for the inequality parameters  $(\beta, \alpha)$ , then creating the impulse matrix based on which the probabilities are computed. In order to clarify the difference between the reference point utilized in Selten and Chmura (2008) (the aspiration level) and that utilized in this paper, it is useful to start by summarizing the mechanics behind the computation of the original version of the IBE.

Let's consider the normal form game depicted in Figure 1 below,

		L ( $Q_l$ )	→	R ( $1-Q_l$ )	
U ( $P_u$ )	↑	$a_l + c_l, b_u$		$a_r, b_u + d_u$	
		$a_l, b_d + d_d$		$a_r + c_r, b_d$	↓
D ( $1 - P_u$ )			←		

**Figure 1: structure of the 2x2 games** (arrows point in the direction of best replies; probabilities in parentheses)

In the above figure,  $a_l, a_r, b_u, b_d \geq 0$  and  $c_l, c_r, d_u, d_d > 0$ .

$c_l$  and  $c_r$  are player 1's payoffs in favor of U,D while  $d_u, d_d$  are player 2's payoffs in favour of L,R respectively. Note that player 1 can secure the higher one of  $a_l, a_r$  by choosing one of his pure strategies, and player 2 can similarly secure the higher one of  $b_u, b_d$ . Therefore, the authors define the aspiration levels for the 2 players as given by:

$$s_i = \max(a_l, a_r) \text{ for } i=1 \text{ and } s_i = \max(b_u, b_d) \text{ for } i=2$$

The transformed game (henceforth TG) is constructed as follows: player i's payoff is left unchanged if it is less or equal to  $s_i$ , while payoffs in excess of  $s_i$  are reduced by half such surplus. Algebraically, calling  $x$  and  $\hat{x}$  the payoffs before and after the transformation, the following obtains:

$$\text{if } x \leq s_i \Rightarrow \hat{x} = x$$

$$\text{if } x > s_i \Rightarrow \hat{x} = x - \frac{1}{2}(x - s_i)$$



If after the play, player  $i$  could have obtained a higher payoff by employing the other strategy, player  $i$  receives an impulse in the direction of the other strategy, of the size of the foregone payoff in the TG.

	L ( $Q_l$ )	R ( $1-Q_l$ )
U ( $P_u$ )	0 , $d_u^*$	$c_r^*$ , 0
D ( $1 - P_u$ )	$c_l^*$ , 0	0 , $d_d^*$

**Figure 2: Impulses in T.G. in the direction of unselected strategy**

The concept of impulse balance equilibrium requires that player one's expected impulse from  $U$  to  $D$  is equal to the expected impulse from  $D$  to  $U$ ; likewise, player two's expected impulse from  $L$  to  $R$  must equal the impulse from  $R$  to  $L$ . Formally,

$$P_u Q_r c_r^* = P_d Q_l c_l^*$$

$$P_u Q_l d_u^* = P_d Q_r d_d^*$$

Which, after some manipulation, can be shown to lead to the following formulae for probabilities:

$$P_u = \frac{\sqrt{c_l^* / c_r^*}}{\sqrt{c_l^* / c_r^*} + \sqrt{d_u^* / d_d^*}} \quad ; \quad Q_l = \frac{1}{1 + \sqrt{\frac{c_l^*}{c_r^*} \frac{d_u^*}{d_d^*}}} \quad (1)$$

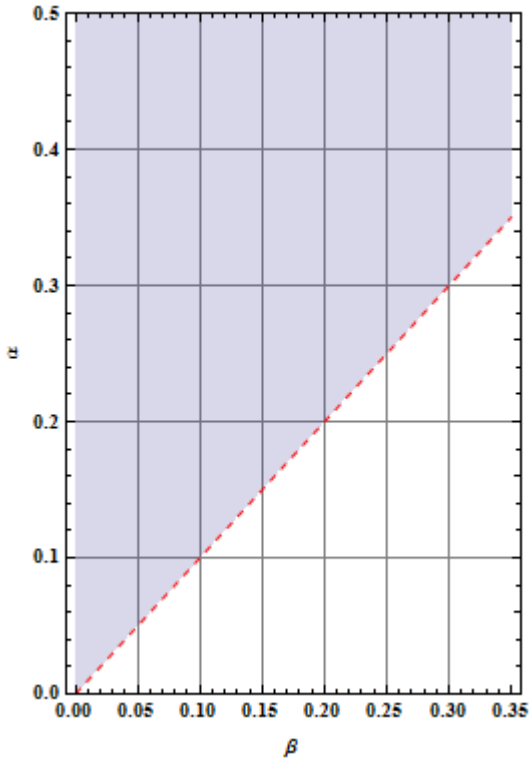
Replacing the aspiration level framework with the inequality aversion one doesn't require the computation of the TG based on aspiration level framing, as the original payoffs are now modified by including the inequality parameters ( $\beta, \alpha$ ).

Formally, recalling that  $\mathcal{U}_i = x_i - \alpha_i \max\{x_j - x_i, 0\} - \beta_i \max\{x_i - x_j, 0\}$ , one can modify the matrix in Figure 1 to replace the (self-centered) psychological reference point represented by the aspiration level with the other-regarding reference considerations embodied in the inequity aversion. Table 1, below, contains the proposed payoff modifications:

	L	R
U	$a_l + c_l - \alpha_i \max\{b_u - a_l - c_l, 0\} - \beta_i \max\{a_l + c_l - b_u, 0\},$ $b_u - \alpha_j \max\{a_l + c_l - b_u, 0\} - \beta_j \max\{b_u - a_l - c_l, 0\}$	$a_r - \alpha_i \max\{b_u + d_u - a_r, 0\} - \beta_i \max\{a_r - b_u - d_u, 0\},$ $b_u + d_u - \alpha_j \max\{a_r - b_u - d_u, 0\} - \beta_j \max\{b_u + d_u - a_r, 0\}$
D	$a_l - \alpha_i \max\{b_d + d_d - a_l, 0\} - \beta_i \max\{-b_d - d_d + a_l, 0\},$ $b_d + d_d - \alpha_j \max\{a_l - b_d - d_d, 0\} - \beta_j \max\{-b_d - d_d + a_l, 0\}$	$a_r + c_r - \alpha_i \max\{b_d - a_r - c_r, 0\} - \beta_i \max\{a_r + c_r - b_d, 0\},$ $b_d - \alpha_j \max\{a_r + c_r - b_d, 0\} - \beta_j \max\{b_d - a_r - c_r, 0\}$

**Table 1: structure of the 2x2 games accounting for inequality aversion**

The inequality aversion parameters used in the proposed equity-driven IBE must satisfy the constraints  $\beta_i \leq \alpha_i$  and  $0 \leq \beta_i \leq 1$ . A cutout of the relevant parameter space is described by the highlighted area in Figure 3 below:



$$\beta_i \leq \alpha_i \text{ and } 0 \leq \beta_i \leq 1$$

**Figure 3: A cutout correspondence between  $\beta$  and  $\alpha$  (grey area) under the inequity aversion restrictions**

Based on these payoffs, the artificial probabilities in (1) can be computed in order to find the mixed strategy equilibrium predictions corresponding to specific values of  $\beta$  and  $\alpha$ .

#### 4. Two measures of the relative performance of EIBE: best fit and predictive power

The preceding analysis served to familiarize us to the mechanics behind the first of the two concepts advanced in this paper, namely the equity-driven impulse balance equilibrium. We are now ready to assess the descriptive and predictive success of the original impulse balance equilibrium in comparison to EIBE.

Following a methodology which has been broadly utilized in the literature to measure the adaptive and predictive success of a point in a Euclidean space, the mean squared distance (MSD) of observed and theoretical values is employed.<sup>6</sup> More precisely, let's first focus on the ability of EIBE to describe the choices of a population playing entirely mixed 2x2 games: for each of the 6 non constant sum games considered, a grid search with a mean squared deviation criterion on the  $(\beta, \alpha)$  parameter space has been conducted to estimate the best fitting parameters, that is those that minimize the distance between the data generated by the model and the observed relative frequencies of play.

With this definition in mind, we say that the best overall fit is given by the parameter configuration that minimizes the mean over all games of the distance between the experimental data and the artificial predictions generated by the model. This amounts to first computing the mean squared deviations independently for each game  $i$  and then finding the  $(\beta, \alpha)_{best\ fit}$  that minimize the average across all games. Algebraically, letting  $\mathbf{f}$  and  $\mathbf{p}$  be the N-length vectors of observed and estimated choice frequencies, respectively, we seek to minimize:

$$MSD = \frac{1}{N} \sum_{i=1}^N MSD_i \quad (2)$$

where  $MSD_i$  is the average of game  $i$ 's squared distances, given by:

$$MSD_i = \frac{(f_{ui} - p_{ui})^2 + (f_{li} - q_{li})^2}{2} \quad (3)$$

and  $f_{ui}$  and  $f_{li}$  are the observed frequencies of playing up and left in game  $i$ , respectively, while  $p_{ui}$  and  $q_{li}$  are the estimated relative choice probabilities in mixed strategy. Note that a smaller MSD indicates better fit, i.e. a smaller distance to the experimental data.

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<sup>6</sup> Cf. Erev & Roth (1998), Selten (1991, 1999), as well as Marchiori & Warglien (2008) for supporting arguments on the suitability of MSD as a measure of the distance between a model's prediction and the experimental data.

Table 2 and Table 3 present complementary results on the relative performances of the examined stationary equilibrium concepts. In Table 2, in addition to the recorded choice frequencies and Nash equilibrium (NE) predictions, a summary of the results of the explanatory power of EIBE relative to IBE is shown for each non constant sum game, utilizing both the transformed (TG) as well as the original payoffs (OG). The comparisons between the two concepts are made both within game class (e.g. by comparing the performance within the class of transformed or original games in column 5), and across game class in the last column (e.g. between the performance of EIBE using original game  $i$  and IBE using transformed game  $i$ ,  $i=7,\dots,12$ ).

The *raison d'être* of the two-fold comparison is that not only it is meaningful to assess whether the proposed model can better approximate the observed frequencies than impulse balance equilibrium can, but it is especially important to answer the question: does EIBE outperform IBE when the former is applied to the original payoffs of game  $i$  and the latter is applied to the corresponding transformed payoffs? In other words, since the inequality aversion concept overlaps to a certain extent to that of having impulses in the direction of the strategy not chosen, applying the inequality aversion adjustment to payoffs that have already been transformed to account for the aspiration level will result in “double counting”.<sup>7</sup> It is therefore more relevant to compare the best fit of EIBE on OG (see rows highlighted in blue in the last column of Table 2) to that obtained by applying impulse balance equilibrium to TG.

	<b>FREQUENCY</b> [ $f_u, f_l$ ]	<b>N.E.</b> [ $P_u, Q_l$ ]	<b>BEST FIT EIBE</b> [ $P_u, Q_l$ ] ( $\beta, \alpha$ )	<b>IBE</b> [ $P_u, Q_l$ ] $\beta=\alpha=0$	<b>MSD EIBE</b> < <b>MSD IBE?</b>	<b>MSD EIBE(OG)</b> < <b>MSD IBE (TG)?</b>
<b>TG7</b>	[.141,.564]		[.104,.634] (0,0)	[.104,.634]	NO	<i>n.a.</i>
<b>OG7</b>	[.141,.564]	[.091,.909]	[.099,.568] (.054,.055)	[.091,.500]	YES	YES
<b>TG8</b>	[.250,.586]		[.270,.586] (.043,.065)	[.258,.561]	YES	<i>n.a.</i>
<b>OG8</b>	[.250,.586]	[.182,.727]	[.257,.584] (.000,.471)	[.224,.435]	YES	YES
<b>TG9</b>	[.254,.827]		[.180,.827] (.07,.10)	[.188,.764]	YES	<i>n.a.</i>
<b>OG9</b>	[.254,.827]	[.273,.909]	[.233,.840] (.330,.330)	[.162,.659]	YES	YES

<sup>7</sup> See TG7 and TG12 in Table 2 for instances where the best fit is achieved when both inequity parameters are 0 (in contrast to the paired original games, which have nonnegative parameters). Moreover,  $(\beta, \alpha)_{TG} < (\beta, \alpha)_{OG}$  for all games, indicating that aspiration level and inequity aversion reference dependence overlap to some extent.

	<b>FREQUENCY</b> [ $f_u, f_l$ ]	<b>N.E.</b> [ $P_u, Q_l$ ]	<b>BEST FIT EIBE</b> [ $P_u, Q_l$ ] ( $\beta, \alpha$ )	<b>IBE</b> [ $P_u, Q_l$ ] $\beta=\alpha=0$	<b>MSD EIBE</b> < MSD IBE	<b>MSD EIBE(OG)</b> < MSD IBE (TG)
<b>TG10</b>	[.366,.699]		[.355,.759] (.089,.134)	[.304,.724]	YES	<i>n.a.</i>
<b>OG10</b>	[.366,.699]	[.364,.818]	[.348,.717] (.253,.253)	[.263,.616]	YES	YES
<b>TG11</b>	[.331,.652]		[.357,.652] (.012,.018)	[.354,.646]	YES	<i>n.a.</i>
<b>OG11</b>	[.331,.652]	[.364,.727]	[.343,.642] (.000,.415)	[.316,.552]	YES	YES
<b>TG12</b>	[.439,.604]		[.496,.575] (0,0)	[.496,.575]	NO	<i>n.a.</i>
<b>OG12</b>	[.439,.604]	[.455,.636]	[.439,.604] (.017,.397)	[.408,.547]	YES	YES

*Table 2: Ex-post (best fit) descriptive power of EIBE vs. IBE*

Inspection of Table 2 suggests a strong positive answer to the following two relevant questions regarding the ability of the proposed concept to fit the observed frequencies of play: within the same class of payoffs (TG or OG), is the descriptive power of EIBE superior to that of the IBE? And, perhaps more importantly, is this still true when the two concepts are applied to their natural payoff matrices, namely the original and the transformed one respectively?

The last two columns of Table 2 show that, based on a comparison of the mean squared deviations of the predicted probabilities from the observed frequencies under the two methods, the EIBE fares better than IBE when the IA parameters are fit to each game separately.

This result, however, may owe, at least in part, to the fact that a parametric concept, such as the one advanced here (as well as equity-driven QRE introduced in Section 5), is compared to a parameter-free one. In order to correct for this advantage, results for the proposed parametric concepts are also reported avoiding to fit them for each game separately. This is done in two ways (as will be further explained below): by utilizing the two parameters that best fit all games to derive each game's predictions (and MSD), or by making out-of-sample predictions for each game based on the two free parameters that minimize the MSD of the remaining 5 games.

Let's take a closer look at the evaluation of the performance of equity-driven impulse balance equilibrium concept by means of an assessment of its predictive power. As mentioned, this is accomplished by partitioning the data into subsets, and simulating each experiment using parameters estimated from the other experiments. By generating the MSD statistic repeatedly on the

data set leaving one data value out each time, a mean estimate is found making it possible to evaluate the predictive power of the model. In other words, the behavior in each of the 6 non-constant sum games is predicted without using that game's data, but using the data of the other 5 games to estimate the probabilities of playing up and down. By this cross-prediction technique, one can evaluate the stability of the parameter estimates, which shouldn't be substantially affected by the removal of any one game from the sample.<sup>8</sup> Erev and Roth (1998) based their conclusions on the predictive success and stability of their learning models by means of this procedure, as well as, more recently, Marchiori and Warglien (2008).

Table 3, below, shows summary MSD scores (100\*Mean-squared Deviation) organized as follows: each of the first 6 columns represents one non-constant sum game, while the last column gives the average MSD over all games, which is a summary statistic by which the models can be roughly compared.<sup>9</sup> The first three rows present the MSDs of the NE and IBE predictions (for  $\beta=0=\alpha$ ) on the transformed and original payoffs respectively. The remaining three rows display MSDs of the EIBE model on the original payoffs: in the fourth row, the parameters are separately estimated for each game (12 parameters in total); in the fifth row, the estimated 2 parameters that best fit the data over all 6 games (and over all but Game 7, the reason will be discussed below), are employed (the same two  $\beta, \alpha$  that minimize the average score over all games are used to compute the MSDs for each game); in the last row the accuracy of the prediction of the hybrid model is showed when behavior in each of the 6 games is predicted based on the 2 parameters that best fit the other 5 games (and excluding Game 7).

Model		G 7	G 8	G 9	G 10	G 11	G 12	Mean (s.d.)
NE (on OG) 0 parameters	All games	6.08	1.23	.354	.708	.422	.064	1.48 (2.29)
	G8-12							.555 (.440)
IBE (on OG) 0 parameters	All games	.330	1.17	1.83	.878	.497	.209	.819 (.610)
	G8-12							.917 (.627)
IBE (on TG) 0 parameters	All games	.315	.035	.416	.224	.094	.205	.215 (.140)
	G8-12							.195 (.134)

<sup>8</sup> Cross-validation (also known as jackknifing) is extensively discussed in Busemeyer et al. (2000).

<sup>9</sup> Note that here we restrict attention to the OGs when considering EIBE.

<b>EIBE by game (on OG)</b>								
12 parameters	All games	.090	.003	.031	.033	.056	.000	<b>.035 (.034)</b>
	G8-12							<b>.025 (.020)</b>
6 par. ( $\beta$ s only)	All games							<b>.058 (.050)</b>
<b>EIBE best fit (on OG)</b>								
2 parameters								
(.157,.157)	All games	.746	.178	.428	.152	.140	.030	<b>.279 (.254)</b>
(.253,.259)	G8-12	-	.042	.098	.033	.173	.034	<b>.076 (.060)</b>
<b>EIBE predict (on OG)</b>								
2 parameters	All games	2.22	.238	.585	.186	.141	.031	<b>.567 (.837)</b>
	G8-12	-	.044	.149	.033	.189	.035	<b>.09 (.074)</b>

*Table 3: MSD scores of the considered equilibrium concepts (standard deviations for the means in parentheses)*

Table 3 summarizes further evidence in favor of the newly developed equity-driven impulse balance equilibrium. One can see from the third row that (as already signaled by Table 2), if the parameters of inequality aversion are allowed to be fit separately in each game, the improvements in terms of reduction of MSD are significant, both with respect to the Nash and impulse balance equilibrium. In order to consider a more parsimonious version of the model evaluated in this section, the aggregate MSD score of a 1-parameter adaptation of EIBE, which one may call envy-driven IBE, is also reported in the fourth row of Table 3. Note that the overall reduction in the number of parameters from 12 to 6 doesn't come at a dear price in terms of MSD, which goes from 0.35 for the full model to 0.58 for the reduced one, signaling the relative importance of the disadvantageous inequity aversion with respect to advantageous inequity aversion.

Let's now restrict the number of parameters to 2 (common to all games, cf. row 5 "EIBE best fit"): the mean MSD is still more than five times smaller than Nash's. If one doesn't include the extremely high MSD reported in both cases for Game 7 (for reasons discussed below), the gap actually increases, as the EIBE's MSD becomes more than seven times smaller than Nash's. With respect to the overall MSD mean of the IBE, when considering all games the proposed concept has a higher MSD, although a similar order of magnitude (.279 and .215 respectively). If one focuses only on games 8-12, again we have a marked superiority of equity-driven IBE over conventional IBE, as the MSD of the latter is more than twice that of the new concept. A similar pattern appears in the last row of the table, concerning the predictive capability: if Game 7 is excluded, the values are in line with the ones obtained in the fifth row, indicating stability of the parameters who survive the cross-validation test. One comforting consideration regarding the appropriateness of the exclusion of Game 7 comes from the widespread anomalous high level of its MSD score in all rows

of the table, which for both Nash and EIBE predict is about four times the corresponding mean level obtained over the six games. It is plausible that this evidence is related to the location of Game 7 in the parameter space. It is in fact located near the border, as previously pointed out, and therefore may be subject to the overvaluation of extreme probabilities by the subjects due to overweighting of small probabilities.

The next two sections consider incorporating fairness motives in the quantal response equilibrium notion, one that has recently attracted considerable attention thanks to its ability to rationalize behavior observed in experimental games. In addition to providing an interesting case for comparison, it should also allow to shed light on the suspected anomalous nature of Game 7.

### 5. Equity-driven Quantal Response Equilibrium

The former analysis has also been conducted utilizing the quantal response equilibrium concept (henceforth QRE) in conjunction with preferences that are again allowed to be affected by the counterparty's fate, via the inequity aversion parameters. The resulting model is called EQRE. Before showing the results, which are given in Table 4 and Table 5 and show an even better overall performance of this concept compared to the one examined in the previous sections, let's briefly describe the QRE. This probabilistic choice model was introduced by McKelvey, Palfrey and Thomas (1995), and concerns games with noisy players that base their choices on quantal best responses to the behavior of the other parties, so that deviations from optimal decisions are negatively correlated with the associated costs. That is to say, individuals are more likely to select better choices than worse choices, but do not necessarily succeed in selecting the very best choice. In the exponential form of quantal response equilibrium, considered here, the probabilities are proportional to an exponential with the expected payoff multiplied by the logit precision parameter ( $\lambda$ ) in the exponent: as  $\lambda$  increases, the response functions become more responsive to payoff differences. Formally,

$$P_{ij} = \frac{e^{\lambda \pi_{ij}(P_{-i})}}{e^{\lambda \pi_{ij}(P_{-i})} + e^{\lambda \pi_{ik}(P_{-i})}} \quad (2)$$

Where  $i, j=1,2$  are the players ( $k \neq j$ ),  $P_{ij}$  is the probability of player  $i$  choosing strategy  $j$  and  $\pi_{ij}$  is player  $i$ 's expected payoff when choosing strategy  $j$  given the other player is playing according to the probability distribution  $P_{-i}$ .



## 6. Two measures of the relative performance of the EQRE: best fit and predictive power

The following is a companion table to Table 2, as it reports the results of comparisons between the new hybrid model and the IBE concept, the former always outperforming the one employing the IBE on the transformed games. Note that the penultimate column now compares the performance of the two proposed concepts, showing that EQRE outperforms EIBE in five of the six games<sup>10</sup>.

	FREQUENCY $[f_u, f_l]$	NE $[P_u, Q_l]$	BEST FIT EQRE $[P_u, Q_l] (\beta, \alpha) \lambda$	IBE $[P_u, Q_l]$ $\beta=\alpha=0$	MSD EQRE < MSD EIBE	MSD EQRE(OG) < MSD IBE(TG)
<b>TG7</b>	[.141,.564]			[.104,.634]		<i>n.a.</i>
<b>OG7</b>	[.141,.564]	[.091,.909]	[.141,.564] (.105,.209) $\lambda=0.335$	[.091,.500]	YES	YES
<b>TG8</b>	[.250,.586]			[.258,.561]		<i>n.a.</i>
<b>OG8</b>	[.250,.586]	[.182,.727]	[.250,.586] (.059,.431) $\lambda=0.31$	[.224,.435]	YES	YES
<b>TG9</b>	[.254,.827]			[.188,.764]		<i>n.a.</i>
<b>OG9</b>	[.254,.827]	[.273,.909]	[.254,.827] (.083,.316) $\lambda=0.6$	[.162,.659]	YES	YES
<b>TG10</b>	[.366,.699]			[.304,.724]		<i>n.a.</i>
<b>OG10</b>	[.366,.699]	[.364,.818]	[.366,.699] (.362,.240) $\lambda=0.31$	[.263,.616]	YES	YES
<b>TG11</b>	[.331,.652]			[.354,.646]		<i>n.a.</i>
<b>OG11</b>	[.331,.652]	[.364,.727]	[.311,.652] (.003,.02) $\lambda=0.91$	[.316,.552]	YES	YES
<b>TG12</b>	[.439,.604]			[.496,.575]		<i>n.a.</i>
<b>OG12</b>	[.439,.604]	[.455,.636]	[.439,.604] (.042,.137) $\lambda=0.55$	[.408,.547]	same	YES

Table 4: Ex-post (best fit) descriptive power of EQRE with respect to IBE and EIBE

<sup>10</sup> in game 12 they achieve a substantially equal equilibrium prediction.

As before, in order to assess the performance of the concepts over multiple games, the parameters are restricted to be the same over all the games, as shown in the penultimate row in Table 5: EQRE displays a better fit than EIBE (smaller mean square deviation) in all but game 11, achieving a mean MSD of .147 as opposed to .279 for the latter. As for the predictive power, measured for each game by fitting parameters estimated on the remaining five, when all games are considered the mean MSD is substantially lower for the equity-driven QRE, averaging .214 vs. a score of .567 for the equity-driven IBE.

Model	G 7	G 8	G 9	G 10	G 11	G 12	Mean (s.d.)
<b>NE (on OG)</b> 0 parameters	6.08	1.23	.354	.708	.422	.064	<b>1.48</b> (2.29)
<b>IBE (on OG)</b> 0 parameters	.330	1.17	1.83	.878	.497	.209	<b>.819</b> (.610)
<b>IBE (on TG)</b> 0 parameters	.315	.035	.416	.224	.094	.205	<b>.215</b> (.140)
<b>EQRE by game (on OG)</b> 18 parameters	5.5* $10^{-6}$	2.4* $10^{-7}$	7.5* $10^{-6}$	6.4* $10^{-7}$	7.4* $10^{-8}$	5.7* $10^{-6}$	<b><math>3.3 \cdot 10^{-6}</math></b> ( $3.0 \cdot 10^{-6}$ )
<b>Parametric best fit (OG)</b>							
EIBE ( $\beta=\alpha=.16$ )	.746	.178	.428	.152	.140	.030	<b>.279</b> (.279)
EQRE ( $\beta=.15, \alpha=.24, \lambda=.43$ )	.251	.012	.397	.036	.163	.027	<b>.147</b> (.154)
<b>EIBE vs. EQRE predict (OG)</b>							
2 par. EIBE	2.220	.238	.585	.186	.141	.031	<b>.567</b> (.831)
3 par. EQRE	.558	.023	.420	.062	.189	.030	<b>.214</b> (.226)

Table 5: MSD scores of the considered equilibrium concepts

Two important considerations should be remarked at this point. Firstly, for what concerns the overall fit, even without excluding the potentially problematic game 7, the EQRE concept outperforms the conventional impulse balance equilibrium applied to the transformed games (MSD scores are .147 and .215, respectively); this is noteworthy, since it wasn't the case for the other hybrid concept<sup>11</sup>. Secondly, the above considerations are confirmed by the predictions obtained with the jackknifing technique: for the EQRE specification the mean MSD score based on cross-predictions is not substantially higher than the one calculated when the parameters that best fit all

<sup>11</sup> In fact, the impulse balance equilibrium obtains dramatically higher MSD scores when the original games are employed in place of the transformed ones, with an almost four-fold increase. The intuition behind this is, loosely speaking, that the IBE is not as parameter-free as it looks: that is, by utilizing transformed payoffs for each game (although based on common definition of aspiration level), it effectively allows for game-specific adjustments similar to those obtained by adding a parameter which can take different values in each game.

games are employed (.214 and .147, respectively). This doesn't hold for the EIBE concept, whose score in the prediction field in the last row is roughly double the one in the best fit row (.567 in place of .279). Note also that the average MSD for equity-driven QRE when cross-predicting is approximately equal to the mean score for IBE on all transformed games (.214 for EQRE as opposed to .215 for IBE), further confirming the stability of the parameters in the other-regarding version of QRE. Again, this cannot be said for EIBE, whose score when using parameters fitted out of sample is substantially higher than the score for the parameter-free impulse balance equilibrium (.567 to be compared to .215).

## 7. Discussion of the results

Based on the above comparisons, the inequity aversion generalization of the quantal response equilibrium concept appears to emerge as the best performing in terms of goodness of fit among the considered stationary concepts. Following the behavioral stationary concept interpretation of mixed equilibrium<sup>12</sup>, the experimental evidence leads to the conclusion that, among the stationary concepts considered here, the proposed other-regarding generalization of the QRE is the behavioral concept that best models the probability of choosing one of two strategies in various non constant-sum games spanning a wide parameter space. More specifically, even when restricting the degrees of freedom of the parametric models and comparing the goodness of fit utilizing the same parameters ( $\beta, \alpha, \lambda$  if any) for all six games, the other-regarding QRE outperforms all of the other stationary concepts considered here.

The explanatory power of the considered models leads to the following ranking, starting with the most successful in terms of fit to the experimental data (and with the goodness of fit decreasing progressively): EQRE, IBE, EIBE, QRE and Nash equilibrium.<sup>13</sup>

Of course, more parsimonious concepts such as NE and IBE, are at a disadvantage when compared to parameterized models such as EIBE and EQRE, due to the parameter-free nature of the former two (see footnote 5 regarding IBE). For this reason the above ranking is based on rows 1, 3 and 5 in Table 5, avoiding to give an unfair advantage to the proposed parametric models due to fitting the parameters to each game separately.

<sup>12</sup> that sees it as the result of evolutionary (or learning) processes in a situation of frequently repeated play with two populations of randomly matched opponents.

<sup>13</sup> See the grey highlighted rows in Table 5.

It is significant to note that the order of the four concepts established under the above comparison, namely EQRE, IBE, EIBE and NE, is confirmed when restricting attention to the MSD obtained with parameters estimated out-of-sample for the parametric concepts (see the last row of Table 5).

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*Appendix A: Games utilized in Selten & Chmura; in the present paper only games 7 to 12 (non-constant sum games) are investigated.*

Constant Sum Games		Non-Constant Sum Games																	
Game 1	<table border="1"> <tr> <td>10</td><td>8</td><td>0</td><td>18</td></tr> <tr> <td>9</td><td>9</td><td>10</td><td>8</td></tr> </table>	10	8	0	18	9	9	10	8	Game 7	<table border="1"> <tr> <td>10</td><td>12</td><td>4</td><td>22</td></tr> <tr> <td>9</td><td>9</td><td>14</td><td>8</td></tr> </table>	10	12	4	22	9	9	14	8
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9	9	10	8																
10	12	4	22																
9	9	14	8																
Game 2	<table border="1"> <tr> <td>9</td><td>4</td><td>0</td><td>13</td></tr> <tr> <td>6</td><td>7</td><td>8</td><td>5</td></tr> </table>	9	4	0	13	6	7	8	5	Game 8	<table border="1"> <tr> <td>9</td><td>7</td><td>3</td><td>16</td></tr> <tr> <td>6</td><td>7</td><td>11</td><td>5</td></tr> </table>	9	7	3	16	6	7	11	5
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Game 3	<table border="1"> <tr> <td>8</td><td>6</td><td>0</td><td>14</td></tr> <tr> <td>7</td><td>7</td><td>10</td><td>4</td></tr> </table>	8	6	0	14	7	7	10	4	Game 9	<table border="1"> <tr> <td>8</td><td>9</td><td>3</td><td>17</td></tr> <tr> <td>7</td><td>7</td><td>13</td><td>4</td></tr> </table>	8	9	3	17	7	7	13	4
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Game 4	<table border="1"> <tr> <td>7</td><td>4</td><td>0</td><td>11</td></tr> <tr> <td>5</td><td>6</td><td>9</td><td>2</td></tr> </table>	7	4	0	11	5	6	9	2	Game 10	<table border="1"> <tr> <td>7</td><td>6</td><td>2</td><td>13</td></tr> <tr> <td>5</td><td>6</td><td>11</td><td>2</td></tr> </table>	7	6	2	13	5	6	11	2
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7	6	2	13																
5	6	11	2																
Game 5	<table border="1"> <tr> <td>7</td><td>2</td><td>0</td><td>9</td></tr> <tr> <td>4</td><td>5</td><td>8</td><td>1</td></tr> </table>	7	2	0	9	4	5	8	1	Game 11	<table border="1"> <tr> <td>7</td><td>4</td><td>2</td><td>11</td></tr> <tr> <td>4</td><td>5</td><td>10</td><td>1</td></tr> </table>	7	4	2	11	4	5	10	1
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Game 6	<table border="1"> <tr> <td>7</td><td>1</td><td>1</td><td>7</td></tr> <tr> <td>3</td><td>5</td><td>8</td><td>0</td></tr> </table>	7	1	1	7	3	5	8	0	Game 12	<table border="1"> <tr> <td>7</td><td>3</td><td>3</td><td>9</td></tr> <tr> <td>3</td><td>5</td><td>10</td><td>0</td></tr> </table>	7	3	3	9	3	5	10	0
7	1	1	7																
3	5	8	0																
7	3	3	9																
3	5	10	0																

L: left R: right  
U: up D: down

Player 1's payoff is shown in the upper left corner  
Player 2's payoff is shown in the lower right corner