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# Increasing Fundraising Success by Decreasing Donor Choice 

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Suggested contributions, membership categories, and discrete, incremental thank-you gifts are devices often used by benevolent associations that provide public goods. Such devices focus donations into discrete levels, thereby effectively limiting the donors' freedom to give. We study the effects on overall donations of the tradeoff between rigid schemes that severely restrict the choices of contribution on the one hand, and flexible membership contracts on the other, taking into account the strategic response of contributors whose values for the public good are private information. We show flexibility dominates when i) the dispersion of donors taste for the public good increases, ii) the number of potential donors increases, and iii) there is greater funding by an external authority. Using the number of default membership categories that National Public Radio stations offer as proxy for flexibility, we document the existence of empirical correlations consistent with our predictions: stations offer a larger number of suggested contribution levels as i) the incomes of the population served become more diverse, ii) the population of the coverage area increases, and iii) there is greater external support from the Corporation for Public Broadcasting.


Keywords: private provision, categories, restricting donations, heterogeneity, crowding out JEL: H41, D61, D82

# Increasing Fundraising Success by Decreasing Donor Choice* 

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Suggested contributions, membership categories, and discrete, incremental thank-you gifts are devices often used by benevolent associations that provide public goods. Such devices focus donations into discrete levels, thereby effectively limiting the donors' freedom to give. We study the effects on overall donations of the tradeoff between rigid schemes that severely restrict the choices of contribution on the one hand, and flexible membership contracts on the other, taking into account the strategic response of contributors whose values for the public good are private information. We show flexibility dominates when i) the dispersion of donors' taste for the public good increases, ii) the number of potential donors increases, and iii) there is greater funding by an external authority. Using the number of default membership categories that National Public Radio stations offer as proxy for flexibility, we document the existence of empirical correlations consistent with our predictions: stations offer a larger number of suggested contribution levels as i) the incomes of the population served become more diverse, ii) the population of the coverage area increases, and iii) there is greater external support from the Corporation for Public Broadcasting.


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## 1 Introduction

Private provision of public goods plays an important role in the US economy. ${ }^{1}$ Beyond familiar charities such as the Red Cross and environmental groups like the Sierra Club, many local associations support orchestras, zoos, community radio stations, and various other endeavors that can, at least in part, be thought of as public goods. It is therefore not surprising that the economics literature provides multiple answers to the question of why people give, including enlightened self-interest and altruism (e.g. Bergstrom et al., 1986), warm-glow (e.g. Andreoni, 1989, 1990), prestige (Harbaugh, 1998a, 1998b), signaling (Glazer and Konrad, 1996), and selective incentives (Olson, 1965). The psychology, sociology, and marketing literatures add many other motivations, including the simple fact of being asked and the "even-a-penny-helps" technique. ${ }^{2}$ Fundraising activities may well take into account all these motivations at various stages of a campaign.

We focus on one of the most common practices association managers and fund raisers use: accepting, recommending, recognizing, or otherwise rewarding donations according to endogenously designed bins or categories. ${ }^{3}$ This practice may take the form of a minimum suggested or accepted donation, of some level that must be reached to publicize a donation, of affixing nicknames to donation categories (e.g., in increasing order of donation, "member," "supporter," "benefactor"), or more generally of offering different combinations of selective incentives at various levels of contributions (for example, a bumper sticker for a $\$ 20$ donation, a bumper sticker and an audio cd for a $\$ 50$ donation, and so on).

A number of questions naturally arise about this practice. What response does it elicit from donors? How should levels that trigger benefits be chosen? Are there any observable characteristics of the donor population that push toward offering a membership scheme with many levels rather than only a few? ${ }^{4}$ First, we provide a simple theoretical framework in which to analyze these questions. Then, using data about the membership schemes offered by National Public Radio stations, we showcase empirical correlations consistent with our findings.

We choose a "positive" theoretical approach, similar to the one of Harbaugh (1998a), who directly targets the relationship between categories and prestige. ${ }^{5}$ Harbaugh (1998a) posits a pure warm-glow motivation

[^1]for giving (donors receive a private benefit from their donations) and shows that, generically, creating one category donations have to fall into to be recognized-and thus rewarded with the additional private good "prestige"-dominates recognizing donations based on their exact amount. The force behind this result is "bunching at the low end" of a category, an effect empirically confirmed in Harbaugh (1998b), and experimentally observed by Andreoni and Petrie (2004) and by Li and Ryanto (2009). Transitioning from exact to categorical recognition, "bunching" refers to donations that end up being clustered at the cutoff value that triggers the beginning of a category, rather than falling in a neighborhood on either side of such cutoff.

Beyond prestige, "bunching at the low end" can be expected for other motivations for giving as well. Indeed, in a purely material selective benefit example, consider a radio station that rewards donations with discounts at various merchants. Suppose first that discounts are directly proportional to the donated amount, a scheme that may be thought of as having very many categories, and consider a donor willing to give $\$ 20$. Consider now a different membership scheme where the reward is a fixed discount level, but only if the donation exceeds $\$ 25$. The donor may then decide to bump up his contribution to $\$ 25$, and a similar behavior would be expected of all donors otherwise willing to donate an amount between, say, $\$ 20$ and $\$ 24.99$, thus creating bunching of contributions (in the absence of other strategic considerations). An effect going in the opposite direction is also possible when the continuous-benefit scheme is replaced by the discrete one, since donations of $\$ 25$ and $\$ 26$ are now rewarded in the same way, thus reinforcing bunching toward $\$ 25$.

From a theoretical point of view, the exact nature of the benefit, whether prestige or a more general selective incentive, is not fundamental to create bunching. Moreover, Croson and Marks (2001) experimentally observe bunching even in the case of simple suggestions of donation levels. Indeed, a very similar effect could be reached, albeit simplistically, by restricting the agents' ability to donate to exactly $\$ 25$ or nothing at all. ${ }^{6}$ We find this is the easiest way to think about the effects of categories. Thus, our earlier questions can be rephrased as: When does it pay for an association to restrict agents' freedom to give? What determines whether an association should offer a rigid, restrictive membership scheme or a more flexible one with many categories?

After this reformulation, a natural place to look for an answer is the experimental literature comparing discrete-level contribution models with those with continuous contributions. There appears to be an expectation that continuous-level contribution schemes perform better. Cadsby and Maynes (1999), Hsu (2003),

[^2]and Suleiman and Rapoport (1992) report finding in experimental situations that allowing continuous contribution possibilities significantly increases contributions over requiring that contributors either contribute nothing or their entire endowment. Authors explain this finding by noting that with continuous contributions there is a symmetric pure strategy equilibrium with provision while no such equilibrium exists when the contribution options are all or nothing (e.g., Cadsby and Maynes, 1999, p. 57). However, as Andreoni and Petrie (2004) point out, the theoretical comparison between total contributions when all donation amounts are possible and when agents' freedom to give is restricted depends on what contribution levels are available. No effort in this direction appears in this literature.

Therefore, at least three aspects appear deserving of more study. First, while the warm-glow model of Harbaugh's is surely interesting, it is worth investigating the effects of restricting agents' freedom to donate in a pure-public-good model, thereby reintroducing strategic considerations and free-riding into the picture. Second, how is the optimal discrete contribution level chosen in this framework? Third, under what conditions does the restricted contribution-level membership scheme perform better than one with unrestricted levels? In particular, is one scheme always better than the other or does the choice reveal a true trade-off? And in this last case, which observable characteristics of the donor population are important for the trade-off?

Our basic theoretical model provides answers to these questions. We cast our analysis in a private-values subscription game framework. As our baseline case, we suppose all donations are welcome, as in Barbieri and Malueg (2009). Next, we consider the alternative policy in which the fundraisers specify a particular contribution that they will accept. This policy imposes bunching of types, and, as a representation of actual membership schemes, it favors simplicity over realism. After characterizing the optimal level for the accepted contribution, we demonstrate the importance of the shape of the cumulative distribution function describing players' private values for the discrete good: if it is convex (concave), the single contribution threshold (unrestricted contribution campaign) always raises greater contributions. While these cases are important for identifying the forces that make the continuous or discrete contribution framework preferred, it is more reasonable to expect the density of these values to be initially increasing and then decreasing if private values for the public good are correlated with income.

In this last case - that is, when the cumulative distribution of values is first convex and then concave - a true trade-off emerges. We showcase the basic forces underlying the decision by the fundraisers whether to restrict the freedom in choosing a contribution level by potential contributors, or to offer a flexible membership scheme in which choices are less constrained. Two such forces are "dispersion of values" and "extent of crowding-out," leading to the following predictions. A membership scheme that restricts contributors' decisions becomes less attractive as

1. the dispersion of values increases,
2. the number of potential contributors increases, or
3. the amount provided by an external authority increases.

Moreover, we show graphically and by example how these predictions remain valid when our basic model is enriched to cope with more realistic membership schemes in which agents are free to donate any amount they desire, but benefits kick in only for donations above a pre-specified amount.

Finally, we turn our attention to the actual behavior of fundraisers and association organizers. Do they appear to behave in a way consistent with the predictions of the model? We analyze the membership schemes offered by all National Public Radio stations in the continental US, proxying flexibility with the number of "default" membership levels offered by a station, and we present empirical relations consistent with the three predictions of our model.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3 we characterize the unique symmetric equilibrium for both restricted- and unrestricted-level schemes, and we calculate the optimal discrete contribution level. Section 4 explores the role of the shape of the distribution of values and presents the implications of "dispersion of values" and "extent of crowding-out" on the choice of restricting agents' flexibility to donate. Section 5 contains the empirical evidence and Section 6 concludes.

## 2 The Model

We study the problem of $n$ players who simultaneously contribute to the funding of a binary public good. Player $i$ 's value for the good is $v_{i}, i=1, \ldots, n$. Players' values are independently and identically distributed random variables with cumulative distribution functions (cdf) $F$, which has support $[0,1]$. A player's realized value is known only to that player. We suppose $F$ is continuous with density function $f$. The cost of the public good is $c$, which we assume is a random variable uniformly distributed over the interval $[0, \bar{c}]$, where $\bar{c} \geq n$, and $c$ is independent of players' values. ${ }^{7}$ The foregoing description is common knowledge.

In the terminology of Admati and Perry (1991), we consider the subscription game: players' contributions are refunded if they are insufficient to cover $c$. If the good is provided, then the payoff to player $i$ is $v_{i}-$ (player $i$ 's contribution). If the good is not provided, then the payoff to player $i$ is 0 .

[^3]
## 3 Equilibria

We look for a symmetric equilibrium strategy $s$. The expected utility of agent $i$ with value $v_{i}$ contributing $x$ when other players use strategy $s(\cdot)$ is

$$
\begin{equation*}
U_{i}\left(x \mid v_{i}\right) \equiv\left(v_{i}-x\right) \operatorname{Pr}\left(c \leq x+\sum_{j \neq i} s\left(v_{j}\right)\right) \tag{1}
\end{equation*}
$$

Because cost is distributed uniformly and independently of players' values, if agent $i$ contributes $x$, then the probability that the good is provided is

$$
\operatorname{Pr}\left(c \leq x+\sum_{j \neq i} s\left(v_{j}\right)\right)=\mathrm{E}\left[\operatorname{Pr}\left(c \leq x+\sum_{j \neq i} s\left(v_{j}\right) \mid v_{j}, j \neq i\right)\right]=\mathrm{E}\left[\frac{x+\sum_{j \neq i} s\left(v_{j}\right)}{\bar{c}}\right]=\frac{x+(n-1) K}{\bar{c}},
$$

where $K \equiv \mathrm{E}\left[s\left(v_{j}\right)\right]$ is the expected contribution of agent $j$ using strategy $s$. Now the expected utility of agent $i,(1)$, becomes

$$
\begin{equation*}
U_{i}\left(x \mid v_{i}\right)=\frac{1}{\bar{c}}\left(v_{i}-x\right)(x+(n-1) K) \tag{2}
\end{equation*}
$$

Note that if each player's expected contribution is $K$, then the probability of provision is $\mathrm{E}\left[\operatorname{Pr}\left(c \leq \sum s\left(v_{i}\right)\right)\right]=$ $n K / \bar{c}$. Thus, any change in the fundraising mechanism that changes the expected contribution will directly affect the probability that the good is provided. Moreover, note that the uncertainty in the cost threshold $c$ makes our framework very close to the more traditional one of Bergstrom et al. (1986) in which the quantity of the public good is variable and contributions are sunk. Indeed, if agents have a simple multiplicative utility function over private and public good consumption, if $v_{i}$ is reinterpreted as income so that private good consumption is $v_{i}-x$, and if the public good is available at constant marginal cost $\bar{c}$ (so the total provided just equals total donations divided by $\bar{c}$ ), then the expected utility of agent $i$ with income $v_{i}$ that makes donation $x$ is given in (2).

### 3.1 Unrestricted contribution possibilities

Here we characterize the unique equilibrium when any nonnegative contributions are allowed. Since $U_{i}$ in (2) is strictly concave in $x$, the first-derivative $\partial U_{i}\left(x \mid v_{i}\right) / \partial x=\left[v_{i}-(n-1) K-2 x\right] / \bar{c}$, along with the non-negativity constraint on $x$, yields the following "best-response" function for player $i$ : $s\left(v_{i}\right)=$ $\max \left\{0,\left[v_{i}-(n-1) K\right] / 2\right\}$. Using this best-response function and the definition of $K$ above, in the sym-
metric equilibrium the following equation must be satisfied by $K$ :

$$
\begin{align*}
K & =\mathrm{E}\left[\max \left\{0, \frac{1}{2}(v-(n-1) K)\right\}\right]=\frac{1}{2} \int_{(n-1) K}^{1}(v-(n-1) K) f(v) d v \\
& =\frac{1}{2} \int_{(n-1) K}^{1}(1-F(v)) d v \tag{3}
\end{align*}
$$

where the final inequality follows from integration by parts. The right-hand side of (3) is continuous and strictly decreasing in $K$ over $[0,1 /(n-1)]$, with value $\mathrm{E}[v] / 2>0$ at $K=0$ and value 0 at $K=1 /(n-1)$. Therefore, there is a unique value of $K$, which we denote by $K^{c}$, that solves (3). Consequently, with unrestricted contributions there is a unique symmetric equilibrium strategy, which is given by

$$
s^{c}(v)= \begin{cases}\frac{1}{2}\left(v-(n-1) K^{c}\right) & \text { if } v \geq(n-1) K^{c}  \tag{4}\\ 0 & \text { otherwise }\end{cases}
$$

where $K^{c}$ solves (3) (it can be shown there are no asymmetric equilibria-see Barbieri and Malueg, 2009). The following two-player example illustrates equilibrium in the subscription game with threshold uncertainty when contributions are not restricted.

Example 1 (Values are distributed between 0 and 1 according to a convex cdf).

Consider two players whose values are independently and identically distributed on $[0,1]$ according to the cdf $F(v)=v^{2}$. Then (3) reduces to

$$
K=\int_{K}^{1} \frac{1}{2}\left(1-v^{2}\right) d v=\frac{1}{3}-\frac{K}{2}+\frac{K^{3}}{6}
$$

the solution to which is $K^{c} \approx 0.223462$, which is also each player's expected contribution.

### 3.2 Binary contribution possibilities

Next we suppose players are restricted to contribution levels of 0 and $x$, where $x \in(0,1)$. The equilibrium strategy will be of the form

$$
s(v)= \begin{cases}x & \text { if } v \geq v^{0} \\ 0 & \text { if } v<v^{0}\end{cases}
$$

for some value $v^{0}$. Suppose all players but player 1 use such a strategy. If player 1 has value $v$, her expected payoff when not contributing is

$$
U^{n c}(v)=v \operatorname{Pr}\left(\sum_{j \neq 1} s\left(v_{j}\right) \geq c\right)=v \times \frac{(n-1)\left(1-F\left(v^{0}\right)\right) x}{\bar{c}}
$$

and her expected payoff when contributing $x$ is

$$
U^{c}(v)=(v-x) \operatorname{Pr}\left(x+\sum_{j \neq 1} s\left(v_{j}\right) \geq c\right)=(v-x) \times \frac{x+(n-1)\left(1-F\left(v^{0}\right)\right) x}{\bar{c}}
$$

Solving the indifference condition $U^{n c}(v)=U^{c}(v)$ yields the threshold value

$$
\begin{equation*}
v^{0}=x\left[1+(n-1)\left(1-F\left(v^{0}\right)\right)\right]=x+(n-1) K \tag{5}
\end{equation*}
$$

where $K=x\left(1-F\left(v^{0}\right)\right)$ is a player's expected contribution. The middle expression in (5) is strictly decreasing in $v^{0}$, with value $n x$ when $v^{0}=0$ and value $x$ when $v^{0}=1$, implying that for each $x \in(0,1)$, there is a unique solution $v^{0}$ to (5). Hence, there is a unique symmetric equilibrium in the subscription game with binary contribution possibilities.

The following example applies the above analysis to show the common intuition favoring unrestricted contributions over discrete contribution possibilities may not be warranted.

Example 2 (The probability of provision: binary or continuous contribution possibilities).

We again suppose there are two players, values are independently and identically distributed on $[0,1]$ according to the $\operatorname{cdf} F(v)=v^{2}$, and cost is uniformly distributed over $[0, \bar{c}]$, where $\bar{c} \geq 2$. From Example 1 we know that each player's expected contribution in the unrestricted-contribution case is $K^{c} \approx 0.223462$.

Next suppose players' contributions are restricted to be either 0 or $x$ (we may assume $x \leq 1$ ). Equilibrium has players use a strategy given by

$$
s(v)= \begin{cases}x & \text { if } v \geq v^{0} \\ 0 & \text { if } v<v^{0}\end{cases}
$$

where solution of the first equality of (5) yields the critical threshold $v^{0}(x)=\left(\sqrt{1+8 x^{2}}-1\right) /(2 x)$. Each player's expected contribution is then $K(x) \equiv x \operatorname{Pr}\left(v \geq v^{0}(x)\right)=\left(\sqrt{1+8 x^{2}}-1-2 x^{2}\right) /(2 x)$. This expected contribution is strictly concave in on $[0,1]$, reaching its maximum at $x^{*}=(\sqrt{7-\sqrt{17}}) / 4 \approx 0.424035$; the resulting expected contribution of each player is $K\left(x^{*}\right) \approx 0.238118$, which exceeds the expected contribution in the unrestricted-contribution model by about $6 \%$. Identical conclusions hold as well for the probability
of provision in the two settings. Obviously, though, for "poor" choices of $x$, the binary-contribution model yields strictly lower contributions than does the unrestricted model.

Example 2 clarifies how the choice of level for the restricted contribution scheme is crucial. For the rest of the analysis, we denote with $K^{d}$ a player's equilibrium expected contribution when the only contributions allowed are $\left\{0, x^{d}\right\}$, where $x^{d}$ is the level that maximizes the equilibrium expected contribution. Thus, $K^{d}=\left[1-F\left(v^{0}\right)\right] x^{d}$, where $v^{0}$ is the threshold value above which a player contributes, and, by $(5), v^{0}=$ $x^{d}+(n-1) K^{d}$. At $x^{d}$, the first-order condition $d K^{d} / d x^{d}=0$ implies

$$
\begin{equation*}
1-F\left(v^{0}\right)=x^{d} f\left(v^{0}\right) \tag{6}
\end{equation*}
$$

The following lemma, building on (5) and (6), is useful for the rest of the analysis.

Lemma 1 (Bounding the best binary contribution). Let $F$ be the common distribution of the $n$ players' independent values.

1. If $F$ is convex on $\left[v^{0}, 1\right]$, then $x^{d} \geq\left[1-(n-1) K^{d}\right] / 2$, with strict inequality if $F$ is strictly convex on $\left[v^{0}, 1\right]$.
2. If $F$ is concave on $\left[v^{0}, 1\right]$, then $x^{d} \leq\left[1-(n-1) K^{d}\right] / 2$, with strict inequality if $F$ is strictly concave on $\left[v^{0}, 1\right]$.

Proof. If $F$ is convex on $\left[v^{0}, 1\right]$, then $1=F(1) \geq F\left(v^{0}\right)+f\left(v^{0}\right)\left(1-v^{0}\right)$, so

$$
\begin{align*}
x^{d} & =\frac{1-F\left(x^{d}+(n-1) K^{d}\right)}{f\left(x^{d}+(n-1) K^{d}\right)} & & (\text { by }(5) \text { and }(6)) \\
& \geq 1-\left(x^{d}+(n-1) K^{d}\right), & & \left(\text { by convexity of } F \text { on }\left[v^{0}, 1\right]\right) \tag{7}
\end{align*}
$$

implying

$$
\begin{equation*}
x^{d} \geq \frac{1}{2}\left(1-(n-1) K^{d}\right) \tag{8}
\end{equation*}
$$

If $F$ is strictly convex on $\left[v^{0}, 1\right]$, then the inequality in (7) holds strictly, and so too does (8). If instead $F$ is concave on $\left[v^{0}, 1\right]$, then the inequality in (7) is reversed as is that in (8), with strict inequality holding in both (7) and (8) if $F$ is strictly concave on $\left[v^{0}, 1\right]$.

## 4 Continuous or discrete contributions?

If instead of allowing all contribution levels, the fundraisers restrict contributions to a finite set, then they face a tradeoff. On the one hand, some who might have preferred to give a positive amount now find themselves unwilling to give the minimum acceptable amount, which may reduce overall contributions. On the other hand, some who had planned to give an "intermediate" amount might now prefer to bump up their contributions to the minimum acceptable level, causing them to contribute more than they might otherwise have done, and this tends to raise contributions. Overall, the effect of setting a target contribution will balance these two effects, causing some potential contributors to drop out while encouraging others to give slightly more.

### 4.1 The cases of convex or concave $F$

Our first proposition shows that, when the density of players' values is either increasing or decreasing, fundraisers have a clear preference for either the continuous or the binary contribution scheme.

Proposition 1 (Continuous versus binary contributions). Let the common distribution of players' independent values be $F$.

1. If $F$ is convex, then $K^{d} \geq K^{c}$, with strict inequality if $F$ is strictly convex.
2. If $F$ is concave, then $K^{d} \leq K^{c}$, with strict inequality if $F$ is strictly concave.

Proof. First suppose $F$ is convex. The equilibrium contribution function in the continuous game is $s^{c}(v)=$ $\max \left\{0,\left[v-(n-1) K^{c}\right] / 2\right\}$. Define $x^{*}=s^{c}(1)=\left[1-(n-1) K^{c}\right] / 2$; in the equilibrium of the binary contribution game with $\left\{0, x^{*}\right\}$ denote by $K^{*}$ a player's expected contribution. Because $x^{d}$ maximizes a player's expected contribution, it must be that $K^{d} \geq K^{*}$. We will show $K^{*} \geq K^{c}$, with strict inequality if $F$ is strictly convex, thereby proving part 1 . Let $\varphi$ denote a uniform probability distribution on the interval $\left[(n-1) K^{c}, 1\right]$ (the use of this distribution will become clear below). The proof is by contradiction, so suppose the proposition is false, that is, $K^{*}<K^{c}$. Then

$$
\begin{align*}
K^{*} & =x^{*}\left[1-F\left(x^{*}+(n-1) K^{*}\right)\right] \\
& >x^{*}\left[1-F\left(x^{*}+(n-1) K^{c}\right)\right]  \tag{9}\\
& =\frac{1}{2}\left[1-(n-1) K^{c}\right]\left[1-F\left(\frac{1}{2}\left[1+(n-1) K^{c}\right]\right)\right] \\
& =\frac{1}{2}\left[1-(n-1) K^{c}\right][1-F(\mathrm{E}[v \mid \varphi])] \\
& \geq \frac{1}{2}\left[1-(n-1) K^{c}\right][1-\mathrm{E}[F(v) \mid \varphi]] \tag{10}
\end{align*}
$$

$$
\begin{aligned}
& =\frac{1}{2}\left[1-(n-1) K^{c}\right]\left[1-\frac{1}{1-(n-1) K^{c}} \int_{(n-1) K^{c}}^{1} F(v) d v\right] \\
& =\frac{1}{2}\left[\left[1-(n-1) K^{c}\right]-\int_{(n-1) K^{c}}^{1} F(v) d v\right] \\
& =\frac{1}{2} \int_{(n-1) K^{c}}^{1}(1-F(v)) d v \\
& =K^{c}
\end{aligned}
$$

contradicting the assumption that $K^{*}<K^{c}$. (The last equality follows from (3).) If, further, $F$ is strictly convex, then the contradiction hypothesis becomes $K^{*} \leq K^{c}$ and the inequality in (9) becomes weak while that in (10) becomes strict, again yielding a contradiction. This establishes part 1.

Next suppose $F$ is concave, and suppose contrary to part 2 that $K^{d}>K^{c}$. Then Lemma 1 implies that $x^{d}<\left(1-(n-1) K^{c}\right) / 2=s^{c}(1)$. We will show that the binary game with positive contribution $x^{d}$ yields expected revenue less than $K^{c}$, contradicting the initial assumption that $K^{d}>K^{c}$.


Figure 1: Comparison of binary and continuous expected contributions when $F$ is concave.

Figure 1 shows the comparison being made, where $v^{*}$ solves $s^{c}(v)=x^{d}$. It is readily checked that $v^{*}=(n-1) K^{c}+2 x^{d}$. By (5), the associated binary-contribution game equilibrium has threshold value $v^{0}=(n-1) K^{d}+x^{d}$. Let $\varphi$ denote a uniform probability distribution on the interval $\left[(n-1) K^{c}, v^{*}\right]$. A player's expected contribution in the continuous game is

$$
\begin{array}{rlr}
K^{c} & =\int_{(n-1) K^{c}}^{v^{*}} s^{c}(v) f(v) d v+\int_{v^{*}}^{1} s^{c}(v) f(v) d v \\
& >\int_{(n-1) K^{c}}^{v^{*}} s^{c}(v) f(v) d v+s^{c}\left(v^{*}\right)\left[1-F\left(v^{*}\right)\right] & \left(\text { by } v^{*}<1\right) \tag{11}
\end{array}
$$

$$
\begin{array}{ll}
=\left.s^{c}(v) F(v)\right|_{(n-1) K^{c}} ^{v^{*}}-\frac{1}{2} \int_{(n-1) K^{c}}^{v^{*}} F(v) d v+s^{c}\left(v^{*}\right)\left[1-F\left(v^{*}\right)\right] & \\
=s^{c}\left(v^{*}\right)-\frac{1}{2} \int_{(n-1) K^{c}}^{v^{*}} F(v) d v & \left(\text { by } s^{c}\left((n-1) K^{c}\right)=0\right) \\
=x^{d}\left[1-\frac{1}{2 x^{d}} \int_{(n-1) K^{c}}^{(n-1) K^{c}+2 x^{d}} F(v) d v\right] & \left(\text { by } s^{c}\left(v^{*}\right)=x^{d}\right) \\
=x^{d}[1-\mathrm{E}[F(v) \mid \varphi]] & \\
\geq x^{d}[1-F(\mathrm{E}[v \mid \varphi])] &  \tag{13}\\
=x^{d}\left[1-F\left((n-1) K^{c}+x^{d}\right)\right] & \text { (by } F \text { concave) } \\
\geq x^{d}\left[1-F\left((n-1) K^{d}+x^{d}\right)\right] & \\
=x^{d}\left[1-F\left(v^{0}\right)\right] & \text { (by contradiction) }) \\
=K^{d}, &
\end{array}
$$

contradicting the assumption that $K^{d}>K^{c}$. If, further, $F$ is strictly concave, then the contradiction hypothesis becomes $K^{d} \geq K^{c}$ and the inequality in (11) becomes weak while that in (13) becomes strict, again yielding a contradiction. This establishes part 2.

Example 2 illustrates the first parts of Lemma 1 and Proposition 1. The intuition for Proposition 1 can be understood with reference to Figure 2, which depicts equilibrium strategies when players' values are uniformly distributed over $[0,1]$. In this case, $x^{d}=s^{c}(1)$ and $K^{c}=K^{d}$. The restriction to contributing either 0 or $x^{d}$ leads types above $v^{0}$ to contribute more than in the continuous case, a benefit represented by region $B$. But this restriction causes types below $v^{0}$ to contribute nothing, and this cost is represented by region $A$. For the uniform distribution, $v^{0}$ is midway between $(n-1) K^{c}$ and 1 , so the areas of regions $A$ and $B$ are equal. And because the distribution of $v$ is uniform, the weighted benefit of region $B$ equals the weighted cost of region $A$. Now suppose the distribution of values deviates from uniform becoming slightly convex (i.e., the density is slightly increasing). Then, ignoring the induced change in strategies as a first-approximation, the weight on region $B$ becomes larger than that on region $A$, so the binarycontribution setting yields greater contributions than the unrestricted setting. The comparison is reversed if the distribution becomes slightly concave, as then region $A$ receives greater weight than region $B$. This accords with the general finding in Proposition 1.


Figure 2: Comparison of binary and continuous strategies when $F$ is concave: $K^{c}=K^{d}$.

### 4.2 The role of heterogeneity and crowding-out

When the cdf of players' values is neither concave nor convex, Proposition 1 does not yield a definitive comparison. It is however possible to obtain insights for the case of a symmetric distribution $F$ that is first convex and then concave. We provide an instance of such distribution in our leading example for this section.

Example 3 (Triangular distribution). Consider the density of $v$ given by $f(v ; a)=(1-(1 / 4) a)+a v$ for $v \in[0,1 / 2]$ and $f(v ; a)=(1-(1 / 4) a)+a(1-v)$ for $v \in(1 / 2,1]$, where $a \in[0,4]$ parameterizes the "peakedeness" of the distribution, as defined in Proschan (1965). When a is zero, we have the usual uniform distribution on $[0,1]$. As a increases, the weight on the tails of the distribution decreases and concentrates around the mean/median of $\frac{1}{2}$.

A first result, very useful for the rest of our analysis, is the following necessary condition for binary contributions to dominate continuous contributions. ${ }^{8}$

Proposition 2 (Comparison of binary and continuous contributions). Suppose the distribution $F$ of players' values is symmetric with mean and median $\mu$. Furthermore, assume $F$ is strictly convex for $v<\mu$ and $F$ is strictly concave for $v>\mu$. If $K^{d} \geq K^{c}$, then $v^{0}<\mu$.

By Proposition 2 (a proof of which is in the Appendix), setting a fixed donation level such that $v^{0}>\mu$ is counterproductive for the fundraiser. This result is intuitive, given the discussion preceding Proposition 1. There is a tradeoff in restricting agents' freedom to give. On the one hand, some who might have preferred to give a positive amount find themselves unwilling to give the minimum acceptable amount. On the other hand, some who had planned to give an "intermediate" amount now prefer to bump up their contributions.

[^4]The cutoff between these two different responses to a restriction is $v^{0}$ : types immediately below $v^{0}$ become non-contributors. If the fundraiser's choice puts $v^{0}$ in the concave part of the distribution, then types immediately below $v^{0}$-those who reduce their contribution-outnumber types immediately above $v^{0}$. We illustrate this reasoning for the all-important case $K^{c}=K^{d}$.

Figure 3 depicts the same comparison between the equilibrium contribution functions as in Figure 1, and we have superimposed a symmetric density function, labeled $f(v)$, for values. In Figure 3 , because of the assumption $K^{c}=K^{d}$, triangles $A$ and $B$ are congruent since $v^{0}-(n-1) K^{c}=v^{0}-(n-1) K^{d}=x^{d}$. The continuous contribution scheme dominates in area $C$. Therefore, according to the density $f$, area $B$ must be weighted more heavily than $A$ to assure $K^{c}=K^{d}$, and that would be impossible if $v^{0}>\mu$.


Figure 3: If $f$ is symmetric and single-peaked and if $K^{c}=K^{d}$, then $v^{0}<\mu$.

We now establish comparative statics for the binary contribution possibilities. The following proposition applies to all symmetric distributions $F(v ; a)$ that are strictly convex for $v$ that goes from zero to the mean/median $\mu$, strictly concave thereafter, and where the parameter $a$ captures peakedness as in Example 3 .

Proposition 3 (Comparative statics for binary contributions). If $v^{0}>\mu$, then $K^{d}$ decreases in $a$. If $v^{0}<\mu$, then $K^{d}$ increases in a.

Proof. Adapting the definition in Proschan (1965), for $a^{1}>a^{0}$, the distribution $F\left(v ; a^{1}\right)$ is more peaked than $F\left(v ; a^{0}\right)$ if, for any $t$

$$
\int_{\mu-t}^{\mu+t} f\left(v ; a^{1}\right) d v \geq \int_{\mu-t}^{\mu+t} f\left(v ; a^{0}\right) d v
$$

which, in our symmetric distribution environment, implies $F\left(v ; a^{0}\right) \geq F\left(v ; a^{1}\right)$ if $v \leq \mu$, and $F\left(v ; a^{0}\right) \leq$
$F\left(v ; a^{1}\right)$ if $v \geq \mu$. Therefore, thinking of a marginal change in $a$, we have

$$
\begin{equation*}
(v-\mu) \frac{\partial F(v ; a)}{\partial a} \geq 0 \tag{14}
\end{equation*}
$$

Applying the implicit function theorem with respect to $a$ to the system composed of equation (5), of equation (6), and of the definition of $K^{d}$ yields, after rearrangement

$$
\left(1+(n-1)\left[1-F\left(v^{0}(a) ; a\right)\right]\right) \frac{d K^{d}}{d a}=-x^{d} \frac{\partial F\left(v^{0}(a) ; a\right)}{\partial a}
$$

Therefore, using (14), $\frac{d K^{d}}{d a}$ has the opposite sign of $(v-\mu)$, thus establishing the proposition.
Proposition 3 is especially interesting in comparison with the continuous-contribution case. From Barbieri and Malueg (2009) we know that $K^{c}$ is always decreasing in $a$ (essentially this follows from the convexity of the contribution strategy in the continuous case). In contrast, the relationship for the binary contribution possibilities case depends on the position of the threshold type $v^{0}$ relative to $\mu$. This dependence is intuitive. If $v^{0}<\mu$ and peakedness increases, then, even leaving $x^{d}$ unchanged, the types above $\mu$ continue to contribute $x^{d}$ and among those types below $\mu$ the number who are contributors increases, so overall contributions increase. Allowing for optimal adjustment of the contribution level $x^{d}$ can further raise donations at the more peaked distribution. When $v^{0}>\mu$, an analogous effect shows that a small increase in peakedness will decrease expected donations in the binary-contribution case.

A consequence of Propositions 2 and 3 is a ranking of total contributions $K^{d}$ and $K^{c}$ that depends on the peakedness of the distribution.

Proposition 4 (Peakedness-induced ordering). As peakedness of the distribution $F(v ; a)$ increases, at most one intersection between $K^{d}$ and $K^{c}$ can occur, at which $K^{d}$ becomes larger than $K^{c}$.

Proof. Assume first, by contradiction, that an intersection between $K^{d}$ and $K^{c}$ occurs at which $K^{c}$ becomes larger than $K^{d}$. Then there exists a point at which $K^{d}=K^{c}$, but $d K^{c} / d a \geq d K^{d} / d a$. However, Proposition 2 implies $v^{0}<\mu$, and Proposition 3 further implies $d K^{d} / d a>0$, thus yielding $d K^{c} / d a>0$, which contradicts the fact that $d K^{c} / d a<0$, as established in Barbieri and Malueg (2009, Proposition 5).

Using the family of distributions in Example 3, Table 1 illustrates Propositions 2-4. In accord with Proposition 3, for $a=0,1,2$, we have $v^{0} \geq \mu$ and increases in peakedness reduce $K^{d}$; but for $a=2,3,4$, we have $v^{0} \leq \mu$ and increases in $a$ increase $K^{d}$. The data also reflect Proposition 4's conclusion that as the distribution becomes more peaked, the binary scheme may come to dominate the continuous-contribution scheme (here the ranking switches for a value of $a$ lying between 3 and 4 ). Note too that $K^{d}>K^{c}$ when

Table 1: Results for optimal continuous or binary contribution schemes, depending on peakedness, $n=2$

| $a$ | $K^{c}$ | $K^{d}$ | $x^{d}$ | $v^{0}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0 | 0.17157 | 0.17157 | 0.41421 | 0.58578 |
| 1 | 0.17058 | 0.16779 | 0.37242 | 0.54021 |
| 2 | 0.16961 | 0.16667 | 0.33333 | 0.5 |
| 3 | 0.16865 | 0.16718 | 0.31415 | 0.48133 |
| 4 | 0.16772 | 0.16837 | 0.29984 | 0.46821 |

$a=4$, so Proposition 2 implies $v^{0}<\mu$, which is indeed the case here ( $v^{0}=0.46821<.5=\mu$ ). More generally, whether the graphs of $K^{d}$ and $K^{c}$ intersect depends on the available range for the peakedness. One may show that if the distribution $F(v, a)$ goes, in order of increasing peakedness, from uniform on $[0, \bar{v}]$, to a degenerate distribution on $\mu=\bar{v} / 2$, then the intersection will happen. ${ }^{9}$

The relationship between the binary vs. continuous comparison and peakedness, as just discussed, is intuitive. Offering only a limited number of alternative contribution levels-the binary contribution possibilities is an extreme case - is a way to target a subset of types (those in a right neighborhood of $v^{0}$ ) and induce them to contribute more than they otherwise would. Clearly, this effect obtains because agents have fewer contribution options. The down side of this restriction of contribution possibilities is that some types may choose to contribute less than they otherwise would. Types smaller than the target may decide to contribute nothing at all while they would have contributed a smaller, but positive amount, if given the opportunity. Similarly, types larger than the target may be constrained to contribute less than they would have done if given more alternatives. The situation is illustrated in Figure 3 for $K^{c}=K^{d}$. Now increase peakedness slightly, and, as a first-approximation, suppose in the two scenarios players continue using the strategies depicted. An increase in peakedness of the distribution of values tends to reduce the significance of regions $A$ and $C$ while increasing that of region $B$, so that the equivalence of donations under the two contribution schemes breaks in favor of $K^{d}$ as $F$ becomes more peaked.

We now hold peakedness constant and consider how changes in the number of potential contributors affect the relationship between $K^{d}$ and $K^{c}$.

Proposition 5 (Number-of-player-induced ordering). As the number of players $n$ increases, at most one intersection between $K^{d}$ and $K^{c}$ can occur, at which $K^{c}$ becomes larger than $K^{d}$. Moreover, for $n$ sufficiently large, $K^{c}>K^{d}$.

Proof. It is immediate to verify that both $K^{c}$ and $K^{d}$ are decreasing in $n$. Therefore, the first part of the

[^5]proof is complete after we establish that if $K^{d} \geq K^{c}$, then $\left|\frac{d K^{d}}{d n}\right|>\left|\frac{d K^{c}}{d n}\right|$. From equations (3)-(6) and the definition of $K^{d}$ we obtain
\[

$$
\begin{aligned}
\left|\frac{d K^{d}}{d n}\right|-\left|\frac{d K^{c}}{d n}\right| & =\frac{1-F\left(v^{0}\right)}{1+(n-1)\left(1-F\left(v^{0}\right)\right)} K^{d}-\frac{1-F\left((n-1) K^{c}\right)}{2+(n-1)\left(1-F\left((n-1) K^{c}\right)\right)} K^{c} \\
& \geq \frac{1-F\left(v^{0}\right)}{1+(n-1)\left(1-F\left(v^{0}\right)\right)} K^{c}-\frac{1-F\left((n-1) K^{c}\right)}{2+(n-1)\left(1-F\left((n-1) K^{c}\right)\right)} K^{c} \quad\left(K^{d} \geq K^{c} \text { is assumed }\right) \\
& =\left(\frac{1-2 F\left(v^{0}\right)+F\left((n-1) K^{c}\right)}{\left[1+(n-1)\left(1-F\left(v^{0}\right)\right)\right]\left[2+(n-1)\left(1-F\left((n-1) K^{c}\right)\right)\right]}\right) K^{c} \\
& >0
\end{aligned}
$$
\]

where the final inequality follows because $v^{0}<\mu$ (which follows from Proposition 2) implies $F\left(v^{0}\right)<1 / 2$.
To show that for $n$ sufficiently large $K^{d}<K^{c}$, proceed by contradiction; that is, suppose that there exists some $N^{\prime}$ such that for all $n>N^{\prime}, K^{d} \geq K^{c}$ (the possibility of multiple intersections is excluded by the analysis of the previous paragraph). By Proposition 2, it must be that $v^{0}<\mu$, for all $n>N^{\prime}$. From the definition of $K^{d}$ we have $K^{d} \geq x^{d} / 2$, and by equation (6) $x^{d} \geq 1 /\left(2 f\left(v^{0}\right)\right) \geq 1 /(2 f(\mu))$, which together imply $\lim _{n \rightarrow \infty} K^{d} \geq 1 /(4 f(\mu))>0$; therefore $\lim _{n \rightarrow \infty} v^{0}=\lim _{n \rightarrow \infty}(n-1) K^{d}+x^{d}=+\infty$, which contradicts $v^{0}<\mu$.

The following example illustrates Proposition 5.

Example 4 (Triangular distribution continued). Consider three players and the same distribution the density $f(v ; a)$ in Example 3. For $a=4$ we have $K^{c}=0.12778$ and $K^{d}=0.125$. A comparison with the previous calculations for the two-player case in Table 1 reveals that in moving from 2 to 3 players, the ranking of $K^{c}$ and $K^{d}$ switches, fixing peakedness at $a=4$, in accordance with Proposition 5.

It turns out that the main force underlying our result on the number of agents is the same we identify in the next proposition about crowding-out. Let $y$ denote the level of contributions that are exogenously provided by an external authority, and consider how players' contributions change as $y$ increases. Replicating the steps leading to (3), we obtain that, in equilibrium, the expected contribution amount with unrestricted contribution solves

$$
\begin{equation*}
K^{c}=\frac{1}{2} \int_{(n-1) K^{c}+y}^{1}(1-F(v)) d v \tag{15}
\end{equation*}
$$

Similarly, when contributions are restricted, the indifferent type $v^{0}$, the optimally chosen level $x^{d}$, and the
expected contribution amount $K^{d}$ solve

$$
\begin{equation*}
v^{0}=x^{d}+(n-1) K^{d}+y, \quad x^{d}=\frac{1-F\left(v^{0}\right)}{f\left(v^{0}\right)}, \quad \text { and } \quad K^{d}=x^{d}\left(1-F\left(v^{0}\right)\right) \tag{16}
\end{equation*}
$$

We now hold constant all other parameters and consider how changes in the amount $y$, exogenously given by an external authority, affect the relationship between $K^{d}$ and $K^{c}$. (The proof of Proposition 6 , which is similar to that for Proposition 5, is given in the Appendix.)

Proposition 6 (Crowding-out-induced ordering). As the amount y increases, at most one intersection between $K^{d}$ and $K^{c}$ can occur, at which $K^{c}$ becomes larger than $K^{d}$.

The following example illustrates Proposition 6.

Example 5 (Triangular distribution continued). Consider two players and the same distribution the density $f(v ; a)$ in Example 3. For $a=4$ and $y=0$, as derived in our previous calculations in Table 1, $K^{c}=0.16772$ and $K^{d}=0.16837$. For $a=4$ and $y=0.1$ we have $K^{c}=0.13626$ and $K^{d}=0.13341$. Thus, the ranking of $K^{c}$ and $K^{d}$ switches as we increase $y$, in accordance with Proposition 6.

Propositions 5 and 6 are two manifestations of the same main force. Contributions from a membership scheme that restricts contributors' decisions are more responsive to changes in the environment (e.g., an increase in the number of potential contributors or an increase in external donations) than contributions from a more flexible scheme, conditional on the fundraiser being indifferent between the two, that is for $K^{c}=K^{d}$. The intuition for the result is that a flexible mechanism allows agents wishing to reduce their contributions to do so in a smooth, measured manner that is largely independent of their value. Indeed, in equilibrium, a given change in the expected donation of one player is achieved because (almost) all types who were contributing a positive amount end up reducing their donation by a common quantity. The adjustment is very different in a rigid membership scheme. The only possibility to reduce one's donation is to stop contributing at all. True to its characterization, a rigid membership scheme "breaks but does not bend" and forces a jerky response from agent types: types sufficiently far from $v^{0}$ do not change their behavior at all, while types sufficiently close to $v^{0}$ precipitously drop their contribution from $x$ to nothing. ${ }^{10}$ Which of the the smooth or the jerky adjustments ends up being larger then depends on the relative importance of types near $v^{0}$. As discussed earlier and as depicted in Figure 3, when the fundraiser is indifferent between the flexible or the rigid scheme, that is for $K^{d}=K^{c}$, it must be the case that types near $v^{0}$ are very important, and the jerky adjustment ends up being larger in expectation. Therefore, the larger the number of potential

[^6]contributors or the amount provided by external sources, the larger the crowding-out for a rigid contribution scheme, relative to a flexible one, up to the point in which flexibility becomes preferred by the fundraisers.

### 4.3 Benefit-induced restrictions

The main objective of this section is showing that our earlier results, in particular Propositions 4, 5, and 6, survive when we consider a more realistic membership scheme in which agents are free to donate any amount they desire. "Restrictions" in donations arise from the package of selective benefits. With respect to the model in Section 2, nothing changes about the way in which agents benefit from the public good. However, we now assume that contributors also enjoy a selective benefit $b(x)$, distributed by the association in exchange for a donation level $x$. We maintain the assumption, typical of the subscription game, that if the public good cannot be produced, then agents receive their contributions back and obtain a payoff of zero. When the public good is produced, for simplicity, we assume $b(x)$ enters additively in the utility function, so that the expected utility of agent $i$ in (2) now becomes

$$
U_{i}\left(x \mid v_{i}\right)=\frac{1}{\bar{c}}\left(v_{i}+b(x)-x\right)(x+(n-1) K+y)
$$

We consider two ways in which fundraisers allocate selective benefits. In the first, $b$ equals an exogenously specified amount $q>0$, but only if the donation $x$ exceeds an endogenously chosen level $x_{b}^{d}$. Otherwise, $b=0$. We label this the "discrete-benefit" scheme. In the second, $b$ is a simple linear function of donations: $b(x)=\alpha x$, with $\alpha>0$. This formulation resembles Harbaugh's (1998a) introduction of "prestige" that results from contributions. If contributions are reported exactly, more prestige is "bought" with larger contributions, and we specify a proportional representation of this. If categories are introduced, as in Harbaugh, then after a donor contributes, the receiver simply reports publicly in which category that donor's contribution fell. ${ }^{11}$ Alternatively, we introduce a single category, where anyone contributing at least $x^{d}$ is reported to be a member of this category and thereby receives prestige benefit of $q$ when the good is also provided. An important difference with Harbaugh's setup remains: A contributors' utility depends on other agents' donations; thus, strategic considerations remain paramount.

For the "continuous-benefit" scheme it is easy to retrace our steps leading to (4) and to show that in the

[^7]only symmetric equilibrium donations are
\[

s_{b}^{c}(v)= $$
\begin{cases}\frac{1}{2}\left(\frac{v}{1-\alpha}-(n-1) K_{b}^{c}-y\right) & \text { if } v \geq\left((n-1) K_{b}^{c}+y\right)(1-\alpha) \\ 0 & \text { otherwise }\end{cases}
$$
\]

where $K_{b}^{c}$ solves $K_{b}^{c}=\mathrm{E}\left[s_{b}^{c}(v)\right]$, and $y$ is the amount exogenously provided by an external authority-as in Proposition 6.

For the discrete-benefit scheme, types donating amounts smaller than $x_{b}^{d}$, that is, types for which $b(x)=0$, either do not contribute, or if they do, they donate $\frac{1}{2}\left(v-(n-1) K_{b}^{d}-y\right)$, where $K_{b}^{d}$ is the equilibrium expected contribution. Proceeding in a similar fashion, types that donate more than $x_{b}^{d}$ (they receive benefit $b(x)=q$ ) contribute $\frac{1}{2}\left(v+q-(n-1) K_{b}^{d}-y\right)$. The previously mentioned agents have very low or very high values. For intermediate values, the possibility of capturing the benefit $b(x)=q$, through an upwards departure from $x=\frac{1}{2}\left(v-(n-1) K_{b}^{d}-y\right)$ to $x=x_{b}^{d}$, may prove attractive. Indeed, after comparing the appropriate utility levels, we find equilibrium donations are

$$
s_{b}^{d}(v)= \begin{cases}0 & \text { if } v \leq(n-1) K_{b}^{d}+y \\ \frac{1}{2}\left(v-(n-1) K_{b}^{d}-y\right) & \text { if }(n-1) K_{b}^{d}+y<v \leq v_{b}^{0} \\ x_{b}^{d} & \text { if } v_{b}^{0}<v \leq(n-1) K_{b}^{d}+y+2 x_{b}^{d}-q \\ \frac{1}{2}\left(v+q-(n-1) K_{b}^{d}-y\right) & \text { if }(n-1) K_{b}^{d}+y+2 x_{b}^{d}-q<v \leq 1\end{cases}
$$

where $v_{b}^{0}$, the type indifferent between donating $\left(v-(n-1) K_{b}^{d}-y\right) / 2$ and $x_{b}^{d}$, is

$$
v_{b}^{0}=(n-1) K_{b}^{d}+y+2 x_{b}^{d}-2 \sqrt{q\left((n-1) K_{b}^{d}+y+x_{b}^{d}\right)} .
$$

(Note also that types larger than $(n-1) K_{b}^{d}+y+2 x_{b}^{d}-q$ contribute more than $x_{b}^{d}$.)
The following figure depicts $s_{b}^{c}(v)$ and $s_{b}^{d}(v)$ and, for ease of notation, we indicate with $\underline{v}_{b}^{c}\left(\underline{v}_{b}^{d}\right)$ the value at which the continuous-benefit (discrete-benefit) equilibrium strategy becomes positive. By the above descriptions, $\underline{v}_{b}^{c}=\left((n-1) K_{b}^{c}+y\right)(1-\alpha)$ and $\underline{v}_{b}^{d}=(n-1) K_{b}^{d}+y$.

Figure 4 shows that the comparison of discrete-benefit vs. continuous-benefit schemes is very similar to the comparison of discrete-level vs. continuous-level schemes in Figure 3: if $K_{b}^{c}=K_{b}^{d}$, then discrete does better in area $B$ while continuous is superior in areas $A$ and $C$. All earlier graphical intuitions about the advantages and pitfalls restricting agents' freedom to donate carry over to a scheme that restricts agents' rewards for a donation, because the relative position of areas $A, B$, and $C$ is the same. Therefore, we may


Figure 4: Comparison of discrete-benefit and continuous-benefit expected contributions.

Table 2: Continuous-benefit vs. discrete-benefit equilibria, $q=0.01$

| Parameter Configuration | $K_{b}^{c}$ | $\alpha$ | $K_{b}^{d}$ | $x_{b}^{d}$ |
| :--- | :---: | :---: | :---: | :---: |
| $(1) n=2, y=0, a=1$ | 0.17607 | 0.03119 | 0.17526 | 0.20759 |
| $(2) n=2, y=0, a=4$ | 0.17361 | 0.03393 | 0.17383 | 0.20826 |
| $(3) n=3, y=0, a=4$ | 0.13348 | 0.04271 | 0.13307 | 0.16482 |
| $(4) n=2, y=0.1, a=4$ | 0.14325 | 0.04014 | 0.14294 | 0.17575 |

expect the comparative statics about $K^{d}$ and $K^{c}$ in Propositions $4-6$ to remain valid for their respective analogues $K_{b}^{d}$ and $K_{b}^{c}$, as confirmed by the next example.

Example 6 (Triangular distribution continued). Consider the same distribution the density $f(v ; a)$ in Example 3. Let the value of selective benefits for the discrete-benefit scheme, $q$, be 0.01 . The marginal value of selective benefits for the continuous-benefit scheme, $\alpha$, is endogenously determined so that the equilibrium average expected benefit is identical for the two schemes. Table 2 summarizes the relevant quantities.

From Table 2 we see the switches in the order of $K_{b}^{c}$ and $K_{b}^{d}$ are all in accordance with Propositions 4-6. Indeed, increasing peakedness, $a$, favors the discrete-benefit scheme, as the comparison of configurations (1) and (2) shows. Moreover, the continuous-benefit scheme is favored by an increase in the number of players, $n$, as configurations (2) and (3) show, and by an increase in the amount $y$ exogenously provided by an external authority, as configurations (2) and (4) show. ${ }^{12}$

The intuitive reason for why the relative position of areas $A, B$, and $C$ is the one depicted in Figure 4 is straightforward. The position of area $A$ is determined by the larger likelihoood of positive contributions

[^8]under the flexible scheme, which arises because even very small contributions are rewarded with selective benefits, in contrast with the rigid scheme. Area $C$ reflects the fact that, under the flexible scheme, the marginal selective benefit is positive, while it is almost always zero under the rigid scheme. Thus, the incentive to contribute more than $x_{b}^{d}$ in the rigid scheme comes only from the increase in the probability of provision. In contrast, under the flexible scheme, there arises the additional prize of a larger selective incentive. Therefore, it is not surprising that contributions larger than $x_{b}^{d}$ are more frequent in the flexible scheme, as depicted in area $C$. Finally, since the flexible mechanism dominates for small and large values and since Figure 4 is drawn under the assumption $K_{b}^{c}=K_{b}^{d}$, it must be the case that the rigid mechanism dominates for intermediate values, as described by area $B$.

## 5 Empirical evidence: default membership levels for NPR stations

Before entering into details, it is worth highlighting our objective for this section. In no way are we considering the model in Section 2 and "bringing it to the data." Our theoretical model is, by design, very stylized. Its purpose is to showcase the basic forces underlying the decision by the fundraisers whether to restrict the freedom in choosing a contribution level by potential contributors, or to offer a flexible membership scheme in which agents' choices are less constrained. Two such forces we identified are "dispersion of values" and "extent of crowding-out," leading to these three predictions:

P1. The larger the dispersion of values, the less attractive becomes a membership scheme that restricts contributors' decisions (Proposition 4).

P2. The larger the number of potential contributors, the less attractive becomes a membership scheme that restricts contributors' decisions (Proposition 5).

P3. The larger the amount provided by an external authority, the less attractive becomes a membership scheme that restricts contributors' decisions (Proposition 6).

While the proofs of the respective propositions depend on the details of our model, we believe the intuitions we provided transcend them. In this section we explore the basic consistency of the membership schemes we see actually used with the three predictions described above.

The main data for this section are information on number of different membership levels offered by National Public Radio (NPR) stations, geographic and socio-economic information about the populations they serve, and funding amounts provided to NPR stations by the Corporation for Public Broadcasting (CPB). While the empirical literature that treats NPR stations as producers of public goods is vast (see, for example, Berry and Waldfogel, 1999; Kingma, 1989; and Manzoor and Straub, 2005), we believe our analysis
is the first to investigate the relationship between the demographics of a station's audience and the structure of the fundraising scheme offered by that station. In our quest to document basic relationships consistent with P1-P3, we proxy flexibility of the membership scheme with the number of different membership levels offered by a station, dispersion of values with income inequality, number of agents with adult population served, and contributions by an external authority with CPB funding.

### 5.1 Data

Data are drawn from a variety of sources. We began with a list of all NPR stations operating in the continental United States, available at http://www.npr.org/stations/pdf/nprstations.pdf. This list provides a first geographical indication of the coverage area in addition to call letters - e.g. KDAQ for the NPR station in Shreveport, LA—that, in combination with the FM or AM frequency, uniquely identify NPR stations. Our information on geographic coverage is integrated with maps from Radio-Locator. From the website http://www.radio-locator.com we obtained descriptions of the predicted coverage area of each NPR station. We organized geographic information as follows. First, we included all metropolitan or micropolitan statistical areas falling within the predicted coverage pattern. ${ }^{13}$ In the few cases in which there are no metropolitan or micropolitan areas within the coverage pattern, we chose to assign as geographic area the counties falling within the coverage pattern. Radio-Locator also provides the website of each station. In many cases, a few different NPR stations all link to the same website. These stations are effectively operating, as far as membership campaigns are concerned, as a single entity, namely, a network of NPR stations (e.g., KDAQ in Shreveport, LA; KLSA in Alexandria, LA; KBSA in El Dorado, AK; and KLDN in Lufkin, TX, all constitute the "Red River Radio" network, with website www.redriverradio.org).

Our information on default membership levels comes from the websites of stand-alone NPR stations and the NPR station networks. In particular, we first followed links such as "donate now" or "support" or "pledge now" and counted the number of different options offered. ${ }^{14}$ Finally, we searched the websites for "leadership circles," "producers' clubs," or other similar monikers for large donations. If a specific level (or levels) were explicitly identified on the website, but not in the regular "pledge now" page, we included it as a separate contribution level. ${ }^{15}$ We accessed these websites between May 22, 2009, and June 7, 2009. At those times, no scheduled membership campaigns were ongoing.

[^9]We then collected aggregate economic and demographic characteristic of each geographic area, including population, income, education, racial makeup, voting percentage, population density and commuting time. ${ }^{16}$ We chose population as a straightforward approximation to the number of potential donors to the public good. We chose income as a proxy for willingness to pay for the public good. We added the other variables in the attempt to control for other factors that may affect the willingness to pay for the public good. In our choices we are guided by our own experience that much radio listening happens in the car, and by what various NPR stations websites say about their listeners in their "Underwriting" or "Business Sponsorship" pages. In particular, beyond high income and high education, civic activism is often mentioned as a distinguishing characteristic of NPR listeners, and we proxy this with voting behavior.

Finally, we collected information on CPB funding from their most recent annual report available on their website, http://www.cpb.org/aboutcpb/reports. We use the information to create a cross section with the following information for each network of NPR stations (we consider a stand-alone NPR station as a network of one): (1) number of different default membership levels; (2) adult population; (3) income inequality and median income; (4) CPB support; and (5) other economic, demographic, and political controls.

Before presenting our basic estimations, we describe the variables of greater interest in more detail, beginning with the number of suggested contribution levels. Their distribution is reported in Table 3, which shows a fair amount of dispersion and where the largest number of levels suggested is 21 . A small percentage of the sample (3.8\%) offered no pre-specified levels. In our estimations, we excluded these few observations for two reasons. The first is theoretical: we believe there is a marked discontinuity between offering only one membership level, and simply leaving any contributor free to donate as much as he wants without any suggestion, guidance, or inducement level. If we were to include the observation where no suggested level appears, we believe we should include them at the other extreme; that is we feel that a membership scheme with no suggested levels is much closer in the flexibility it allows the contributor to a scheme with 21 possible levels, rather than to one with only one possible membership level. The second reason is that a large fraction of the stations offering no specified membership level appear to be very tightly connected to a higher-level institution, such as a university. This connection is so tight that, sometimes, following the "pledge now" link, one is sent to the university donation webpage. It is not entirely clear to us how to view such strict linkage and the effects it may have on the membership policy of the NPR station.

The adult population (persons over 18) for each geographic area served by a network are reported in Table 4. Here, too, there is fair dispersion in the data, with the largest population in excess of 13 million for stations covering the New York metropolitan area and a small percentage of stations ( $5.8 \%$ of the sample) covering a population less than 50,000 . In our estimations we excluded the 15 NPR networks serving a

[^10]Table 3: The distribution of the number of suggested contribution levels

| Number of default contribution levels <br> suggested by NPR network | Frequency | Percent | Cumulative |
| :---: | :---: | ---: | ---: |
| None | 10 | 3.86 | 3.86 |
| 1 | 4 | 1.54 | 5.41 |
| 2 | 1 | 0.39 | 5.79 |
| 3 | 5 | 1.93 | 7.72 |
| 4 | 4 | 1.54 | 9.27 |
| 5 | 8 | 3.09 | 12.36 |
| 6 | 31 | 11.97 | 24.32 |
| 7 | 19 | 7.34 | 31.66 |
| 8 | 28 | 10.81 | 42.47 |
| 9 | 30 | 11.58 | 54.05 |
| 10 | 26 | 10.04 | 64.09 |
| 11 | 25 | 9.65 | 73.75 |
| 12 | 23 | 8.88 | 82.63 |
| 13 | 19 | 7.34 | 89.96 |
| 14 | 7 | 2.70 | 92.66 |
| 15 | 5 | 1.93 | 94.59 |
| 16 | 4 | 1.54 | 96.14 |
| 17 | 3 | 1.16 | 97.30 |
| 18 | 1 | 0.39 | 97.68 |
| 19 | 3 | 1.16 | 98.84 |
| 20 | 1 | 0.39 | 99.23 |
| 21 | 2 | 0.77 | 100.00 |
| Total | 259 |  |  |

population less than 50,000 . They cover remote parts of the country, with no metropolitan or micropolitan areas, or very small ones. Moreover, they tend to be very tightly connected to institutions like Native American Nations or Reservations. In our estimations we also excluded "statewide" NPR networks (e.g., Georgia Public Broadcasting). We have 15 statewide networks in our sample. We are especially concerned about the influence of state legislatures or state boards of education in their creation and operation. ${ }^{17}$

The figures for the amount of CPB support, excluding the "statewide" NPR networks, are reported in Table 5. We have only included the total of "Radio Community Service Grants" and "Radio Programming Grants," leaving out other forms of grants-for example, "Digital Support Grants"-that appear much less common and permanent. The notable feature in the data is that the overwhelming majority of stations receive some funding but less than $\$ 800,000$. Of the 9 NPR networks out of 244 we could not find an amount for, three offer no membership levels and one has population smaller than than 50,000 ; therefore they are excluded based on earlier considerations. We feel it is better to eliminate also the remaining five rather than

[^11]Table 4: Adult populations of NPR networks' audiences

| Total adult population served |  |  |  |
| ---: | :---: | ---: | ---: |
| by an NPR network | Frequency | Percent | Cumulative |
| less than 50,000 | 15 | 5.79 | 5.79 |
| between 50,000 and 100,000 | 10 | 3.86 | 9.65 |
| between 100,000 and 200,000 | 26 | 10.04 | 19.69 |
| between 200,000 and 300,000 | 23 | 8.88 | 28.57 |
| between 300,000 and 400,000 | 26 | 10.04 | 38.61 |
| between 400,000 and 500,000 | 23 | 8.88 | 47.49 |
| between 500,000 and 600,000 | 10 | 3.86 | 51.35 |
| between 600,000 and 700,000 | 9 | 3.47 | 54.83 |
| between 700,000 and 800,000 | 6 | 2.32 | 57.14 |
| between 800,000 and 900,000 | 7 | 2.70 | 59.85 |
| between 900,000 and 1000,000 | 13 | 5.02 | 64.86 |
| between 1,000,000 and 1,500,000 | 24 | 9.27 | 74.13 |
| between 1,500,000 and 2,000,000 | 12 | 4.63 | 78.76 |
| between 2,000,000 and 3,000,000 | 16 | 6.18 | 84.94 |
| between 3,000,000 and 4,000,000 | 22 | 8.49 | 93.44 |
| between 4,000,000 and 8,000,000 | 10 | 3.86 | 97.30 |
| more than 8,000,000 | 7 | 2.70 | 100.00 |
|  | Total | 259 |  |

assigning them a value of zero. ${ }^{18}$
We constructed our income variables as follows. The census data provides number of households with income falling in different intervals (e.g., $\$ 15,000$ to $\$ 20,000$ ) by county or by metropolitan or micropolitan area. Because our geographic unit of interest is the area covered by an NPR network, which may be a combination of all three - county, metropolitan area, and micropolitan area-we first aggregated these numbers. We then calculated the percentage of households in the area covered by the NPR network with incomes below $\$ 15,000$ and with incomes above $\$ 150,000$. Finally, as our measure of median income we used the mid-point of the median income interval in the network coverage area. ${ }^{19}$ After all the eliminations described above, we are left with 216 out of 259 observations. ${ }^{20}$

The unconditioned relation between number of default contribution levels and adult population turns out to be non-linear, nicely accommodated with a logarithmic transformation of population as depicted in Figure 6. (This and the following figures are in the Appendix.) The unconditioned relation between contribution levels and proportion of households with income above $\$ 150,000$ is depicted in Figure 7, while the relation for households with income below $\$ 15,000$ is depicted in Figure 8. While Figures 6 and 7 display

[^12]Table 5: NPR network support from the Corporation for Public Broadcasting

| CPB support in dollars for <br> non-statewide NPR networks | Frequency | Percent | Cumulative |
| ---: | :---: | ---: | ---: |
| nothing | 9 | 3.69 | 3.69 |
| less than 50,000 | 5 | 2.05 | 5.74 |
| between 50,000 and 100,000 | 27 | 11.07 | 16.80 |
| between 100,000 and 150,000 | 57 | 23.36 | 40.16 |
| between 150,000 and 200,000 | 49 | 20.08 | 60.25 |
| between 200,000 and 250,000 | 28 | 11.48 | 71.72 |
| between 250,000 and 300,000 | 22 | 9.02 | 80.74 |
| between 300,000 and 400,000 | 14 | 5.74 | 86.48 |
| between 400,000 and 500,000 | 14 | 5.74 | 92.21 |
| between 500,000 and 600,000 | 8 | 3.28 | 95.49 |
| between 600,000 and 700,000 | 4 | 1.64 | 97.13 |
| between 700,000 and 800,000 | 1 | 0.41 | 97.54 |
| between 800,000 and 900,000 | 0 | 0.00 | 97.54 |
| between 900,000 and 1000,000 | 0 | 0.00 | 97.54 |
| between 1,000,000 and 1,500,000 | 4 | 1.64 | 99.18 |
| between 1,500,000 and 2,000,000 | 1 | 0.41 | 99.59 |
| between $2,000,000$ and $3,000,000$ | 1 | 0.41 | 100.00 |
|  | Total | 244 |  |

patterns that conform to our theoretical intuitions, the relation for poorer households is far less obvious-a linear specification is not statistically significant-without conditioning on the other variables of interest. Finally, the unconditioned relation between contribution levels and CPB support is depicted in Figure 9a. Our theoretical intuitions about crowding out are consistent with the pattern displayed for all observations but the six "super" NPR networks with CPB support larger than $\$ 800,000$, as Figure 9b, which excludes them, clearly shows.

### 5.2 Results

We present in Table 6 the results of the multivariate OLS regressions. We estimate 4 different equations. On the left-hand side they have the number of default membership levels. On the right-hand side of each equation, we place

F1. Weights for the upper and the lower tails of the income distribution conditional on median income,

F2. Natural logarithm of adult population,

F3. The amount provided by the CPB;
along with the control variables described in Table 6. For item F1, income, we investigate a "restrictive" definition of income tails, that is $\$ 150,000$ and $\$ 15,000$, and a more "expansive" definition, namely, $\$ 125,000$

Table 6: Dependent variable: number of default contribution levels proposed by an NPR network


[^13]and $\$ 20,000$. For item F3, CPB support, we investigate a "restrictive" linear estimate that excludes the six observations with CPB support larger than $\$ 800,000$, and an "expansive" quadratic estimate that includes these observations. The different combinations give rise to the 4 columns in Table 6.

The basic relationships reported in Table 6 are in broad agreement with our theoretical hypotheses. For income, item F1, both the coefficient for the upper tail and the coefficient for the lower tail of the income distribution are positive and statistically significant, in accordance with prediction P1. The coefficient on the natural logarithm of adult population is positive and significant, so F2 is in accordance with P2. The coefficients on the amount provided by the CPB, item F3, imply a positive relationship between number of default level and CPB support for all but the six "super" networks with CPB support larger than $\$ 800,000 .{ }^{21}$ This broad pattern is confirmed as well using a quadratic specification for population, especially if one excludes the six "super" networks. Therefore, we feel justified in claiming that empirical patterns are consistent with our theoretical intuitions. ${ }^{22}$

## 6 Conclusion

Fundraisers may profit from restricting donors' possible levels of contribution because such restrictions can induce some people to contribute more than they otherwise would. But this benefit must be weighed against the cost that these restrictions can also induce some people to give less than they otherwise would. The relative importance of these two effects determines whether such restrictions are indeed profitable.

Using a subscription game framework to study the private provision of a discrete public good, we have identified several factors militating in favor of greater flexibility for contributors. If the distribution of players' values is concave, then the flexible (continuous) contribution framework yields greater revenue. For symmetric distributions of players' values having a density that is first increasing and then decreasing, the flexible scheme is again preferred as i) the dispersion of donors' taste for the public good increases, ii) the number of potential donors increases, and iii) there is greater funding by an external authority. These predictions of the model are supported by fundraising practices of NPR stations in the US. We found that these stations offer a larger number of suggested contribution levels as i) the incomes of the population served become more diverse, ii) the population of the coverage area increases, and iii) there is greater external support from the Corporation for Public Broadcasting.

[^14]The more direct implications of our results concern fund raising: We identify easily obtainable characteristic of the target donor population that should be taken into account in the practical design of a campaign. Moreover, the forces behind our results appear to be relevant for more widely defined collective effort problems, such as team production. Our most general message is that the design of a campaign affects the responses of contributors in ways that at times are predictable. Exploiting such responses may prove valuable for further research. For instance, our model suggests that, when faced with a rigid donation scheme, contributions from very high or very low value donors are less easily crowded-out than those from donors with intermediate values. This observation may be of help to the large literature on crowding out. ${ }^{23}$

Finally, it is worth pointing out that the value of our empirical exercise lies in the novelty of the data about membership levels and in the simultaneous match of various empirical correlations with theoretical predictions. More empirical research is needed in the directions that our model points out. Our results on the determination of the best discrete-contribution levels may also offer guidance to experimental analysis, making it possible to design experiments that have the potential to actually test whether offering membership categories can raise total contributions.

[^15]
## Appendix

Proof of Proposition 2. The proof is by contradiction: given $K^{d} \geq K^{c}$ we assume $v^{0} \geq \mu$ and show this leads to the contradictory conclusion that $K^{d}<K^{c}$. Now suppose $K^{d} \geq K^{c}$ and $v^{0} \geq \mu$. Because $v^{0} \geq \mu$ and $F$ is strictly concave $\forall v>v^{0}$, part 2 of Lemma 1 implies $x^{d}<\left[1-(n-1) K^{d}\right] / 2 \leq\left[1-(n-1) K^{c}\right] / 2$, where the second inequality follows from the assumption $K^{d} \geq K^{c}$. Proceeding as in the proof of Proposition 1 part 2, we obtain (11) and reach (12'):

$$
K^{c}>x^{d}\left[1-\frac{1}{2 x^{d}} \int_{(n-1) K^{c}}^{(n-1) K^{c}+2 x^{d}} F(v) d v\right]=x^{d}(1-\mathrm{E}[F(v) \mid \varphi])
$$

where $\varphi$ denotes a uniform probability distribution on the interval $\left[(n-1) K^{c},(n-1) K^{c}+2 x^{d}\right]$.
We now separate the parameter space into three exhaustive regions and show that, in all three, ( $12^{\prime}$ ) implies $K^{c}>K^{d}$. In the first region $\mu \leq(n-1) K^{c}$, so that $F$ is strictly concave for the relevant range of the integral in $\left(12^{\prime}\right)$, so that

$$
\begin{aligned}
K^{c} & >x^{d}\left(1-F(\mathrm{E}[v \mid \varphi])=x^{d}\left[1-F\left((n-1) K^{c}+x^{d}\right)\right]\right. \\
& \geq x^{d}\left[1-F\left((n-1) K^{d}+x^{d}\right)\right] \\
& =K^{d} .
\end{aligned}
$$

In the second region $\mu \geq(n-1) K^{c}+2 x^{d}$, so from (12') we obtain

$$
\begin{aligned}
K^{c} & >x^{d}(1-\mathrm{E}[F(v) \mid \varphi]) \\
& \geq x^{d}\left[1-F\left(v^{0}\right)\right] \\
& =K^{d}
\end{aligned}
$$

where the second inequality follows because $F(v) \leq F\left(v^{0}\right)$ for every $v$ in the support of $\varphi$ by the assumption $v^{0} \geq \mu$.

We proceed to the analysis of the third and final region, $(n-1) K^{c}<\mu<(n-1) K^{c}+2 x^{d}$, with the help of Figure 5. We first define a new distribution function $H$ that agrees with $F$ for $v<(n-1) K^{c}$ and $v>2 \mu-(n-1) K^{c}$, but, for $(n-1) K^{c} \leq v \leq 2 \mu-(n-1) K^{c}, H$ is equal to the straight line connecting points $A$ and $B$ :

$$
H(v)=\frac{F\left(2 \mu-(n-1) K^{c}\right)-F\left((n-1) K^{c}\right)}{2\left(\mu-(n-1) K^{c}\right)}\left(v-(n-1) K^{c}\right)+F\left((n-1) K^{c}\right)
$$

One can easily verify that $H$ inherits symmetry around $\mu$ from $F$, using $H(\mu+z)+H(\mu-z)=1, \forall z \in[0, \mu]$. Moreover, the curvature properties of $F$ imply $H(\mu)=F(\mu), H(v) \geq F(v)$ if $v<\mu$, and $H(v) \leq F(v)$ if $v>\mu$. In other words, $F$ second-order stochastically dominates $H$, so

$$
\begin{equation*}
\int_{(n-1) K^{c}}^{(n-1) K^{c}+2 x^{d}} F(v) d v \leq \int_{(n-1) K^{c}}^{(n-1) K^{c}+2 x^{d}} H(v) d v \tag{17}
\end{equation*}
$$

because $F$ and $H$ agree for $v<(n-1) K^{c}$. Finally, note that by construction $H$ is concave for $v \geq(n-1) K^{c}$.


Figure 5: Linearization of $F$ about its mean/median.

Using $\left(12^{\prime}\right),(17)$, and concavity of $H$ on $\left[(n-1) K^{c}, 1\right]$, we now have

$$
\begin{align*}
K^{c} & >x^{d}\left[1-\frac{1}{2 x^{d}} \int_{(n-1) K^{c}}^{(n-1) K^{c}+2 x^{d}} H(v) d v\right] \\
& =x^{d}[1-\mathrm{E}[H(v) \mid \varphi]] \\
& \geq x^{d}[1-H(\mathrm{E}[v \mid \varphi])] . \tag{18}
\end{align*}
$$

If $\mathrm{E}[v \mid \varphi]<\mu$, then (18) implies $K^{c}>x^{d} / 2 \geq x^{d}\left(1-F\left(v^{0}\right)\right)=K^{d}$. If $\mathrm{E}[v \mid \varphi] \geq \mu$, then, because
$F(v) \geq H(v) \forall v \geq \mu,(18)$ implies

$$
K^{c}>x^{d}[1-F(\mathrm{E}[v \mid \varphi])]=x^{d}\left[1-F\left((n-1) K^{c}+x^{d}\right)\right] \geq x^{d}\left[1-F\left((n-1) K^{d}+x^{d}\right)\right]=K^{d}
$$

Thus, in all cases, if $K^{d} \geq K^{c}$, then $v^{0} \geq \mu$ is impossible, so it must be that $v^{0}<\mu$.

Proof of Proposition 6. Making explicit our focus on the amount $y$, we denote with $K^{c}(y)$ the solution to (15) and with $K^{d}(y)$ the solution for $K^{d}$ of (16). It is immediate to verify that both $K^{c}(y)$ and $K^{d}(y)$ are decreasing in $y$. Therefore, as in the proof of Proposition 5 , the proof is complete if $K^{d} \geq K^{c}$ implies $\left|\frac{d K^{d}}{d y}\right|>\left|\frac{d K^{c}}{d y}\right|$. From equation (15) we obtain

$$
\begin{equation*}
\left|\frac{d K^{c}}{d y}\right|=\frac{1-F\left((n-1) K^{c}\right)+y}{2+(n-1)\left(1-F\left((n-1) K^{c}+y\right)\right)} \tag{19}
\end{equation*}
$$

while from (16) we have

$$
\begin{equation*}
\left|\frac{d K^{d}}{d n}\right|=\frac{1-F\left(v^{0}\right)}{1+(n-1)\left(1-F\left(v^{0}\right)\right)} \tag{20}
\end{equation*}
$$

Moreover, we can replicate the same steps leading to Proposition 2 for the case $y>0$ and reach the same conclusion: $K^{d} \geq K^{c}$ implies $v^{0}<\mu$. Simple algebra on equations (19) and (20) shows that the result follows if

$$
1-2 F\left(v^{0}\right)+F\left((n-1) K^{c}+y\right)>0
$$

which is ensured by $v^{0}<\mu$.




Figure 9a: Number of suggested contribution levels and CPB support (expansive)


## References

Admati, Anat R., and Motty Perry (1991), Joint projects without commitment, Review of Economic Studies 58, 259-276.

Andreoni, James (1989), Giving with impure altruism: applications to charity and Ricardian equivalence, Journal of Political Economy 97, 1447-1458.

Andreoni, James (1990), Impure altruism and donations to public goods: a theory of warm glow giving, Economic Journal 100, 464-477.

Andreoni, James, (1998), Toward a theory of charitable fund-raising, Journal of Political Economy 106, 1186-1203.

Andreoni, James, (2006), Philanthropy, Handbook of Giving, Altruism and Reciprocity Volume 2. Edited by Serge-Christophe Kolm and Jean Mercier Ythier, Elsevier B. V.

Andreoni, James, and Ragan Petrie (2004), Public goods experiments without confidentiality: a glimpse into fund-raising, Journal of Public Economics 88, 1605-1623.

Andreoni, James, and Lise Vesterlund (2001), Which is the fair sex? Gender differences in altruism, The Quarterly Journal of Economics 116, 293-312.

Barbieri, Stefano, and David A. Malueg (2009), Threshold uncertainty in the private-information subscription game, mimeo.

Bekkers, Rene, and Pamala Wiepking (2007), Understanding philanthropy: a review of 50 years of theories and research, working paper, Vrije Universiteit Amsterdam.

Bergstrom, Theodore, Lawrence Blume, and Hal Varian (1986), On the private provision of public goods, Journal of Public Economics 29, 25-49.

Berry, Steven T., and Joel Waldfogel (1999), Public radio in the United States: does it correct market failure or cannibalize commercial stations? Journal of Public Economics 71: 189-211.

Brush, Jesse (2007), Does income inequality lead to more crime? A comparison of cross-sectional and time-series analyses of United States counties, Economics Letters 26, 264-268.

Cadsby, Charles B., and Elizabeth Maynes (1999), Voluntary provision of threshold public goods with continuous contributions: experimental evidence, Journal of Public Economics 71, 53-73.

Cornelli, Francesca, (1996), Optimal selling procedures with fixed costs, Journal of Economic Theory 71, 1-30.

Croson, Rachel, and Melanie Marks (2001), The effect of recommended contributions in the voluntary provision of public goods, Economic Inquiry 39, 238-249.

Glazer, Amihai, and Kai A. Konrad (1996), A signaling explanation for charity, The American Economic Review 86, 1019-1028.

Harbaugh, William T. (1998a), What do donations buy? A model of philanthropy based on prestige and warm glow, Journal of Public Economics 67, 269-284.

Harbaugh, William T. (1998b), The prestige motive for making charitable transfers, The American Economic Review 88, 277-282.

Hsu, Li-Chen (2003), Effects of framing, group size, and the contribution mechanism on cooperation in threshold public goods and common resources experiments, Academia Economic Papers 31, 1-30.

Kingma, Bruce (1989), An accurate measurement of the crowd-out effect, income effect, and price effect of charitable contributions, Journal of Political Economy 97, 1197-1207.

Li, Jingping, and Yohanes E. Ryanto (2009), Category reporting in charitable giving: an experimental analysis, mimeo.

Manzoor, Sonia H., and John D. Straub (2005), The robustness of Kingma's crowd-out estimate: evidence from new data on contributions to public radio, Public Choice 123, 463-476.

Martimort, David, and Humberto Moreira, Common agency and public good provision under asymmetric information, Theoretical Economics, forthcoming.

McBride, Michael (2006), Discrete public goods under threshold uncertainty, Journal of Public Economics 90, 1181-1199.

Nitzan, Shmuel, and Richard E. Romano (1990), Private provision of a discrete public good with uncertain cost, Journal of Public Economics 42, 357-370.

Olson, Mancur (1965), The logic of collective action, Harvard University Press.
Proschan, Frank (1965), Peakedness of distributions of convex combinations, Annals of Mathematical Statistics 36, 1703-1706.

Suleiman, Ramzi, and Amnon Rapoport (1992), Provision of step-level public goods with continuous contribution, Journal of Behavioral Decision Making 5, 133-153.


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[^1]:    ${ }^{1}$ For example, Andreoni (2006) reports that private giving hovers between $1.5 \%$ and $2.1 \%$ of personal income in the US.
    ${ }^{2}$ Bekkers and Wiepking (2007), in their review of the literature on philanthropy, state that "Many people have developed cognitive strategies to reject responsibility for the welfare of others. One such strategy is the argument that 'one cannot afford a donation.' Legitimizing paltry contributions by adding the phrase 'even a penny helps' in a solicitation for contributions may neutralize these strategies [citations omitted]."
    ${ }^{3}$ In our dataset, almost all NPR stations offer "default" membership categories. For more details, see Table 3.
    ${ }^{4}$ See Table 3 for the distribution of "default" membership categories for NPR stations.
    ${ }^{5}$ Another theoretical approach to answer these questions is mechanism design. Indeed, Cornelli (1996), albeit tangentially to her main goal of characterizing the optimal direct mechanism, suggests how implementation can occur through a scheme with categories. However, extending the results in Cornelli (1996) to answer the questions we are interested in appears complicated. Moreover, some authors describe the mechanism design approach as too complicated, too abstract, and too organizationally taxing to provide a realistic account of a situation with very many potential donors (see, e.g., Andreoni, 1998). Others, like Martimort and Moreira (forthcoming), comment on how a full-fledged mechanism design approach requires a degree of commitment power that may be excessive in a variety of situations.

[^2]:    ${ }^{6}$ Beyond creating bunching, there are of course other reasons to discourage small but positive donations. For example, extremely small donations may entail relatively large processing costs. Moreover, one may run the risk of legitimizing too small a donation. Indeed, in describing the limitation of the "even-a-penny-helps" technique, Bekkers and Wiepking (2007) state "...the phrase may even decrease the amount donated, exactly because it legitimizes paltry contributions [citation omitted]." Also see footnote 2 above.

[^3]:    ${ }^{7}$ Beyond Barbieri and Malueg (2009), uncertainty in the cost appears in Nitzan and Romano (1990) and McBride (2006).

[^4]:    ${ }^{8}$ Symmetry is not essential for the result. For example, the proof and figure (in the Appendix) are easily adapted to a positively skewed distribution $\tilde{F}$ that coincides with $F$ for values less than the median $\mu$, but $\tilde{F}$ first-order stochastically dominates $F$ for $v>\mu$.

[^5]:    ${ }^{9}$ When $F$ is a degenerate distribution at $\mu$, players contribute some common fixed amount $s_{0}$ in the Nash equilibrium with unrestricted contributions. In the best discrete contribution scheme, the fundraiser will generally choose a level $x^{d} \neq s_{0}$; for this reason, $K^{d}>K^{c}$ when $F$ is a degenerate distribution.

[^6]:    ${ }^{10}$ In the words of Bergstrom et al. (1986), the jerky response happens only at the "extensive" margin, while the smooth response happens mostly at the "intensive" margin.

[^7]:    ${ }^{11}$ For example, the New Orleans Preservation Resource Center identifies major donors by their membership in categories designated as Italianate ( $\$ 25,000$ and above), Greek Revival ( $\$ 15,000-\$ 24,999$ ), Romanesque Revival ( $\$ 10,000-\$ 14,999$ ), Steamboat Gothic ( $\$ 5,000-\$ 9,999$ ), Queen Anne ( $\$ 2,500-\$ 4,999$ ), Landmark ( $\$ 1,000-\$ 2,499$ ), and Conservator ( $\$ 500-\$ 999$ ).

[^8]:    ${ }^{12}$ We have also performed numerical comparisons between a scheme with one benefit level and a two-level scheme, and we find the same patterns as in Table 2. Indeed, the relatively more flexible two-level scheme generates larger expected contributions than the rigid one-level scheme when there are increases i) in the dispersion of values, ii) in the number of players, and iii) in the amount exogenously provided by an external authority. Full details are available upon request.

[^9]:    ${ }^{13}$ We use Table 2a available at http://www.census.gov/population/www/cen2000/briefs/phc-t29/index.html for our definitions of metropolitan and micropolitan statistical areas.
    ${ }^{14}$ Often, two kinds of membership are offered, one without gift and one with gifts. In case the levels with gift did not correspond to the level without gift, we considered the level with gift as an additional level, as long as it did not fall within five dollars of any level without gift. To deal with different payment plans, e.g. monthly or once for the whole year, we annualized all amounts without discounting.
    ${ }^{15}$ We did not include donations in kind, donations of stocks, charitable gift annuities and donations of other financial instruments. Whenever the website belongs to an organization operating jointly a TV station and a NPR station, we excluded TV-related gifts from our calculations.

[^10]:    ${ }^{16}$ These are derived from from "USA Counties" data files, available at http://www.census.gov/support/DataDownload.htm.

[^11]:    ${ }^{17}$ It is also somewhat problematic to obtain reliable and complete geographical coverage data. For example, Georgia Public Broadcasting does not have a station in Atlanta, but its radio programming can be heard on the second audio program of its TV station. To avoid such complications, we exclude these observations.

[^12]:    ${ }^{18}$ The six "super" NPR networks that receive more than $\$ 800,000$ deserve a special advance mention as well (see Figures 9 a and 9 b ).
    ${ }^{19}$ Brush (2007) uses similar measures of income when analyzing income inequality and crime.
    ${ }^{20}$ Our results are not qualitatively affected by marginally different inclusion rules, with the caveat described above for stations that do not offer pre-specified membership levels, in the sense that the variables of interest remain statistically significant, albeit sometimes less precisely estimated.

[^13]:    * Significant at the $5 \%$ level.
    ** Significant at the $1 \%$ level.
    Note: All independent variables are for the year 2000 with the exception of income variables, which are for the year 1999, and
    CPB support, which is for the year 2007.

[^14]:    ${ }^{21}$ The estimated maximum for the quadratic is about $\$ 950,000$.
    ${ }^{22}$ Among control variables, the male-to-female ratio is deserving of mention. The regressions indicate that the larger the percentage of males, the more flexible the membership contract offered. Andreoni and Vesterlund (2001) provide evidence from dictator games that "...men are more likely to be either perfectly selfish or perfectly selfless, whereas women tend to be 'equalitarians' who prefer to share evenly." Should the larger heterogeneity in men's preferences extend to the public good "NPR," then the positive coefficient in our regressions for male-to-female ratio would be consistent with the theoretical prediction in Proposition 4.

[^15]:    ${ }^{23}$ See, e.g., Manzoor and Straub (2005) and references therein.

