Indexed Sovereign Debt: An Applied Framework*

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February 2007 - UNDER REVISION

Abstract

A small number of countries have issued real indexed sovereign debt in recent years. This type of contracts could improve risk sharing between debtor countries and international creditors and diminish the probability of occurrence of debt crises. However, the structure of the optimal GDP indexed contracts is not obvious and neither is the magnitude of its welfare effects.

This paper addresses these issues. We characterize the optimal features of indexed debt contracts in a dynamic stochastic equilibrium framework with incomplete markets and compare these features to existing ones. We show that the optimal indexed debt contract can not be studied abstracting from the total portfolio of the issuing country. We conclude obtaining a quantitative approximation of the welfare effects of indexation: the optimal indexed debt contract improves welfare and features payments increasing in the state of the economy. Calibrating our model to Argentina's economy we find that the welfare gains from introducing "optimal" indexed debt are equivalent to an increase between 0.6% and 2% in certainty equivalent aggregate consumption. Although existing real indexed contracts typically entail payments increasing in the state of the economy, they also feature thresholds of the chosen real variable that trigger payments. We argue that the latter are typically suboptimal.

**This paper is undergoing a thorough revision regarding the quantitative results* We thank Tito Cordella, Alberto Martin, Eduardo Levy Yeyati and participants to the Pre-conference on Country Insurance at the World Bank and LACEA for their comments and suggestions. All remaining errors are our responsibility.

1 Introduction

In March 2005 Argentina finished the debt restructuring process that followed the sovereign default and financial crisis of 2001. The restructured debt included a relatively novel component, some of the payments were linked to the evolution of Argentina's GDP. The innovation was not well received by investors at the time.[1] However, it captured the spirit of some of the proposals under discussion since the string of sovereign debt crises in Russia, Ukraine, Pakistan, Ecuador and Argentina renewed the debate on the institutional framework of sovereign credit markets and the design of sovereign debt contracts.[2] This paper focuses on the latter problem.

There exists some consensus that indexing sovereign debt payments to real variables could in principle improve risk sharing among debtor countries and international creditors while it would diminish the probability of financial crises.[3] However, there are two issues that deserve further attention: first, it is not clear whether the design of existing real indexed sovereign debt is optimal and how it could be improved; second, it is interesting to have some sense about the magnitude of its welfare effect.

The goal of this paper is to analyze some features of the design of existing indexed debt contracts and to provide a first quantitative approximation of the potential welfare gains that the introduction of real indexed debt could generate. In order to do so, we begin by characterizing the optimal design of real indexed sovereign debt in a dynamic stochastic small open economy with limited commitment. We then compare these optimal contracts with the existing real indexed debt contracts and quantify their welfare implications through a calibration of our model to the Argentine economy from 1983 to 2000.

We find that indexed debt contracts improve welfare and the optimal indexed debt contract has the features of an insurance contract with payments increasing in the state of the economy. In general, existing real indexed contracts, such as Argentina's GDP-warrants or Bulgaria's GDP-indexed bonds feature payments increasing in the state of the economy, but also include a threshold that triggers them. We argue that these thresholds are usually suboptimal.

[1] See Fernandez et al. (2006) or Balston and Ghezzi (2004).

[2] See IMF (2002) for a summary of some of these proposals.

[3] See Borensztein and Mauro (2002).

Our preliminary calibration results suggest that the welfare improvement of substituting standard defaultable debt with GDP-indexed debt without thresholds in the Argentine economy is equivalent to an increase in average aggregate consumption ranging from 0.6% to 2%, where the size of the effect increases in the magnitude of the direct costs of default.[4]

We develop a model of sovereign borrowing related to Arellano (2003) and Yue (2004) adaptation of Eaton and Gersowitz (1981) and we extend it to analyze real indexed debt. In our model the government tries to maximize the welfare of an infinitely lived risk averse representative agent facing random income shocks. In order to do so, the government can borrow from homogenous and risk neutral international creditors using one period bonds and has access to a non-contingent savings technology. Consistently with the weak legal framework of sovereign borrowing, we assume that the government cannot credibly commit to repay its debts to foreign creditors. Every period it chooses whether to fully repay, renegotiate or repudiate its outstanding debts. The last two options entail direct costs or penalties.[5] In addition, repudiation triggers the exclusion from credit markets.[6] These direct costs of renegotiating and repudiating are not appropriable by creditors and are assumed to be increasing in the level of income. The intuition behind this assumption can be related to Grossman and Van Huyck (1988) excusable and non-excusable defaults argument. As the state of the economy gets worse (i.e. the income level decreases) the penalty or direct cost of defaulting is smaller since the default becomes "more excusable". Thus, with non-indexed debt contracts if defaults (renegotiation or repudiation) were to occur, they are more likely to happen when the government faces low income shocks. Government borrowing will be endogenously constrained by the presence of limited commitment.

In this framework, indexed debt contracts improve welfare. They may do so through two channels. First, as it is standard, they improve the amount of risk-sharing between a risk-averse sovereign borrower and risk-neutral lenders; second, they reduce the probability of default and, therefore, diminish or eliminate the deadweight losses that default creates.

[4] These estimates assume a discount factor of 0.8 and direct costs of defaults (or penalties) ranging from 2% to 4 % of GDP during the years (1 on average) in which the economy is excluded from credit markets.

[5] These direct costs of default or penalties have been interpreted as trade sanctions by Bulow and Rogoff (1989), as reputation spill-overs by Cole and Kehoe (1997) and as informational costs by Sandleris (2005).

[6] See Gelos, Sahay, and Sandleris (2004) for the different impact on market access of a settled and an unsettled default.

Indexed debt is welfare improving even if we take into account that non-indexed contracts also have some degree of contingency. In effect, the presence of limited commitment allows the government to default in some states and, as Zame (1993) pointed out in the context of corporate debt, this introduces some degree of contingency in the non-indexed contract. However, it typically does so in a costly way due to the existence of direct costs of default. Moreover, indexed debt improves welfare even considering that, through reborrowing, the government can smooth consumption issuing non-indexed debtonly. This is the case because, as long as the stochastic process of the shock is not mean reverting, the best consumption path that reborrowing can generate will follow that of the shock, though possibly with a smaller dispersion. The reason is that smoothing through reborrowing is affected by the wealth effects created by the shocks.

The optimal indexed debt contract is one that allows the government portfolio of assets and liabilities to generate payoffs similar to that of an insurance contract. That is, the portfolio payoffs should involve negative returns to creditors and net payments to the government in the presence of bad income shocks to the country. This contract cannot rule out renegotiations in some states, but it can guarantee that repudiation will never occur. The crucial ingredient for this result is the assumption that repudiation entails no payments to creditors and a direct cost for the government (the "penalty") that is not appropriable. In a state in which the government was choosing to repudiate non-indexed debt, we can design an interest payment of indexed debt smaller than the "penalty" of repudiation. The government would then be better off making this small payment and avoiding the costs of repudiation. Creditors in turn would benefit receiving an (even small) positive payment instead of nothing.

The assumptions that the government is risk averse and creditors risk neutral are crucial for our results. Alternative assumptions on this regard may affect and even reverse them. If creditors were more risk averse than the government, non-indexed debt might fail to be the best option. Alternatively, the presence of heterogenous creditors with different preferences over instruments types, may call for the presence of a mix of indexed and non-indexed debts in the optimal borrowing structure.

There are concerns that sovereign debt contracts indexed to real variables partially under the control of the government, such as GDP, may create or worsen moral hazard.[7] In effect, the government may have incentives to undertake less growth-oriented policies if interest payments increase in GDP. However, the extent and magnitude of this moral hazard is not clear in practise.[8] Thus, given that the only source of uncertainty in our model is shocks to income, GDP is the preferred indexation variable when we introduce indexed debt. Note also that the presence of moral hazard is not exclusive of indexed contracts. In effect, given that the costs of renegotiation and repudiation are affected by the level of income, distorsions on the government policies might potentially also arise with non-indexed debt too, though limited to those states in which the government defaults.

The few existing cases of real indexed sovereign debt contracts share common features. They usually include a threshold level of the chosen indexation variable above which payments to creditors are triggered. Then, the magnitude of their interest payments increases as the chosen indexation variable increases. Payments increasing on the chosen variable are compatible with our characterization of the optimal indexed contract. However, explaining the presence of thresholds is more challenging. We explore environments under which threshold to interest payments could be optimal. First, thresholds are compatible with the optimal debt contract if the government had access to contingent assets.[9] Without contingent assets, thresholds would be optimal if we introduce the constraint that the government cannot build a portfolio of assets and liabilities that generates net payments to the government from credit markets (excluding reborrowing). The intuition is that in those states in which the optimal "unconstrained" indexed contract calls for negative returns for creditors and net payments to the government, the best that the "constrained" contract can do is to ensure zero coupon payments to creditors. Notice that in this latter case the government can achieve *more insurance* across good states than across bad ones. Finally, we also analyze the specific thresholds chosen in the latest experience with real indexed debt: Argentina's GDP warrant. This warrant includes a threshold on the level of GDP and on its growth rate. We conclude that

[7] Krugman (1988) was the first one to suggest this. In order to address this issue it has been suggested to index the contracts to variables beyond the control of the government such as commodity prices or trading partners' growth rates.

[8] There are many examples of inflation indexed debt in reality that are activey traded. It is easy to argue that government have a stronger incentive to misreport inflation than GDP. First, lower inflation is perceived to be a good signal while the opposite happens with GDP growth rates; second, lower inflation decreases interest payments on inflation indexed debt as it would happen with GDP indexed debt. Still we do observe inflation indexed debt being traded and issued. Thus the moral hazard issue does not seem to be crucial and we will abstract from it in our discussion.

[9] Furthermore, if the set of assets allows the government to complete the market, then the structure of the debt contracts may become irrelevant.

they can hardly be considered optimal.

Our paper bridges two branches of sovereign debt literature that have, surprisingly, remained separate. The first strand emphasizes the presence of limited commitment in a dynamic framework. Our model is related, within this group, to Eaton and Gersowitz (1981) and, in particular, to the adaptation of their model made by Arellano (2004) and Yue (2005). However, we extend this framework to tackle the issue of the optimal design of real-indexed sovereign debt and its quantitative welfare effect. The second strand of literature, which is more policy oriented and best represented by Borensztein and Mauro (2004), analyzes the issues regarding real-indexed sovereign debt contracts abstracting from a thorough modelling of its limited commitment structure.

The paper is organized as follows. Section 2 presents our baseline model. Section 3 compares non-indexed and indexed debt contracts and analyzes their optimal design. Section 4 analyzes the design of existing experiences with real-indexed sovereign debt. Section 5 presents some numerical simulations that quantify the welfare effect of real-indexed debt. Finally, Section 6 concludes.

2 The Baseline Model

2.1 Environment

Consider a small open economy in the spirit of Eaton and Gersowitz (1981) where a government tries to maximize the welfare of an infinitely lived representative agent. In each period $t = 0, 1, 2, 3, ...$ the agent receives a stochastic endowment, $y_t(s)$, contingent on the state of the economy, $s = 1, ..., S$ where $y_t(s) > y_t(s')$ if $s > s'$. The representative agent preferences are given by the following sum of instantaneous utility functions:

$$U = \sum_{t=0}^{+\infty} \beta^t E_0 \left[u(c_t(s)) \right] \tag{1}$$

where $\beta < 1$ represents the intertemporal discount factor, $u(.)$ is a strictly concave utility function increasing in $c_t(s)$, national consumption in period t and contingency s, and E_0 is the time 0 expected value of the utility from consumption in future periods.

In each period t, after observing the state of the economy s, a benevolent government that has no commitment technology makes two decisions in order to maximize the welfare of the representative agent: whether to fully repay, renegotiate or repudiate its outstanding debt with foreign creditors, and how much new debt to issue and assets to purchase. The government can issue one-period debt in international credit markets and has access to a risk free international asset delivering the world interest rate R_t^w.

Since in this paper we want to focus on the welfare effect of government debt indexation, it is convenient to assume that the international asset the government can purchase has a price of 1 and pays R_t^w in every state, i.e. it is risk free. Government debt instead can be indexed (or not) to the state of the economy. The promised state contingent payment of the government portfolio of assets and liabilities can be written as:

$$b_t(s) = x_t(s)d_t - R_t^w a_t \tag{2}$$

where $b_t(s)$ is the payoff of the portfolio in state s, and d_t and a_t are, respectively, the debt and asset position of the country while $x_t(s)$ is the state contingent promised payment at time t of one unit of debt issued at time $t-1$ as described by the vector $\overrightarrow{x}_t$:

$$\overrightarrow{x}_t = \begin{bmatrix} x_t(1) \\ ... \\ x_t(s) \\ ... \\ x_t(S) \end{bmatrix}, \forall t$$

For the sake of realism we assume that the debt contract entails $x_t(s) \geq 0$, $\forall s$. For instance, a standard non-indexed discount debt contract is characterized by $x_{t+1}(s) = 1$, $\forall s$. With a little abuse in definition, we will sometimes refer to $b_t(s)$ as the level of net debt of the economy. The economy state contingent budget constraint at time t in contingency s then becomes:

$$y_t(s)+q_t(s, x_{t+1}(s')d_{t+1}, R_t^w a_{t+1})d_{t+1}(s)+R_t^w a_t-a_{t+1}(s) = c_t(s)+h_t(s)\cdot x_t(s)d_t+\lambda(h_t, y_t(s)) \tag{3}$$

where $y_t(s)$ is the state contingent level of national income; $a_{t+1}(s)$ is the amount of assets purchased at time t and delivering at time $t+1$; d_{t+1} is the amount of debt issued at time t and maturing at time $t+1$; $q_t(s, x_{t+1}(s')d_{t+1}, R_t^w a_{t+1})$ is the endogenously determined unit price of debt issued at time t, which depends on the government's incentive to default (affected by the level of debt and assets in its portfolio and the state of the economy, s). $\lambda(h_t, y_t(s))$ represents the direct cost (or penalty) to the country for not meeting its current payment obligations, $b_t(s)$. We assume that $\lambda(h_t(s), y_t(s))$ is a continuos and differentiable function of the level of income, $y_t(s)$, and the choice of the country between repaying, renegotiating and repudiating its debt at time t, i.e. $h_t(s)$:

$$h_t(s) = \begin{cases} 1 \text{ if the country repays} \\ 0 < \varphi\left(x_t(s)d_t, y_t(s)\right) < 1 \text{ if the country renegotiates} \\ 0 \text{ if the country repudiates} \end{cases}$$

where $\varphi_t(s)$ is a continuous and differentiable function of $x_t(s)d_t$ and $y_t(s)$ and represents the share of outstanding payments that the government will have to make if it chooses to renegotiate.[10][11]

For the sake of realism we assume that, for any given level of income y, the penalty to a country for repudiating its debt is larger than the penalty when the country renegotiates:

$$0 < \lambda\left(\varphi, y\right) < \lambda\left(0, y\right) < y; \forall y > 0 \tag{4}$$

There is no penalty when the country pays back its debt obligations in full:

$$\lambda\left(1, y\right) = 0, \ \forall y$$

We remain agnostic regarding the exact nature of these costs of default[12]. The only assumption we make is that the penalty function $\lambda\left(.,.\right)$ is monotonically increasing (but less than one to one) in the level of GDP, y. The intuition behind this assumption can be related to Grossman and Van Huyck (1988) argument about excusable and non-excusable defaults. As the state of the economy gets worse (i.e. the income level decreases) for a given level of debt, the penalty that would follow is likely to be smaller as the default would be "more excusable". We also assume that, the amount to be repaid following a renegotiation $\varphi\left(x_t(s)d_t, y_t(s)\right)$ is decreasing in the level of payments due and is increasing in income. The idea is that, for any given amount of debt, when the state of the economy worsens, creditors will have a harder time making their case for large payments (i.e.

[10] We are assuming that foreign assets held by the government can not be seized in the event of a default.

[11] In order to solve the model we will assume that φ_t is the repayment requested by creditors to settle the renegotiation.

[12] As explained above, direct costs evaluated in the sovereign debt literature include trade sanctions (Bulow and Rogoff (1989)), informational costs (Cole and Kehoe (1997) and Sandleris (2005)), and legal and adminitrative costs.

their bargaining power will be diminished)[13]. Similarly, given the level of income, the empirical evidence seems to suggest that as the level of debt increases a smaller share of debt is repaid upon renegotiation, though total payments increase in debt.[14] The following conditions formalize these assumptions:

Penalty from defaulting and income

$$0 < \frac{\partial \lambda \left(h, y\right)}{\partial y} < 1 \text{ if } h < 1$$

Renegotiation outcome, debt and GDP

$$\begin{gathered} \frac{\partial \varphi(x(s)d, y(s))}{\partial x(s)d} < 0 \\ \frac{\partial [\varphi(x(s)d, y(s)) \cdot x(s)d]}{\partial x(s)d} > 0 \\ \frac{\partial \varphi(x(s)d, y(s))}{\partial y(s)} > 0 \end{gathered}$$

In our setup any default choice involves a penalty (or direct cost) to be paid by the defaulting country. While the debtor country pays in contingency s a cost equal to:

$$D(h_t, s) = \begin{cases} h_t = 1 \Leftrightarrow repay & d_t(s) \\ h_t = \varphi_t(s) \Leftrightarrow renegotiate & \varphi_t(s)x_t(s)d_t + \lambda(\varphi_t(s), y_t(s)) \\ h_t = 0 \Leftrightarrow repudiate & \lambda(0, y_t(s)) > 0 \end{cases} \tag{5}$$

international lenders only receive:

$$L(h_t, s) = \begin{cases} h_t = 1 \Leftrightarrow repay & d_t(s) \\ h_t = \varphi_t(s) \Leftrightarrow renegotiate & \varphi_t(s)d_t(s) \\ h_t = 0 \Leftrightarrow repudiate & 0 \end{cases} \tag{6}$$

thus it is transparent that

$$D(h_t, s) > L(h_t, s);\ \forall h_t(s) < 1, \forall s \tag{7}$$

(7) results from the assumption that the direct costs of defaults suffered by the borrowing country are not appropiable by creditors.

[13] See Yue (2005) for a formal derivation of these results as the outcome of a Nash bargaining game between creditors and the government.

[14] See Sturzenegger and Zettlemeyer (2004).

Also, consistently with the empirical evidence, we assume that countries are excluded from credit markets when they repudiate their debts. The exclusion from international credit markets is captured by setting $d_{t+1} = 0$ and $a_{t+1} = 0$ for an exogenously given period of time. In this paper we take a reduced form approach assuming that, once excluded from credit markets, the probability that the country re-enters international financial markets in any subsequent period is determined at the level θ. Notice that the exclusion from credit markets is particularly costly for the government as we assume that, once excluded, it cannot save.[15]

Since international lenders are risk neutral (or, alternatively, able to fully diversify country risk) they only consider the expected rate of return paid by the country's securities. Therefore, the country debt must guarantee a (possibly state contingent) interest rate equal - in expected value - to the world interest rate:[16]

$$E_{t-1}[R_t] = \frac{\sum_s |_{h_t(s)=1} \pi_t(s) \cdot x_t(s) + \sum_s |_{h_t(s)=\varphi_t(s)} \pi_t(s) \cdot \varphi_t(s) x_t(s)}{q_{t-1}(s)} = R_t^w \tag{8}$$

where $\pi_t(s)$ labels the probability that contingency s realizes. Equation (8) captures the probability of repayment and renegotiation and their respective payoffs to creditors. These probabilities will be endogenously determined by the equilibrium. Creditors extract no value from those states where the economy repudiates its debt.

[15] The role of this assumption, widely used in the sovereign debt literature, is to make the exclusion from future borrowing costly. As shown by Bulow and Rogoff (1989), the necessary condition for this to happen is that the government cannot replicate the payoffs of the contract from which it is excluded. Suboptimal savings as in Amador (2003) or a restricted set of assets available to the government following a default as in Kletzer and Wright (2000) and Wright (2002) would suffice for this to happen. Although the "no savings" assumption is clearly extreme, it is the simplest way to achieve this result.

[16] An equivalent way of thinking about this choice is in the following competitive framework. International creditors can observe the total amount of debt that the government wishes to issue and the interest rate it offers to pay. On the basis of this observation, creditors form expectations about the probability of repayment, renegotiation and repudiation. Then, they compare the expected rate of return they receive on these bonds with the world interest rate. If the former is larger or equal than the latter, then the issuance takes place and it is randomly allocated among all applying creditors. Otherwise the issuance does not take place.

2.2 Optimization Problem and Equilibrium

The benevolent government optimizes as follows:

$$\max_{c_t(s),h_t(s)=\{0,\varphi_t(s),1\},d_{t+1}(s),a_{t+1}(s);t=0,...\infty} U=\sum_{t=0}^{+\infty}\beta^t E_0\left[u(c_t(s))\right]=$$

$$=\sum_{t=0}^{+\infty}\beta^t E_0\left\{u\left[y_t(s)+q_t(s,x_{t+1}(s')d_{t+1},R_t^w a_{t+1})d_{t+1}(s)-a_{t+1}(s)-h_t\cdot(x_t(s)d_t)+R_t^w a_t-\lambda(h_t,y_t(s))\right]\right\} \tag{9}$$

where the choice variables of the government - $c_t(s)$, $h_t(s)$, a_{t+1} and $d_{t+1}(s)$ - are indexed by s whenever we want to make explicit their state contingency. Our assumption on credit markets exclusion implies $a_{t+1}=d_{t+1}=0$ with an exogenously given positive probability if the country has an unresolved repudiation on its debt.

The utility of the representative agent in (9) suggests that the government is not indifferent among all combinations of assets and liabilities that yield the same portfolio payoffs. The reason is that debt payments and the outcome of renegotiation depend upon the level of gross debt while the repudiation penalty depends on the level of income only. Therefore - as we will see in section 3.1 - the design of the debt contract will be affected by the amount of assets in the portfolio. Since both assets and liabilities determine consumption through repayment/default decision, the equilibrium market price of debt is a function $q_t(s,x_{t+1}(s')d_{t+1},R_{t+1}^w a_{t+1})$ of the entire country portfolio.

Our setup allows for a recursive formulation. The state variables of the recursive problem are three: the gross amount of repayments to be made, $x_t(s')d_t$,the gross amount of payments to be received, $R_t^w a_t$, and the existence of an unresolved repudiation, $H_t=\{0,1\}$, where $H_t=0$ if the government has an unresolved default at time t, and $H_t=1$ when it does not. We can write the value function for the government problem as:

$$V(H,x(s)d,a|\,s)=\max_{h,c,a\prime,d\prime}\left\{u\left(c\right)+\beta\cdot E\left[V(H',x(s')d',a'|\,s')\right]\right\} \tag{10}$$

where we use the standard convention of omitting the time "t" label and denoting by a "$'$" a variable referred to "$t+1$". (10) can be specified in greater details as follows under repayment, renegotiation and repudiation, and when there is no preexistent unresolved default:

$$V(1,x(s)d,a|\,s)|_{h=1}=$$

$$=\max_{a',d'}\left\{u(y+qd'+R^w a-x(s)d-a')+\beta\cdot E\left[V(1,x(s')d',a'|\,s')\right]\right\}$$

$$V(1, x(s)d, a|\, s)|_{h=\varphi} =$$
$$= \max_{a'_\varphi, d'_\varphi} \left\{ u(y + q[x(s')d'_\varphi, R^w a'_\varphi]d'_\varphi + R^w a - \varphi \cdot x(s)d - \lambda(\varphi, y) - a'_\varphi) + \beta \cdot E\left[V(1, x(s')d'_\varphi, a'_\varphi|\, s')\right] \right\}$$

$$V(1, x(s)d, a|\, s)|_{h=0} =$$
$$= \{u(y + R^w a - \lambda(0, y)) + \beta E\left[(1-\theta)V(0, 0|\, s) + \theta V(1, 0|\, s')\right]\} \tag{11}$$

while the value function with a preexistent unsettled repudiation is given by:

$$V(0, 0|\, s) = u(y(s)) + \beta E\left[(1-\theta)V(0, 0|\, s) + \theta V(1, 0|\, s')\right] \tag{12}$$

where θ represents the (exogenous) probability of being able to re-enter capital markets following repudiation.

Note that the decision of a country to repudiate, renegotiate or repay its debt, is made period-by-period. Thus the expected value from next period onwards incorporates the fact that the government could choose to default in the future. The government default policy can be characterized by repayment, renegotiation and repudiation sets. For a given level of assets and liabilities:

$$I(y(s)) = \{y : V(1, x(s)d, a|\, s)|_{h=1} \geq \max_{h\in\{0,\varphi(s)\}} V(1, x(s)d, a|\, s)|_{h=\varphi}, V(1, x(s)d, a|\, s)|_{h=0}\}\}$$
$$\Phi(y(s)) = \{y : V(1, x(s)d, a|\, s)|_{h=\varphi} \geq \max_{h\in\{0,1\}} \{V(1, x(s)d, a|\, s)|_{h=1}, V(1, x(s)d, a|\, s)|_{h=0}\}\}$$
$$O(y(s)) = \{y : V(1, x(s)d, a|\, s)|_{h=0} \geq \max_{h\in\{\varphi(s),1\}} \{V(1, x(s)d, a|\, s)|_{h=1}, V(1, x(s)d, a|\, s)|_{h=\varphi}\}$$

are the repayment, renegotiation and repudiation sets respectively.

Having analyzed the agents' optimization problems we are ready to define the equilibrium of this economy:

Definition 1 *The recursive equilibrium of the economy is defined as: (i) a set of policy functions for consumption allocations $c(s)$, government international debt $d(s)$ and asset $a(s)$, and repayment $I(y(s))$, renegotiation $A(y(s))$ and repudiation $O(y(s))$ sets; and (ii) a bond price $q(x(s')d', a')$ such that:*

1. *taking the bond price $q(b'(s))$ as given, the consumption allocations $c(s)$, government international debt $d(s)$ and asset $a(s)$, repayment $I(y(s))$, renegotiation $A(y(s))$ and repudiation $O(y(s))$ sets satisfy the government optimization problem;*

2. *$q(x(s')d', a')$ reflects the government default probabilities given government international debt $d'(s)$ and asset $a'(s)$, repayment $I(y(s))$, renegotiation $A(y(s))$ and repudiation $O(y(s))$ sets, and is consistent with creditors' zero expected profit condition (8).*

2.3 Income, Debt and the Default Decision

We are now ready to further characterize the government repayment decision. The following lemmas characterize how the repayment, renegotiation and repudiation sets are affected by changes in the level of initial debt. We assume that the government issues standard non indexed discount bond, i.e. a debt contract designed with payoffs $x_t(s) = 1$, $\forall s, t.$:

Lemma 2 *There exists d' such that the government optimally repudiates its debt. Furthermore, repudiation is also the optimal choice when $d \geq d'$*

Proof. By (11), we may note that $V(1, d, a|\, s)|_{h=0}$ is constant on d while both $V(1, d, a|\, s)|_{h=1}$ and $V(1, d, a|\, s)|_{h=\varphi}$ are decreasing on d. Thus there always exists a level of d, d',rendering repudiation optimal. Then, if repudiation is optimal for d' it must be optimal for any $b \geq b'$ ■

Lemma 3 *If renegotiation is preferred to repayment for a given debt level d^*, then renegotiation is also preferred to repayment $\forall d \geq d^*$.*

Proof. Taking the derivative of the value functions of repayment and renegotiation with respect to the level of gross debt d and applying the envelope theorem we obtain:

$$\frac{\partial\, V(1, d, a|\, s)|_{h=1}}{\partial d} = -u'(c|_{h=1})$$

and

$$\frac{\partial\, V(1, d, a|\, s)|_{h=\varphi}}{\partial d} = -u'(c|_{h=\varphi})[\varphi + \frac{\partial \varphi}{\partial d} d]$$

Note that $0 < [\varphi + \frac{\partial \varphi}{\partial d} d] < 1$ as $0 < \varphi \leq 1$ and $\frac{\partial \varphi}{\partial d} < 0$ by assumption. Furthermore, if renegotiation is preferred to full repayment at d', it must be the case that then from concavity $c|_{h=\varphi} > c|_{h=1}$of utility function we have that $u'(c|_{h=1}) > u'(c|_{h=\varphi})$ (as the continuation value must be the same). This implies that at d'

$$\frac{\partial\, V(1, d, a|\, s)|_{h=1}}{\partial d} > \frac{\partial\, V(1, d, a|\, s)|_{h=\varphi}}{\partial d}$$

in absolute value. Therefore, an increase in the level of debt will generate a larger decline in the value function of repayment. As a result, $\forall d \geq d^*$

renegotiation will be preferred to full repayment. ■

Although the possibility of default introduces some degree of contingency in an otherwise non-contingent debt contract, it does so at the cost of a deadweight loss: the default penalty defined in (4). The presence of these penalties makes the government willing to repay its debt in some states of the world:

Lemma 4 *If for a given level of debt, b, the probability of repudiation,* $\sum_s|_{h(s)=0}\pi(s)$*, is positive, then this probability increases with the level of debt.*

Proof. Let $\widehat{d}$ be such that for a given level of income $\widehat{y}$ the following condition holds:

$$V(1,\widehat{d},a\big|\,s)\Big|_{h=0} = \max_{h\in\{\varphi(s),1\}}\{V(1,\widehat{d},a\big|\,s)\Big|_{h=1}, V(1,\widehat{d},a\big|\,s)\Big|_{h=\varphi}\}$$

We know that such $\widehat{d}$ exists from Lemma 2. Upon indifference the country was repaying or renegotiating at income level $\widehat{y}$ with debt payments $\widehat{d}$. From Lemma 2 we know that for debt level $\widehat{d}+\varepsilon$, $V(1,\widehat{d}+\varepsilon,a\big|\,s)\Big|_{h=0} > \max\{V^1(1,\widehat{d}+\varepsilon,R^w a\big|\,s),V^\varphi(1,\widehat{d}+\varepsilon,a\big|\,s)\}$, then the probability of repudiation is increasing in the level of debt repayment. ■

We can summarize the results of the lemmas above in the following proposition that characterizes how the probabilities of repudiation, renegotiation and full repayment are affected by changes in the level of debt.

Proposition 5 *If, at a given level of gross debt d, both the probability of repudiation,* $\sum_s|_{h(s)=0}\pi(s)$*, and of repayment,* $\sum_s|_{h_t(s)=1}\pi_t(s)$*, are positive, then the probability of repudiation is increasing in the level of debt, while the probability of repayment decreases in it.*

Corollary 6 *If for a given level of gross debt, d, the probability of repudiation,* $\sum_s|_{h(s)=0}\pi(s)$*, or the probability of renegotiation,* $\sum_s|_{h(s)=\varphi}\pi(s)$*,are positive, then the interest rate paid by non-indexed debt,* $R=\frac{1}{q}$*, is increasing in the level of both gross and net debt, d.*

As in Eaton and Gersowitz (1981) and Yue (2005) the probability of repudiation is increasing in the level of debt, and so is the equilibrium interest rate under non-indexed debt. This means that there will be a threshold level of debt payments above which the repudiation set will be equal to the whole set of income levels. Creditors will never choose to lend such an amount. The level of government borrowing is therefore restricted by its inability to commit to repay.

We now analyze how the repayment, renegotiation and default sets are affected by changes in the level of income when we fix the debt level, b. In order to facilitate our discussion, we make the additional assumption that the direct cost of repudiation is more sensitive to changes in the level of income than the total cost of renegotiation. Formally:

$$\frac{\partial\lambda(0,y)}{\partial y} > \frac{\partial\lambda(\varphi,y)}{\partial y} + \frac{\partial\varphi(b,y)}{\partial y} \tag{13}$$

So we can state the following Lemma:

Lemma 7 *Under assumption (13) and given the level of debt d, if the government chooses to repudiate at y', it will do so $\forall\ y < y'$*

Proof. Taking the derivative of the value functions of repayment, renegotiation and repudiation with respect to y and applying the envelope theorem we obtain:

$$\frac{\partial\, V(1,d,a|\, s)|_{h=1}}{\partial y} = u'(c_{h=1}),$$
$$\frac{\partial\, V(1,d,a|\, s)|_{h=\varphi}}{\partial y} = u'(c_{h=\varphi})[1 - \frac{\partial\varphi}{\partial y} - \frac{\partial\lambda(\varphi,y)}{\partial y}]$$
$$\frac{\partial\, V(1,d,a|\, s)|_{h=0}}{\partial y} = u'(c_{h=0})[1 - \frac{\partial\lambda(0,y)}{\partial y}]$$

If for y' repudiation was the optimal choice for given level of debt b, then assumption (13) ensures that at y' :

$$\frac{\partial\, V(1,d,a|\, s)|_{h=0}}{\partial y} < \frac{\partial\, V(1,d,a|\, s)|_{h=1}}{\partial y}$$

and

$$\frac{\partial\, V(1,d,a|\, s)|_{h=0}}{\partial y} < \frac{\partial\, V(1,d,a|\, s)|_{h=\varphi}}{\partial y}$$

Thus repudiation must be optimal for $y < y'$ ■

The intuition for this lemma is relatively simple. First, we observe that all value functions are increasing in the level of income. Then, if we find that at income y' repudiation is optimal, as income goes below y' the value function of repudiation declines less than those of repayment and renegotiation. But as income decreases so does the direct cost of repudiation, and so the government continue to choose repudiation. The following lemma obtains a similar implication when comparing repayment and renegotiation:

Lemma 8 *If for a given level of debt d and income y' the government prefers to renegotiate rather than repay its debt, then it will do so $\forall y < y'$*

Proof. If renegotiation is preferred to repayment then it must be the case that $c_{h=\varphi} > c_{h=1}$. If this is the case, we have that

$$\frac{\partial\, V(1, b|\, s)|_{h=1}}{\partial y} > \frac{\partial\, V(1, b|\, s)|_{h=\varphi}}{\partial y}$$

from the proof to Lemma 7. But then the government will prefer renegotiation over repayment if $y < y'$ ■

The intuition for this result is related to the fact that, if at income y' renegotiation is preferred to full repayment, then as income falls the decline in the value function of repayment will be larger than that of renegotiation. As a result the government would still prefer renegotiation to repayment. The following lemma proves that the inverse result also holds in the case of repayment.

Lemma 9 *If for a given level of debt d and income y' full repayment is optimal, then it will also be optimal when $y > y'$.*

Proof. The proof is in two steps:

i) if repayment is preferred to repudiation at given debt d and income y', then repayment must be preferred to repudiation for $y > y'$. We prove this by contradiction. Assume that full repayment is preferred to repudiation at y' for given level of debt d and there exists $y > y'$ for which repudiation is preferred to repayment. Then from Lemma 7 repudiation must be preferred to repayment also for y', which is a contradiction;

ii) if repayment is preferred to renegotiation at given level of debt d for income level y', then repayment must be preferred to renegotiation when $y > y'$. We prove this by contradiction.

Assume that full repayment is preferred to renegotiation at y' for a given level of debt and there exists $y > y'$ for which renegotiation is preferred to repayment. Then from Lemma 8 renegotiation must be preferred to repayment also for y', which is a contradiction. ■

We can summarize the results of the above Lemmas by the following proposition:

Proposition 10 *Assume that for a given level of debt d the government chooses to repay at some levels of income and repudiate in others. Let* $y' = \max\{y\}$ *at which the government finds optimal to repudiate and let* $y" = \min y$ *for which the government finds optimal to fully repay. Then:*

i) the lowest level of income at which the government repays is higher than the highest level of income at which it repudiates $y" \geq y'$*;*

ii) if income is low, i.e. $y \leq y'$*, the government repudiates;*

iii) if income is high, i.e. $y \geq y"$*, the government repays;*

iv) $\forall y \in (y', y")$ *the government renegotiates.*

Therefore the choice of the government to repay, renegotiate or repudiate is income dependent.

3 Indexation Rules

3.1 On the Optimality of Indexed Debt Contracts: Main Features

In this section we characterize the optimal indexed debt contract and we compare consumption allocations of the economy under two different debt instruments - non-indexed (NID) and indexed (ID) debt . NID is defined by the fact that promised payments on issued gross debt are constant across contingencies, $x(s) = 1$, $\forall s$,[17] while ID allows for state contingent promised payments on issued debt, i.e. $x(s) \neq x(s') >> 0$ for at least two states s, s'.

Also NID entails some degree of contingency through the possibility of default. Moreover, whenever the country defaults, the cost of doing so is also state contingent. However, the degree of contingency allowed by defaults comes at a cost: the cost of all direct penalties enforced against the borrowing country - $\lambda(.)$ functional of our analysis - whenever its government renegotiates or repudiates.

[17] Making payments of NID equal to 1 is without loss of generality.

ID improves welfare in our framework. It does so through two channels: first, it increases the amount of risk-sharing between a risk-averse borrowing country and risk-neutral lenders; second, it reduces the probability of default and so the magnitude of the direct costs that default entails for debtor countries. The crucial assumption for this second channel to operate is that the direct penalty costs - reflecting what seems realistic - constitute deadweight losses that are not appropiable by creditors[18]. The following Lemma formalizes the intuition that ID contracts improve welfare:

Lemma 11 *For any given level of debt d for which there exists an equilibrium NID contract that entails renegotiation or repudiation in at least one state, we can construct the ID contract $x(s) > 0, \forall s$ satisfying creditors' zero profit condition, (8), and such that:*

1. *it is preferred by the government to the equilibrium NID contract, $x(s) = 1\ \forall s$;*
2. *it does not involve default in any state of the world;*
3. *if there is perfect competion among creditors, it allows debtor countries to appropriate all surplus generated by the indexed contract.*

It is instructive to construct the indexed debt contract, $x(s)$, and so we do not relegate the argument in a proof. In the following discussion, we take as given the level of debt d: since our reasoning holds for any level of debt at which there is positive borrowing, doing so does not affect the generality of the argument.

Pick any state s^* in which the government chose to renegotiate its outstanding NID. We can always design the ID by setting $x(s^*)$ so that the government prefers to repay its debt d rather than renegotiate it:

$$x\left(s^*\right) d \leq \varphi\left(x\left(s^*\right) d, y\left(s^*\right)\right) \cdot x\left(s^*\right) d + \lambda\left(\varphi, y\right)$$

$$\Leftrightarrow$$

$$x\left(s^*\right) d \leq \frac{\lambda(\varphi,y)}{\left[1-\varphi(x(s^*)d,y(s^*))\right]}$$

and so that creditors receive more payments by ID than by the renegotiation of NID:

$$\varphi\left(x\left(s^*\right) d, y\left(s^*\right)\right) x\left(s^*\right) d \geq \varphi\left(d, y\left(s^*\right)\right) d$$

Pick now any state $\widehat{s}$ in which the government chose to repudiate the NID contract. The choice of repudiating debt has two effects that deserve our attention: the default penalty and

[18] See equations (6), (5) and (7) for reference.

the exclusion from credit markets. These facts can be formally studied by comparing the value functions for repayments and repudiation in (11). It is simple to observe that the value function for repayment is a decreasing function of debt payments $x(s)d$

$$V(1, x(\widehat{s})d, a|\,\widehat{s})|_{h=1}$$

while the value function of repudiation is independent from the level of debt d and depends instead on the duration of exclusion from credit markets, the size of the penaly, $\lambda(0, y(\widehat{s}))$, and the level of assets held by the country.

$$V(1, a|\,\widehat{s})|_{h=0}$$

It is therefore possible to design the ID contract by picking $x(\widehat{s}) > 0$ so that:

$$V(1, x(\widehat{s})d, a|\,\widehat{s})|_{h=1} = V(1, a|\,\widehat{s})|_{h=0} \tag{14}$$

ID designed according to equation (14) makes the borrower indifferent between defaulting on its NID and repaying the ID obligations while it is strictly preferred by international lenders because they receive $x(\widehat{s}) > 0$ instead of facing a repudiation. It is also important to observe that the design of Id through the level of payments $x(\widehat{s})$ depends on the stock of assets held by the country, a. This suggests that any financial design of the country a borrowing instruments that is more able to accomodate the country's smoothing needs can not abstract from the country entire portfolio, including assets as well as liabilities.

In conclusion, this argument shows that creditors are receiving more under the ID contract than under the NID contract even when the cost for the debtor country is the same in all states in which the country was defaulting. This also implies, because of the competition among creditors - equation (8) - that the country will be able to borrow at better terms, i.e. at a higher price $q(.)$, or decrease its payments in the states with larger income. In any case the country welfare will be increased. The argument above also allows to give an exact characterization of the ID contract. We summarize it here through the following proposition that highlights the main parts of the argument already presented:

Lemma 12 *For any given level of gross debt d and gross asset a, we can construct the optimal NID contract such that*

1. $x(s) = 1$, *in all states s in which the NID contract, $x(s) = 1$, entailed no default*

2. $x(s^*)\,d \le \frac{\lambda(\varphi,y)}{[1-\varphi(x(s^*)d,y(s^*))]}$, *in all states s^* in which the NID contract, $x(s) = 1$, entailed renegotiation*

3. $x(\widehat{s})\,d$ *such that*

$$V(1, x(\widehat{s})\,d, a|\,\widehat{s})|_{h=1} = V(1, a|\,\widehat{s})|_{h=0}$$

in all states $\widehat{s}$ in which the NID contract, $x(s) = 1$, entailed repudiation

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We now characterize the optimal ID contract focusing again our discussion in the pattern of payments that it should generate for the portfolio. As the government is assumed to be risk averse and creditors risk neutral, the optimal contract will be the one that provides the highest possible level of insurance to the country for a given level of net debt.[19] It is important to notice that such debt contract should usually generate $b(s) < 0$ in some state s, i.e. it requires that the country becomes a net recipient of funds from international credit markets in some states. That is, it does not require that the debt contract itself generates payments to the government, but that the whole portfolio does it in net terms. A portfolio generating such a flow of payments closely resembles an insurance product more than a pure debt contract like the ones we observe in reality.

Although countries do not typically have access to explicit insurance contracts, they have an alternative way to become net recipients of funds in some states (i.e. $b(s) < 0$): governments can potentially build an optimal portfolio of assets and liabilities able to produce the desired combination of payments across contingencies "to and from" international financial markets. These are exactly the flow of funds on which we focus in our model. The following example clarifies this observation:

Example 13 *Assume without loss of generality that there are two equally probable states and the world interest rate $R^w = 1$. The government finds it optimal to receive 10 units in the bad state and pay 10 units in the good one in order to smooth the effects of income shocks. If the government can not write a simple insurance contract, it could issue ID promising to pay 0 in the bad state and 20 in the good one. Disregarding default risk, the competitive price of such a contract at time of issuance would be 10. If the government is also able to save this 10 units purchasing a non-contingent asset in international financial markets, the payoffs from the whole portfolio to the government would be +10 in the bad and -10 in the good state.*

[19] Full insurance is not always feasible due to the presence of limited commitment.

As this example makes clear the combination of asset and debt generates the same net payoffs of the optimal insurance contract. As long as the government can save, it is irrelevant whether the debt contract is restricted to promise only positive payments *from* the issuing government. Therefore in our discussion we will focus only on the net payoffs of the country portfolio, disregarding the exact portfolio strategy that generates them.

The following proposition formalizes our discussion on the optimal indexed contract.

Proposition 14 *Assume the best ID contract $b(s) \geq 0, \forall s$ is not enough to deliver full insurance and $b(s') \leq \bar{b}(s')$ for at least one state s'. Then there exists a portfolio combination af assets and debt delivering payments $\overrightarrow{b^*}$, preferred to $\overrightarrow{b}$, that entails net payments to the government in at least one state $\widehat{s}$, i.e. $b^*(\widehat{s}) < 0$, where the marginal utility of consumption is larger than in s', i.e.*

$$\pi(s')\frac{du(c(s'))}{dc(s')} < \pi(\widehat{s})\frac{du(c(\widehat{s}))}{dc(\widehat{s})}$$

Proof. Pick the best ID debt contract with $b(s) \geq 0, \forall s$ not achieving, by assumption, full insurance. Under this debt contract marginal utilities differ across states:

$$\pi(s')\frac{du(c(s^*))}{dc(s^*)} < \pi(\widehat{s})\frac{du(c(s))}{dc(s)}, \ \forall s, s^*$$

It is immediate to observe that debt contract $b(s)$ must imply $b(\widetilde{s}) = 0$ in contingency $\widetilde{s}$ where marginal utility is the highest across states. The country welfare can then be improved by transferring interest payments away from state $\widetilde{s}$, i.e. making $b(\widetilde{s}) < 0$. But this is exactly what portfolio $\overrightarrow{b^*}$ guarantees when there is a state s' where the interest rate payment can be increased, i.e. $b(s') \leq \bar{b}(s')$, and where marginal utilities is lower, i.e.

$$\pi(s')\frac{du(c(s'))}{dc(s')} < \pi(\widehat{s})\frac{du(c(\widehat{s}))}{dc(\widehat{s})}$$

■

There is a (potentially) large set of economies for which the maximum amount of insurance compatible with limited commitment requires the borrowing country to become a recipient of net payments from the rest of the world. Proposition 14 makes clear that even the best indexed *debt* contract may fail to achieve this result by itself. The highest country insurance can only be achieved if the country constructs a portofolio - i.e. *net* debt $b(s)$ of our discussion- through which it may receive net payments from international financial markets in the states when it is most needed.

4 An Evaluation of Existing Real Indexation Rules for Sovereign Debt

4.1 Existing Experiences

In broad terms, proposals to improve the design or legal framework of sovereign debt contracts can be grouped in two categories: those focused on diminishing the costs of debt crises and those aiming at reducing their frequency. The former include suggestions to introduce collective action clauses in the contracts or sovereign bankruptcy institutions that may simplify and reduce the costs of the restructuring process once a default occurs. The latter, on which we focus in this paper, emphasize the potential benefits of making sovereign debt contracts explicitly contingent, indexing debt payments to real variables related to the economic performance of the debtor country.[20]

Proposals to index sovereign debt payments to real variables have been around for nearly 25 years when the debt crisis of the 1980s triggered interest in the issue. Around that time, Bailey (1983) suggested that debt should be converted into claims proportional to exports, and Lessard and Williamson (1985) made the case for real indexation of debt claims. A few years later, Shiller (1993) discussed the importance of creating “macro markets” for perpetuities linked to GDP.

The recent string of sovereign debt crises in Russia, Ecuador, Pakistan, Ukraine and Argentina generated a second wave of interest in contingent sovereign debt contracts for emerging countries. Haldane (1999), Daniel (2001) and Caballero (2003) suggested that countries would benefit from issuing debt indexed to some relevant commodity price. Drèze (2000) suggested the use of GDP-indexed bonds as part of a strategy to restructure the debt of the poorest countries and Borensztein and Mauro (2002, 2004) tried to revive the case for GDP-indexed bonds for emerging countries.[21]

The basic idea behind all these proposals is to use indexed sovereign debt as a way to improve risk sharing between debtor countries and international creditors and, in doing so, to reduce the probability of occurrence of debt crises. One important difference across proposals is whether they suggest indexing the debt instruments to variables partially under the control of the government or

[20] Many of these proposals try to address specific issues that arise with bond borrowing, which has become the preponderant form of government financing in emerging countries since the exchange of bank loans for tradable bonds resolved the widespread debt crisis of the eighties.

[21] See Borensztein and Mauro (2002) and Borensztein et al. (2004) for a detailed analysis of indexing proposals for sovereign debt.

beyond it. While indexation to broader measures such as GDP or exports that are partially under the control of the government would likely provide greater insurance benefits, potential investors might be concerned about the authorities' incentives to tamper with data or undertake less growth-oriented policies. These concerns regarding the potential risks of moral hazard were first discussed in this context by Krugman (1988) and Foot, Scharfstein and Stein (1989).

It is not clear how relevant are these moral hazard issues in reality. However, if they were, the option of indexing debt contracts to commodity prices outside the control of the government would only be useful to a small group of emerging countries as GDP growth is poorly correlated with these variables for most of them.

Although more than 20 developing and developed countries have issued inflation indexed debt, the experience with bonds indexed to real variables is much more limited. Table 1 summarizes the experiences with each of them.

Table 1: Countries Issuing Real Indexed Bonds

Type	Country
GDP-Indexed Debt	Costa Rica (1990), Bosnia and Herzegovina (1990s), Bulgaria (1994), Argentina (2
Commity Indexed Debt	US (1864), France (1970s), Mexico (1990s), Nigeria (1990s) Venezuela (1990s)

Source: Borensztein et al. (2004)

4.1.1 Argentina GDP-warrant

The most recent experience with sovereign bonds indexed to real variables is Argentina's GDP-warrant. In March 2005, Argentina finished its debt restructuring process that followed the default and financial crisis of 2001. Each new bond issued in the restructuring included a unit of GDP-linked warrants. These warrants were tied to the bonds for the first 180 days, and became detachable thereafter.

Given the magnitude of the restructuring, Argentina's GDP-warrants are the first sovereign debt instruments indexed to real variables for which there is a sizable market. The GDP-linked securities have a notional amount equal to the corresponding defaulted debt tendered and accepted in the 2005 restructuring, converted to the corresponding currency using the exchange rate as of December 31, 2003 (roughly US$62 bn, of which AR$88 bn, US$14.5 bn external and US$2.9bn internal, EUR11.9 bn).

Payments on the GDP-linked securities take place only if the following three conditions are met:[22]

(i) Actual real GDP exceeds the base case GDP for each reference year

(ii) Annual growth in actual real GDP is higher than the growth rate in the

base case GDP for the reference year (base case GDP real annual growth rate is 3.5% per year initially and gradually converging to 3%)

(iii) Total payments made on the security do not exceed the payment cap

of 48% of the notional amount during the life of the security.[23]

Whenever these three conditions are met the formula used to calculate the payments for each notional unit of the warrants is the following:

$$Payment = 0.05 \cdot ExcessGDP \cdot UnitofCurrencyCoefficient$$

where Excess GDP is the amount by which actual real GDP converted into nominal GDP using the GDP deflator exceeds the base case nominal GDP, and the Unit of currency coefficient is defined as: USD: 1/81.8 = 0.012225, Euro: 1/81.8 x (1/0.7945) = 0.015387, ARS: 1/81.8 x (1/2.91750) = 0.004190.

As GDP data is usually published with a lag of a couple of months and it is usually revised in the following months, payments are calculated on the November following the relevant reference year, and made effective a month later. This creates a lag of a year between the economic performance that might trigger a payment and the payment itself. Thus, potentially, troublesome situations may arise for the government. For example, assume that after a year of very high growth that meets all

[22] In all cases calculations are based on the data published by Argentina's Bureau of Statistitcs (INDEC).

[23] The GDP-warrants expire when the 48 cents per dollar cap is reached, and no later than December 15th, 2035.

the conditions for the payment, an adverse shock drives the economy into a recession. The lag in the payments of the GDP warrant implies that the government will have to make a large payment precisely during the recession.

Trading of GDP-warrants began in a "when and if" market before they were detached. In May 2005 the "consensus" value of the GDP-warrant with dollar coupon among investors was 2 cents per dollar. In July 21st, the first available date with data from the "when and if" market, the bid price for the GDP warrants was 3 cents per dollar coupon (50% increase in two months). Furthermore, by the end of 2005 the Argentina's outstanding growth rates and a better understanding of the instrument led the markets to reevaluate their assessment of the value of the GDP-warrant. The price almost tripled when compared with the consensus value upon issuance. By the end of 2006 its price reached 13 cents per dollar coupon, six times higher than the "consensus" value at the time of the exchange and four times higher than the first available trading price. This remarkable increase in price has caught the attention of investors, authorities and the public in Argentina.

The first payment of the GDP-warrant took place in December 2006 and amounted to US$387 mn. In fact, given current consensus forecast for GDP growth in Argentina in the next couple of years, payments on the indexed component are expected to triple in the next two years. In 2008 payments of the indexed component are expected to be roughly equivalent to total coupon payments (plus capitalization) on the three new bonds issued in the restructuring considered together.

Summarizing, despite little interest shown initially by investors, the market for Argentina's GDP-warrant took off fueled by the excellent performance of Argentina's economy in recent years. This is good news for GDP-indexed bonds as it would be the first successful case of such an instrument. However, it is inevitable to wonder whether it was a good idea to include them in the exchange from the point of view of Argentina, given that they did not seem to have any significant impact in the level of acceptance of the proposal.

4.1.2 Brady bonds with Value Recovery Rights (VRRs)

Some years before Argentina's experience with GDP-warrants, a handful of emerging market economies issued bonds with elements of real indexation in the past. Various Brady bonds issued by Bosnia and Herzegovina, Bulgaria, Costa Rica, Mexico, Nigeria, and Venezuela in exchange for defaulted loans in the early 1990s included Value Recovery Rights (VRRs). The VRRs were designed to provide the banks with a partial recovery of value lost, as a result of the debt and debt-service reduction contemplated in the Brady exchange, in the event of a significant increase in the debtor country's capacity to service its external debt. Mexico's VRRs, for example, provided for the possibility of quarterly payments, beginning in 1996, based upon certain increases in the price of oil.

Brady bonds issued by Bulgaria, Bosnia & Herzegovina and Costa Rica contained elements of indexation to GDP. In the case of Bulgaria for example, its Discount Brady bonds had a component named Additional Interest Payments (AIP) that was indexed to GDP. The AIP was triggered when two conditions were met: (1) Bulgaria's GDP surpasses 125% of its 1993 level, and (2) there is a year-over-year increase in GDP. For these years (not including the year in which the threshold is reached) the semi-annual interest supplement was defined as half of that year's GDP growth. The outlays themselves were scheduled to occur "as soon as practically possible" and were to coincide with regular interest payment dates. The AIP were not warrants, detachable or otherwise, though they were intrinsically equivalent.

Bulgaria's GDP-linked bond was generally viewed as a failure as a result of two factors. First, the bonds were callable at par. This meant that the government could decide to repurchase the bonds rather than pay out when faced with onerous GDP-linked payments, and as a result investors would miss out on the lucrative upside. In fact this is exactly what happened. A second problem with Bulgaria's bonds was that the conditions were fairly vague. In effect, "GDP" itself was not well defined, so it was open to interpretation the exact measure of GDP to be used. The government made use of this ambiguity for a while choosing definitions of GDP that prevented the AIP from being triggered.

Bosnia and Herzegovina's GDP-linked Brady bonds included additional interest payments whenever GDP growth rates exceeded for two years a predetermined growth rate and GDP per-capita rise above $2,800 (1997 USD, adjusted by German CPI). These bonds have also suffered with problems in the definition of GDP and their trading activity has been very limited according to Bear Stearns.

In general, the experience with VRRs has not been positive. Indexation formulas were complex and often ambiguous. There were restrictions on their tradability and many times were not detachable, and some of the bonds were callable.

4.1.3 Commodity-Linked Bonds

The main advantages of bonds indexed to commodity prices are that the data is available without a time lag and is not subject to manipulation by governments. Compared with GDP-linked bonds, however, their main disadvantage is that for most countries the correlation between economic performance and commodity prices is relatively low.

Bonds whose repayments are indexed to commodity prices have been used, although rarely, since the 1700s. In 1782, the State of Virginia issued bonds linked to the price of land and slaves. In 1863 the Confederate States of America issued "cotton bonds", whose payments increased with the price of cotton. "Gold clauses," effectively indexing payments to the price of gold, were widespread in the United States in

the 19th century through 1933. France also experimented with gold-price-indexed bonds, the "Giscard," in 1973, but the losses caused by the depreciation of the French Franc caused the government to cease offering this instrument.

Oil-backed bonds appeared in the financial markets during the late 1970s. Mexico is considered the first country to offer oil-linked bonds in April 1977. The "Petrobonos" were issued domestically on behalf of the government by NAFINSA, a development bank owned by the Mexican government. They had a relatively active domestic secondary market developed in which most investors were Mexican. The bond promised to pay an annual rate of 12.65823% and had a three years maturity. Upon maturity, the Petrobonos were redeemed at a value equal to the maximum of the face value or the market value of the referenced units of oil plus all coupons received during the life of the bond.

Other countries and private companies have also experimented with commodity-linked bonds. For instance, Venezuela issued oil linked-bonds as part of its Brady agreement. India issued oil linked bonds to oil companies in April 1998 in payment for debts it had incurred by receiving oil products below market cost. Malaysia accepted a loan from Citibank indexed to palm oil.

More recently, loans combined with protection (through swaps) from commodity price fluctuations have also been made available by the World Bank to member countries, beginning in September 1999, though interest has thus far been limited.

4.2 On the design of existing real indexation rules

Our model can help us evaluate some of the proposals regarding alternative indexation rules for sovereign debt contracts. First, note that we have abstracted from moral hazard in our analysis of indexed debt. In this perspective the only source of uncertainty for borrowing countries is income shocks. It follows therefore that GDP may be employed as the optimal indexation variable when we discuss indexed debt.[24]

Moreover, it is important to emphasize that our analysis is performed under the assumption that creditors are homogenous and risk neutral. Alternative assumptions on this regard may affect the welfare implications that we obtain. For instance, if creditors were more risk averse than the government, non-indexed debt might become the best option for the government. Alternatively, the presence of heterogenous creditors with different preferences over instruments types, may call for the presence of a mix of ID and NID in the optimal borrowing structure.

As our brief survey shows the few existing cases of real indexed sovereign debt contracts share a common features. They usually include, as in the Bulgaria and Argentina GDP-indexed bonds, a threshold level of the chosen variable above which payments are triggered, the magnitude of the payments increases on the chosen variable after that threshold.

As Proposition 14 shows, the optimal real indexation rule that is, the one that provides the maximum amount of insurance does not involve thresholds.[25] In fact, if the space of the shock is continuous, it requires continuous increases in payments as the state of the economy improves. Furthermore, the optimal real indexation rule should generate net payments from creditors to the government for the portfolio as a whole in some states. That is should be similar to an insurance

[24] In our framework it is equivalent to index the contract to GDP and to the state of the economy.

[25] The presence of limited commitment may make full insurance unfeasible though.

contract.[26]

The presence of threshold levels of income that trigger payments is then clearly suboptimal. An interesting issue to analyze is under what conditions the presence of these thresholds levels are optimal. These thresholds would be irrelevant if the government can exactly offset them through contingent assets.[27] Even if this were not the case, they could still be "constrained" optimal if the government were unable to save optimally and debt contracts were restricted not to involve payments to the government ($R(s) \geq 0, \forall s$). In effect, as we argued above, the "unconstrained" optimal contract requires that the government makes net payments in high income states and receives net payments from creditors in low income ones. If we constrain the contract structure or the government ability to save so that the portfolio cannot generate net payments to the goverment in any state, then the so constrained optimal contract calls for zero payments to creditors in those states in which the government was receiving payments under the "unconstrained" optimal contract. This implies that the "constrained" optimal contract will have some threshold level of income, y_N, below which there should be no payments to creditors.

In the case of the Argentina's GDP warrant the threshold level of income that triggers payments of the variable component involves two conditions:

i. real growth rate of GDP in the period higher than the trend growth rate of GDP (approximately 3.5%);

ii. GDP of the period higher than trend GDP (calculated with a real annual growth rate of approximately 3.5% starting from the initial year)

The "constrained" optimality of each of these conditions and whether both are necessary depends on the stochastic process that Argentina's income is believed to follow.[28] In effect, assume that shocks to income are i.i.d. and normally distributed around a real growth rate of 3.5% (i.e.:

[26] It is important to bear in mind that, as we explained above, even if the real world sovereign contracts do not contemplate the possibility of payments to the government from international credit markets in some states, the government may still be able to achieve net payments from creditors for the portfolio as a whole through the purchase of assets.

[27] In fact, even having non-indexed debt would not be a problem if the government were to have access to fully indexed assets. The important issue is how the portfolio of the government is indexed, regardless if that is achieved through indexed debt or indexed assets.

[28] We are assuming that Argentina cannot offset these thresholds through its holdings of indexed assets.

this assumption implies that shocks are permanent). Then the first condition might be optimal given the non-negative constraint in net payments to the government. The intuition is straightforward. As shocks are i.i.d. around a 3.5% real growth rate, then, regardless of the previous outcome, on expectation the growth rate should be 3.5%. Any growth rate below that would then entail net payments from creditors to the government under the optimal "unconstrained" indexed contract, but given the non-negative constraint, the best that can be done is to make payments equal to 0 in those states, which is precisely what the first condition does.[29] Assume instead that the trend growth rate of income is 3.5% and that shocks are temporary, more of the business cycle type, normally distributed around the trend. Under these assumptions the second condition would be "constrained" optimal as long as it does not entail defaultit would entail zero payments when income is below its expected level in any given period.

However, it is not clear that putting both conditions simultaneously gives you the contraint optimal contract. Not even if the Argentine economy is believed to be subject to more than one type of shock.

5 The Welfare Effects of Real Indexation: Quantitative Results

In this section we present a quantitative exercise on the effects on welfare of introducing indexed debt contracts. Our exercise will compare welfare under two type of situations, one in which the payoffs of the portfolio (or net debt payments) are set to be constant across states, and another in which these payoffs are assumed to be indexed to the state of the economy. In both cases we assume that in all the states the government makes positive net payments to creditors and holds no assets. The indexed debt contract we use is similar to the optimal under these constraints.

Argentina is used as the benchmark economy for the quantitative exercise, since it represents the mayor case of sovereign default in recent history and has issued GDP-indexed sovereign debt. The GDP statistics for Argentina are from Aguiar and Gopinath (2006) and correspond to the seasonally adjusted quarterly real GDP series for 1983.1-2000.2 obtained from the Ministry of Economy and Production (MECON) of Argentina. The series is filtered with the Hodrick-Prescott

[29] In order to be sure that is "constrained" optimal it would be necessary to check also whether it induces renegotiation in some high income state.

filter. Output, debt and private consumption are expressed as a percentage of GDP.

5.1 Calibration

The calibration involves choosing the functional forms and the parameter values. The parameters are chosen based on existing empirical work on emerging markets, if available. Otherwise they are set to mimic empirical regularities of emerging markets.

The per period utility function of households is assumed to have a standard CRRA specification:

$$U(c) = \frac{(c)^{1-\sigma}}{1-\sigma} \tag{15}$$

with a standard value of σ=2. The quarterly risk-free world interest rate is set at 1%. Consistent with the findings of Gelos, Sahay and Sandleris (2003) for the 90s, the reentry probability is set to 0.2, which corresponds approximately to one year of exclusion from international credit markets after a default event. The discount factor β is set at 0.8, as in Aguiar and Gopinath (2006). Yue (2006) considers a value of 0.74. This high degree of impatience has been linked to political factors in emerging economies (see Cuadra and Sapriza (2006) and Hatchondo, Martinez and Sapriza (2006)).

The parameters characterizing the income process in the model are set to approximately match the observed volatility of 4.08% of the cyclical component of GDP in Argentina for the period under study. The output follows an AR(1) process with an autocorrelation coefficient of $\rho = 0.9$ and $\sigma_z = 3.4\%$. The mean of log output equals 0 so that average detrended output in levels is standardized to one. We solve the model numerically using the discrete state-space method. Each period represents a quarter. The process of output is approximated by a discrete first order Markov chain with 25 values using Hussey and Tauchen's (1991) procedure.

Consistent with Chuhan and Sturzenegger (2003), while the country is in autarky after defaulting, it experiences an average quarterly output loss λ of 4%. Arteta and Hale (2006), Sandleris (2005) and Dooley (2000) among others, provide a rationale for the loss of output when countries face debt crises. The parameters for the model are shown in Table 2 in the Appendix.

The indexation rule corresponds to a linear function of output for income levels below the mean, with a range from 80% of the bond face value for income 20% below trend, to 100% for the mean value. For income realizations above the mean, the country pays the face value. The welfare comparisons of the indexed vs. non-indexed bond economies, follow Lucas' methodology of equivalent constant consumption: the equivalent constant stream of consumption is calculated for each state of the value function for each economy and then the average percentage difference of the equivalent consumption is obtained.

5.2 Results

This section describes some key properties of the model and presents the welfare comparison results derived from the indexed vs. non indexed debt economies. A sensitivity analysis is conducted with respect to some key parameter values.

As observed in the data, the incentives to default in the model are higher for highly indebted countries and default risk is countercyclical.

Figure 1 shows the default region for the calibrated economy, i.e., the combinations of productivity shocks and foreign debt levels for which default is the optimal decision. Given an income realization, if default is optimal for a certain level of foreign debt, it will be optimal for all higher levels of debt. Analogously, if repayment is optimal for a given debt level, it will be optimal for all higher levels. Thus, for each realization of s, default incentives are stronger for highly indebted governments. This result arises because the value of paying back and staying in good credit standing is strictly increasing in foreign assets, while the value of defaulting and going to autarky does not depend on the amount of foreign assets. Thus, if default is the optimal decision for some level of assets b given s, then the value of staying in the contract is lower than the value of defaulting. A higher amount of foreign assets increases the value of the contract without affecting the value of default. Thus, for each value of s there exists a threshold $b^* (s)$ for which the value of not default is equal to the value of default. For all levels of assets higher than $b^* (s)$ the government optimally honors its debt.

Figure 2 plots the discount bond price schedule as a function of assets for the highest, middle and lowest values of the productivity shock. As the figure shows, for a given income realization, the bond price is a decreasing function of foreign debt. For small levels of foreign debt, the government always pays back its debt, so it borrows from international markets at the world risk free interest rate. In this debt range, the bond price is simply the inverse of the gross risk free rate. However, as the foreign debt increases, at a certain level the bond price starts to decrease because the incentives to default are stronger for highly indebted governments. At a sufficiently large debt level the government always defaults regardless of the output realization, so the probability of default is one and consequently the bond price is zero.

The bond price schedules also illustrate that for all levels of debt, the bond price is lower (higher interest rate) when the economy is hit by an adverse output realization. This result derives from the presence of incomplete asset markets in the model. This market structure makes defaulting on foreign debt more attractive in bad times when output is low since the repayment of non contingent loans is more costly in recessions. Since productivity shocks are persistent, lending resources to the government in times of low output involves a higher default risk. Thus, risk neutral lenders are willing to lend resources to the government by charging a higher interest rate.

Figures 3 and 4 depict the corresponding default region and bond price schedules when the output loss in autarky is 4%. Since the relative cost of the default option is now higher, the default region shifts to the left reflecting a higher amount of sustainable debt as well as a higher threshold for the risk free debt. Since the bond price reflects the risk premium, the price schedule also shifts to the left.

Similar plots for the indexed benchmark economy are presented in figures 5 and 6. Figure 5 shows that when foreign debt payments are reduced when the economy experiences adverse output realizations, the default region shifts significantly to the left. This implies that for any given income realization, the level of debt required to induce the government to default is much larger. Alternatively, for any given level of current debt, the country would default now only under much harsher conditions. While the entire default region is shifted to the left, the displacement is significantly larger for low values of income, since when those states are realized, the country has to pay a proportionately smaller fraction of the bond face value. In the extreme case when income is close to 5% below trend, the country incentives to default tend to become negligible under this indexation rule. Figure 6 shows the bond price schedules for the highest and lowest income shocks. The figure is consistent with this default region in Figure 5, and shows that for these extreme income realizations the bond prices are the reverse of the non indexed case: intuitively, for any given borrowing, the higher the shock today, the higher it will be next period, and since the incentive to default increases in the shock, the higher the premium required by the foreign lender. This relationship is not monotic, though, as the default region shows. This means that mildly adverse shocks may lead the economy to face lower bond prices compared to extremely positive shocks. This is because in some cases, the discount obtained by the country due to the bond indexation is not sufficient to offset the increase in wealth due to an extremely positive output realization. Thus, the optimal design of the indexation rule is a key determinant of the default incentives of the country.

As shown in Table 3, the welfare gains from indexation for the benchmark economy are around 2% of consumption. That is, our simulated economy would be able to achieve a constant level of consumption 2% higher with an indexed debt contract. The intuition for this result is the fact that the indexed debt contract reduces the default regions and therefore allows for higher levels of debt to be supported in equilibrium. The increase in consumption is reduced to about 0.6% when the output cost decreases to 2%. The intuition is that the gain from avoiding defaults through an indexed debt contract becomes smaller. The sensitivity analysis with respect to the discount rate could respond to a similar logic in terms of changes in the default risk, though as the table illustrates, the welfare changes are slightly more sensitive to variations in the output cost parameter value than to the discount factor in the model economy.

6 Conclusions

Proposals on real indexation of sovereign debt contracts have been around for some time. Furthermore, a number of countries have issued these type of instruments. However, research on the characteristics of the optimal real indexed debt contract in a dynamic equilibrium framework is scant, which makes extremely to formulate policy recommendations regarding the design of these contracts. Furthermore, little has been done in terms of quantifying the welfare effects that they may have. This paper fills this void. We characterize the optimal real indexed debt contract in a dynamic equilibrium framework showing that it has the features of an insurance contract with payments increasing in the state of the economy. We compare these optimal contracts with existing ones. Existing contracts usually display payments increasing in the state of the economy, but also include thresholds levels of the chosen real variable that trigger payments. We argue that this last feature is in general suboptimal as it reduces the amount of insurance that can be achieved.

We try to quantify the welfare effects of real indexed debt contracts by calibrating our model to the Argentina economy. In a very simplified exercise we compare two scenarios one in which the country can only issue non-indexed debt and one in which only indexed debt can be issued (in both cases assuming that the country holds no assets), we show that welfare gains with indexed debt are equivalent to an increase of between 0.6% and 2% in aggregate average consumption. The magnitude of the welfare gain is positively correlated with the direct cost or penalty caused by a default.

Our analysis has relevant policy implications for the design of real indexed debt contracts. .Furthermore, it provides the first model-based quantitative assessment of their welfare effects. However, the latter is done under a number of restrictions such as not allowing for indexed and non-indexed debt to coexist or strictly limiting the set of assets available to the country to build its portfolio. Furthermore, we disregard the issue of moral hazard that has received some attention in the literature on indexed debt. Relaxing these assumptions is part of our ongoing research project on the topic of real indexed sovereign debt as it could help to improve our quantitative assessment and may even bring new insights for the design of real indexed debt contracts.

7 References

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8 Appendix

8.1 Algorithm

1. Assume an initial function for the price of the bond $q_0(b', s)$. To calculate the initial value of the bond, use the inverse of the risk free rate.

2. Use q_0 and the initial values of the value functions (eg. start with 0 matrices) to iterate on the Bellman equations and solve for the value functions and the policy functions for assets and default.

3. Given the default function $d_0(xb, s)$and the default sets, compute the probability of default $\lambda_1(b', s)$ to update the price of the bond using the following equation:

$$q_1 = \frac{(1 - \lambda_1(xb, s))}{1 + r_f} \tag{16}$$

4. Use the updated price of the bond, q_1, to repeat steps 2 and 3 until the following condition is satisfied:

$$\max \left\{ q_i \left(b', s \right) - q_{i-1} \left(b', s \right) \right\} < \epsilon \tag{17}$$

where i represents the number of iterations and ϵ is a small number.

8.2 Tables and Figures

Table 1. Benchmark Parameter Values

Discount Factor	β	0.8
Risk Aversion	σ	2
Re-entry Probability	θ	0.20
Output Loss	ϕ	0.98
Risk Free Interest Rate	r_f	0.01
Autocorrelation coefficient of innovations	ρ	0.90
Standard deviation of innovations	σ_z	0.034

Table 2. Welfare gains from indexation

Sensitivity Analysis

	Discount factor	
	0.95	0.80
Output cost		
2	0.27	0.61
4	0.96	1.94

Note: output cost and welfare figures are in %.