Individual versus systemic risk and the Regulator's Dilemma

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The global financial crisis of 2007–2009 exposed critical weaknesses in the financial system. Many proposals for financial reform address the need for systemic regulation—that is, regulation focused on the soundness of the whole financial system and not just that of individual institutions. In this paper, we study one particular problem faced by a systemic regulator: the tension between the distribution of assets that individual banks would like to hold and the distribution across banks that best supports system stability if greater weight is given to avoiding multiple bank failures. By diversifying its risks, a bank lowers its own probability of failure. However, if many banks diversify their risks in similar ways, then the probability of multiple failures can increase. As more banks fail simultaneously, the economic disruption tends to increase disproportionately. We show that, in model systems, the expected systemic cost of multiple failures can be largely explained by two global parameters of risk exposure and diversity, which can be assessed in terms of the risk exposures of individual actors. This observation hints at the possibility of regulatory intervention to promote systemic stability by incentivizing a more diverse diversification among banks. Such intervention offers the prospect of an additional lever in the armory of regulators, potentially allowing some combination of improved system stability and reduced need for additional capital.

financial stability | global financial markets | financial regulation

The recent financial crises have led to worldwide efforts to analyze and reform banking regulation. Although debate continues as to the causes of the crises, a number of potentially relevant factors have been identified. Financial regulation was unable to keep pace with financial innovation (1, 2), was fragmented in its nature (2), and did not address important conflicts of interest (1, 3–7). More generally, an issue raised by the crises is that of individual vs. systemic risk: regulation was focused on the health of individual firms rather than the stability of the financial system as a whole (1, 2, 4, 8–10). In this paper, we investigate a particular issue that, although not necessarily at the heart of the recent crises, is of great relevance given the newly found interest in systemic regulation. Specifically, we explore the relationship between the risks taken by individual banks and the systemic risk of essentially simultaneous failure of multiple banks.

In this context, we use a deliberately oversimplified toy model to illuminate the tensions between what is best for individual banks and what is best for the system as a whole. Any bank can generally lower its probability of failure by diversifying its risks. However, when many banks diversify in similar ways, they are more likely to fail jointly. This joint failure creates a problem given the tendency for systemic costs of failure to grow disproportionately with the number of banks that fail. The financial system can tolerate isolated failures, but when many banks fail at one time, the economy struggles to absorb the impact, with serious consequences (11–13). Thus, the regulator faces a dilemma: should she allow banks to maximize individual stability, or should she require some specified degree of differentiation for the sake of greater system stability? In banking, as in many other settings, choices that may be optimal for the individual actors may be costly for the system as a whole (14), creating excessive systemic fragility.

Our work complements an existing theoretical literature on externalities (or spillovers) across financial institutions that impact systemic risk (15–32). Much of this literature has focused on exploring liability-side interconnections and how, although these facilitate risk-sharing, they can also create the conditions for contagion and fragility. For instance, some researchers have shown the potential for bankruptcy cascades to take hold, destabilizing the system by creating a contagion of failure (20, 26). When one firm fails, this failure has an adverse impact on those firms to whom it is connected in the network, potentially rendering some of these susceptible to failure. Most obviously affected are those firms to whom the failed institution owes money, but also, the firm's suppliers and even those companies that depend on it for supplies can be put in vulnerable positions. Another insightful strand of research has emphasized the potential for other forms of interdependence to undermine systemic stability, irrespective of financial interconnections: fire sales of assets by distressed institutions can lead to liquidity crises (28). In a very recent approach, the financial crisis is understood as a banking panic in the "sale and repurchase agreement" (repo) market (33). Other recent studies have drawn insights from areas such as ecology, epidemiology, and engineering (34–39).

The present paper builds on the early work by Shaffer (22) and Acharya (23) to explore the systemic costs that attend asset-side herding behavior. Other recent contributions in this direction have considered situations where assets seem uncorrelated in normal times but can suddenly become correlated as a result of margin requirements (refs. 29 and 32 have comprehensive reviews of relevant contributions). In the current work, we use the simplest possible model to investigate other systemic and regulatory implications of asset-side herding, thereby knowingly side-stepping these and many other potential features of real world financial networks. We do not claim that asset-side externalities were at the center of the recent crisis or were more important than other contributory factors. Also, we do not take any position on the extent to which the asset price fluctuations that we consider are because of external economic conditions altering the fair value of certain assets, fire sale effects temporarily depressing the value of assets, price bubbles leading to banks overpaying for assets whose prices subsequently collapse when

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the bubble bursts, general loss of confidence because of uncertainty, global economic imbalances, or other factors. Rather, we study asset-side herding, because it can have very important and not fully explored implications. Possible extensions of our work are discussed in *SI Text*.

We present a framework for understanding the tradeoffs between individual and systemic risk, quantifying the potential costs of herding and benefits of diverse diversification. We then show how systemic risk can be largely captured by two directly observable features of a set of bank allocations: the average distance between the banks in the allocation space and the balance of the allocations (i.e., distance from the average allocation to the individually optimal allocation). We hope that our work may offer insight to policy makers by providing a set of tools for exploring this particular facet of systemic risk.

Model

Consider a highly stylized world, with N banks and M assets. An asset here can be considered as something in which a bank can invest and that can inflict losses or gains proportional to the level of investment. At time $t = 0$, each bank chooses how to allocate its investments across the asset classes. At some later time, $t = 1$, the change in value of each asset is drawn randomly from some distribution. All assets are assumed to be independent and identi-

cally distributed. A bank has then failed if its total losses exceed a given threshold. We recognize that many other factors may cause bank failures, including fire sale effects, interconnections between banks, liquidity issues, and general loss of confidence, but these issues are not the focus of the present work.

For illustrative simplicity, we will take the asset price fluctuations to be drawn from a student t distribution with 1.5 degrees of freedom, a long-tailed distribution often used in financial models (40, 41). The distribution is additionally specified by a probability p that a bank will fail if all its investments are in a single asset class. As we will show, our main findings seem remarkably robust to changes in the detailed assumptions used, including the choice of distribution and the probability p.

We define X_{ij} as the allocation of bank i to asset j. We also define V_j as the loss in value between time $t = 0$ and $t = 1$ of asset j (with negative losses representing profits) drawn from a student t distribution as described above. The total loss incurred by bank i at time $t = 1$ is, thus, $Y_i = \sum_{j=1}^{M} X_{ij} V_j$. Bank i is then said to have failed if $Y_i > \gamma_i$ (that is, if its total losses exceed a given threshold γ_i set by its capital buffer). Additional model details are in *SI Text*.

Results

We now examine the outcomes of this system. Fig. 1A illustrates how the probability of individual bank failure depends on the
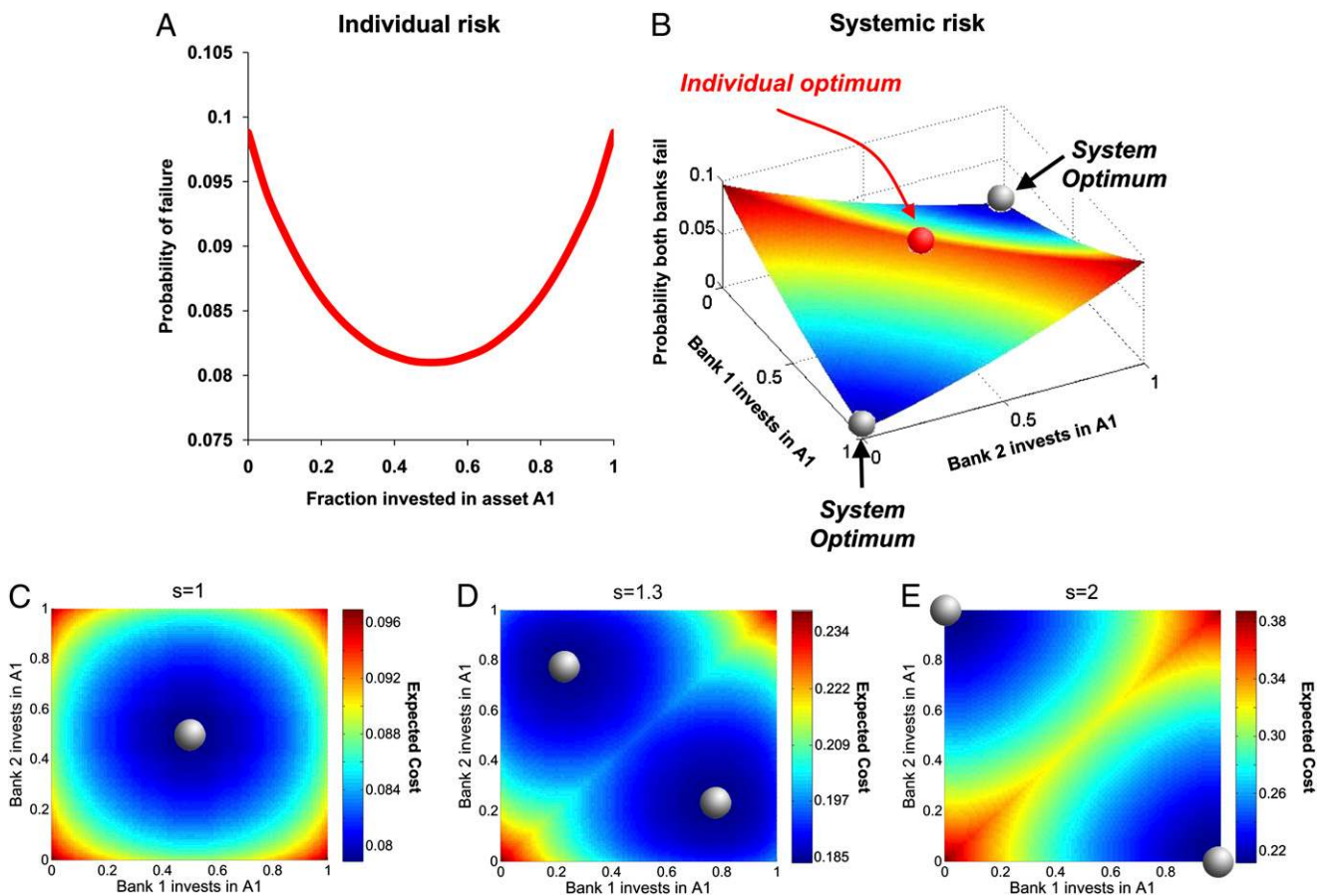

Fig. 1. Probability of bank failure with two banks and two asset classes, A1 and A2. A fundamental tension exists between individual and system risk. Shown are the results of simulations in which the initial value of each asset is one; the loss incurred by each asset after some time t is sampled from a student t distribution with 1.5 degrees of freedom with mean = 0 and a 10% chance of being great than 1; and both banks have capital buffers such that a total loss greater than 1 causes failure. Shown is the average fraction of failures over 10^6 loss samplings. Each bank's individual probability of failure is minimized by investing equally in A1 and A2 (i.e., diversifying uniformly) (A). Uniform diversification, however, does not minimize systemic risk. Instead, the probability of joint failure is minimized by having one bank invest entirely in A1, whereas the other invests entirely in A2 (B). We next consider the cost function $c = k^s$, where k is the number of failed banks, and with s moving from (C) a linear system cost of bank failure ($s = 1$) to (D and E) a system cost that is progressively convex ($s = 1.3$ in D; $s = 2$ in E). The lowest cost configurations are marked by a gray sphere.

allocation between two asset classes when $p = 10\%$. The individually optimal allocation for any given bank, in the sense of minimizing risk for expected return, is to distribute equal amounts into each asset class. We call this individually optimal allocation O^*, and we call the associated probability of individual failure p^*. When all banks are at the individual optimum, we call the configuration uniform diversification, because all banks adopt a common diversification strategy. Uniform diversification, thus, represents a state of the banks maximally herding together in the sense of adopting the same set of exposures. Readers familiar with the standard finance literature will recognize these allocations as those allocations selected under modern portfolio theory (42).

Fig. 1*B* illustrates the probability of total system failure in this system of two banks, p_{SF} (i.e., the probability that both banks fail simultaneously). Unlike individual failure, we find that the probability of joint failure is not minimized by uniform diversification. Instead, a reduction in the probability of joint failure can be achieved by moving the banks away from each other in the space of assets. Indeed, the minimal probability of joint failure is achieved by having each bank invest solely in its own unique asset, which we will call full specialization. Thus, we observe a tension between what is best for an individual bank and what is safest for the system as a whole. The regulator faces a dilemma: should she allow institutions to maximize their individual stability or regulate to safeguard stability of the system as a whole?

To explore this dilemma, we introduce a stylized systemic cost function $c = k^s$, where k is the number of banks that fail and $s \geq 1$ is a parameter describing the degree to which systemic costs escalate nonlinearly as the number of failed banks increases. When many banks fail simultaneously, private markets struggle to absorb the impact. Instead, society incurs real losses, and the economy's long-term potential may be affected (13). Our particular choice of cost function is, of course, an illustrative simplification, but as we show below, our results are robust to considering alternative nonlinear cost functions, and our model is easily extendable to consider any particular cost function of interest.

Fig. 1 *C–E* shows the expected systemic cost of failure C for two banks and two asset classes using various values of s. For a linear cost function ($s = 1$), expected cost is minimized under uniform diversification. In this special case, individual and systemic incentives are aligned. However, when we consider more realistic cases where the cost function is convex (so that the marginal systemic cost of bank failure is increasing), the configuration that minimizes C is no longer uniform diversification but rather, a configuration with diverse diversification. As s increases, an increasingly larger departure from uniform diversification is required to minimize C.

In Fig. 2, we illustrate a more general case of five banks investing in three assets, randomly sampling 10^5 asset allocations. For varying degrees of nonlinearity s, we show the configuration with the lowest expected cost C. When the cost function is linear, the lowest cost configuration is again uniform diversification O^*, where each bank allocates one-third of its investments to each asset. As we increase s, we find that pushing the banks away from uniform diversification to diverse diversification reduces C.

To further explore the relationship between the positioning of banks in asset space and the expected systemic cost, we define D as the average distance between the asset allocations of each pair of banks, scaled so that the distance between banks exposed to nonoverlapping sets of assets is one. We also define a second parameter G to describe how unbalanced the allocations are on average, which is defined as the distance between the average allocation across banks and the individually optimum allocation O^*. *SI Text* has more detailed specifications of D and G. Note that, if all banks adopt the individually optimum allocation, both D and G are zero. Thus, in this case, all banks either survive or fail together, and the system behaves as if there were only a single representative bank. This finding is true regardless of assumptions about how the asset values fluctuate, but of course, it may not extend to more complex models with features such as stochastic heterogeneity across banks.
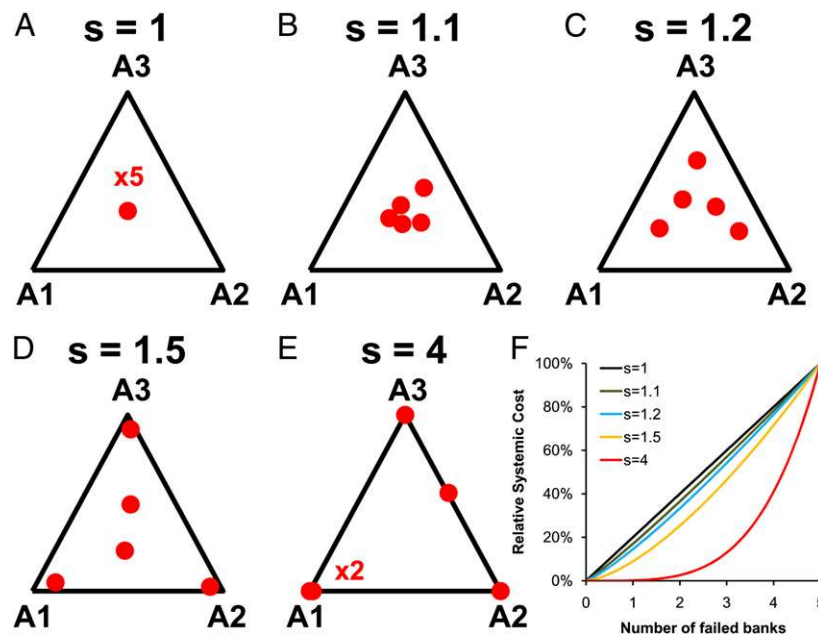

Fig. 2. Lowest expected cost configurations for different levels of cost function nonlinearity s. (*A–E*) We consider five banks investing in three assets, with losses drawn from a student t distribution with 1.5 degrees of freedom having a mean = 0 and a 10% chance of being great than the banks' failure threshold of 1. Shown is the lowest expected cost allocation of 10^5 randomly selected allocations over 10^6 loss samplings. As s increases, the lowest expected cost configuration moves farther from uniform diversification. The cost function for various values of s is shown in *F*.

In Fig. 3, we show expected cost C as a function of D and G across 10^5 random allocations of five banks on three assets. As we have already seen in Fig. 2, in the special case of $s = 1$, expected cost is minimized by uniform diversification at $D = G = 0$; thus, expected cost is increasing in both distance D and imbalance G. At larger values of s, expected cost remains consistently increasing in imbalance G, but the relationship between cost and distance D changes. At $s = 1.2$, cost is large for distances that are either too small or too large. The relationship between distance and cost is clearly nonlinear, and cost is lowest at an intermediate value of D. As s increases to $s = 4$, cost is now lowest when distance is large, and thus, cost is decreasing in D. Providing additional evidence for the ability of D and G to characterize systemic cost, regression analysis finds that D, D^2, and G together explain over 90% of the variation in $\log(C)$.

All of this information suggests that it may be possible in principle, and it could provide a useful guide in practice, to regulate expected systemic cost. For a given level of capital, regulators might set a lower bound on distance D and an upper bound on imbalance G. As shown in Fig. 4, fixing $G = 0$ and requiring D to exceed some value of D_{Min} results in a substantial reduction in the capital buffer needed to ensure that the worst-case expected cost remains below a given level. We particularly consider the worst-case expected cost to take into account potential strategic behavior on the part of the banks. This most pessimistic case shows that, even if the banks are colluding to purposely maximize the probability of systemic failure, regulating D and G creates substantial benefit for the system. Fig. 4 also illustrates the robustness of our results to model details. We observe similar results when varying model parameter values, including the number of banks and assets (Fig. 4A), the nonlinearity of the cost function (provided that s is not too low) (Fig. 4B), and the value of p (Fig. 4C). We also observe similar results when varying the distribution of the asset prices (provided that the tails of the distribution are heavy enough) (Fig. 4D) and when considering assets with a substantial degree of correlation (Fig. 4E and *SI Text*). Furthermore, Fig. 4F shows that our results continue to hold when considering alternate cost functions in which (*i*) the system can absorb the first i bank failures without incurring any cost, with systematic cost then increasing linearly for subsequent failure ($i = 2$ in our simulations), and (*ii*) each of the first i failures causes a systemic cost C_1, whereas each

additional failure above i causes a larger systemic cost C_2 ($i = 2$, $C_1 = 5$, and $C_2 = 30$ in our simulations; *SI Text* has discussion of the various cost functions). This robustness is extremely important, because many of these features are difficult to determine precisely in reality. Because our results do not depend on the details of these assumptions, the importance of diverse diversification may extend beyond the simple model that we consider here.

Regulatory changes under discussion are estimated to require banks to increase their Core Tier One capital substantially in the major developed economies (43). In this context, the potential ability of diverse diversification to reduce capital buffers is of great economic significance. Estimates suggest that, for each 1% reduction that does not compromise system stability, sums in excess of \$10 billion would be released for other productive purposes, with the economic benefits likely to be substantial (43, 44).

Discussion

There is a growing appreciation that prudent financial regulation must consider not only how a bank's activities affect its individual chances of failure but also how these individual-level choices impact the system at large. The analysis presented in this paper highlights a particular aspect of the problem that a systemic regulator will face: when the marginal social cost of bank failures is increasing in the numbers of banks that fail, systemic risk may be reduced by diverse diversification. This nonlinearity of the systemic cost is a natural assumption. The societal costs of dealing with bank failures grow disproportionately with the numbers that fail. Hence, the regulator may wish to give banks incentives to adopt differentiated strategies of diversification.

These results also have implications beyond the financial system. For example, the tension between individually optimal herding and systemically optimal diversification is a powerful theme in ecological systems (45, 46). Natural selection pressures organisms in a given species to adapt (in the same way) to their shared environment. However, maintenance of diversity is essential for protecting the species as a whole from extinction in the face of fluctuating environments and emergent threats such as new parasite species. Herding is also an issue for human societies in domains other than banking. In the context of innovation, for example, people often herd around popular ideas
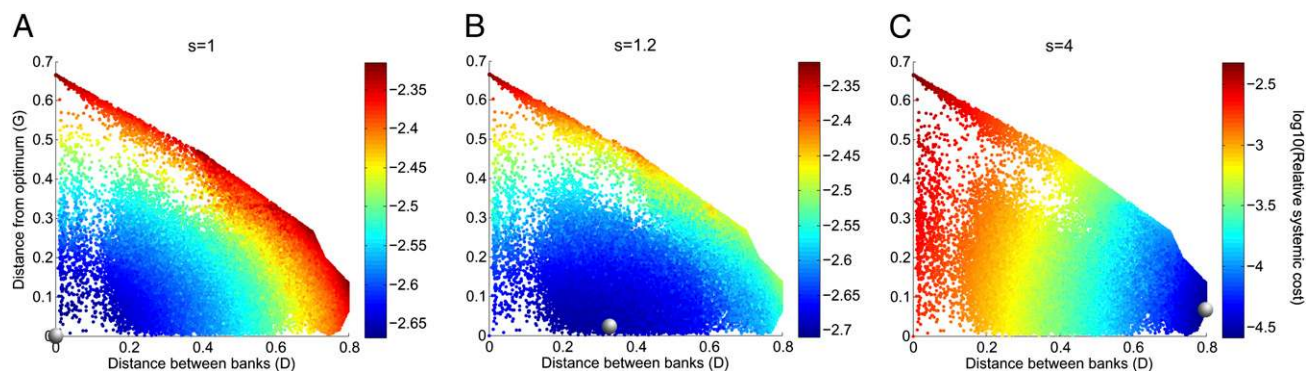

Fig. 3. The systemic risk presented by a given set of allocations is largely characterized by two distinct factors: (*i*) the distance between the banks' allocations D and (*ii*) the imbalance of the average allocation G, defined as the distance between the average allocation and the individually optimal allocation. Shown is the expected cost C associated with 10^5 randomly chosen allocations as described in Fig. 2. When the cost function is linear ($s = 1$), the configuration that minimizes system cost has the banks herding in selecting the portfolio that minimizes individual risk of failure, $\left(\text{that is, } \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ (*A*). As the cost function becomes more nonlinear ($s = 1.2$), the cost-minimizing distance between the banks becomes larger. Here, the configurations that minimize system cost are associated with having banks at an intermediate distance from each other, while still having low imbalance G (*B*). With stronger nonlinearity ($s = 4$), the cost-minimizing configuration puts banks as far apart from each other as possible in asset space—large D (although still keeping the average location as close as possible to the individual optimum, i.e., small G) (*C*). Regressing $\log(C)$ against D, D^2, and G explains 97% of the variation in cost at $s = 1$, 90% of the variation in cost at $s = 1.2$, and 99% of the variation in cost at $s = 4$.
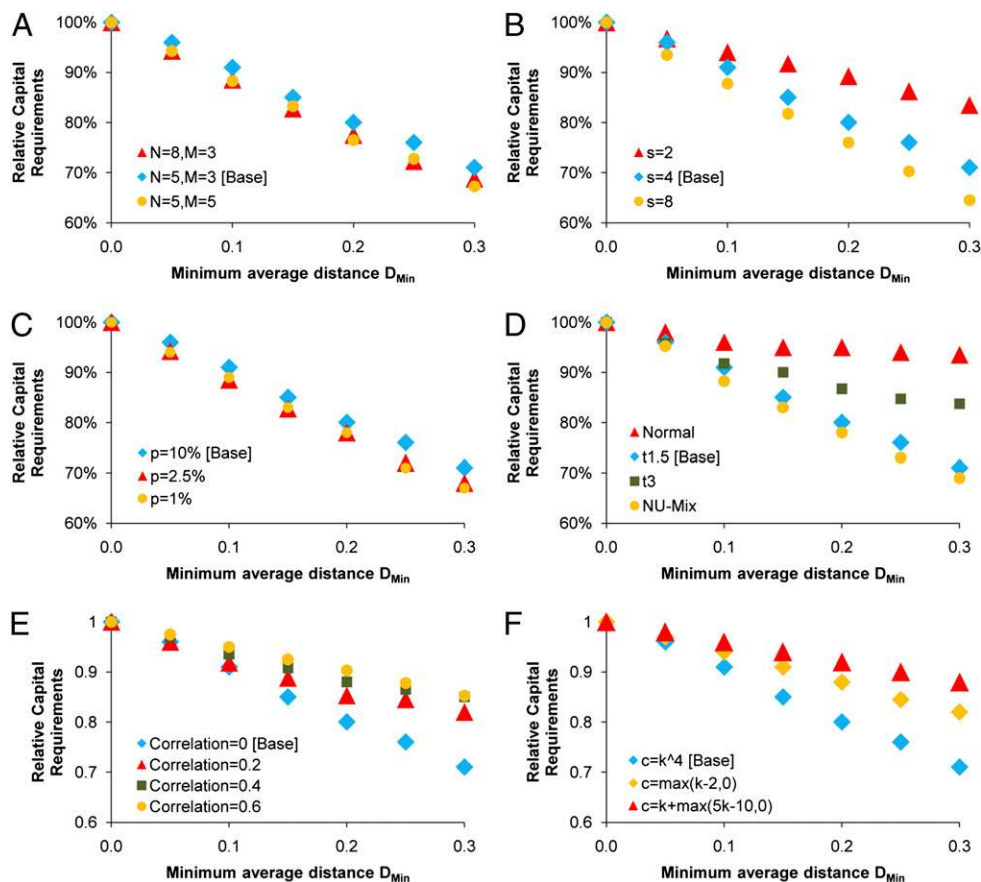
Fig. 4. Imposing a minimum on the distance D can appreciably reduce the capital needed to ensure a given maximum expected cost in our model. For a given set of parameters M, N, and p and a given asset price distribution, we calculate the expected systemic cost C when all banks act to minimize their individual risk (uniform diversification with $G = 0$ and $D = 0$). We then impose a minimum average distance D_{Min}, while keeping $G = 0$. Forcing the banks apart from each other lowers the expected cost for a given level of capital. Thus, for each value of D_{Min}, we find the level of capital for which the worst case (highest expected cost) of 10^6 random allocations still gives an equivalent expected cost (within 2%) to that incurred under uniform diversification. Simulation results were also verified using nonlinear optimization. Shown in blue is the result for a base case of five banks, three asset classes, $s = 4$, and the asset prices each generated independently from a student t distribution with 1.5 degrees of freedom having $p = 10\%$ probability of failure for a bank invested only in one asset. We see that, as D_{Min} increases, banks need to hold less capital in reserve to ensure the same level of system stability. We then show that this result is qualitatively robust to varying the model parameters M and N (A), the nonlinearity of the cost function s (B), the type of distribution (student t with 1.5 degrees of freedom, student t with 3 degrees of freedom, normal distribution, or a mix with the loss having a 5% probability of being from a uniform distribution in the range 0–10 and a 95% probability of being from a normal distribution; D), the degree of correlation between the asset price fluctuations (E), and the choice of cost function, where k is the number of failed banks (F). The alternative cost functions are discussed in greater detail in *SI Text*. In all of the above cases, the loss distributions on a single asset have a mean = 0 and a $p = 10\%$ chance of being greater than the failure threshold of 1. Our results are also robust to changing this failure probability p (C).

and fads, creating systemic costs by making it difficult for new ideas to be appreciated (47).

In our model, the expected systemic cost of bank failures is largely explained by two global parameters of risk exposure and diversity. Both these parameters can be derived by the regulator without the need for complicated calculations of systemic risk, and they can be decomposed into their contributions from individual actors. We also show that a given level of expected systemic cost can be achieved with a more efficient use of capital if the regulator is able to encourage a suitable level of diversity between banks in the system. Thus, this framework presents a potentially useful tool for systemic regulation; our analysis points to the possibility of regulation that combines knowledge of system aggregates and individual bank positions to identify and induce the desired degree of diverse diversification. The practical design of this aspect of regulatory strategy can only emerge from a fuller program of research.

In the meantime, it is our hope that the insights developed in this paper can weigh on the deliberations that are gathering pace surrounding the reform of financial regulation. Active discussion

is under way regarding the design of capital surcharges based on an individual bank's contribution to systemic risk (4, 10, 48). Meanwhile, it is increasingly recognized that financial reporting must improve significantly to support the function of the systemic regulator, and discussion has turned to the practical details of data gathering and analysis (1, 4, 8–10). The basic notion that common diversification strategies can increase systemic risk is not entirely absent from current policy thinking (7), and it predates the recent crisis (49); however, it has received relatively little attention in the literature. A priority for future research is to convert theoretical insights into practical approaches for regulators.

1. French KR, et al. (2010) *The Squam Lake Report: Fixing the Financial System* (Princeton University Press, Princeton).
2. Hillman R (2009) *Financial Regulation: Recent Crisis Reaffirms the Need to Overhaul the U.S. Regulatory System* (United States Government Accountability Office, Washington, DC), Testimony Before the Committee on Banking, Housing, and Urban Affairs GAO-09-1049T.
3. Makin JH (2008) *Risk and Systemic Risk* (American Enterprise Institute for Public Policy Research, Washington, DC).
4. Bank of England (2008) *The Role of Macroprudential Policy*. Bank of England Discussion Paper (Bank of England, London, UK).
5. Bolton P, Freixas X, Shapiro J (2009) *The Credit Ratings Game*. National Bureau of Economic Research Working Paper 14712 (National Bureau of Economic Research, Cambridge, MA).
6. Thanassoulis J (2009) This is the right time to regulate bankers' pay. *Economist's Voice* Vol 6, Issue 5, Article 2.
7. Haldane A (2009) Banking on the state. *BIS Rev* 139:1–20.
8. Lo A (2009) *The Feasibility of Systemic Risk Measurements*. Written Testimony for the House Financial Services Committee on Systemic Risk Regulation. Available at http://ssrn.com/abstract=1497682.
9. Sibert A (2010) A systemic risk warning system. *Vox*, http://voxeu.org/index.php?q=node/4495.
10. International Monetary Fund (2010) *Global Financial Stability Report: Meeting New Challenges to Stability and Building a Safer System.* (International Monetary Fund, Washington, DC).
11. Brunnermeier M (2009) Deciphering the liquidity and credit crunch 2007–2008. *J Econ Perspect* 23:77–100.
12. Reinhart CM, Rogoff KS (2009) The aftermath of financial crises. *Am Econ Rev* 99:466–472.
13. Acharya VV, Richardson M (2009) *Restoring Financial Stability: How to Repair a Failed System* (New York University Stern School of Business, New York).
14. Nowak MA (2006) Five rules for the evolution of cooperation. *Science* 314:1560–1563.
15. Bhattacharya S, Gale D (1987) Preference shocks, liquidity and central bank policy. *New Approaches to Monetary Economics*, eds Barnett WA, Singleton KJ (Cambridge University Press, Cambridge, UK).
16. Rochet J-C, Tirole J (1996) Interbank lending and systemic risk. *J Money Credit Bank* 28:733–762.
17. Kiyotaki N, Moore J (1997) Credit cycles. *J Polit Econ* 105:211–248.
18. Freixas X, Parigi B (1998) Contagion and efficiency in gross and net interbank payment systems. *J Financ Intermed* 7:3–31.
19. Freixas X, Parigi B, Rochet J-C (1999) *Systemic Risk, Interbank Relations and Liquidity Provision by the Central Bank*. Working Paper of the Universitat Pompeu Fabra (Universitat Pompeu Fabra, Barcelona).
20. Allen F, Gale D (2000) Financial contagion. *J Polit Econ* 108:1–33.
21. Gai P, Kapadia S (2010) Contagion in financial networks. *Proc R Soc Lond A Math Phys Sci* 466:2401–2423.
22. Shaffer S (1994) Pooling intensifies joint failure risk. *Res Financ Serv* 6:249–280.
23. Acharya VV (2009) A theory of systemic risk and design of prudential bank regulation. *J Financ Stab* 5:224–255.
24. Stiglitz JE (2010) Risk and global economic architecture: Why full financial integration may be undesirable. *Am Econ Rev* 100:388–392.
25. Battiston S, Delli Gatti D, Gallegati M (2009) *Liaisons Dangereuses: Increasing Connectivity, Risk Sharing, and Systemic Risk*. National Bureau of Economic Research Working Paper (National Bureau of Economic Research, Cambridge, MA).
26. Castiglionesi F, Navarro N (2007) *Optimal Fragile Financial Networks*. Discussion Paper of the Tilburg University Center for Economic Research (Tilburg University, Tilburg, The Netherlands).
27. Lorenz J, Battiston S (2008) Systemic risk in a network fragility model analyzed with probability density evolution of persistent random walks. *Networks Heterogeneous Media* 3:185–200.
28. Diamond DW, Rajan R (2007) *Fear of Fire Sales and Credit Freeze*. National Bureau of Economic Research Working Paper, 86:479–512 (National Bureau of Economic Research, Cambridge, MA).
29. Shin HY (2010) *Risk and Liquidity, Clarendon Lectures in Finance.* (Oxford University Press, Oxford).
30. Allen F, Babus A, Carletti E (2010) *Financial Connections and Systemic Risk*. National Bureau of Economic Research Working Paper (National Bureau of Economic Research, Cambridge, MA).
31. Greenwald B, Stiglitz JE (1986) Externalities in economies with imperfect information and incomplete markets. *Q J Econ* 101:229–264.
32. Allen F, Gale D (2007) *Understanding Financial Crises, Clarendon Lecture Series in Finance* (Oxford University Press, Oxford).
33. Gorton G, Metrick A (2010) *Haircuts*. National Bureau of Economic Research Working Paper (National Bureau of Economic Research, Cambridge, MA).
34. May RM (2006) Network structure and the biology of populations. *Trends Ecol Evol* 21:394–399.
35. May RM, Levin SA, Sugihara G (2008) Complex systems: Ecology for bankers. *Nature* 451:893–895.
36. May RM, Arinaminpathy N (2010) Systemic risk: The dynamics of model banking systems. *J R Soc Interface* 7:823–838.
37. Haldane A (2009) *Rethinking the Financial Network*. Speech delivered at the Financial Student Association, Amsterdam, April 2009 (Bank of England, London, UK).
38. Haldane AG, May RM (2011) Systemic risk in banking ecosystems. *Nature* 469:351–355.
39. Sheng A (2010) Financial crisis and global governance: A network analysis. *Globalization and Growth Implications for a Post-Crisis World*, eds Spence M, Leipziger D (World Bank, Washington, DC), pp 69–93.
40. Owen J, Rabinovitch R (1983) On the class of elliptical distributions and their applications to the theory of portfolio choice. *J Finance* 38:745–752.
41. Blattberg RC, Gonedes NJ (1974) A comparison of the stable and student distributions as statistical models of stock prices. *J Bus* 47:244–280.
42. Elton EJ, Gruber MJ (1995) *Modern Portfolio Theory and Investment Analysis* (Wiley, New York).
43. Institute of International Finance (2010) *Interim Report on the Cumulative Impact on the Global Economy of Proposed Changes in the Banking Regulatory Framework, June 2010.* (Institute of International Finance, Washington, DC).
44. Basel Committee on Banking Supervision (2010) *An Assessment of the Long-Term Economic Impact of Stronger Capital and Liquidity Requirements, August 2010* (Basel Committee on Banking Supervision, Basel, Switzerland).
45. Levin S (1998) Ecosystems and the biosphere as complex adaptive systems. *Ecosystems* 5:431–436.
46. Elmqvist T, et al. (2003) Response diversity, ecosystem change, and resilience. *Front Ecol Environ* 9:488–494.
47. Pech R (2001) Reflections: Termites, group behaviour, and the loss of innovation: Conformity rules! *J Manag Psychol* 7:559–574.
48. Adrian T, Brunnermeier M (2009) *CoVar*. Federal Reserve Bank of New York Staff Report 348 (Federal Reserve Bank of New York, New York).
49. Gieve J (2006) *Financial System Risks in the UK: Issues and Challenges*. Centre for the Study of Financial Innovation Roundtable, Tuesday 25 July 2006 (Bank of England, London, UK).

Supporting Information

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SI Text

Details of the Simplified Model. At time $t = 0$, each of the N banks chooses how to allocate its investments across the M asset classes. Let X_{ij} denote the exposure of the ith bank to the jth asset, noting that, in practice, these assets might represent whole asset classes or indeed, levels of exposure to particular risk factors. Our interest is in how the structure of the $(N \times M)$ exposure matrix $X = [X_{ij}]$ affects system stability. After period $t = 1$, the total loss faced by the ith bank is (Eq. **S1**)

$$Y_i = \sum_{m=1}^{M} X_{ij} V_j, \qquad \textbf{[S1]}$$

where $(V_j)_{j=1}^{M}$ are the losses that the M asset classes (with negative losses representing profits); they are assumed, in our simplified model, to be drawn independently from the same arbitrary mean zero distribution. In our base case, each of these losses is distributed as $\left(\dfrac{1}{a}\right)T$, where T is a random variable from a student t distribution with 1.5 degrees of freedom and a is the critical value of the t distribution corresponding to the probability p that a bank will fail if all its investments are in a single asset. In the base case, we take $p = 10\%$ and use the empirical value of a, which was obtained from 1 million independent random draws from a t distribution with 1.5 degrees of freedom.

With each bank endowed with—for simplicity—the same capital buffer $\gamma > 0$, we define a bank failure according to (Eq. **S2**)

$$f_i = \begin{cases} 1 \text{ if } Y_i > \gamma, \\ 0 \text{ otherwise} \end{cases}, \qquad \textbf{[S2]}$$

and therefore, observing a realization of f_i as $f_i = 1$ corresponds to failure of the ith bank.

Our main discussion is concerned with the base case in which the cost function is defined as $C_s = k^s$, where k is the number of bank failures. Letting q_k denote the probability that exactly k banks fail [i.e., $q_k := \Pr(\sum_{i=1}^{N} f_i = k)$], the expected cost is given by (Eq. **S3**)

$$C := \mathbb{E}[c(s)] = \sum_{k=1}^{N} q_k k^s, \qquad \textbf{[S3]}$$

and we note that this equation reduces to $\mathbb{E}\left[\sum_{i=1}^{N} f_i\right]$ (i.e., the expected number of failures) when $s = 1$.

We study the relationship between the expected cost and two variables obtained from the exposure matrix X, and these relationships are defined as (Eq. **S4**)

$$D = \frac{1}{2N(N-1)} \sum_i \sum_l \sum_j |X_{ij} - X_{lj}| \qquad \textbf{[S4]}$$

and (Eq. **S5**)

$$G = \frac{1}{N} \sum_{j=1}^{M} \left| \sum_{i=1}^{N} (X_{ij} - 1/M) \right|. \qquad \textbf{[S5]}$$

D is the average of the l_1 distances between each pair of banks' asset allocations, and G is the l_1 distance of the center of gravity, O^*, of the banks' allocations from the N vector of $1/M$ asset holdings, with the latter being the optimal center of gravity around

which the banks should diversify from one another. Note that the center of gravity and this N vector of $1/M$ asset holdings coincide in the case where each bank diversifies its investments in an individually optimal fashion using standard portfolio theory.

Economic Rationale for a Superlinear Cost Function. There are some quite compelling reasons why the cost to the government and society may be superlinear. Relevant factors include:

- Many countries have funds set aside for the protection of depositors from bank failures (e.g., Financial Services Compensation Scheme), but these funds have finite resources. Costs over and above these funds then become the responsibility of the governments. We might then think of the systemic costs in terms of an elbow function, where costs up to a certain level rise slowly, and above this level, they rise sharply (Fig S1).
  Formally, the elbow function is defined as (Eq. **S6**)

$$\text{elbow}[a,b](x) = \frac{ax}{b} + \max\left\{ 0, \frac{(1-a/b)(x-b)}{(1-b)} \right\} \text{ if } b \geq a. \quad \textbf{[S6]}$$

- The effect on the real economy of a single bank failure can generally be contained. In practice, the preferred option of the authorities is usually to encourage the purchase of the failed or weakened bank by a stronger competitor (e.g., Bear Sterns, Merrill Lynch, Wachovia, and HBOS), which is often assisted by government guarantees. However, multiple failures will tend to lead to serious reduction in competition or disruption in financial services markets that are important to the real economy.
- After the market considers the risk of multiple bank failures to be serious, then the cost of interbank credit increases substantially, and the availability decreases. During the 2008 and 2009 crisis, there were days when even very large banks had great difficulty in borrowing to raise liquidity, and they became enormously reliant on central bank funding. The price of lending to banks sets a floor on the prices that banks can charge on their loans to the real economy, and therefore, if the banks' cost of funds increases, a very substantial frictional cost is imposed on the real economy. Similarly, the marginal (non)availability of funds to the banks constrains the marginal (non)availability of funds to the real economy.
- Banks perform socially and economically useful functions in the real economy. These functions will be reduced by bank failures in three stages.

    i) If a bank fails, then firms and individuals that use that bank for a service will generally suffer an interruption of service. Even if deposits are in practice, guaranteed lines of credit will not be in practice. It may be almost as disruptive for a firm to lose a $1 million line of credit as for the firm to lose $1 million of cash. The impact will be mitigated if the firm or individual has other banks that it can also use.

    ii) A bank failure also reduces the competition for providing a given service and thus, will tend to push up the price. This occurrence becomes particularly serious when all banks but one that provide a service fail. Banks will also have capacity constraints, and therefore, when more than one competitor exits a market segment, it

may be genuinely difficult for banks to serve the disappointed customers at an economically attractive price (for the customers).

  iii) If all of the banks providing a service fail, then clearly, there will be considerable disruption to the real economy.

One simplified way of thinking about this problem is to assume that each economically important function (EIF; which may be financing a firm or a group of individuals) is provided by r of the N banks, and these functions are evenly distributed in the sense that each set of r of the N banks provides the same number of EIFs (for example, if $n = 4$ and $r = 2$, each of the six possible two-element subsets might provide, say, 5 of a total of 30 EIFs). In which case, if k banks fail, the proportion of EIFs that will no longer be provided at all (because all of the banks providing them had failed) would be ${}^{k}C_r/{}^{N}C_r$. We could use this ratio as the cost function, making the stylized simplifying assumption that the only serious systemic costs are when an entire EIF is not provided. If r/k is small, which is possible when r/N is small, then this cost function is very similar to k^s with $s = r$. In fact, as a numerical coincidence, when $n = 5$ and $r = 3$, the ${}^{k}C_r$ cost function is very similar to k^s with $s = 4$. Fig. S2 illustrates these similarities.

To see why this case is true, note that that ${}^{k}C_r/{}^{N}C_r$ is proportional to $k!/(k-r)!$, and also note that (Eq. S7)

$$\frac{(k!/(k-r)!)}{(k-1)!/(k-r-1)!} = (1-r/k)^{-1} = 1 + r/k - O\left((r/k)^2\right) \quad \textbf{[S7]}$$

and (Eq. S8)

$$k^s/(k-1)^s = (1-1/k)^{-s} = 1 + s/k - O\left((s/k)^2\right), \quad \textbf{[S8]}$$

which shows how the observed similarity between $c = k^s$ and ${}^{k}C_r$ arises with $s = r$.

The above analysis suggests an approach to actually estimating the cost function by considering five points.

  i) What are the EIFs that are provided by the banks at a suitable level of granularity?
  ii) What would be the economic cost of each EIF not being provided?
  iii) How much of that economic cost would be incurred if various percentages of capacity in this EIF were taken out because of bank failure? Because of the considerations in *iv* above, this finding is likely to be superlinear in the capacity that was removed.
  iv) Whether there are any important interaction effects so that the cumulative economic cost is even greater than the sum of the economic costs associated with partial or total loss of each EIF.
  v) Hence, the total economic cost of failure of any given subset of the banks.

Additional research is certainly needed in this area, but we believe that considerations such as the ones listed above provide a sufficiently compelling rationale for considering a superlinear cost function such as the one in our paper.

Extensions to Base Case. *Correlated assets.* In Fig. 4*E*, we considered the effect of introducing correlation between the asset price fluctuations. More specifically, we drew 10^6 random numbers from an M variate student t distribution with 1.5 degrees of freedom and correlation coefficients all equal to ρ, where $\rho = 0.2, 0.4, 0.6$. These numbers were obtained using Matlab's built-in function for generating numbers from a multivariate student t distribution.

Cost functions. Despite the natural economic arguments that justify the use of $c = k^s \ s > 1$ (*SI Text, Economic Rationale for a Superlinear Cost Function*), it is important to convince ourselves that the results remain qualitatively robust to the choice of cost function. For this reason, we also consider the flexible functional form introduced in *SI Text, Economic Rationale for a Superlinear Cost Function* (Eq. S9):

$$c_{a,b,N}(k) := c_{a,b}(k/N) = \max\left\{0, \frac{(1-a/b)(k/N-b)}{(1-b)}\right\}, \quad \textbf{[S9]}$$

where we restrict b to be in the set of positive integers $\leq N$. The above cost function is zero when $k/n = 0$, one when $k/n = 1$, and rising linearly to $c_{a,b} = 1 - a/b$ when $k/n = b$.

Wider extension to risk factors. Although the present model is formulated in terms of fluctuations in asset prices, the essential features can be used in any situation where these parameters occur.

  i) A number of distinct risk factors that influence the failure of an entity.
  ii) The exposure of each entity to these risk factors can vary.
  iii) The contribution of these risk factors can be thought of as additive (or by taking logs, multiplicative).
  iv) An entity has failed if the sum of the contributions from each risk factor exceeds a certain threshold.

For example, in many cases, the value of a firm's liabilities as well as the value of its assets vary. Even banks, which are traditionally thought of as having deposits whose value is fixed, generally issue bonds or other securities whose values vary in certain circumstances. This variability could be modeled by considering the variation of such values as contributing to the losses or profits of the bank in the natural way, although the constraint that the sum of the exposures must add to one would be relaxed appropriately.

Other extensions will readily occur to the readers (and the authors) but are beyond the scope of this present paper.

Robustness of the Dependence Between *C*, *D*, and *G*. It is reassuring to note that the dependence observed in Fig. 2 Seems to not just be a feature of low-dimensional cases (i.e., small N and M). To investigate this issue we observed the coefficient of determination (R^2) between the expected systemic cost C and the best-fit ordinary lest squares projection onto D, D^2, and G for larger values of M and N. The R^2 seems broadly constant as N increases from 3 to 12 and M increases from 3 to 5 (Fig. S3). This finding is reassuringly suggestive that the observed correlation is not simply an artifact of low dimensionality.

Fig. S1. Several superlinear cost functions.

Fig. S2. Comparison of $c = k^s$ and ^kC_r.

Fig. S3. Robustness of the coefficient of determination to increasing dimensionality in an ordinary lest squares regression of C on D, D^2, and G.