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Individuality, distinguishability, and (non-)entanglement: A defense of Leibniz's principle

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ABSTRACT

The paper provides a philosophical interpretation of Ghirardi, Marinatto, and Weber's physical criterion of (non-)entanglement in terms of individuality and distinguishability. It firstly clarifies the relation between ontology and labeling, and then defends the non-standard view that non-similar particles and similar fermions are individuated by a traditional version of Leibniz's principle of the identity of indiscernibles. It will be argued that Leibniz's principle is satisfied explicitly in non-entangled states, whereas in entangled states it can be defended via the summing defense.

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1. Introduction

In a series of physics papers Ghirardi, Marinatto, and Weber (GMW)[1] have argued that there always are (at least) *two different* types of states accessible for many particles in quantum mechanics.[2] Particles with different state-independent properties such as mass and charge (non-similar particles) are in

1. "Non-entangled" states if and only if the state vector is factorizable, i.e., it is expressible as a (tensor) product vector, such as

$$|non\rangle = |\downarrow_z\rangle_1 |\uparrow_z\rangle_2 \quad (1)$$

2. "Entangled" states otherwise, i.e., if and only if the state vector cannot be factorized, such as

$$|ent\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2] \quad (2)$$

Particles having all the state-independent properties in common (similar particles) are in

1. "Non-entangled", (anti)symmetric states if and only if the state vector could be obtained by (anti)symmetrizing a product state, such as[3]

$$|non\rangle = \frac{1}{\sqrt{2}}[|R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 - |L\rangle_1|\uparrow_z\rangle_1|R\rangle_2|\downarrow_z\rangle_2] \quad (3)$$

2. "Entangled", (anti)symmetric states if and only if they could not be obtained by (anti)symmetrizing a product state, such as

$$|ent\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2] \otimes [|R\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2] \quad (4)$$

The aim of my paper is to provide a philosophical interpretation of this physical result, i.e., to explain the differences of all four types of states in terms of individuality and (in)distinguishability.[4]

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[1] See Ghirardi, Marinatto, & Weber (2002) and Ghirardi & Marinatto (2003, 2005).

[2] For simplicity, I will focus on the two-particle case. Quantum mechanics is taken to be non-relativistic, and the completeness assumption is made.

[3] $|R\rangle$ and $|L\rangle$ are two orthogonal spatial locations. According to GMW, there is a slight difference between fermions and bosons, besides the symmetry sign, which doesn't matter for my purposes.

[4] Up to now, the philosophical debate on these issues only has considered the singlet state of pure spin space which threatens to confound antisymmetry with

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Firstly, I will sketch why such a philosophical interpretation is needed. In general, I would say that a philosophical interpretation is needed when a physical result is conceptually unclear or even inconsistent. In the case at hand, however, one might think that it *is* sufficiently clear: the notion of individuality is used in an ontologically rather neutral sense, namely in the way that a particle has an individuality or is an individual if and only if it is numerically distinct from other things. So, to consider a particle as an individual leaves open whether and how it is individuated: its individuality might be grounded – for instance, it could be numerically distinct from other things in virtue of distinguishing qualitative facts – or ungrounded, i.e., taken to be primitive. Since GMW assume that there are numerically distinct particles in all four types of states, there always are individuals in this sense. The notion of (in)distinguishability is used in the way that numerically distinct particles are (in)distinguishable if and only if they (share or) don't share all the state-independent properties. Thus, there always are distinguishable individuals in states of types (1) and (2), and there always are indistinguishable individuals in states of types (3) and (4). The distinction between non-entangled and entangled states seemingly doesn't matter for the individuality and (in)distinguishability of quantum particles.

However, then, GMW state that similar particles, such as two electrons, are "truly indistinguishable" so that one cannot pretend that "a particular one of them" has properties, and that the set of observables has to be restricted to the symmetric ones, such as $\hat{S}_z = \hat{s}_z \otimes \hat{1} + \hat{1} \otimes \hat{s}_z$ (see Ghirardi & Marinatto, 2003, p. 383). Indistinguishability has, hence, something to do with the requirement of permutation invariance which concerns the state-*dependent* properties. It seems that similar particles would be distinguishable in some sense if non-symmetric product states, such as $|R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2$, were allowed for them. Correspondingly, it seems that non-similar particles are indistinguishable in some sense when they are in (anti)symmetric states, in a state such as $|\uparrow_z\rangle_1|\uparrow_z\rangle_2$ or in some state of type (2).[5]

This ambiguity becomes crucial when one takes into account a *complete* set of state-dependent properties also for non-similar particles (which is required, since GMW define "non-entanglement" to be the case in which numerically distinct subsystems possess complete sets of properties; see Ghirardi & Marinatto, 2003, p. 381). Then, one has to distinguish three types of states available for non-similar particles:

1. Product states, such as

$$|product\rangle = |R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 \tag{5}$$

2. (Partially) Entangled states, e.g., those that could be obtained by (anti)symmetrizing a product state, such as

$$|part\rangle = \frac{1}{\sqrt{2}}[|R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 - |L\rangle_1|\uparrow_z\rangle_1|R\rangle_2|\downarrow_z\rangle_2] \tag{6}$$

3. (Completely) Entangled states, such as

$$|comp\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_1|\uparrow\rangle_2 + |\downarrow\rangle_1|\downarrow\rangle_2] \otimes [|R\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2] \tag{7}$$

According to GMW (see Ghirardi & Marinatto, 2003, p. 382), states of type (6) are under the same heading as states of type (7): both (6) and (7) represent (types of) *entangled* states with respect to non-similar particles. The difference only is that in a state of type (6) non-similar particles are not *completely* entangled, not in full generality, since the range of the reduced statistical operator does not coincide with the full single-particle Hilbert space[6]; in contrast to (7). So, the particles objectively possess some state-dependent properties – all operators which have the respective subspaces as eigenmanifolds have precise values – , whereas in a state of type (7) the particles do not possess any state-dependent property. However, in a state of type (6) they do not possess a *complete* set of properties, therefore they are entangled also in such states.

In sharp contrast, similar particles in a state of type (3) *possess* a complete set of properties, according to GMW (see Ghirardi & Marinatto, 2003, p. 383). In this case, one can attribute a complete set of properties to the "subsystems", although one cannot attribute the possessed properties "to one rather than to the other constituent". Thus, one can take a very same state – one of type (3) and one of type (6) – which counts as a non-entangled state, when similar particles are therein, but as an entangled state when non-similar particles are therein. This obviously has something to do with the individuality and (in)distinguishability of non-similar vs. similar particles so that (non-)entanglement in fact matters for these philosophical concepts.

There is, of course, a clear sense in which states of type (6) represent entangled states for non-similar particles. Since non-symmetric observables, such as $\hat{s}_z \otimes \hat{1}$, are physically meaningful for them, a state of type (6) can be reduced: by an EPR-like measurement – and the eigenvalue–eigenvector link to be assumed – the system can collapse onto a product state of type (5). A situation is, hence, available in which one can correctly state that "particle 1 has a complete set of properties distinguishable from particle 2 also having a complete set of properties". In contrast, due to the requirement of permutation invariance, no EPR-like measurement can collapse a state of type (3) onto a product state which is, therefore, not available for similar particles. However, from this it neither follows that states of type (3) are non-entangled nor that states of type (6) are entangled.

For, one could firstly state that similar particles are always entangled, also in states of type (3), since it is never possible to attribute complete sets of properties to them, *exactly because* even for such a state it is meaningless to speak of particle 1 as distinguishable from particle 2. GMW, however, claim that "*[e] ven though* for such a state [of type (3)] it is meaningless to speak of particle 1 as distinguishable from particle 2, we can correctly state that there is a particle with spin up along *z*-axis and located in region R and […] there is a particle with spin down along *z*-axis and located in region L" (Ghirardi & Marinatto, 2003, p. 384; italics mine). Secondly, one could state that in some states of type (6) we *can* attribute complete sets of state-dependent properties to non-similar particles, namely to the subsystems, although one cannot attribute the possessed properties to one rather than to the other constituent. That a state of type (5) is available in which one can state that "particle 1 has a complete set of properties distinguishable from particle 2 also having a complete set of properties" does

(*footnote continued*)
entanglement. Recently, Ladyman, Linnebo, & Bigaj (2013) have accepted and reformulated GMW's physical result but without drawing the philosophical consequences.

[5] As becomes clear in the first part of Section 2, sharing all state-dependent properties can consistently be combined with an ontology of individuality because of different state-independent properties. Usually, however, the lack of impenetrability has been combined with primitive individuation, i.e., with indistinguishable bare particulars – and not with Leibniz-individuals. At least at first view, entanglement (and symmetric product states) is hence problematic also with respect to non-similar particles.

[6] In my example, the range of the reduced statistical operator coincides with the subspace spanned by $|R\rangle|\downarrow_z\rangle$ and $|L\rangle|\uparrow_z\rangle$.

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apparently not exclude that, within a state of type (6), one can state that "there is a particle with a complete set of properties and there is another particle also having a complete set of properties". In (5) one can say which is which but in (6) one cannot; nevertheless both particles, it seems, can have complete set of properties both in (5) and in (6). GMW, however, (rightly) strike against it: the talk of subsystems *simpliciter* – one particle vs. another particle – as distinguished from a talk of *particular* subsystems – one of them rather than the other; particle 1 vs. particle 2 – only is adequate with respect to similar particles. Since in states of type (6) no *particular* subsystem possesses a complete set of properties – not: particle 1 as opposed to particle 2 – , no subsystem at all possesses a complete set of state-dependent properties. In contrast, in states of type (3), although no particular subsystem possesses a complete set of properties, the subsystems *simpliciter* possess complete sets of properties. The tensor product indexes "1" and "2" apparently represent labels that are closest related to the physical subsystems, in the case of non-similar particles, but only loosely so connected in the case of similar particles. Labeling apparently has something to do with the individuality and (in) distinguishability of particles *and* with (non-)entanglement.

The purpose of my paper is, in particular, to spell out the difference between states of type (3) and states of type (6), hence to clarify why some very same states are entangled, when non-similar particles are therein, but non-entangled when similar particles are therein. The solution will be that non-similar quantum particles and similar fermions are always individuated by distinguishing state-independent *or* state-dependent properties, i.e., they are individuals – numerically distinct from other things – in virtue of distinguishing complete sets of state-independent and state-dependent properties. In order to be consistent, I then have to show that within the entangled states of similar fermions, i.e., in states of type (4), there are *no* numerically distinct particles. A past interaction has unified the numerically distinct systems into a currently undivided whole (which can be divided by EPR measurement). So, I will defend Leibniz's Principle of the Identity of Indiscernibles (PII), according to which there are no indistinguishable individuals, for non-similar particles and similar fermions, and I will use the "summing defense" for PII (see Hawley, 2009, 111ff.) concerning the entangled states of type (4).[7] Before I will develop my defense of PII in Sections 4 and 5, I will analyze the shortcomings or inconsistencies of the underlying ontology in the GMW approach (Section 2) and the ambiguous interpretation of the tensor product labels by GMW (Section 3). Labeling and ontology will be connected with PII in a way that justifies my alternative direction.

2. The bundle theory of substance and bare particularity

As being said, a philosophical interpretation of a physical result is needed when it is conceptually unclear or even inconsistent. Let me now make explicit the underlying ontology in the reasoning of GMW and let me show that it is inconsistent or at least not straightforward. Compare firstly an entangled state (of full generality)

$$|ent\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2] \otimes [|R\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2] \tag{8}$$

with the paradigmatic state in question which is allegedly non-entangled with respect to similar fermions but (partially) entangled with respect to non-similar particles:

$$|???\rangle = \frac{1}{\sqrt{2}}[|R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 - |L\rangle_1|\uparrow_z\rangle_1|R\rangle_2|\downarrow_z\rangle_2] \tag{9}$$

An obvious difference consists in the fact that in the (completely) entangled state all the properties are mixed, whereas in the crucial state some properties are coupled, namely, in the given example, the spatial locations with spin along z-axis. Dissolving (some) entanglement apparently means *combining properties*.

Look, then, at an undoubtedly non-entangled state for non-similar particles, a factorized state, such as

$$|non\rangle = |R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 \tag{10}$$

Within this state, not only are coupled the spatial locations with spin along z-axis but also with the distinguishing state-independent properties of charge, mass, and spin. In state (9), by contrast, the coupled state-dependent properties are disconnected from the different bundles of state-independent properties. In this line of reasoning, state (9) is still entangled, exactly because some properties are *still mixed*. A further disentanglement is possible by combining the complete sets[8] of state-dependent properties with the complete sets of state-independent properties. It seems that within a completely disentangled state particles possess precise combinations of all (available) properties, complete sets of state-dependent and state-independent properties. This idea of disentanglement by combining properties is in full accordance with the *bundle theory of substance*, according to which empirical objects are nothing but bundles of properties.

In my understanding, GMW assume the bundle theory of substance with respect to non-similar particles. Such a particle, the (unique) electron for example, which in the given factorized state can physically be understood as being spatially located at R with spin-down along z-axis is ontologically understood as being nothing other than the bundle: $[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$. The bundle theory of substance is closely related to PII (see O'Leary-Hawthorne, 1995) so that non-similar particles apparently satisfy Leibniz's principle; they are numerically distinct in virtue of distinguishing qualitative facts. Further, GMW modify the simple bundle theory in a well-known way,[9] namely by distinguishing an essential kernel (a *nucleus*) – composed of the state-independent properties – from a peripheral cloud (an accidental *covering*) with state-dependent properties. (Partial or complete) Entanglement makes the covering porous, so to speak, but does not affect the individuality and the distinguishability of the particles. Consequently, the *principium individuationis* of non-similar particles is a strong version of PII

$$\forall F,\ (F(x) \leftrightarrow F(y)) \Rightarrow x = y \tag{11}$$

with only state-independent properties in the scope of the universal quantifier.

Finally, GMW consistently combine strong-PII with a lack of *impenetrability*, since they metaphysically allow for non-similar particles to *share* all state-dependent properties, as within: $|sym\rangle = |R\rangle_1|\downarrow_z\rangle_1|R\rangle_2|\downarrow_z\rangle_2$.[10] This way, one always has distinguishable individuals in all states of type (1) or (2) – and (5), (6), or (7) – independently of whether the non-similar particles are within

[7] Symmetric product states with similar bosons therein provide peculiar difficulties for my approach. With respect to bosons, I will defend PII in a follow-up paper.

[8] According to many philosophers, sets are abstract objects which is obviously not intended here. I will use "set" in the sense of *bundle*.

[9] For analogously different variants of the bundle theory of substance see Simons (1994). Note that I assume – in line with Armstrong (1978) and O'Leary-Hawthorne (1995), and apparently in line with GMW – that properties are (immanent) universals, not tropes. It is controversial in which way the trope ontology is related to PII.

[10] Such a state is "metaphysically" allowed, since no principle, such as Pauli-exclusion, forbids the symmetric product states. It might not be physically realizable to prepare a Bose condensate with non-similar particles, but this doesn't matter for my ontological purposes.

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non-entangled, partially, or completely entangled states. All this fits nicely with GMW's claim that with respect to non-similar particles one cannot speak of subsystems or particles *simpliciter* but has to speak of particle 1 as opposed to particle 2, because a particle *essentially is* a particular nucleus composed of characteristic state-independent properties. Therefore, if one cannot say which is which – as in a state of type (6) – , i.e., if one cannot say with which nucleus the state-dependent properties are connected, one cannot say that a particle possesses a complete bundle of state-dependent properties.

However, GMW drastically change their underlying ontology when they turn to similar particles. To see this, compare the alleged non-entangled state

$$|non-ent\rangle = \frac{1}{\sqrt{2}}[|R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 - |L\rangle_1|\uparrow_z\rangle_1|R\rangle_2|\downarrow_z\rangle_2] \tag{12}$$

with the hypothetical, indeed forbidden, non-symmetric product state:

$$|non-sym\rangle = |R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 \tag{13}$$

As said, from permutation invariance follows that no EPR-like measurement can collapse state (12) onto state (13) so that no further disentanglement is possible. In terms of property-combination, state (12) is no more entangled than the undoubtedly non-entangled product state, since in both states there are the very same bundles of properties, namely, in the given example, $[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$ and $[q_e; m_e; s=\frac{1}{2}; L; \uparrow_z]$. In contrast to a state of type (6), in which the coupled state-dependent properties are disconnected from the distinguishing state-independent properties – the clouds are disconnected from the nuclei, i.e., the particles –, no indeterminacy of property-combination is present within state (12). As long as properties are concerned, state (12) is on a par with state (13). Therefore, if particles are nothing but complete bundles of properties, one can correctly state that in a state like (12) "there is a particle with spin down along z-axis and located in region R and there is a particle with spin up along z-axis and located in region L", since this means nothing other than that there are such two complete bundles of properties as given.

In accordance with the bundle theory of substance, it would, hence, be straightforward if GMW concluded that this is all that can be said. They could re-modify their bundle theory, giving up the nucleus-covering distinction, and claim that quantum particles *essentially are* complete bundles of state-independent *and* state-dependent properties. State-dependent properties might be time-indexed, while state-independent properties are not, but this difference no longer implies the ontological distinction of nucleus and covering, in light of similar particles. One can no longer hold that a particle is constituted by the not-time-indexed properties alone so that a particle could no longer be a mere nucleus,[11] but has to be the whole bundle composed of both types of properties.[12] Consequently, GMW could weaken PII by broadening the scope of the universal quantifier. Quantum particles would be numerically distinct in virtue of distinguishing qualitative facts, namely by being characteristic complete bundles of properties. They all would satisfy PII

$$\forall F,\ (F(x) \leftrightarrow F(y)) \Rightarrow x = y \tag{14}$$

with both state-independent as well as state-dependent properties in the scope of the universal quantifier. As a (perhaps undesired) consequence, however, one has to accept that in the *entangled* states of type (4) there are no numerically distinct particles. For, if it is true that in such a state "it is not possible, for example, to attribute any definite spin property to the particle located in R and equivalently no definite spatial property can be attributed to the particle with spin up" (Ghirardi & Marinatto, 2003, p. 384), no definite bundle of properties is given so that no particle is constituted according to the re-modified bundle theory of substance and according to weaker-PII.

GMW, however, do not stop at the point when they say that there is a particle with one complete set of state-dependent properties and another particle with a different set. They continue to talk about something missing in state (12), in contrast to state (13). Thus, they introduce some (further?) particles, called "1" and "2", which are *something other* than bundles of properties, something other than, for instance, $[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$ or $[q_e; m_e; s=\frac{1}{2}; L; \uparrow_z]$. For, although the bundles *are* distinguished in a state like (12) – simply by having *different* elements – , GMW claim that it is not possible "in the considered peculiar situation [...], both conceptually and practically, to distinguish the particles" (Ghirardi & Marinatto, 2005, p. 387); one cannot say which is which. Here, "distinguishing" particles and the which-is-which clause have a very different meaning as before: with respect to non-similar particles, one distinguishes a particle from others by picking out the characteristic bundle of (state-independent) properties, and one can say which is which if one can say which nucleus is connected with which set of state-dependent properties. In the case of similar particles, however, the alleged nucleus is one and the same so that GMW obviously do not have in mind the triviality that one cannot distinguish the nuclei and that one cannot say which is which when there is only one. What GMW apparently mean, instead, is that telling which is which requires to pick out the right *bearer* of the right whole bundle of properties.

Not only are the particles 1 and 2 not identical with certain bundles of properties, by being their bearers they are, considered for themselves, rather *bare* of any property. They are bare particulars, located on the metaphysical ground floor. Similar particles 1 and 2, conceived of in this way, are "truly indistinguishable", not because they share some properties but exactly because they are bare, i.e., indistinguishable *in principle* by distinguishing qualitative facts. This way, disentanglement is still the complete coupling of properties, but the difference between non-symmetric product states and the peculiar states in question no longer is a qualitative difference. The very same property-bundles are present in (12) and (13) but with a metaphysical difference: within the (inaccessible) product state the first bundle is connected with the bare particular 1, while the second bundle is coupled with the bare particular 2. Within (12), by contrast, all the properties are disconnected from their bare bearers; therefore one cannot state that "particle 1 has a complete set of properties distinguishable from particle 2 having a different complete set of properties". So, interpreting GMW, a physical subsystem (particle) *simpliciter* is the (whole) bundle of properties, whereas a *particular* subsystem (particle) is the underlying bare particular. This is the distinction that is, now obviously, inappropriate for non-similar particles, since they are not and do not have bare bearers but are nuclei composed of state-independent properties.[13]

With respect to similar particles, GMW assume the ontology of bare particularity, they consider the particles 1 and 2 as Lockean substances which individualities (numerical distinctnesses) are ungrounded. Quantum particles would then *violate* any version of

[11] Similar particles would have the very same nucleus and would, therefore, be the very same, numerically identical particle.

[12] This makes each property to be an essential element of the bundle.

[13] One might object that GMW could consistently assume primitive individuality in a world with only non-similar particles. However, in order to avoid the strange view that in such a world the particles are "truly indistinguishable" by being bare particulars, the primitive individuals 1 and 2 must be connected with state-independent properties being essential to them. In this way, the (only) advantage of primitive individuation gets lost, namely that it avoids (perhaps unattractive) essentialism.

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PII, in the sense that PII is not the *principium individuationis* but the objects are individuated primitively. All this sounds inconsistent. According to GMW, a world in which there only are non-similar particles contains numerically distinct objects individuated by strong-PII. From the moment on at which a second, similar particle emerges, these two similar particles are individuated primitively. I have a further worry: according to Armstrong's theory of substance, there are *three* fundamental kinds of items in the world – bare particulars, immanent universals, and state of affairs.[14] Bare particulars are GMW's similar particles 1 and 2, immanent universals are GMW's (complete sets of) properties, and states of affairs are what GMW mean by "distinguishing" (similar) particles, the which-is-which clause. The crucial difference between an ontology with Lockean substances and the bundle theory of substance consists in the view how empirical objects are constituted: either by combining properties (bundle theory) or by combining properties with a bare particular. A state of affair, the connection of some immanent universals with some bare particular, is therefore crucial for the constitution of empirical objects. There would be no empirical particles, according to Armstrong, within a non-entangled, (anti)symmetric state in which the immanent universals are disconnected from the bare particulars. So, the view that in all accessible states of similar fermions the particles, called "1" and "2", are disconnected from the complete sets of properties strikes against the spirit of an(y) ontology involving bare particulars. I, therefore, suggest to cancel the particles 1 and 2 from the ontology of quantum mechanics.

3. Labeling and (in)distinguishability

Before I will defend the bundle theory of substance also for similar fermions, let me consider GMW's ambiguous interpretation of the tensor product space indexes 1 and 2. It can be shown that the talk of "distinguishing" particles by telling "which is which" is ambiguous, because GMW use two *different sorts* of labels, namely labels according to the description theory of proper names, in the case of non-similar particles, and labels according to the direct reference theory of proper names in the case of non-similar particles.[15] I will spell out how this fits to the different ontologies of PII-individuated non-similar particles, on the one hand, and primitively individuated similar particles, on the other hand. Consequently, for my purpose to defend PII also for similar fermions, I need descriptivist labels also for similar fermions.

Both descriptivist and directly referential names have a unique referent which cannot change in a given context. According to the description theory of proper names, such a label picks out its referent via a description of its characteristic, qualitative facts. Take, for instance, wisdom as a universal, i.e., as being a unique entity which exists only once, though perhaps being multiply instantiated. One may call it "a" or "b", "1" or "2", or "Ike" or "Mike" – labeling is somewhat arbitrary, conventional – but its metaphysically adequate label is the descriptivist one: "Wisdom". Take, secondly, an abstract mathematical object such as the (unique) number six. Once again, one may call it "a" or "b", "1" or "2", or "Ike" or "Mike" – but its metaphysically adequate label is again descriptivistic: "6" or "3+3". The same holds for concrete, empirical substances if (and only if) the bundle theory of substance is assumed. If Socrates is empirically human, wise, and beautiful, and ontologically nothing other than the bundle of the (universal) properties Man, Wisdom, and Beauty, then "Socrates" is the metaphysically adequate label only if it is descriptivistic; explicit: "[M; W; B]".

Now, my claim is that according to GMW, the indexes 1 and 2, which mathematically refer to the subspaces of the tensor product Hilbert space, are proper names in the sense of the description theory when applied to the physical subsystems of many non-similar quantum particles. In this case, these labels express the characteristic nuclei of their state-independent properties, so that, for example, "1" means "$[q_e; m_e; s=\frac{1}{2}]$" when referring to the (unique) electron. Take, then, the electron and the proton within a non-entangled, non-symmetric product state, such as

$$|non-sym\rangle = |R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 \tag{15}$$

GMW's formula that, in this case, one can correctly state that particle 1 is located in region R with spin down along z-axis and particle 2 is located in region L with spin up along z-axis could be translated without change of meaning into the sentence: "$[q_e; m_e; s=\frac{1}{2}]$ possesses $[R; \downarrow_z]$, and $[q_p; m_p; s=\frac{1}{2}]$ possesses $[L; \uparrow_z]$".[16]

In line with the description theory of proper names, one is justified in assuming that one can label two numerically different things only if one can conceptually distinguish one from the other. To suppose that the objects under consideration are nameable is equivalent to assuming that they are distinguishable in some specified respect.[17] Consequently, telling which is which is equivalent to distinguishing one particular subsystem from the other, by using descriptivist labels. The striking example which shows that GMW use "1" and "2" in this sense is a symmetric product state with non-similar particles therein:

$$|sym\rangle = |R\rangle_1|\downarrow_z\rangle_1|R\rangle_2|\downarrow_z\rangle_2 \tag{16}$$

Although a legion of classical philosophers such as Aristotle, Leibniz, and Kant endorsed impenetrability as a necessary condition for the numerical distinctness of things, and although within such a state the particles are empirically indistinguishable in some specified respect, they are still essentially distinguishable, according to GMW, and one can tell which is which. Take the proton and the neutron, telling them apart and distinguishing the one from the other are the very same thing, expressible by the sentence: "$[q_p; m_p; s=\frac{1}{2}]$ possesses $[R; \downarrow_z]$, and so does $[q=0; m_n; s=\frac{1}{2}]$".

In sharp contrast, according to (a radical version of) the direct reference theory of proper names, referring goes *directly*, i.e., without description, without mentioning any property of the referent. Directly referential names have no content, are empty, or do not express any qualitative fact concerning its referent. In this line, one is justified in assuming that one can label two numerically different things, *simply because* they are *two*. To suppose that the objects under consideration are nameable *no longer* is equivalent to assuming that they are distinguishable in some specified respect. Labeling and distinguishing are two very different things, according to anti-descriptivism.

With respect to similar particles, GMW drastically change their theory of labeling. Rather obvious, at first, is that the very same indexes 1 and 2, which mathematically refer to the subspaces of the tensor product Hilbert space, no longer can be proper names of the physical subsystems such that they express some characteristic nucleus, such as "$[q_e; m_e; s=\frac{1}{2}]$". For, similar particles have the

[14] For a comparison of Armstrong's theory with the bundle theory of substance see O'Leary-Hawthorne (1995, p. 192).

[15] The distinction between descriptivist and directly referential labels is by now standard in the philosophy of language. It traces back to Kripke (1980). It should be emphasized that Kripke's own variant, i.e., the causal theory of reference is not appropriate for quantum particles due to their lack of spatiotemporal trajectories (according to the standard interpretation). GMW adopt a more radical variant of directly referential proper names for similar quantum particles.

[16] Note that "$[q_e; m_e; s=\frac{1}{2}]$" and "$[q_p; m_p; s=\frac{1}{2}]$" are singular terms, while "$[R; \downarrow_z]$" and "$[L; \uparrow_z]$" are general terms: there could be different particles possessing the same sets of state-dependent properties.

[17] By contrast, see Cortes (1976, p. 498) who holds that, *in general*, labeling and distinguishing are equivalent. However, in 1976, Frege's and Russell's description theory was the dominant view, and Cortes was obviously unaware of a Kripkean, opposing alternative.

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same nucleus, if any, but bear still different labels. Moreover, "1" or "2" as being labels of the physical particles does not express any property of them but are empty. In the case of similar particles, GMW use the labels as proper names of (the radical version of) the direct reference theory. This can strikingly be shown by considering a symmetric product state with two similar bosons therein:

$$|sym\rangle = |R\rangle_1|\downarrow_z\rangle_1|R\rangle_2|\downarrow_z\rangle_2 \quad (17)$$

This state is conceived of, by GMW, as being *non-entangled* so that one can attribute a complete set of state-dependent properties to each of the constituents. In contrast to a likewise non-entangled, symmetric non-product state, such as

$$|non-ent\rangle = \frac{1}{\sqrt{2}}[|R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 + |L\rangle_1|\uparrow_z\rangle_1|R\rangle_2|\downarrow_z\rangle_2], \quad (18)$$

one can even connect the sets of properties with the particles 1 and 2. GMW apparently have no worries with bosonic products; they do not doubt that in these states there are numerically distinct, indistinguishable particles. Further, they do not question that it is possible to talk about one of the bosons, although one cannot distinguish it from the other. In slight modification of an above quoted claim, one allegedly has to say that "[e]ven though for such a state [(17)] it is meaningless to speak of particle 1 *as distinguishable* from particle 2, we can correctly state that particle 1 is located in region R with spin down along z-axis, and so is particle 2". Thus, even though they are indistinguishable, they are nameable – which only is possible with anti-descriptivist labels in hand.

Consequently, GMW's claim that one cannot tell which is which in a non-entangled, antisymmetric fermion state, such as

$$|non-ent\rangle = \frac{1}{\sqrt{2}}[|R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 - |L\rangle_1|\uparrow_z\rangle_1|R\rangle_2|\downarrow_z\rangle_2] \quad (19)$$

turns out to have a peculiar meaning: one cannot refer to particle 1 or 2 *via the different complete sets of properties*. However, this is rather trivial, since with directly referential labels one refers just directly and not via properties. Used in the right way, referring with strong-directly referential labels is justified simply when there are numerically distinct things and, thus, *cannot fail*. What, rather, fails, in the given case, is that when one uniquely picks out a particular particle, to be conventionally called "1" or "2", one cannot say with which set of state-dependent properties it is coupled.

Correspondingly, it is not the case that in an inaccessible, non-symmetric product state, such as

$$|non-sym\rangle = |R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 \quad (20)$$

one can tell which is which by referring to particles 1 and 2 via the different sets of properties. Once again, one directly refers to a particle, conventionally be called "1" or "2", but then one can, in this hypothetical case, couple the particle picked out with a specific bundle of properties. (No wonder that quantum mechanics forbids it.)

Though it is strange, GMW's sudden change of the sort of labels is in perfect accordance with their sudden change of the ontology. While Fregean descriptivist labels are closely related to (some version of) PII, the radical version of directly referential labels fits nicely with bare particularity. Objects that are numerically distinct in virtue of distinguishing qualitative facts, metaphysically bear labels which express these qualitative characteristics, whereas objects that are individuated primitively, most reasonably bear labels without content. Hence, bare particulars on the ontological ground floor metaphysically bear directly referential labels.[18] My worry with this sudden change is the same as before: so long as the world only contains non-similar particles, all the labels are Fregean, but once one second, similar particle emerges, for these two similar particles directly referential labels are introduced.

I will, therefore, strike against GMW with the consistent view, according to which also similar fermions bear descriptivist labels.[19] One *can* tell them apart, one can say which is which, but only in the non-entangled states. Within the given, non-entangled state, the (PII-distinguishable) similar fermions bear the descriptivist labels "$[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$" and "$[q_e; m_e; s=\frac{1}{2}; L; \uparrow_z]$" so that GMW's claim – there is a particle with spin down along z-axis and located in region R and there is a particle with spin up along z-axis and located in region L – has to be translated into the sentence: "there is $[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$, and there is $[q_e; m_e; s=\frac{1}{2}; L; \uparrow_z]$".[20] No further which-is-which question has to be answered, simply because there are no particles labeled with the empty names "1" or "2". To be consistent, within the entangled, antisymmetric fermion states there cannot be numerically distinct particles.

4. Establishing PII for similar fermions

Take, once again, a non-entangled, antisymmetric fermion state, such as

$$|non\rangle = \frac{1}{\sqrt{2}}[|R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 - |L\rangle_1|\uparrow_z\rangle_1|R\rangle_2|\downarrow_z\rangle_2] \quad (21)$$

According to my interpretation, in such a state there are numerically distinct particles in virtue of distinguishing qualitative facts, namely due to different elements within the complete sets of state-dependent properties. Similar fermions satisfy PII, in non-entangled states. Consequently, they are (nothing other than) bundles of state-independent and state-dependent properties, namely $[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$ and $[q_e; m_e; s=\frac{1}{2}; L; \uparrow_z]$ in the given example. Correspondingly, they are metaphysically adequately labeled according to the description theory of proper names, namely in the given case by "$[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$" and "$[q_e; m_e; s=\frac{1}{2}; L; \uparrow_z]$". Again consequently, instead of (synthetic) predicative sentences – such as "particle 1 is located in region R and has spin down along z-axis" – (synthetic) identifying sentences express judgments about similar fermions, such as "there is $[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$".[21]

The opponent view is that similar fermions are individuated primitively, i.e., their numerical distinctness is a brute fact. Most reasonably, such particles are essentially bare particulars and the metaphysically most adequate labels are of the directly referential sort. The tensor product Hilbert space indexes "1" and "2" directly refer to physical subsystems, i.e., independently of allegedly distinguishing properties and independently of whether the particles are in non-entangled states or in entangled ones. Support for this view comes from the entangled states, such as

$$|ent\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2] \otimes [|R\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2] \quad (22)$$

Within such a state, "it is not possible, for example, to attribute any definite spin property to the particle located in R and

[18] Troublesome, in this respect, is Black's famous paper. Unaware of this distinction, Black let his *opponent* of PII only operate with descriptivist names, while the *proponent* of PII intuitively allows anti-descriptivist labels (see Black, 1952, pp. 157, 159). It should be the other way around.

[19] Obviously, symmetric boson products are problematic in this respect. As being said, how to treat bosons is up to a further paper.

[20] Note that this sentence is not predicative but the copula *is* is identifying. This fits to the bundle theory without nucleus/covering-distinction.

[21] Further, analytic predications such as "$[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$ possesses $[R; \downarrow_z]$" are also meaningful.

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equivalently no definite spatial property can be attributed to the particle with spin up" (Ghirardi & Marinatto, 2003, p. 384). No specified bundle of properties is given and, so, the particles cannot be labeled with descriptivist proper names. Therein, however, is something which has twice the charge and twice the mass of an elementary fermion so that, seemingly, numerically distinct particles are present. The ontology of bare particulars being labeled by the directly referential proper names "1" and "2" would be in accordance with this physicist's talk.

However, similar fermions are not always entangled. Quantum particles, in general, are within an entangled state when they are bounded by physical forces – in an atom, for example – or due to a past interaction like in the EPR case. Interaction-free particles generally are in non-entangled states, and so are similar fermions, although their non-entangled states are not product states but antisymmetric ones. Thus, the crucial difference between non-similar particles and similar fermions is not that the latter are always entangled, while the former can be in product states. The crucial difference is, rather, that similar fermions always satisfy the requirement of permutation invariance, while the others do not. If, then, it is not questionable that non-similar particles satisfy (a version of) PII so that a world that only contains unique, non-similar particles confirms the bundle theory of substance (in some specified way), then the alleged violation of PII by similar fermions crucially depends on permutation invariance. It is, therefore, reasonable to consider at first those states which are purely induced by the requirement of permutation invariance. These states are not the entangled, antisymmetric fermion states but the non-entangled ones.

I will now present two arguments against primitive individuation of similar fermions in non-entangled states which justifies my alternative view of PII-individuation. The first argument runs as follows: assume that within such a non-entangled, antisymmetric fermion state were the primitively individuated particles 1 and 2. Undoubtedly, not even for such a non-entangled state it is possible to attribute (a complete set of) state-dependent properties to these particles 1 and 2. What one can say, at best, is that both particles are together in the same global, two-particle state and/or they are in the same reduced mixed state, namely, in the given example, within

$$\hat{\rho}_{1;2} = \frac{1}{2}(|R,\downarrow_z\rangle\langle\downarrow_z,R| + |L,\uparrow_z\rangle\langle\uparrow_z,L|) \tag{23}$$

Not only are the primitively individuated, similar fermions "truly indistinguishable" by being bare, i.e., not only are distinguishing qualitative facts irrelevant for their individuality, but also is it never possible to combine them with specific, different properties. Particles 1 and 2 are always, even within the non-entangled states, connected with the *same* state, with the same reduced density matrix.

Let now act the following symmetric operator on the composite system: $(\hat{R}\hat{s}_y) \otimes \hat{1} + \hat{1} \otimes (\hat{R}\hat{s}_y)$. In the spatial region R – and only in R – a spin measurement along a different axis, in the y-direction, will be performed. The resulting vector is necessarily antisymmetric in virtue of permutation invariance and will be, with probability $\frac{1}{2}$,

$$|non.2\rangle = \frac{1}{\sqrt{2}}[|R,\downarrow_y\rangle|L,\uparrow_z\rangle - |L,\uparrow_z\rangle|R,\downarrow_y\rangle] \tag{24}$$

According to the anti-Leibniz view, both particles 1 and 2 are now together in the new composite state and/or in the new reduced density matrix:

$$\hat{\rho}_{1;2} = \frac{1}{2}(|R,\downarrow_y\rangle\langle\downarrow_y,R| + |L,\uparrow_z\rangle\langle\uparrow_z,L|) \tag{25}$$

Thus, after the measurement in R, both primitively individuated particles 1 and 2 are, again, combined with the same mixed state. Before the measurement, both particles have been connected with the reduced mixed state (23), and afterwards both particles are connected with the reduced mixed state (25). The given operator, therefore, which operates only in R, changes the situation for *both* particles 1 and 2 as if the particles were still entangled.

In case of entanglement, one would indeed expect that every (symmetric) operator acts on the whole and not on a single particle separately. An EPR-like measurement that collapses an entangled state onto a less entangled or non-entangled state breaks some symmetry of the whole, and the resulting state shows some correlation between its parts. With physically meaningful entangled states, Bell inequalities can be violated in certain circumstances. All this strongly suggests that one cannot manipulate a particular constituent of an entangled state alone; an EPR measurement does not act on a single particle alone. In my example, however, the non-entangled, antisymmetric state is in every instrumental sense really non-entangled: one cannot violate any Bell inequality. From this perspective, it sounds very strange that an operator, such as the given one, acts on both particles 1 and 2 and changes both their connections to certain mixed states. With this operator, to recall, one performs a spin measurement in a spatial region which is possibly far away from the other region. Nevertheless, the particles 1 and 2 are somehow both located therein, although the composite state in which the particles are is non-entangled.

According to my defense of PII, in contrast, the operator $(\hat{R}\hat{s}_y) \otimes \hat{1} + \hat{1} \otimes (\hat{R}\hat{s}_y)$ acts on the right particle alone – namely on "$[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$" – and leaves the left particle untouched. Correspondingly, in the resulting state (24) there are two numerically distinct, distinguishable particles – namely "$[q_e; m_e; s=\frac{1}{2}; R; \downarrow_y]$" and "$[q_e; m_e; s=\frac{1}{2}; L; \uparrow_z]$" – one of which is still the same as before. Analogously, GMW have to say that in (24) there is a particle with spin down along y-axis and located in region R and there is a particle with spin up along z-axis and located in region L. They should agree that the particle with spin up along z-axis and located in region L has been untouched by the spin measurement in R and is still the same as before. Accordingly, my interpretation is closest to the spirit of their understanding of non-entanglement.

My second argument against primitive individuation of similar fermions in non-entangled states concerns *diachronic* identity (persistence). Suppose that GMW really agree that in case of non-entanglement a measurement can act on a single particle separately, without touching the other constituent. One complete bundle of properties would still be the same. Following GMW, however, one has to say that the subsystem is the same even though it is meaningless to speak of particle 1 as distinguishable from particle 2. In accordance with many physicists, they would emphasize that one cannot re-identify particle 1 or 2. The problem with this lack of re-identification is that it turns out to be a logical consequence from their ontology which is immune against pragmatic considerations. The particles 1 and 2 cannot be re-identified over time, simply because it is never possible to "identify" them *via properties*. With respect to similar fermions, they *always* are disconnected from the property-bundles – non-symmetric product states are inaccessible – so that they persist independently of any property. There are *in principle* no qualitative criteria for temporal identity.

The following example suggests that this immunity for qualitative identification is (at least) pragmatically unsatisfactory. Take, once again, the running example to be the state at some initial time t_0:

$$|non\rangle = \frac{1}{\sqrt{2}}[|R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 - |L\rangle_1|\uparrow_z\rangle_1|R\rangle_2|\downarrow_z\rangle_2] \tag{26}$$

At a later time t_1, the state might be the following:

$$|non.3\rangle = \frac{1}{\sqrt{2}}[|R\rangle_1|\uparrow_z\rangle_1|L\rangle_2|\downarrow_z\rangle_2 - |L\rangle_1|\downarrow_z\rangle_1|R\rangle_2|\uparrow_z\rangle_2] \tag{27}$$

There is now a particle with spin *up* along z-axis located in region R and there is now a particle with spin *down* along z-axis located in L.

Again, two different complete sets of properties are present, but in this case *both* property-bundles differ from both previous bundles: namely, for short, $[R; \downarrow_z]/[L; \uparrow_z]$ vs. $[R; \uparrow_z]/[L; \downarrow_z]$.

Reasonably, the question arises whether the two particles with their respective spin values have exchanged their spatial locations or whether the two particles are still located in the same regions but have flipped their spins. The two states tell nothing about this. Though one might have good empirical or pragmatic reasons for one or the other alternative, the lesson from quantum mechanics might best be understood that in such a case both particles differ in individuality from the two particles at the initial time. According to (my understanding of) GMW, however, the question is *a priori* meaningless, since it is impossible, for metaphysical reasons, to make a difference between spatial exchange and spin-flip: the (persisting) similar particles 1 and 2 are *in principle* disconnected from their properties. According to my defense of PII, in contrast, the answer depends (more liberally) on pragmatic reasons. In line with some theory of persistence, state-dependent properties could be considered as being time-indexed – such as $R_{t_0}, R_{t_1}\dots$ – so that the bundle of a persisting substance extends to, for example $[q_e; m_e; s=\frac{1}{2}; R_{t_0}; R_{t_1}; \downarrow_z(t_0); \uparrow_z(t_1)]$.[22] This way,[23] one can both model the spatial exchange vs. spin-flip alternative as well as the radical no-persistence view. It no longer is the metaphysics which is decisive for the pragmatic alternatives of temporal change.

5. The summing defense for PII

Granted that (some traditional version of) PII could be established for similar fermions in non-entangled, anti-symmetric states, the other type of fermion states turns out to be an alleged counterexample to such a PII. For, in the entangled, antisymmetric states, such as my running example

$$|ent\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2] \otimes [|R\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2] \tag{28}$$

"it is not possible, for example, to attribute any definite spin property to the particle located in R and equivalently no definite spatial property can be attributed to the particle with spin up" (Ghirardi & Marinatto, 2003, p. 384). No specified bundle of properties is given, and the particles cannot be labeled with descriptivist proper names. They apparently violate any interesting version of PII, hence, they are numerically distinct *not* in virtue of distinguishing qualitative facts.

Similar fermions apparently satisfy and violate PII depending on the contingent fact in which type of state they are, namely whether they are in non-entangled states or in entangled states. This is inconsistent, for PII is taken to be the alleged *principium individuationis* and deeply connected with the bundle theory of substance so that similar fermions would be bundles of different properties, when in non-entangled states, and primitively individuated, bare particulars when in entangled states. It is imperative, therefore, to have a defense strategy for PII with respect to entanglement.

According to Hawley (2009), there are, in general, three different strategies available for defenders of PII, in light of some alleged counterexample:

1. The identity defense, according to which defenders of PII argue that the alleged numerically distinct (but qualitatively indistinguishable) entities are in fact numerically identical. One famous example of this strategy is O'Leary-Hawthorne's (1995): Black's spheres are numerically identical if conceived of as Russellian bundles of universals.
2. The discerning defense, according to which defenders of PII argue that there is a disguised, overlooked qualitative discernibility for the numerically distinct entities. The relevant example of this strategy is Saunders' (2006), according to which fermions in singlet states are weakly discernible by having opposite spin.[24]
3. The summing defense, according to which defenders of PII argue, once again, that there are no numerically distinct entities but, this time, in the way that they have been unified into one single, undivided whole without numerically distinct parts. With respect to the quantum case, this strategy has been performed, for example, by Dieks, recently recalled in Dieks and Versteegh (2008).

It is obvious that the identity strategy doesn't work in the quantum case: the whole has twice the mass, charge, and spin. The discerning defense leads to weak discernibility at best, in case of entanglement, which threatens that the individuation principle differs depending on the contingent fact in which state the particles are. They allegedly are numerically distinct in virtue of an irreflexive (but symmetric) relation, in case of entanglement, but in virtue of different, complete sets of properties in case of non-entanglement. Moreover, Hawley and others have convincingly argued that weak discernibility cannot ground the numerical distinctness:

> When two objects are weakly discernible, this fact is grounded in the fact that the objects in question are distinct; weak discernibility cannot itself be the ground of distinctness. (Hawley, 2009, p. 110)

In my terms one can say that it might be true that particles 1 and 2 have opposite spin, but if "1" and "2" are their metaphysically adequate labels, then they have been labeled independently of the irreflexive relation and are, hence, numerically distinct independently of that relation. If, by contrast, the particles are conceived of as bundles of properties, they cannot be distinguished by the symmetric relation; both bundles would be one and the same: $[q_e; m_e; s=\frac{1}{2};$ R–L apart from the other; opposite spin]. Finally, O'Leary-Hawthorne (1995) has shown that Black's sphere is spatially several meters apart from *itself* if it is conceived of as a bundle of universals. In this case, the respective relation would *not* be irreflexive. So, if one assumes that such relations are irreflexive, one presupposes that their relata are particulars, i.e., numerically distinct things. Thus, only the summing strategy might do the job.

In contrast to Hawley, who suggests that an advocate of the summing defense could, in Black's scenario, simply state that his universe contains only a simple, partless object which extends through a disconnected spatial region (see Hawley, 2009, p. 106), I will argue, in the spirit of scientific metaphysics, that one needs a plausible physical explanation. An entangled, antisymmetric fermion state represents a partless whole, because a past interaction has unified two PII-individuated fermions and can be divided into two PII-individuated fermions via an EPR-like measurement. Adequate conceptions are needed: of "unifying", according to which previously separated fermions produce an undivided whole, and of "dividing", according to which an undivided whole becomes an internal structure, numerically distinct and qualitatively distinguished part(icle)s are produced.

[22] Correspondingly, its Fregean, descriptivist label extends to "$[q_e; m_e; s=\frac{1}{2}; R_{t_0}; R_{t_1}; \downarrow_z(t_0); \uparrow_z(t_1)]$".

[23] If one opts for perdurantism to be the best conception of persistence, then time-indexing the whole bundles is required. They are, then, the temporal parts of an even larger, four-dimensional bundle. Let aside how the presentists would model the persistence of similar fermions.

[24] Likewise, Black's proponent of PII holds that the two spheres in the symmetric universe are weakly discernible: "Each of the sphere will surely differ from the other in being at some distance from that other one, but at no distance from itself […] And this will serve to distinguish it from the other" (Black, 1952, p. 157).

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The idea is, roughly, the following: within an entangled state, there is one single, physical object which essentially is nothing but a complete bundle of state-independent and state-dependent properties, in the given example: $[2q_e; 2m_e; R-L; \hat{S}^2=0; \hat{S}_z=0]$. The whole has twice the charge and twice the mass of an elementary electron; it has a spatial extension of magnitude $R-L$, and both (in fact: all) global spin observables show the eigenvalue equal to 0. No physical subsystem has any sharp or unsharp value. Now, an EPR-like measurement can be performed: the entangled state will collapse onto a non-entangled, antisymmetric state. Two different bundles emerge, namely in the given example: $[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$ and $[q_e; m_e; s=\frac{1}{2}; L; \uparrow_z]$. A crucial difference between the entangled state

$$|ent\rangle = \frac{1}{\sqrt{2}}[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2] \otimes [|R\rangle_1|L\rangle_2 + |L\rangle_1|R\rangle_2] \tag{29}$$

and the non-entangled state

$$|non\rangle = \frac{1}{\sqrt{2}}[|R\rangle_1|\downarrow_z\rangle_1|L\rangle_2|\uparrow_z\rangle_2 - |L\rangle_1|\uparrow_z\rangle_1|R\rangle_2|\downarrow_z\rangle_2] \tag{30}$$

is that the former system has global, non-supervenient properties which the latter has not. The entangled state is an eigenstate of the global, non-supervenient spin-operator $\hat{S}^2$ (with eigenvalue 0), while the non-entangled state is not. Analogously, the global spatial operator (with eigenvalue $R-L$) represents the non-supervenient property of spatial extension for the entangled state, whereas in the non-entangled state particle-locations emerge. The global spinprojection-operator $\hat{S}_z$ still has eigenvalue 0 which expresses the identity of the whole, but this whole now is composed of parts: the global spin-value supervenes on the single-particle spins (and analogously for the spatial property). Correspondingly, I suggest the following definition for EPR-like measurements as dividing an undivided whole:

Definition 1 (*dividing*). An undivided whole divides into part (icle)s – it becomes a divided whole – iff

- some relevant, state-dependent property of the whole survives which could stand for its identity;
- some relevant, non-supervenient, state-dependent property of the whole vanishes without substitution;
- some relevant, state-dependent properties emerge within the whole without replacing one there.[25]

The opposite direction is unifying: an interaction, for instance, a measurement of the total spin operator $\hat{S}^2$ lets a global, non-supervenient spin value to emerge, while every single-particle spin value vanishes.[26] Analogously, particle locations vanish and the global, non-supervenient property of spatial extension emerges, instead. By unifying, numerically distinct and qualitatively distinguished particles vanish in favor of a single object, an undivided whole:

Definition 2 (*unifying*). Numerically different particles unify to an undivided whole iff

- all state-dependent properties of the single particles vanish without substitution;
- one (or more) global, non-supervenient, state-dependent property of the whole emerges.

[25] Of course, the spatial sense of "there" or "within" is not intended.

[26] Note that firstly dividing a partless whole with eigenvalue 0 of $\hat{S}^2$ and then unifying the resulting parts by a measurement of that operator might lead to a new whole with eigenvalue equal to 1. Dividing is, hence, irreversible.

This way, the summing defense for PII could be successful. Within the entangled, antisymmetric states there are no numerically distinct fermions but only one single quantum object, i.e., only one single, complete bundle of state-independent and state-dependent properties, such as $[2q_e; 2m_e; R-L; \hat{S}^2=0; \hat{S}_z=0]$.

6. Conclusion

The present paper has defended the view, according to which numerically distinct particles arise in virtue of distinguishing qualitative facts. Traditional PII is the individuation principle for (at least) non-similar particles and similar fermions. They ontologically are nothing other than complete bundles of properties and their metaphysically adequate labels are of the Fregean descriptivist sort. The only difference between non-similar and similar particles is that in a universe which contains only non-similar particles a strong version of PII-as-individuation-principle would be satisfied; all particles would then be individuated by different state-independent properties. In the actual universe, however, which contains more than one particle with same state-independent properties, a weaker version of PII is the *principium individuationis*.

The distinction between non-entangled and entangled states of similar particles, established by GMW, is crucial for the defense of PII for similar particles. It is explicitly satisfied when similar particles are in non-entangled states. In these states there always are numerically distinct particles in virtue of distinguishing, qualitative facts; they essentially are nothing but complete bundles of state-independent and state-dependent properties, such as $[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$ and $[q_e; m_e; s=\frac{1}{2}; L; \uparrow_z]$, in the given example. These particles are not labeled by the tensor product indexes 1 and 2 which turn out to be names of the directly referential sort with which one would refer to primitively individuated objects, i.e., to bare particulars. Instead, the metaphysically adequate labels for similar particles in non-entangled states are proper names according to the description theory, namely in the given example: "$[q_e; m_e; s=\frac{1}{2}; R; \downarrow_z]$" and "$[q_e; m_e; s=\frac{1}{2}; L; \uparrow_z]$".

Entangled states challenge this view, since in such states no specified bundles of properties are present which could stand for physical subsystems. Entangled states, but only such states, represent an alleged counterexample to PII. However, PII can be defended also in light of entanglement via the summing defense, according to which there are no numerically distinct things at all, but only one single, complete bundle of state-independent and state-dependent properties is present, namely, e.g., $[2q_e; 2m_e; R-L; \hat{S}^2=0; \hat{S}_z=0]$, labeled by the descriptivist proper name: "$[2q_e; 2m_e; R-L; \hat{S}^2=0; \hat{S}_z=0]$". An EPR-like measurement would divide this partless whole and produce numerically distinct parts which are different bundles of properties satisfying PII explicitly. It previously has been produced by the unification of likewise PII-individuated, numerically distinct bundles of properties.

With respect to similar bosons, the same ontology apparently holds as long as the entangled, symmetric states and the non-entangled, symmetric, non-product states are concerned. However, symmetric product states are non-entangled, according to GMW, so that within a Bose condensate there seemingly are numerically distinct particles with the same complete bundle of properties which *violates* any version of PII. Bosonic product wholes provide a new, more radical counterexample of PII. A subsequent paper will show that with a crucially different summing defense a traditional version of PII can be defended also in light of symmetric product states: PII-individuated, similar bosons can be unified in a way in which similar fermions cannot. Correspondingly, such partless whole (the Bose condensate) can

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only be divided in a crucially different way, not by an EPR-like measurement.

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