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Induced Gauge Theory on a Noncommutative Space[1]

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Abstract. We discuss the calculation of the 1-loop effective action on four dimensional, canonically deformed Euclidean space. The theory under consideration is a scalar ϕ^4 model with an additional oscillator potential. This model is known to be re normalisable. Furthermore, we couple an exterior gauge field to the scalar field and extract the dynamics for the gauge field from the divergent terms of the 1-loop effective action using a matrix basis. This results in proposing an action for noncommutative gauge theory, which is a candidate for a renormalisable model.

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1. Introduction

This talk is based on a joint work with H. Grosse. For more details see [1]. The two dimensional case has been discussed in [2].

Feynman rules for Quantum Field Theory over noncommutative spaces reveal new structures. They stem from the modification of space-time at small length scales. Planar contributions show the standard singularities which can be handled by the usual renormalisation procedure. The non-planar one loop contributions are finite for generic momenta. However, they become logarithmically divergent at exceptional momenta. The usual UV divergences are then reflected in new singularities in the infrared, which is called UV/IR mixing. This spoils the usual renormalisation procedure: Inserting such loops to a higher order diagram generates singularities of any inverse power. In [3], H. Grosse and R. Wulkenhaar were able to give a solution of this problem for the special case of a scalar theory defined on the canonically deformed Euclidean space $\mathbb{R}^4_\theta$ with commutation relation for the coordinates:

$$[x^\mu \stackrel{\star}{,} x^\nu] = i\theta^{\mu\nu},$$

where $\theta^{ij} = -\theta^{ji} \in \mathbb{R}$, and the $\star$-product is given by the Weyl-Moyal product

$$f \star g\,(x) = e^{i/2\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}} f(x)g(y)\big|_{y\to x}. \tag{1}$$

For simplicity, we use the the following parametrisation of $\theta_{\mu\nu}$:

$$(\theta_{\mu\nu}) = \begin{pmatrix} 0 & \theta & & \\ -\theta & 0 & & \\ & & 0 & \theta \\ & & -\theta & 0 \end{pmatrix}, \quad (\theta^{-1}_{\mu\nu}) = \begin{pmatrix} 0 & -1/\theta & & \\ 1/\theta & 0 & & \\ & & 0 & -1/\theta \\ & & 1/\theta & 0 \end{pmatrix}.$$

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The UV/IR mixing contributions were taken into account through a modification of the free Lagrangian by adding an oscillator term with parameter Ω,

$$S_0 = \int d^4x \quad \Big(\tfrac{1}{2}\phi \star [\tilde{x}_\nu,\, [\tilde{x}^\nu, \phi]_\star]_\star + \tfrac{\Omega^2}{2}\phi \star \{\tilde{x}^\nu, \{\tilde{x}_\nu, \phi\}_\star\}_\star \\ + \tfrac{\mu^2}{2}\phi \star \phi + \tfrac{\lambda}{4!}\phi \star \phi \star \phi \star \phi\Big)(x)\,, \tag{2}$$

where $\tilde{x}_\nu = \theta^{-1}_{\nu\alpha} x^\alpha$ and $i\partial_\mu f = [\tilde{x}_\mu, f]_\star$. The spectrum of the free Hamiltonian is modified. The harmonic oscillator term was obtained as a result of the renormalisation proof. The model fulfills the Langmann-Szabo duality [4] relating short distance and long distance behaviour. There are indications that a constructive procedure might be possible and give a nontrivial ϕ^4 model, which is currently under investigation [5].

In a different interesting approach, the UV/IR singularities are interpreted in terms of an induced gravity action [6].

In order to obtain the action for a gauge theory, which hopefully is renormalisable, we extract the divergent terms of the heat kernel expansion. Such a procedure leads in the commutative case to a renormalisable gauge field action. We introduce the local, unitary gauge group $\mathcal{G}$ under which the scalar field ϕ transforms covariantly like

$$\phi \mapsto u^* \star \phi \star u, \;\; u \in \mathcal{G}. \tag{3}$$

The approach employed here makes use of two basic ideas. First, it is well known that the $\star$-multiplication of a coordinate - and also of a function, of course - with a field is not a covariant process. The product $x^\mu \star \phi$ will not transform covariantly,

$$x^\mu \star \phi \nrightarrow u^* \star x^\mu \star \phi \star u\,.$$

Functions of the coordinates are not effected by the gauge group. Fields are taken to be elements of a module [7]. The introduction of covariant coordinates

$$\tilde{X}_\nu = \tilde{x}_\nu + A_\nu \tag{4}$$

finds a remedy to this situation [8]. The gauge field A_μ transforms such that we have for the covariant coordinates:

$$\begin{aligned} \tilde{X}_\mu &\mapsto u^* \star \tilde{X}_\mu \star u\,; \\ A_\mu &\mapsto \mathrm{i}u^* \star \partial_\mu u + u^* \star A_\mu \star u\,. \end{aligned} \tag{5}$$

This leads to the definition of a gauge invariant model, which is the starting point of our investigations. This model is given by the following action:

$$\begin{aligned} S &= \int d^4x \left(\frac{1}{2}\phi \star [\tilde{X}_\nu,\, [\tilde{X}^\nu,\, \phi]_\star]_\star + \frac{\Omega^2}{2}\phi \star \{\tilde{X}^\nu, \{\tilde{X}_\nu, \phi\}_\star\}_\star \right. \\ &\quad \left. + \frac{\mu^2}{2}\phi \star \phi + \frac{\lambda}{4!}\phi \star \phi \star \phi \star \phi\right)(x)\,. \end{aligned} \tag{6}$$

Secondly, we apply the heat kernel formalism. The gauge field A_μ is an external, classical gauge field coupled to ϕ. In the following sections, we will explicitly calculate the divergent terms of the one-loop effective action. In the classical case, the divergent terms determine the dynamics of the gauge field [9–11]. There have already been attempts to generalise this approach to the non-commutative realm; for non-commutative ϕ^4 theory see [12, 13]. First steps towards gauge kinetic models have been done in [14–16]. However, the results there are not completely comparable. Our action contains an oscillator term

$$\frac{\Omega^2}{2}\phi\star\{\tilde{X}^\nu,\{\tilde{X}_\nu,\phi\}_\star\}_\star\,.$$

This term is crucial, it alters the free theory. Therefore, we expand around the free action $-\Delta+\Omega^2\tilde{x}^2$ rather than $-\Delta$. As a consequence, the Seeley-de Witt coefficients cannot be used.

In the following sections, we describe our model and the employed method of extracting the singular contributions of the one-loop action in some detail. The results are summarised and discussed in the final Section.

2. The Model

The regularised one loop effective action for the model defined by the classical action (6) is given by

$$\Gamma^\epsilon_{1l}[\phi]=-\frac{1}{2}\int_\epsilon^\infty\frac{dt}{t}\,\mathrm{Tr}\left(e^{-tH}-e^{-tH^0}\right)\,. \tag{7}$$

For the effective potential H we have the expression

$$\frac{\delta^2 S}{\delta\phi^2}\equiv H=\frac{2}{\theta}H^0+V\,. \tag{8}$$

The field independent contributions are contained in the potential H_0, whereas V involves linear and quadratic terms in the gauge and matter field. The method is not manifestly gauge invariant, contributions from different orders need to add up to a gauge invariant result.

The effective action is calculated as a power series in the potential V. In order to do so we employ the Duhamel expansion which is an iteration of the identity

$$\begin{aligned}e^{-tH}-e^{-tH^0}&=\int_0^t d\sigma\,\frac{d}{d\sigma}\left(e^{-\sigma H}e^{-(t-\sigma)H^0}\right)\\&=-\int_0^t d\sigma\,e^{-\sigma H}\,\frac{\theta}{2}V\,e^{-(t-\sigma)H^0}\,,\end{aligned} \tag{9}$$

yielding

$$\begin{aligned}e^{-tH}&=e^{-tH^0}-\frac{\theta}{2}\int_0^t dt_1 e^{-t_1H^0}Ve^{-(t-t_1)H^0}\\&\quad+\Big(\frac{\theta}{2}\Big)^2\int_0^t dt_1\int_0^{t_1}dt_2 e^{-t_2H^0}Ve^{-(t_1-t_2)H^0}Ve^{-(t-t_1)H^0}+\ldots\end{aligned} \tag{10}$$

Therefore, we get for the 1-loop effective action the following formula:

$$\begin{aligned}\Gamma^\epsilon_{1l}&=-\frac{1}{2}\int_\epsilon^\infty\frac{dt}{t}\mathrm{Tr}\Big(e^{-tH}-e^{-tH^0}\Big)\\&=\frac{\theta}{4}\int_\epsilon^\infty dt\;\mathrm{Tr}\;Ve^{-tH^0}-\frac{\theta^2}{8}\int_\epsilon^\infty\frac{dt}{t}\int_0^t dt'\,t'\;\mathrm{Tr}\;Ve^{-t'H^0}Ve^{-(t-t')H^0}\\&\quad+\frac{\theta^3}{16}\int_\epsilon^\infty\frac{dt}{t}\int_0^t dt'\int_0^{t'}dt''\,t''\;\mathrm{Tr}\;Ve^{-t''H^0}Ve^{-(t'-t'')H^0}Ve^{-(t-t')H^0}\\&-\frac{\theta^4}{32}\int_\epsilon^\infty\frac{dt}{t}\int_0^t dt'\int_0^{t'}dt''\int_0^{t''}dt'''\,t'''\;\mathrm{Tr}\;Ve^{-t'''H^0}Ve^{-(t''-t''')H^0}Ve^{-(t'-t'')H^0}Ve^{-(t-t')H^0}\\&\quad+\mathcal{O}(\theta^5)\,.\end{aligned} \tag{11}$$

The calculations are performed in the matrix basis, where the star product is just a matrix product:

$$A^\nu(x) = \sum_{p,q\in\mathbb{N}^2} A^\nu_{pq} f_{pq}(x)\ , \phi(x) = \sum_{p,q\in\mathbb{N}^2} \phi_{pq} f_{pq}(x)$$

and

$$\begin{aligned} f_{pq}\star f_{mn} &= \delta_{qm} f_{pn}, &(12)\\ f_0\star f_0 &= f_0\,. &(13)\end{aligned}$$

This choice of basis simplifies the calculations. In the end, we will again represent the results in the x-basis. From the coordinates we can build two oscillators:

$$a^{(1)} = \frac{1}{\sqrt{2}}(x^1+ix^2),\ \ a^{(2)} = \frac{1}{\sqrt{2}}(x^3+ix^4). \tag{14}$$

The ground state f_0 is a idempotent under star multiplication and is given by a Gaußian,

$$f_0(x) = 4\,e^{-\frac{1}{\theta}\sum_i x_i^2}.$$

All the other basis elements are obtained by acting with creation and annihilation operators from the left and right, respectively:

$$\begin{aligned} f_{\substack{m^1n^1\\ m^2n^2}} &= \alpha(n,m,\theta)\,\bar a^{(2)\star m^2}\star\bar a^{(1)\star m^1}\star f_0\star a^{(1)\star n^1}\star a^{(2)\star n^2}, &(15)\\ \bar a^{(1)}\star f_{\substack{m^1n^1\\ m^2n^2}} &= \sqrt{\theta(m^1+1)}f_{\substack{m^1+1\,n^1\\ m^2\ \ n^2}}, &(16)\\ a^{(1)}\star f_{\substack{m^1n^1\\ m^2n^2}} &= \sqrt{\theta m^1}f_{\substack{m^1-1\,n^1\\ m^2\ \ n^2}}\,. &(17)\end{aligned}$$

In the next step, we have to apply the above method to the gauge invariant model (6). After a suitable rescaling, all the operators depend, beside on θ, only on the following three parameters:

$$\rho = \frac{1-\Omega^2}{1+\Omega^2},\ \tilde\epsilon = \epsilon(1+\Omega^2),\ \tilde\mu^2 = \frac{\mu^2\theta}{1+\Omega^2}\,. \tag{18}$$

The part of the effective potential independent of the gauge field in the matrix basis is given by

$$\begin{aligned} \frac{H^0_{mn;kl}}{1+\Omega^2} &= \Big(\frac{\tilde\mu^2}{2}+(n^1+m^1+1)+(n^2+m^2+1)\Big)\delta_{n^1k^1}\delta_{m^1l^1}\delta_{n^2k^2}\delta_{m^2l^2}\\ &-\rho\Big(\sqrt{k^1l^1}\,\delta_{n^1+1,k^1}\delta_{m^1+1,l^1}+\sqrt{m^1n^1}\,\delta_{n^1-1,k^1}\delta_{m^1-1,l^1}\Big)\delta_{n^2k^2}\delta_{m^2l^2}\\ &-\rho\Big(\sqrt{k^2l^2}\,\delta_{n^2+1,k^2}\delta_{m^2+1,l^2}+\sqrt{m^2n^2}\,\delta_{n^2-1,k^2}\delta_{m^2-1,l^2}\Big)\delta_{n^1k^1}\delta_{m^1l^1}\,. \qquad (19)\end{aligned}$$

For the field dependent potential V we obtain

$$
\begin{aligned}
V_{kl;mn}(1+\Omega^2) \;=\;& \Big(\frac{\lambda}{3!(1+\Omega^2)}\phi\star\phi + \big(\tilde{X}_\nu \star \tilde{X}^\nu - \tilde{x}^2\big)\Big)_{lm}\delta_{nk} \\
&+\Big(\frac{\lambda}{3!(1+\Omega^2)}\phi\star\phi + \big(\tilde{X}_\nu \star \tilde{X}^\nu - \tilde{x}^2\big)\Big)_{nk}\delta_{lm} \\
&+\Big(\frac{\lambda}{3!(1+\Omega^2)}\phi_{lm}\phi_{nk} - 2\rho A_{\nu,lm}A^\nu_{nk}\Big) \\
&+\rho\mathrm{i}\sqrt{\frac{2}{\theta}}\Big(\sqrt{n^1}A^{(1+)}_{\substack{l^1 m^1\\ l^2 m^2}}\delta_{\substack{n^1-1\, k^1\\ n^2\; k^2}} - \sqrt{n^1+1}A^{(1-)}_{\substack{l^1 m^1\\ l^2 m^2}}\delta_{\substack{n^1+1\, k^1\\ n^2\; k^2}} \\
&\qquad\qquad +\sqrt{n^2}A^{(2+)}_{\substack{l^1 m^1\\ l^2 m^2}}\delta_{\substack{n^1\; k^1\\ n^2-1\, k^2}} - \sqrt{n^2+1}A^{(2-)}_{\substack{l^1 m^1\\ l^2 m^2}}\delta_{\substack{n^1\; k^1\\ n^2+1\, k^2}}\Big) \\
&-\rho\mathrm{i}\sqrt{\frac{2}{\theta}}\Big(-\sqrt{m^1+1}A^{(1+)}_{\substack{n^1 k^1\\ n^2 k^2}}\delta_{\substack{l^1\, m^1+1\\ l^2\; m^2}} + \sqrt{m^1}A^{(1-)}_{\substack{n^1 k^1\\ n^2 k^2}}\delta_{\substack{l^1\, m^1-1\\ l^2\; m^2}} \\
&\qquad\qquad -\sqrt{m^2+1}A^{(2+)}_{\substack{n^1 k^1\\ n^2 k^2}}\delta_{\substack{l^1\; m^1\\ l^2\, m^2+1}} + \sqrt{m^2}A^{(2-)}_{\substack{n^1 k^1\\ n^2 k^2}}\delta_{\substack{l^1\; m^1\\ l^2\, m^2-1}}\Big)
\end{aligned}
\tag{20}
$$

with the definition

$$A^{(1\pm)} = A^1 \pm iA^2, \; A^{(2\pm)} = A^3 \pm iA^4 \,.$$

The heat kernel e^{-tH^0} of the Schrödinger operator can be calculated from the propagator given in [3]. In the matrix base of the Moyal plane, it has the following representation:

$$\Big(e^{-tH^0}\Big)_{mn;kl} = e^{-t(\mu^2\theta/2+\Omega D)}\delta_{m+k,n+l}\prod_{i=1}^{D/2} K_{m^i n^i;k^i l^i}(t)\,, \tag{21}$$

$$
\begin{aligned}
K_{m,m+\alpha;l+\alpha,l}(t) =& \sum_{u=0}^{\min(m,l)} \sqrt{\binom{m}{u}\binom{l}{u}\binom{\alpha+m}{m-u}\binom{\alpha+l}{l-u}} \\
&\times e^{2\Omega t}\left(\frac{1-\Omega^2}{2\Omega}\sinh(2\Omega t)\right)^{m+l-2u} X_\Omega(t)^{\alpha+m+l+1}\,,
\end{aligned}
\tag{22}
$$

where

$$X_\Omega(t) = \frac{4\Omega}{(1+\Omega)^2 e^{2\Omega t} - (1-\Omega)^2 e^{-2\Omega t}}\,. \tag{23}$$

The above expressions have to be inserted into the Duhamel expansion (11). Here, we are only interested in gauge theory. Therefore, we concentrate on the divergent terms involving only the gauge field and assume $\lambda = 0$.

3. Some Remarks on the Calculation

In order to extract the divergent contributions we employ the following method:

- First, expand the integrands of the Duhamel expansion (11) for small auxiliary parameters $t, t', t'', \ldots$.
- Expand the infinite sums over indices occuring in the heat kernel but not in the gauge field; divergences stem from these infinite sums. The other contractions are finite assuming that A is a traceclass operator.
- Integrate over the auxiliary parameters.

- Convert the results to x-space using

$$\sum_m T_{mm} = \frac{1}{(2\pi\theta)^2} \int d^4x T(x) \,. \tag{24}$$

To first and second order in the potential V, the effective action contains both, logarithmic and quadratic divergences. To third and fourth order, only logarithmic ones occur. Higher powers in the potential are already finite. This can easily be seen from a power counting argument in the auxiliary parameters. Let us consider the contribution to the effective action of order k. Due to Eq. (11), there are k auxiliary parameters. They for themselves produce a factor t^{k-1}. The infinite sums over the integral kernels contribute inverse powers of t. For example, we have in first order:

$$\sum_{n=0}^{\infty} K_{mn;nm}(t) \sim \sum_n X_\Omega(t)^n \sim \frac{1}{t} + \mathcal{O}(t^0) \tag{25}$$

$$\sum_{n=0}^{\infty} \sqrt{n+1} K_{m+1,n+1;n,m}(t) \sim \frac{\sqrt{m+1}}{t} + \mathcal{O}(t^0) \,; \tag{26}$$

and in second order:

$$\sum_{n=0}^{\infty} K_{nm;mn}(t') K_{n+1,c;c,n+1}(t-t') \sim \sum_n X_\Omega(t')^n X_\Omega(t-t')^n \sim \frac{1}{t} + \mathcal{O}(t^0, t'^0)$$

$$\sum_{n=0}^{\infty} \sqrt{n+1} K_{nm;m+1,n+1}(t') K_{n+1,c;c,n+1}(t-t') \sim \sqrt{m+1}\frac{t'}{t^2} + \mathcal{O}(t'^0, t^0) \,. \tag{27}$$

The potential V may contribute in the worst case a factor $\sqrt{n}^k$ to the infinite sums of order k. Therefore, these sums contribute a factor

$$\sum_n n^{k/2} X_\Omega(t^{(k)})^n X_\Omega(t^{(k-1)} - t^{(k)})^n \dots X_\Omega(t)^n \sim \left(\frac{1}{t}\right)^{\lfloor k/2 \rfloor + 2} \,, \tag{28}$$

where $\lfloor l \rfloor$ is the greatest integer function (see e.g. Mathematica for an exact definition). Hence, the contribution to order k is given by

$$\left(\frac{1}{t}\right)^{\lfloor k/2 \rfloor + 3 - k} \,. \tag{29}$$

For $k = 1$, the exponent is 2, which means that quadratic divergences occur. In the case of $k = 5$, the exponent is 0 and the integration yields a finite result.

Details of the calculations are provided in [1].

4. Results and Conclusions

Let us summarise the results. In the selfdual case, $\Omega = 1$ the divergent contributions are of an especially simple form. The matrix base expressions for the effective potential and the heat kernel simplify a lot. The effective action describes a pure matrix model. The one-loop effective action is given by

$$\Gamma^\epsilon_{1l} = \frac{1}{16\pi^2} \int d^4x \Bigg(\frac{1}{\epsilon\theta}(\tilde{X}_\nu \star \tilde{X}^\nu - \tilde{x}^2) + \left(\frac{\mu^2}{2}(\tilde{X}_\nu \star \tilde{X}^\nu - \tilde{x}^2) + \frac{1}{2}\Big((\tilde{X}_\mu \star \tilde{X}^\mu) \star (\tilde{X}_\nu \star \tilde{X}^\nu) - (\tilde{x}^2)^2 \Big) \right) \ln \epsilon \Bigg) \,. \tag{30}$$

In this case, we propose the logarithmically divergent part as action for the gauge field:

$$S = \frac{1}{16\pi^2}\int d^4x\left(\frac{\mu^2}{2}(\tilde{X}_\nu \star \tilde{X}^\nu - \tilde{x}^2) + \frac{1}{2}\left((\tilde{X}_\mu \star \tilde{X}^\mu)\star(\tilde{X}_\nu \star \tilde{X}^\nu) - (\tilde{x}^2)^2\right)\right). \tag{31}$$

In the case $\Omega \neq 0$, we obtain much more structure and a dynamics:

$$\begin{aligned}\Gamma^\epsilon_{1l} &= \frac{1}{192\pi^2}\int d^4x\left\{\frac{24}{\tilde{\epsilon}\,\theta}(1-\rho^2)(\tilde{X}_\nu \star \tilde{X}^\nu - \tilde{x}^2)\right. \\ &\quad + \ln\epsilon\left(\frac{12}{\theta}(1-\rho^2)(\tilde{\mu}^2-\rho^2)(\tilde{X}_\nu \star \tilde{X}^\nu - \tilde{x}^2)\right. \\ &\quad \left.\left. + 6(1-\rho^2)^2\left((\tilde{X}_\mu \star \tilde{X}^\mu)^{\star 2} - (\tilde{x}^2)^2\right) - \rho^4 F_{\mu\nu}F^{\mu\nu}\right)\right\},\end{aligned} \tag{32}$$

where $F_{\mu\nu} = -i[\tilde{x}_\mu, A_\nu]_\star + i[\tilde{x}_\nu, A_\mu]_\star - i[A_\mu, A_\nu]_\star$. Again, we propose the logarithmically divergent part as an action describing the dynamics of the gauge field,

$$\begin{aligned}S &= \frac{1}{192\pi^2}\int d^4x\left\{\frac{12}{\theta}(1-\rho^2)(\tilde{\mu}^2-\rho^2)(\tilde{X}_\nu \star \tilde{X}^\nu - \tilde{x}^2)\right. \\ &\quad \left. + 6(1-\rho^2)^2\left((\tilde{X}_\mu \star \tilde{X}^\mu)^{\star 2} - (\tilde{x}^2)^2\right) - \rho^4 F_{\mu\nu}F^{\mu\nu}\right\}.\end{aligned} \tag{33}$$

Both, the linear in ϵ and the logarithmic in ϵ divergent term of the one-loop effective action turn out to be gauge invariant. The logarithmically divergent part is an interesting candidate for a renormalisable gauge interaction. The sign of the term quadratic in the covariant coordinates may change depending on whether $\tilde{\mu}^2 \lesseqgtr \rho^2$. This reflects the structure of a phase transition. The case $\Omega = 1$ ($\rho = 0$) is of course of particular interest. One obtains a pure matrix model. In the limit $\Omega \to 0$, we obtain just the standard deformed Yang-Mills action. Furthermore, the action (32) allows to study the limit $\theta \to \infty$.

In addition, we will attempt to study the perturbative quantisation. One of the problems of quantising action (32) is connected to the tadpole contribution, which is non-vanishing and hard to eliminate. The Paris group also considered the 1-loop effective action in the case $\Omega \neq 0$. They calculated the divergent contributions in x-space by evaluating Feynman diagrams and arrived at the same result [17, 18].

Solutions of the equations of motion for similar models have already been considered in [19,20]. An appropriate rescaling of the covariant coordinates $\tilde{X}_\alpha \to \frac{\sqrt{2\sqrt{3}}}{\sqrt{\theta}}\tilde{X}_\alpha$ and the identification $\tau \equiv -\sqrt{3}\,\frac{1-\rho^2}{\rho^2}$ leads to the equations of motion

$$D_\nu F^{\sigma\nu} = \tau\tilde{X}^\sigma + \tau^2\{\tilde{X}^\sigma, \tilde{X}_\nu \star \tilde{X}^\nu\}_\star\,, \tag{34}$$

where we have assumed for simplicity $\tilde{\mu} = 0$ and used

$$D_\nu F^{\sigma\nu} = -i[\tilde{X}_\nu, -i[\tilde{X}^\sigma, \tilde{X}^\nu]_\star + \theta^{-1\,\mu\nu}]_\star = -[\tilde{X}_\nu, [\tilde{X}^\sigma, \tilde{X}^\nu]_\star]_\star\,.$$

In [20], the matter fields have been included in order to find some solutions. However, the gauge part (34) alone also exhibits a number of solutions which are currently under investigation.

For noncommutative $U(1)$ gauge theory a similar model has been discussed in [21]. This model includes an oscillator potential for the gauge fields, $\tilde{x}^2 A^2$. Other terms occuring here are missing. Hence, the considered action is not gauge invariant, but a BRST invariance could be established. These terms may nevertheless come into the game through one loop corrections. In this approach the tadpole contribution does not vanish, but turns out to be finite which is remarkable indeed.

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