

Induction in the oceans

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Summary. The induction of electric currents in the oceans is considered to be of major interest and may also throw light upon the conductivity structure of the mantle. Two aspects are considered: (1) induction by the solar quiet variations which have principal periods of 24, 12, 8 and 6 hr, and (2) induction by the ocean tides which principally have a semi-diurnal lunar period of 12.45 hr. Two classes of model may be devised: those in which the oceans are considered to be insulated from the mantle, and those in which there is a true electrical connection between the two. It appears that methods of calculating the electric currents are outstripping conductivity models. State of the art recommendations are made on what appears to be the best methods to use and these are demonstrated by using simple analogies. Particular attention is called to the Hewson-Browne technique for solving the integral equations which often emerge. It is noted that the formulations of the integral equations are variations of Weaver's method for plane configurations. Problems in which the ocean is isolated can be solved adequately by the zeroth order of the method of matched asymptotic expansions. Curiously this latter technique is simple in principle, involves only the numerical solution of a partial differential equation on the surface of the globe, and has been fecund in producing solutions for a variety of models. Induction by the ocean tides is also considered. Recommendations are made on the most promising lines appearing in present-day theories.

1 Introduction

William Gilbert of Colchester announced in the year 1600 that the Earth itself is a great magnet. Had he possessed the powers of Nostradamus he might have added that even the oceans have a magnetic effect, for the Earth is immersed in a fluctuating electromagnetic field which comes from space, from the ionosphere, from its own metallic liquid core and from man-made sources. The high electrical conductivity of seawater causes the oceans to interfere with incident electromagnetic radiation and also enables the sea and ocean tides to generate magnetic and electrical effects observable far inland. In this review we shall be concerned mainly with the effects of the ocean on the daily variations of the Earth's

magnetic field known as the solar quiet (S_q) variations. These fluctuations have principal components with periods of 24, 12, 8 and 6 hr and arise from ionospheric electric currents. We shall also be interested in the dynamo action of the oceans and sea tides which have a lunar semi-diurnal period of 12.45 hr. Oceanic phenomena stand out from others because of the so-called coast effect. As the oceans constitute a conductor whose depth is negligible on a global scale, electromagnetic effects peculiar to the edge of a thin conductor will be observed on or near the coastline. In fact many observations were placed on the coast, or moved there, in order to escape man-made electrical interference. Although the coast effect (Cox 1960) depends on the geometry of the coastline, its extent and distribution might be able to tell us something about the electrical conductivity of the rocks beneath the ocean and the land.

Early studies were made by Chapman & Whitehead (1922), and the global advances made by Lahiri & Price (1939) which followed have probably not yet been superseded in principle: see Jady (1975). It is worth pointing out that Gilbert (1600), Gauss (1839), Chapman (*cf.* Chapman & Bartels 1940) and Price (1939) were concerned with the Earth as a globe and that calculations using a flat model of the Earth may be ill-posed. Although much valuable information has been derived from flat earth models, the true inducing magnetic field will remain unknown unless the calculations encompass the spherical form of the Earth. It is easy to demonstrate this by means of a thought experiment. Consider a thin conductor (I) in the form of a spherical shell as in Fig. 1. Induce currents in it by means of an oscillating uniform magnetic field $\mathbf{B}_\infty \exp(i\omega t)$, where ω is the frequency and \mathbf{B}_∞ is a constant vector. Compare this with the similar configuration (II) from which a lunar segment has been removed. The induced electric currents will be globally different in the two cases, showing that in general we should treat with due reservation the results from flat earth models. The global nature of oceanic induction was recognized by Bullard & Parker (1970) who carried out the first realistic computations of electric currents induced in the oceans by S_q .

The tidal motions of the oceans cutting across the permanent magnetic field of the Earth also induce electric currents in the oceans and drive electric currents beneath the land. By data processing techniques Malin (1970, 1973) separated the ocean's magnetic tidal effect from the more dominant ionospheric effect. This was difficult as the main components have

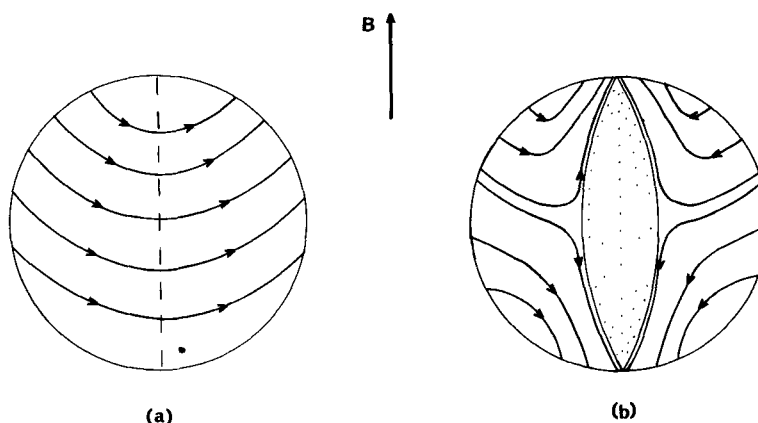


Figure 1. Comparing the currents induced in (a) a complete spherical shell with those when (b) a (shaded) segment of the shell is removed. The time periodic inducing field \mathbf{B} is parallel to the axis (broken) in both cases.

equal lunar periods of 12.45 hr. Different techniques developed by Schlapp & Weekes (*cf.* Schlapp 1977) confirm the reality of this effect. Thus the theorists are faced by a double challenge:

- (1) improve on Price's work on the mantle's conductivity,
- (2) include the dynamo effect of the ocean and sea tides.

Fainberg (1980) describes global induction to be one of the principal problems in modern geomagnetism.

2 Induction and coast effects

If the mantle–ocean system were an insulator the magnetic signals observed would be the same as those of the source field, thought of as being imposed from infinity. Yet, although the electric currents are weak, they flow on such a vast scale that their inductive and self-inductive effects are significant. For electromagnetic disturbances with periods of one day or less one must be prepared to take into account self-induction when solving oceanic induction problems. Even for lower periods the skin depth of the mantle tends to be much less than the radius of the Earth. Under such circumstances the magnetic field of the source will be largely excluded from the Earth's interior. Taking the roughest approximation, the effect upon the source magnetic field would be the same as an infinitely conducting sphere. Thus in geocentric spherical polar coordinates (r, θ, ϕ) , where θ is the co-latitude, and ϕ the longitude a uniform magnetic induction field $B_\infty \exp(i\omega t)$ parallel to the Earth's axis of rotation would give rise to inducing, induced and observed magnetic induction fields on the Earth's surface as shown in Table 1.

Table 1. The inductive effects of an infinitely conducting sphere.

	Inducing	Induced	Observed
Horizontal component	$B_\infty \sin \theta$	$0.5 B_\infty \sin \theta$	$1.5 B_\infty \sin \theta$
Vertical component	$B_\infty \cos \theta$	$-B_\infty \cos \theta$	0

(The time factor $\exp(i\omega t)$ is omitted and will be so henceforth).

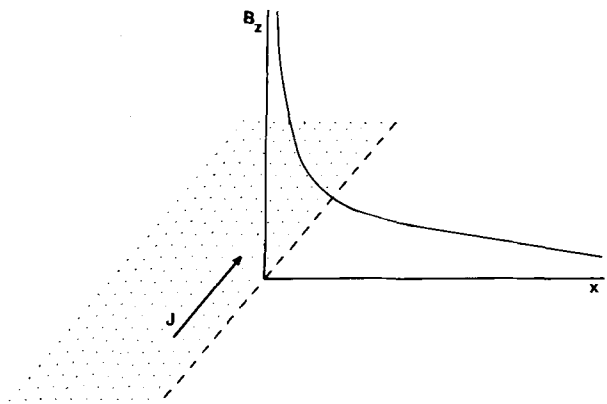


Figure 2. Showing the graph of the downward vertical magnetic field component B_z on the land as a function of x , where B_z arises from an oceanic electric current J in the (shaded) half-plane $z = 0, x < 0$. At $x = 0$ B_z is infinite.

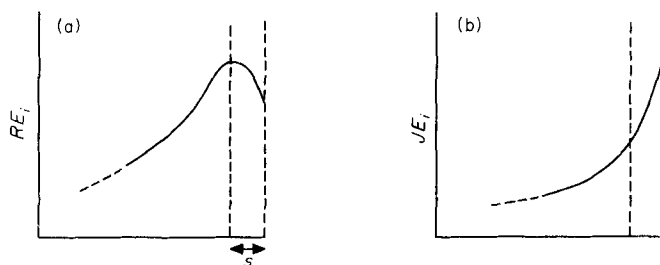


Figure 3. The in-phase hookover (a) and the out-of-phase enhancement (b) in the electric field in the ocean near the coast (see text).

Table 1 shows that we expect the observed magnetic field to be reduced in its vertical component and enhanced in its horizontal component. This is usually true except near the edges of the oceans or near any other sharp contrasts of electrical conductivity. Fig. 2 shows what happens near the edge of the ocean when it is modelled by an infinitely thin sheet electric current, J . The vertical magnetic field component becomes infinite at the coastline, indicating that we must expect a coast effect from this phenomenon if nothing else.

In fact the oceanic electric current can itself suffer an edge effect caused by induction. Hewson-Browne & Kendall (1973) have called this the in-phase hookover and the out-of-phase enhancement.

In addition there will be three other coastal effects: (1) for the so-called *H*-polarization mode of induction in which electromotive forces at right angles to the coastline drive electric currents inland through the conducting rocks; (2) for the so-called *E*-polarization mode of induction in which the oceanic electric currents induce image currents to flow in the conducting rocks beneath the ocean; and (3) physical differences associated with the coastline such as sedimentary rocks which create their own coastal effect. The first and second of these effects are shown in Fig. 4 and, along with the third effect, have inspired many calculations. (See for example: Schmucker 1970; Bailey 1977; Nicoll & Weaver 1977; Parker 1968; Weidelt 1971; Hewson-Browne 1973a; Quinney 1979; Green & Weaver 1978; Dawson & Weaver 1979; Cox & Filloux 1974; Camfield & Gough 1975.)

For very rapid variations, of periods 1 hr or less, a nearby ocean tends to become prominent in the data. The magnetic disturbance vector for periods of the order of one second tends to lie in a fixed plane and to point in a direction parallel to its line of greatest slope towards the ocean (Parkinson 1959, see also Lawrie 1965).

We therefore see that although most problems are global in character they will give rise to apparently local effects.

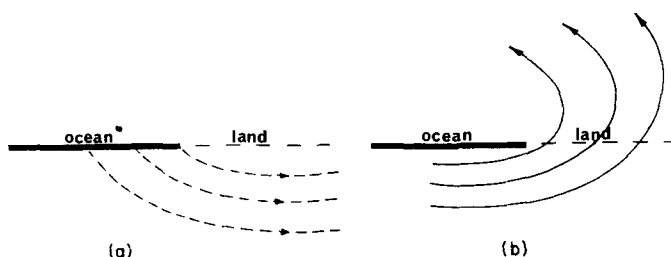


Figure 4. (a) Electric currents in the *H*-polarization mode. (b) Magnetic field in the *E*-polarization mode.

3 New methods for simple models

In view of the difficulty of the problem progress has been possible only through mathematical modelling. The approximation of taking an infinitely conducting sphere $r = a$ as a model, where a is the Earth's radius, has been known for 50 yr or so. Indeed, the skin effect was well established in physics by that time (see Jordan 1982). The model consisting of a uniform spherical shell at $r = a$, of finite electrical conductivity, dates back to Chapman & Whitehead (1922). However, even the simplest advance on this model, such as the infinitely conducting, isolated spherical cap, is very difficult mathematically. Rikitake (1961) and Ashour (1965) dealt with the hemispherical cap on $r = a$ as a model for the Pacific Ocean though the former also included an infinitely conducting sphere at $r = b$. They successfully explained features of the data which had been noticed by Parkinson (1959). Rikitake provided the model and Ashour an exact solution. Unfortunately, further progress proved to be slow until the arrival of modern electronic computers.

The most advanced model used to date has been a realistic ocean in contact with a spherically stratified conducting globe. Throwing everything at the computer in this way does, however, create its own difficulties. The model with which most progress has been made consists of oceans of realistic shape and surface conductivity on the sphere $r = a$. Everything else has zero electrical conductivity except for a sphere $r = b$, $b < a$, whose conductivity is infinite. This model, used by Rikitake (1961), and inspired by the work of Price, envisages a rapid increase in electrical conductivity at a depth of about 500 km (Lahiri & Price, 1939), see Fig. 5. The model allows one, by using the well-known method of images, to approximate the currents induced in the mantle.

Suitable equations have been developed by Bullard & Parker (1970) and solved by them using the Legendre coefficients of Benkova (1940) for S_q variations which have period 24 hr. However, the method used to solve the equations breaks down at periods below a critical value of about 14 hr (for an ocean of depth 4 km). The paper by Bullard & Parker sets a bench mark for subsequent calculations. In Fig. 6 we show calculations by Beamish *et al.* (1980a,b) which reproduce the earlier results by an independent method. The diagram shows the in-phase and out-of-phase parts of the electrical currents flowing in the oceans. Their global nature is evident. It is also clear that a station on the coast would require detailed calculations involving local bathymetry before the magnetic effects of the ocean's electrical currents could be computed there. The global versus local nature of the induction problem is amply illustrated.

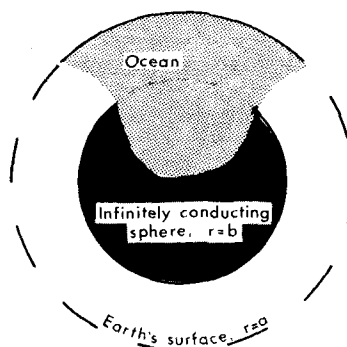


Figure 5. A realistic model ocean on $r = a$ above an infinitely conducting sphere $r = b$. The conductivity for $b \leq r \leq a$ is taken to be zero. (Diagram taken from Beamish *et al.* 1980a).

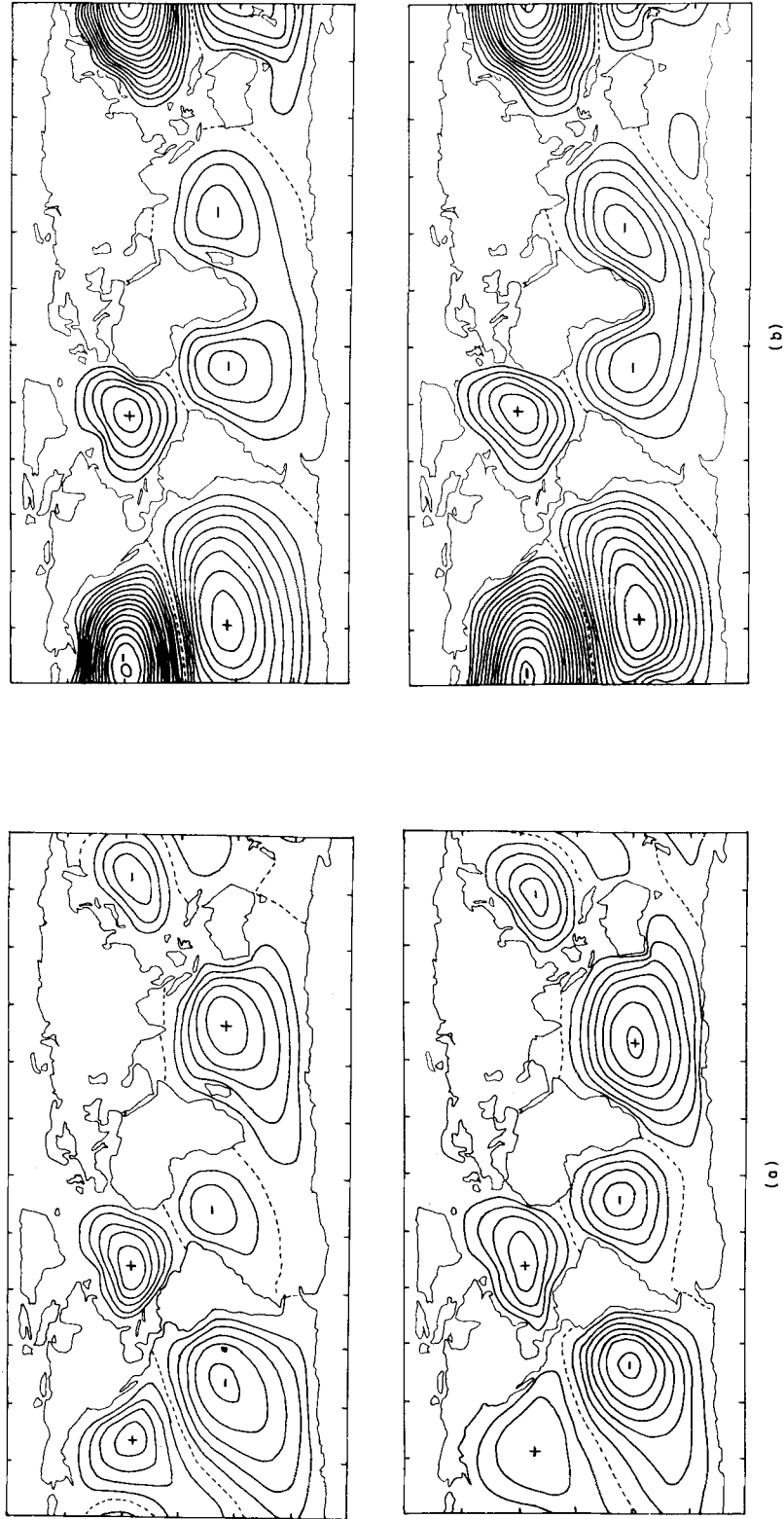


Figure 6. The Bullard-Parker calculations of the 24 hr component of the electric currents induced in the oceans by the S_q variations compared with recalculations by Beamish *et al.* (1980a). Contour interval 1000 A. (a) In-phase part. (b) Out-of-phase part.

To extend the Bullard & Parker calculations to higher frequencies required the implementation of methods new to geomagnetism. These methods are best explained for the general reader by using examples. In fact the geomagnetic equation we wish to solve looks like

$$(1 + i\omega\mathcal{L})f = g, \quad (1)$$

where g is known, f is to be found, $i = \sqrt{-1}$, and \mathcal{L} is some mathematical operator. The required solution of (1) may then, at least formally, be written as

$$f = (1 + i\omega\mathcal{L})^{-1}g, \quad (2)$$

which is equivalent to

$$f = g + \sum_{n=1}^{\infty} (-i\omega\mathcal{L})^n g, \quad (3)$$

provided the infinite series converges. The radius of convergence ω_{crit} , in the complex ω -plane corresponds to the critical period of about 14 hr mentioned earlier.

It might be thought, by the unwary, that an approximate solution for equation (1) for large values of ω could be obtained by solving

$$\mathcal{L}f = g/(i\omega).$$

Unfortunately, this corresponds to the case of infinite electrical conductivity and the solution, f , is infinite at the coastline. For finite conductivities such a result is physically impossible, so this solution cannot be used as a starting point for an expansion in powers of ω^{-1} . Therefore, in general for $\omega > \omega_{\text{crit}}$ we shall not find it possible to expand the solution as a convergent power series in either ω or ω^{-1} , as was originally suggested by Price (1949). This implies that the standard iterative methods will fail to converge, as various authors have discovered.

The clue to a different method lay in a comment by Parker (1968) that analytic continuation in the complex ω -plane would be required to continue his work further. The method to use has been attributed to Sommerfeld and was introduced to geomagnetism by Hutson, Kendall & Malin (1972) and Hewson-Browne *et al.* (1973) as *the method of shifting the spectrum*. Because of its sound physical basis it is worthwhile explaining and using to solve equation (1). See Kendall (1978) for other references and information. The standard Neumann method for solving (1) is to generate a sequence of approximations $f_0, f_1, f_2, \dots, f_n, f_{n+1}, \dots$ where f_0 is chosen as conveniently as possible (often f_0 is put equal to zero) and where

$$f_{n+1} = g - i\omega\mathcal{L}f_n. \quad (4)$$

However, if $\omega > \omega_{\text{crit}}$ this iteration will not converge. Nevertheless, it can be shown that there exists a constant α such that the iteration

$$(1 + i\omega\alpha)f_{n+1} = g + i\omega(\alpha f_n - \mathcal{L}f_n) \quad (5)$$

does converge. [We have only added the latest approximation for $i\omega\mathcal{L}f$ to each side of (4)]. It seems intuitively obvious that α can be chosen so that $\alpha f_n - \mathcal{L}f_n$ is reasonably small. This can be proved rigorously for simple configurations and inferred to be so for realistic oceans. Although the only accurate computations have been for a spherical cap the method is still applicable for general use. We also strongly recommend using the later method of Hewson-Brown (see Hewson-Browne & Kendall 1981 and Hewson-Browne 1981), which replaces

equation (4) by

$$f_{n+1} = \frac{gf_n}{(f_n + i\omega \mathcal{L}f_n)}. \quad (6)$$

Using a good initial guess this iterative procedure will often converge in very few iterations. This is illustrated by Fig. 7.

In spite of the elegance of the functional analysis methods and some controversy arising from the method of shifting the spectrum the best approach to the simple model has arisen from physical intuition. In a series of papers Hewson-Browne & Kendall (1978, 1980, papers I and II), Hewson-Browne (1978, paper III) and Beamish *et al.* (1980a, b, papers IV and V) a very simple approach has been developed. The source magnetic field, in the model used in this section, can pass *beneath* the oceans. The magnetic potential Ω_∞ of the source field on the coast can be regarded as driving the magnetic flux underneath the oceans between the spheres $r=a$ and $r=b$, but as $b \approx 0.9a$ the magnetic induction vector \mathbf{B} is nearly horizontal for $b \leq r \leq a$. In an obvious notation $\mathbf{B} \approx \mathbf{B}_H = \nabla_H^2 \Omega$, where Ω is the magnetic potential. As $\text{div } \mathbf{B} = 0$ we have

$$\nabla_H^2 \Omega = 0 \quad (7)$$

between the oceans on $r=a$ and the sphere $r=b$ with $\Omega = \Omega_\infty$ on the coastline. Under such conditions Ω may be found by solving equations such as (7) for realistic oceans with a fine two-dimensional mesh on $r=a$. Comparison with Bullard & Parker's results proves to be extremely good and is shown in Fig. 6. In Fig. 8 we show the induced effects of S_q for a globe comprising five land masses for an S_q component with period of 12 h. For this case the frequency is just beyond the circle of convergence of the straightforward power-series expansion in powers of ω . Moreover, in this series of papers the authors have combined a full, modern set of S_q coefficients as derived by Malin & Gupta (1977) with a 2° square mesh.

This concludes the state of the art summary for the simple model in which the oceans are electrically isolated from the land. Other global calculations for an ocean isolated from the mantle have been carried out by Hobbs & Dawes (1979). They used the method of shifting the spectrum (rediscovered and renamed) to solve the equations for a considerably restricted

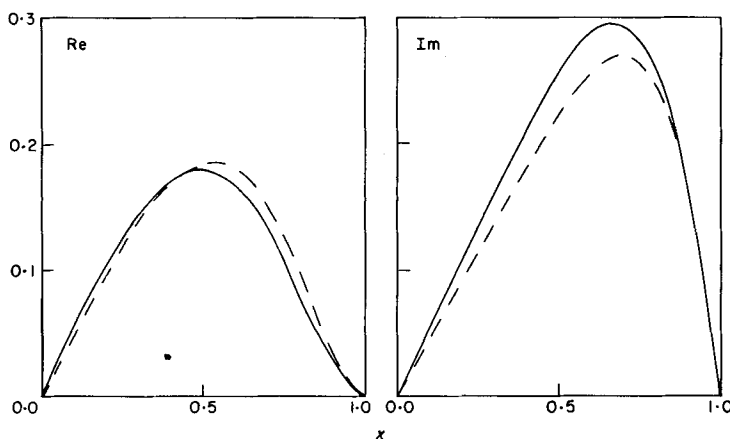


Figure 7. The convergence of the Hewson-Browne (1981) method for a surface current J in a strip of finite conductivity. The first iterate is shown as a broken line. The second iterate is indistinguishable from the exact solution which is shown as a solid line. Both real and imaginary parts are given.

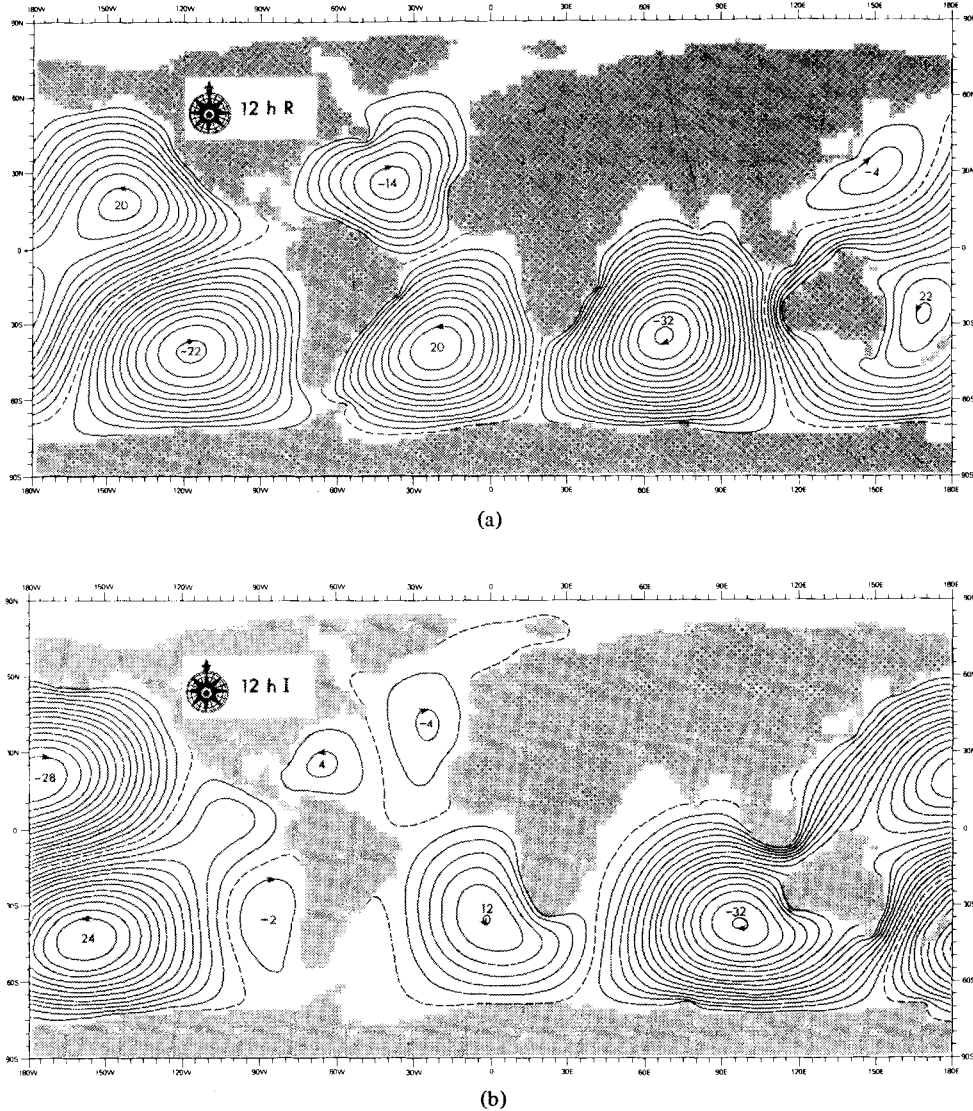


Figure 8. From Beamish *et al.* (1980b), showing for a 12 hr period (a) in-phase and (b) out-of-phase parts of the induced currents. The interval between the contours is 2000 A.

problem comprising a uniform ocean, on a $5^\circ \times 5^\circ$ mesh, using only three S_q coefficients and two land masses. It should be recognized that although a full solution of the problem may be attempted all that has been obtained in practice corresponds to the 'outer solution' of papers I to V mentioned above, which is much easier to obtain. Similar remarks carry forward to the next section.

4 New methods of difficult models

Allowing the oceans to be in electrical contact with a conducting sphere of non-zero electrical conductivity creates a much more difficult type of model. There are at present two successful methods for resolving these complexities, the Biharmonic Green's Function

method and the Multiple Integral Equation Method. Both methods involve Green's functions and can be made to work fairly easily for the simple models of the previous section; it is somewhat arbitrary to put them in this separate section. These methods both constitute ways of getting around the obstacle that the electrical contact between the oceans and the mantle expresses itself through the presence of radial derivatives in the main surface equations. These radial derivatives may be expressed as integrals by utilizing Green's functions, a technique which has been used to great effect by Weaver and his colleagues for a variety of plane configurations (see Dawson 1983).

4.1 THE BIHARMONIC GREEN'S FUNCTION METHOD

As a suitable analogy we shall illustrate the method by solving the equation

$$\frac{\partial f}{\partial r} + \nabla_H^2 f = g, \quad (8)$$

on the surface of the Earth, where f is to be found and g is a given function. We take the radius of the Earth as the unit of length. Let C_n^m and S_n^m be the cosine and sine surface harmonics normalized so that the integrals of their squares over the Earth's surface are equal to one. Then

$$\nabla_H^2 C_n^m = -n(n+1) C_n^m \quad (9)$$

and

$$\nabla_H^2 S_n^m = -n(n+1) S_n^m.$$

We may define the Fourier–Legendre coefficients of any quantity ψ , say, by

$$\psi = \sum_{n=0}^{\infty} \sum_{m=0}^n [\psi_{mn}^c C_n^m + \psi_{mn}^s S_n^m]. \quad (10)$$

Fainberg & Zinger (1980) observe that above a mantle whose conductivity is radially symmetric we may define a response function β_n independent of m such that

$$\partial f_{mn}^{c,s} / \partial r = -\beta_n f_{mn}^{c,s}. \quad (11)$$

Then using equations (9) and (11), from (8) we obtain

$$f_{mn}^{c,s} = -g_{mn}^{c,s} / [n(n+1) + \beta_n]. \quad (12)$$

Reconstructing f gives

$$f = - \iint_{S'} G g' dS' \quad (13)$$

where

$$G = \sum_n \sum_m (C_n^m C_n'^m + S_n^m S_n'^m) / [n(n+1) + \beta_n]. \quad (14)$$

Here the prime denotes quantities evaluated in the primed integration variables. For example,

$$C_n^m C_n'^m + S_n^m S_n'^m = P_n^m(\cos \theta) P_n^m(\cos \theta') \cos m(\phi' - \phi), \quad (15)$$

where P_n^m is the associated Legendre coefficient, and

$$dS' = \sin \theta' d\theta' d\phi'.$$

The expression (14) represents the Biharmonic Green's function for the operator on the right-hand side of equation (8). The presence of $n(n+1)$ in the denominator of the terms in the series (14) will help its convergence. The complex constants β_n must be calculated for the model of the mantle being used.

In fact, for convenience, equation (8) has been over-simplified. There is more than one unknown, and a more realistic equation on which to illustrate the method of Fainberg & Zinger would be a vector equation written as a pair of scalar equations such as

$$\begin{aligned} \frac{\partial f_1}{\partial r} + \nabla_H^2 f_1 + f_2 &= g_1 \\ \frac{\partial f_2}{\partial r} + \nabla_H^2 f_2 - f_1 &= g_2, \end{aligned} \quad (16)$$

where f_1 and f_2 are to be found and g_1 and g_2 are known. Applying equation (13) to the pair of equations (16) gives a coupled pair of integral equations

$$f_1 - \iint_{S'} G f_2' dS' = - \iint_{S'} G g_1' dS' \quad (17)$$

and

$$f_2 + \iint_{S'} G f_1' dS' = - \iint_{S'} G g_2' dS'.$$

Thus equations (16) have been converted to a pair of integral equations. It must be noted that Fainberg & Zinger's equations are much more complicated than the above. Even the set corresponding to the coupled integral equations (17) is more difficult. Fainberg & Zinger adopt an iterative method of solution and prove that such a method will converge under certain circumstances. In 1981 they validated their method on a model similar to that of the previous section using a uniform inducing field with a period of 1 hr (Fainberg & Zinger 1981).

4.2 THE MULTIPLE INTEGRAL EQUATION METHOD

Weaver (1979) has shown how equations such as (8) and (16) may be converted to integral equations by using the Green's Function $\exp(-kR)/R$ for the operator $(\nabla^2 + k^2)$ though he has confined his attention to planar problems. Hewson-Browne & Kendall (1981) have used a similar technique, but find it useful to put an infinitely conducting sphere at $r = b$, where $b < a$. They also work in terms of six variables B_r, B_θ and B_ϕ for the magnetic field components and E_r, E_θ, E_ϕ for the electric field, together with their six radial derivatives, all evaluated on either the sphere $r = a$ or $r = b$. In principle the problem is reduced to solving six simultaneous integral equations in six unknowns. In an obvious notation the unknowns are the six components of B_H^b, B_H^a and E_H^a . Using the Hewson-Browne (1981) technique the integral equations are soluble by iteration, and although 13 surface integrals are required at each sweep, the convergence is rapid.

The technique is easy to explain. Consider, for example, a pair of integral equations such as (17), where the Green's function G is selective. Using the notation

$$\begin{aligned} f_1 - \mathcal{L}f_2 &= \alpha_1 \\ f_2 + \mathcal{L}f_1 &= \alpha_2, \end{aligned} \quad (18)$$

Then at the n th iteration we shall have that

$$\begin{aligned} [f_1]_{n+1} - [f_2]_{n+1} \mathcal{L}[f_2]_n/[f_2]_n &= \alpha_1 \\ [f_2]_{n+1} + [f_1]_{n+1} \mathcal{L}[f_1]_n/[f_1]_n &= \alpha_2. \end{aligned} \quad (19)$$

Providing we are willing to solve two equations in two unknowns at each step and choose $[f_1]_0 \neq 0$ and $[f_2]_0 \neq 0$ the Hewson-Browne iterative procedure (19) can be made to yield solutions; with a good initial guess convergence is very rapid. Equations (78), (79) and (80) of Hewson-Browne & Kendall (1981) provide such an initial estimate for the problem of induction. This method has been validated on a spherical cap problem.

4.3 REMARKS

The new methods described in this section are obviously high powered. They could as easily be applied to the isolated problems of the previous section or to the tidal problems of the next section, but with *no less work* than would be involved than in the electrically conducting case. At present there appears to be no alternative to these two methods.

5 Ocean tidal effects

The flow of conducting seawater across the Earth's magnetic field causes potential differences which drive electric currents through both sea and land. These potential differences are readily detected inland. Longuet-Higgins (1949) summarized early work in this field and formulated a useful model in which seawater flows with uniform velocity up a channel of elliptical cross-section. This was later used by Osgood, Rosser & Webber (1970) in further investigations of the electrical fields induced in the Channel. Brown & Woods (1971) have also detected the strong influence at Aberystwyth of the tides in Cardigan Bay. The electrical field tends to be dominated by local effects and it is chiefly to the magnetic field that we must look for the effects of tides in the deep oceans. The simple channel model has to be made more complicated to give any help, for otherwise it has no observable magnetic field inland or at sea. The electric currents induced in the deep ocean may also leak into the shallower seas. Calculations by Windle, Kendall & Gretton (1971) estimate this effect to be about 0.2 nT for the British Isles.

In the shallow seas the tidal velocity runs high, and it is not surprising that the dynamo effect is observed inland near the coast. The Bay of Fundy in Canada and the Channel deserve special mention as they are tidally resonant and have a large observable effect. An adequate survey has been given by Cochrane & Srivastava (1974). The electromagnetic effects of the deep sea tides may be masked by these local tides.

Measurements at sea or beneath the oceans are difficult and expensive. The lunar semi-diurnal component L_2 of the magnetic variation at sea over the boundary of the continental shelf was obtained by Hill & Mason (1962). Its amplitude of about 20 nT was obtained by subtracting the daily variation at Plymouth from observations at the buoy stations. Later, Larsen & Cox (1966) made observations of the magnetic field on the bed of the Pacific Ocean. Larsen's (1968) follow-up is distinguished for its comparison of these observations

with calculations for a model Kelvin wave passing up the Pacific coast of America. The equations of the problem were reduced to an integro-differential equation in one variable. This was then solved by finite difference techniques and direct matrix inversion. The comparison between theory and observation was good.

Chapman & Kendall (1970) ignored self-induction and used a channel model for the Atlantic Ocean as shown in Fig. 9. The tidal velocity was assumed to be non-uniform and thus produced a magnetic field inland (Kendall & Chapman 1970). They concluded that their results were consistent with the midnight values of L_2 separated earlier by Malin (1973) from the stronger ionospheric variations. Typical values are of the order of 1 or 2 nT. In fact, a channel model of the Atlantic was used again by Edwards, Law & White (1971) in their comprehensive study of geomagnetic induction in the British Isles.

In Sections 3 and 4 we noted that iterative methods of solution of the world wide ocean induction problem need special techniques when the period is less than 14 hr. As the main component of the ocean tide has a semi-diurnal lunar period of about 12.45 hr convergence is not straightforward; the tools for dealing with this problem have not been long available. Tidal electric currents are driven by the motion of seawater with velocity \mathbf{v} across the Earth's permanent magnetic induction field \mathbf{B} . Ohm's law then becomes

$$\mathbf{J} = \sigma (\mathbf{E} + \mathbf{v} \wedge \mathbf{B}), \quad (20)$$

where σ is the conductivity of the seawater and \mathbf{E} the electric field. Hewson-Browne (1973b) shows that for a tidal frequency ω the electric currents will be the same as those induced by a magnetic field B_n normal to the ocean and of magnitude

$$B_n = -\text{curl}_n (\mathbf{v} \wedge \mathbf{B}) / i\omega, \quad (21)$$

in a self-evident notation. So the ocean tidal induction problem is basically of the same type as that of induction by an external source.

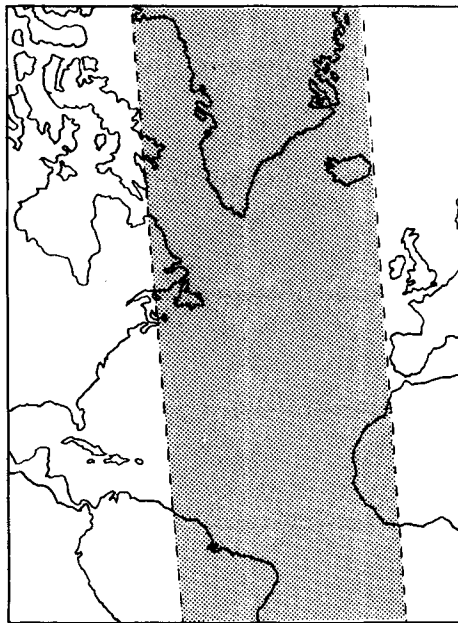


Figure 9. A model channel representing the Atlantic Ocean after Chapman & Kendall (1970).

The only application of advanced methods of solution to tidal induction problems have been by Hewson-Browne (1973b) to the simple models of Section 3 and by Hewson-Browne & Kendall (1981) to the difficult model of Section 4. The former applied the method of shifting the spectrum to a channel model of the Atlantic and obtained good qualitative agreement with Larsen's (1968) Pacific model, the latter used an oscillating spherical cap as a model to show that the Hewson-Browne (1981) method converged quickly for their more general set of equations.

It would appear that there is room for more theoretical work on the tidal induction problem.

6 Conclusion

Geomagnetic induction in the oceans is one subject where theory has a chance of catching up with experiment. The geophysical structure beneath the land and oceans is so obscure that in the end all the models used may prove to be incorrect, and only the theory left to deal with a corrected global model.

Both methods of Green's functions proposed by Fainberg & Zinger (1980) and Hewson-Browne & Kendall (1981) seem to be preferred directions in which to advance. The former may well be speeded up by using the method of Hewson-Browne (1981); while the latter would not work very well without it. As most methods are likely to reduce to operator equations of some kind we may, without prejudice, point the reader in the direction of the Hewson-Browne technique. It is also probably worth considering the method of shifting the spectrum (Hutson *et al.* 1972; Kendall 1978).

For simple problems in which the mantle is isolated from the oceans, the methods used in papers I–V (see earlier) offer the best chance of combining fine resolution with a full array of S_q Legendre coefficients. The diagrams reproduced herein used the full set derived by Malin & Gupta (1977).

No-one has satisfactorily calculated the magnetic field of the electric currents flowing in realistic oceans. The attempt by Hobbs & Dawes (1979) shows some of the difficulties. The inland spread of the magnetic field arising from oceanic currents has been discussed by Hobbs (1981). In fact the effect of the ocean on the induced magnetic field will extend inland to an approximate distance of either one skin depth of the mantle or the depth at which the conductivity begins to increase: whichever is the less. Even then the stronger effects will be found in B_z , in so far as it is important to distinguish that component. The skin depth may be as large as 1000 km, but at that distance from the edge of the continental shelf any edge effect from the ocean will be somewhat diminished. Complicated regions involving shallow seas, such as those around the British Isles, fall well within the expected edge effect zone but their complex conductivity structure hinders simple interpretation. The edge effect upon the horizontal magnetic field component is less than that upon B_z . Thus, one ought to sort out those stations lying on the coast and use their B_z component. As a corollary, one would not expect the induced S_q variation on the land to have a great deal to do with that over the ocean. The analysis of global data has more to do with the conductivity structure of the mantle onshore: analysis of particular results from coastal stations is likely to tell us more about oceanic induction than would general analyses. Calculations from present models would then serve as the boundary conditions for the local model. In the models of Section 3 any stations far inland would see only a smooth sphere beneath them and the models are therefore inappropriate for such stations. Models with no ocean and a radially varying conductivity distribution would be better for such inland stations. Indeed, if such models are generated as an inversion of surface data they have a well-defined 'fit'.

The literature contains a number of confusing geophysical analyses. The global analysis of Lahiri & Price (1939) deduced from S_q and storm variations formed the classical view of a low conductivity of $10^{-3} \text{ mho m}^{-1}$ to a depth of about 500 km with a considerable rapid increase beyond. This model led Chapman & Kendall (1970) to a qualitative evaluation of the effects of tides in the Atlantic on the magnetic field at stations within the British Isles. Furthermore, it is in essence the model used by Larsen (1968) with such success. Against these must be set the conductivity profile derived by Parker (1970) and Larsen (1975) which obtain a different conductivity profile by inversion techniques. Their mantle conductivities do not seem to be entirely compatible with Price-type models.

Paradoxically, when considering the magnetic effects of the ocean tides, a higher value for the mantle conductivity might hinder the flow of electric currents beneath the land. One might argue that the skin effect would constrain the electric currents to short-circuit through the more highly conducting oceans. On the other hand it is possible to argue that the geometry of the non-conducting atmosphere of the Earth would carry the magnetic field of the ocean tides far inland. This much is certain, that Malin (1973) has deduced magnetic sea-tide effects at great distance inland and that Larsen (1980) has observed electric sea-tidal effects at Tucson, which is far enough inland. It would seem that some effect is carrying the sea-tidal magnetic field long distances inland from the oceans. This would be compatible with Price-type models for the electrical conductivity of the mantle. It is clear that further work on global models is desirable.

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