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Riccardo Marino · Patrizio Tomei
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Induction Motor Control Design

Springer

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To Rebecca and Rosanna

R. Marino

To my family and the memory of my parents

P. Tomei

To Paola, my family and the memory of my grandparents

C.M. Verrelli

Series Editors' Foreword

The series *Advances in Industrial Control* aims to report and encourage technology transfer in control engineering. The rapid development of control technology has an impact on all areas of the control discipline. New theory, new controllers, actuators, sensors, new industrial processes, computer methods, new applications, new philosophies…, new challenges. Much of this development work resides in industrial reports, feasibility study papers and the reports of advanced collaborative projects. The series offers an opportunity for researchers to present an extended exposition of such new work in all aspects of industrial control for wider and rapid dissemination.

Over recent years there has been considerable interest in trying to understand and quantify the potential benefits that nonlinear control could bring to industrial applications. One obstacle to the widespread use of nonlinear control has been the issue of finding appropriate nonlinear system models easily. This obstacle is commonly avoided by finding a linear model of limited validity and then designing a robust control able to deliver satisfactory system performance for a wider range of system parameter variations. Finding analytical dynamical nonlinear models for routine industrial application to allow the straightforward development of nonlinear control designs has been a little more problematic; however, there are some industrial areas, such as electrical machines, marine systems, and chemical processes, where nonlinear system models are more readily available for use in nonlinear control designs.

In the field of chemical processes, K.M. Hangos, J. Bokor, and G. Szederkényi exploited nonlinear system models and used nonlinear control techniques in their textbook *Analysis and Control of Nonlinear Process Systems* (ISBN 978-1-85233-600-4, 2004) that was published in our related series: *Advanced Textbooks in Control and Signal Processing*. Marine systems is another field where there are well-known nonlinear models, and researchers K. D. Do and J. Pan have recently developed a whole series of new nonlinear control algorithms for the different control tasks demanded of such systems. A comprehensive presentation of their work can be found in the *Advances in Industrial Control* monograph *Control of Ships and Underwater Vehicles* (ISBN 978-1-84882-729-5, 2009).

This *Advances in Industrial Control* monograph by R. Marino, P. Tomei and G.M. Verrelli is devoted to the control of induction motors from the industrial field of electrical machines. In the monograph the authors report the systematic application of nonlinear control techniques to develop a sequence of sophisticated control algorithms. The key facilitator in this development is the availability of a set of analytical dynamical state space models for induction motor behavior. The authors then exploit the structure of these models in a variety of ingenious ways to develop the increasingly complex nonlinear control algorithms. Despite the closely argued theoretical presentation in the monograph, the basic outline of the approach taken should be easily recognizable to any industrial engineer familiar with the modern control paradigm, namely:

- modelling in this case basic electrical equations leading to nonlinear state space models (Chapter 1);
- open-loop inverse model-based control (Chapter 1);
- feedback control based on a full state vector, including states that are unmeasurable (Chapter 2);
- observers to reconstruct unmeasurable states and observers to estimate uncertain system parameter values (Chapter 3). State observers driven by measurable outputs will facilitate output feedback designs and parameter observers will facilitate adaptive control designs;
- general output feedback control designs (Chapter 4); and
- specialized output feedback designs in this case speed sensorless feedback control (Chapter 5).

The authors use this agenda for induction motor control, carefully absorbing more and more realistic practical assumptions to develop increasingly general control algorithms. At each step of the way, useful validating simulation results are presented. These use the same induction motor parameters so it is possible to compare results within and indeed across chapters as the various control schemes evolve. Surprisingly, not too many different analysis tools from Lyapunov and nonlinear control theory are used in the development and the context and explanation of those that are used can be found in two useful reference appendices.

The *Advances in Industrial Control* monograph series has not seen many entries that present a wholly nonlinear viewpoint for industrial control system design so it is a pleasure to welcome this exhaustive volume by R. Marino, P. Tomei and G.M. Verrelli to the series. The authors have stated that one of their aims in writing this monograph was to give a unified presentation of these nonlinear induction motor controls that subsumes and archives the last thirty or more years of development since the engineers at Siemens (1971) and Toshiba (1980) first developed the nonlinear control method called the direct field-oriented control algorithm. Also threaded into the volume is their own significant research contribution to the field. Clearly, the monograph will be of great interest to electrical and control engineers working in the electrical machines field. Academics, postgraduate students and researchers working in the nonlinear control paradigm will also find new inspiration from the work of the authors and much transferable knowledge for tackling nonlinear control problems in other industrial applications fields.

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Preface

The control of induction motors has attracted much attention from researchers and engineers since 1971. More than 4,000 journal papers have been published on induction motor control and more than 500 specifically on the adaptive control of induction motors: it is still a very active research area since more than 300 journal papers appeared in 2008. The industrial interest in induction motor control is documented by over 80,000 patents on this subject. The availability of low cost powerful digital signal processors and significant advances in power electronics motivated the design of complex induction motor controls. The aim is to achieve the same, or even superior, performance on speed tracking and power efficiency which are obtained by more sophisticated and expensive, but less reliable, electric motors such as direct current or permanent magnet ones. Direct current motors are extensively used in variable speed applications since their flux and torque are independently controlled by the field and the armature current. However, they have disadvantages due to the mechanical commutator and the brushes so that they are limited in high-speed, high-voltage operating conditions. Induction motors are much more difficult to control but have definite advantages since they have no commutator, no brushes, no rotor windings in squirrel cage motors, they have a simple rugged structure, can tolerate heavy overloading, and can produce higher torques with a lower weight, smaller size, and lower rotating mass than direct current motors.

The design of control algorithms for induction motors is, however, very complex for many reasons. It is a multivariable control problem since there are two independent control inputs and two outputs to be controlled: the primary output is the rotor speed to achieve the required dynamic performance, while the secondary output is the rotor flux modulus for power efficiency maximization. It is an intrinsic nonlinear problem since the electromagnetic torque, which controls the rotor speed, is a nonlinear function of stator currents and rotor fluxes, and the operating conditions of interest are away from the equilibrium points so that linear approximation techniques do not apply. It involves parameters such as load torque and rotor resistance which may vary widely during operation; they are critical in the control design and should be identified online to maximize power efficiency. The control design cannot rely on state variable feedback since rotor flux measurements are not easily available. If rotor speed is not measured in order to reduce costs or due to sensor failures and only stator currents and voltages are available from measurements, the control problem is called speed-sensorless. In this case the desired reference signals for rotor speed and flux modulus are to be tracked in spite of parameter perturbations, while both tracking errors are not available for feedback to the controller. The feedforward control which solves the tracking problem in open-loop may be explicitly obtained by computing the induction motor nonlinear inverse dynamics. The stability of the resulting open-loop controlled motor is, however, not always guaranteed since it depends both on the reference trajectories to be tracked and on motor parameters. Even in stable operating conditions the dynamic responses may be unsatisfactory and poorly damped. Hence, feedback from stator currents, and from rotor speed when available, has the goal of enhancing both stability and robustness with respect to parameter perturbations; moreover, it should improve transient behaviors and power efficiency. This book is focused on the nonlinear feedback control design techniques, including adaptive ones, which are required to achieve high speed tracking performance along with high power efficiency in induction motor control.

Besides its technological motivations for electric traction and electric drives, the control of induction motors has an intrinsic interest from the view point of nonlinear control theory, since it involves clearly modeled nonlinear terms such as electromagnetic torque and two critical parameters; the appropriate tools belong to the theory of adaptive output feedback for multi-input, multi-output nonlinear systems. Such a theory started to be developed in 1992 for special classes of single-input, single-output nonlinear systems but it does not encompass the induction motor models. Hence, the control of induction motors constitutes a very interesting case study which evolved into a benchmark nonlinear control problem. In fact, most of the fundamental concepts of nonlinear control theory apply in a nontrivial way. Induction motors are not feedback linearizable by static state feedback but they are feedback linearizable by a dynamic state feedback. It is enough to add one integrator to achieve feedback linearization; this can be done in many different ways even though they all lead to singularities that make it inadvisable to render the closed-loop linear in all operating conditions. Induction motors are input–output feedback linearizable but the input–output feedback linearizing control, which makes the rotor flux angle unobservable, is singular when the rotor flux is zero and it is not power efficient at low rotor flux levels. Induction motors are observable from rotor speed and stator current measurements so that flux observers, including adaptive ones, can be designed. Observer-based output feedback controls can also be designed using Lyapunov techniques. The steady-state dynamics of induction motors are very intriguing: in the case of constant reference signals for rotor speed and rotor flux modulus, they constitute a limit cycle in the state space where the rotating speed of the flux vector is equal to the sum of the desired rotor speed and the so-called slip speed, which depends on the load torque, the rotor resistance, and the flux modulus. In the more general case of bounded reference signals the steady-state dynamics may be very complex. They remain bounded but their stability and attractivity are in general difficult to study. In many cases the attractivity is not global and the stability is not exponential, depending on the reference signals and physical parameters, and instabilities or poor dynamic responses may arise.

Engineers at Siemens and Toshiba developed *ante litteram* in 1971 and 1980, respectively, nonlinear feedback control algorithms which are now called direct field-oriented control and indirect field-oriented control. At that time nonlinear control theory was just at its beginning: researchers were investigating basic controllability (1972) and observability (1977) properties. In fact, the proof that indirect field-oriented control is globally stable was published in 1996. Using today's terminology we can say that direct field-oriented control is an asymptotic state feedback linearizing control which has a singularity when rotor flux is zero, while indirect field-oriented control is a global dynamic output feedback control which has no singularities and allows the motor to start from any initial condition. Field-oriented controls were originally conceived for current-fed motors in which the stator currents can be controlled very rapidly by stator voltages, so that they may be considered control inputs by neglecting the stator currents dynamics; they were then extended to general induction motor models. During the 1980s new important tools in nonlinear state feedback design were developed: feedback linearization and input–output decoupling along with their adaptive generalizations. Good theories proved once again very useful in applications since they led to very innovative control algorithms for induction motors, with superior performance with respect to field-oriented controls. An adaptive feedback linearizing control with online identification of load torque and rotor resistance was developed in 1991. The goals of field-oriented controls and feedback linearizing controls are indeed very similar: they both use nonlinear feedback and nonlinear change of coordinates so that the feedback systems have a simpler structure.

Since both direct field-oriented control and feedback linearizing control make use of rotor flux signals there was a strong motivation to design rotor flux observers. At that time nonlinear observer theory was not fully developed. Nevertheless nonlinear observers for induction motors were designed in 1978: they were called bilinear observers. A complete theory for rotor flux observers, including observers with arbitrary rate of convergence, was successively developed. Adaptive flux observers were also designed which are adaptive with respect to rotor resistance, since rotor flux observers were found to be very sensitive with respect to rotor resistance variations. Identifiability questions naturally arose and were answered using the concepts of persistency of excitation and nonlinear observability. Since 1991 the problem of designing a global output feedback tracking control which does not require rotor flux measurements, is adaptive with respect to load torque and rotor resistance variations, and has no singularities was posed and finally solved in 1999 following the indirect field-oriented approach.

More recently, an important line of research was focused on the design of feedback control algorithms based on stator current measurements only. The absence of rotor speed measurements, which improves the reliability of the motor and reduces its cost, forced the redesign of those control algorithms which make use of rotor speed measurements in many crucial steps. The question itself of speed and rotor flux observability from stator current measurements is rather delicate and leads to the discovery of operating conditions in which observability fails: of course, the concept of nonlinear observability has to be used since the motor model is nonlinear. The study of identifiability of rotor resistance and load torque from stator current measurements leads to the discovery of persistently exciting reference signals for the flux modulus, which is required to be time-varying. Several speed sensorless control algorithms were recently developed which show superior performance with respect to inverse system based controls but are of course inferior to controls which make use of speed sensors.

At the present stage of research on induction motor control a coherent collection of estimation and control algorithms is available, including the most recent speed sensorless controllers. This book collects and discusses, using a unified notation and a modern nonlinear control terminology, the most important steps and issues in the design of estimation and control algorithms for induction motors. Many estimation and control algorithms are reported: their stability is analyzed and their performance is illustrated by simulations and experiments on the same induction motor. An intense and challenging collective research effort (which also involved at various stages the authors of this book) is carefully documented and analyzed, with the aim of providing and clarifying the basic intuition and tools required in the analysis and design of nonlinear adaptive feedback control algorithms. This material should be of specific interest to engineers who are engaged in the design of control algorithms for electric motors and, more generally, to a broader audience interested in nonlinear control design. In fact, induction motor dynamics are surprisingly rich and their control is challenging even to engineers with a strong nonlinear control background. The induction motor is an excellent source of projects, examples, and exercises for courses in nonlinear control design since they can be physically and experimentally tested. The book can be used for graduate courses on the control of induction motors and for independent study.

This book is divided into six chapters and two appendices. Since the stability of controlled induction motors is carefully analyzed throughout the book, the basic definitions and tools from Lyapunov stability theory are recalled in Appendix A. Since in many instances the basic concepts and tools from nonlinear control theory (nonlinear change of coordinates, observability, feedback linearization, input–output decoupling) are used, they are recalled in Appendix B. In Chapter 1, the modeling issues and the basic assumptions are discussed; moreover the structural properties of the models such as observability, parameter identifiability, linearizability, inverse dynamics and steady-state behaviors, including power losses minimization, are analyzed. Chapter 2 is devoted to state feedback control to explore the performance which can be obtained using full state variables measurements, and to examine those controllers which could tolerate the replacements of sensors by asymptotic observers and those which can be made adaptive with respect to uncertain parameters. The estimation of state variables, in particular rotor fluxes and rotor currents, and the identification of physical parameters such as load torque and resistances are discussed in Chapter 3: adaptive rotor flux observers are also presented. In Chapter 4 output feedback controls based on rotor speed, stator current, and stator voltage measurements are presented: some algorithms incorporate flux observers to improve performance, while the most complex algorithm is adaptive with respect to both torque load and rotor resistance. In Chapter 5 speed-sensorless output feedback controls which are based only on stator current measurements are discussed along with their adaptive versions. Chapter 6 contains some concluding remarks. All the control schemes are numerically simulated for the same motor with similar references to illustrate their performance, so that they can be compared: advantages and drawbacks of each scheme are pointed out. Some estimation and control algorithms are validated by experiments. Experimental tests are also presented which validate the motor model and the parameters used. The bibliography collects more than 200 journal papers and books on induction motor control from 1971 to 2009; it is, however, far from being complete and only contains all the material which was actually used during the preparation of this book. Finally, frequent exchanges of ideas and fruitful collaborations with Professor Sergei Peresada on induction motor control are acknowledged with pleasure.

Rome,
December 2009

Riccardo Marino
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Contents

Chapter 1
Dynamical Models and Structural Properties

Abstract Starting from the three physical stator and rotor windings, several state space dynamical models for the induction motor are introduced in this chapter. Each model clarifies specific dynamical properties. Their steady-state operating conditions are determined and analyzed: in particular the steady-state torque–speed characteristic curve is computed when sinusoidal voltages with constant amplitude and frequency are applied. This curve reveals many important nonlinear features: for instance, for a given load torque there may be two operating conditions, a stable one and an unstable one; they become closer and closer as the load torque increases up to a load torque bifurcation value. More generally, the dynamic inverse system is explicitly computed: it generates the voltage inputs which are required to track a desired time-varying rotor speed profile with the desired rotor flux modulus. The flux modulus parameterizes the control input: it may be chosen to minimize the power losses or to keep the voltage modulus constant or below a desired level (field weakening). The corresponding tracking dynamics are also computed: they determine limit cycles in the state space whose speed depends on the load torque and the desired rotor speed and flux. The structural properties of the motor from the control view point are studied: observability from stator currents and rotor speed measurements; observability from stator currents and rotor fluxes; observability from stator current measurements only; feedback linearizability, *i.e.* the possibility of transforming the motor model into a linear and controllable system by state feedback (either static or dynamic), which implies the controllability property; the identifiability, from different set of measurements, of critical parameters such as load torque and rotor resistance which may vary during operations. The induction motor turns out to be feedback linearizable by a dynamic state feedback; it is observable for any voltage input if stator currents and rotor speed are measured but it is not observable if only stator currents are measured and rotor speed and rotor fluxes are kept constant.

1.1 Modeling Assumptions

Consider a two-pole, three-phase symmetrical induction machine (see Figure 1.1). The stator windings are assumed to be identical with resistance R_s and equivalent

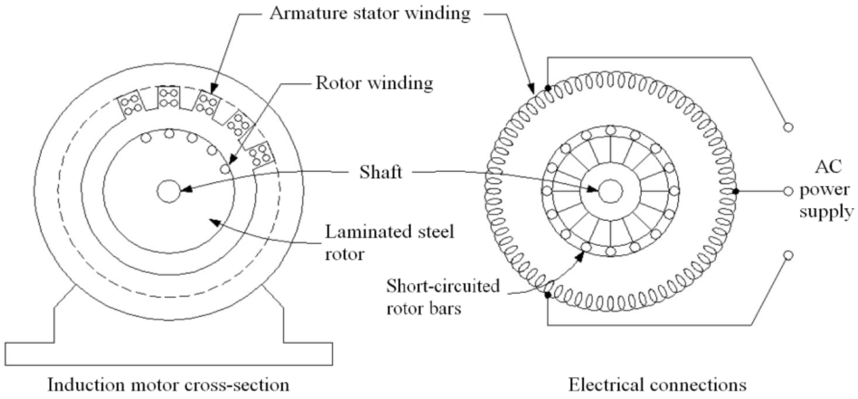

Fig. 1.1 Three-phase induction motor

turns N_s. The rotor windings are also assumed to be identical with resistance R_r and equivalent turns N_r. The air gap is assumed to be uniform. Stator and rotor windings are assumed to be approximated as sinusoidally distributed windings. The angle δ represents the rotor position with respect to the stator. We assume that the induction machine is operated as a motor, that is the rotor speed

$$\omega = \frac{\mathrm{d}\delta}{\mathrm{d}t}$$

and the load torque T_L have opposite signs. The rotor windings are short circuited while the stator windings are connected to a balanced three-phase source. When a balanced three-phase current is flowing in the stator windings an air gap magneto-motive force rotates about the air gap at a speed determined by the frequency of the stator currents and the number of poles. If the speed of the rotating magneto-motive force is different from the rotor speed, balanced three-phase currents will be induced in the short circuited rotor windings; the names of induction motor or asynchronous motor are due to this physical principle. The difference between the speed of the rotating magneto-motive force due to stator currents and the speed of the rotor determines the frequency of the induced rotor currents. If this speed difference is zero, that is the rotor rotates at the same speed as the magneto-motive force, no rotor currents are induced. Let

$$\Psi_s = [\psi_{s1}, \psi_{s2}, \psi_{s3}]^{\mathrm{T}}$$

$$\Psi_r = [\psi_{r1}, \psi_{r2}, \psi_{r3}]^{\mathrm{T}}$$

be the vectors whose components are the stator and rotor flux linkages, respectively, with $1,2,3$ denoting the three phases. Similarly, let

$$I_s = [i_{s1}, i_{s2}, i_{s3}]^{\mathrm{T}}$$
$$I_r = [i_{r1}, i_{r2}, i_{r3}]^{\mathrm{T}}$$

be the vectors whose components are the stator and rotor currents. Then, for an induction motor with one pole pair, we can write

$$\begin{aligned}
\frac{\mathrm{d}\psi_{s1}}{\mathrm{d}t} + R_s i_{s1} &= u_{s1} \\
\frac{\mathrm{d}\psi_{s2}}{\mathrm{d}t} + R_s i_{s2} &= u_{s2} \\
\frac{\mathrm{d}\psi_{s3}}{\mathrm{d}t} + R_s i_{s3} &= u_{s3} \\
\frac{\mathrm{d}\psi_{r1}}{\mathrm{d}t} + R_r i_{r1} &= 0 \\
\frac{\mathrm{d}\psi_{r2}}{\mathrm{d}t} + R_r i_{r2} &= 0 \\
\frac{\mathrm{d}\psi_{r3}}{\mathrm{d}t} + R_r i_{r3} &= 0
\end{aligned} \tag{1.1}$$

where the stator and rotor fluxes, under the assumption of linear magnetic circuits, satisfy the linear relation

$$\begin{bmatrix} \Psi_s \\ \Psi_r \end{bmatrix} = \begin{bmatrix} l_s & l_{s,r} \\ l_{s,r}^{\mathrm{T}} & l_r \end{bmatrix} \begin{bmatrix} I_s \\ I_r \end{bmatrix} \tag{1.2}$$

with

$$\begin{aligned}
l_s &= \begin{bmatrix} l_{sl} + l_{sm} & -\frac{l_{sm}}{2} & -\frac{l_{sm}}{2} \\ -\frac{l_{sm}}{2} & l_{sl} + l_{sm} & -\frac{l_{sm}}{2} \\ -\frac{l_{sm}}{2} & -\frac{l_{sm}}{2} & l_{sl} + l_{sm} \end{bmatrix} \\
l_{s,r} &= l_{sr} \begin{bmatrix} \cos(\delta) & \cos\left(\delta + \frac{2}{3}\pi\right) & \cos\left(\delta - \frac{2}{3}\pi\right) \\ \cos\left(\delta - \frac{2}{3}\pi\right) & \cos(\delta) & \cos\left(\delta + \frac{2}{3}\pi\right) \\ \cos\left(\delta + \frac{2}{3}\pi\right) & \cos\left(\delta - \frac{2}{3}\pi\right) & \cos(\delta) \end{bmatrix} \\
l_r &= \begin{bmatrix} l_{rl} + l_{rm} & -\frac{l_{rm}}{2} & -\frac{l_{rm}}{2} \\ -\frac{l_{rm}}{2} & l_{rl} + l_{rm} & -\frac{l_{rm}}{2} \\ -\frac{l_{rm}}{2} & -\frac{l_{rm}}{2} & l_{rl} + l_{rm} \end{bmatrix} .
\end{aligned} \tag{1.3}$$

In (1.3) l_{sl} denotes the leakage inductance of the stator windings, l_{sm} denotes the magnetizing inductance of the stator windings, l_{rl} denotes the leakage inductance of the rotor windings, l_{rm} denotes the magnetizing inductance of the rotor windings, and l_{sr} denotes the amplitude of the mutual inductance between stator and rotor windings. Neglecting iron losses we set $l_{sl} = 0$ and $l_{rl} = 0$ so that (1.3) becomes

$$l_s = l_{sm} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}$$
$$l_r = l_{rm} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} .$$

Denote by

$$L_s = \frac{3}{2} l_{sm}$$
$$L_r = \frac{3}{2} l_{rm}$$
$$M = \frac{3}{2} l_{sr}$$

the stator, rotor, and mutual inductances, respectively. Hence, from (1.3) we have

$$l_s = \frac{2}{3} L_s \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix}$$
$$l_{s,r} = \frac{2}{3} M \begin{bmatrix} \cos(\delta) & \cos\left(\delta + \frac{2}{3}\pi\right) & \cos\left(\delta - \frac{2}{3}\pi\right) \\ \cos\left(\delta - \frac{2}{3}\pi\right) & \cos(\delta) & \cos\left(\delta + \frac{2}{3}\pi\right) \\ \cos\left(\delta + \frac{2}{3}\pi\right) & \cos\left(\delta - \frac{2}{3}\pi\right) & \cos(\delta) \end{bmatrix}$$
$$l_r = \frac{2}{3} L_r \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} . \tag{1.4}$$

When the motor is operating in balanced conditions we have the constraints

$$\begin{aligned} i_{s1} + i_{s2} + i_{s3} &= 0 \\ i_{r1} + i_{r2} + i_{r3} &= 0 \\ u_{s1} + u_{s2} + u_{s3} &= 0 \end{aligned} \tag{1.5}$$

and, therefore, it is convenient to introduce the new variables

$$\begin{bmatrix} i_{s0} \\ i_{sa} \\ i_{sb} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{s1} \\ i_{s2} \\ i_{s3} \end{bmatrix} \triangleq U \begin{bmatrix} i_{s1} \\ i_{s2} \\ i_{s3} \end{bmatrix} \tag{1.6}$$

in which U is a unitary matrix ($U^{-1} = U^{\mathrm{T}}$ and $\det[U] = 1$); similarly we define

$$\begin{bmatrix} i_{r0} \\ i_{rd'} \\ i_{rq'} \end{bmatrix} = U \begin{bmatrix} i_{r1} \\ i_{r2} \\ i_{r3} \end{bmatrix}$$
$$\begin{bmatrix} \psi_{s0} \\ \psi_{sa} \\ \psi_{sb} \end{bmatrix} = U \begin{bmatrix} \psi_{s1} \\ \psi_{s2} \\ \psi_{s3} \end{bmatrix}$$
$$\begin{bmatrix} \psi_{r0} \\ \psi_{rd'} \\ \psi_{rq'} \end{bmatrix} = U \begin{bmatrix} \psi_{r1} \\ \psi_{r2} \\ \psi_{r3} \end{bmatrix}$$
$$\begin{bmatrix} u_{s0} \\ u_{sa} \\ u_{sb} \end{bmatrix} = U \begin{bmatrix} u_{s1} \\ u_{s2} \\ u_{s3} \end{bmatrix} \tag{1.7}$$

where $(\psi_{rd'}, \psi_{rq'})$ and $(i_{rd'}, i_{rq'})$ denote the (d', q')-components of the rotor flux and current vectors in the (d', q') reference frame attached to the rotor, rotating at rotor speed $\omega = \dot{\delta}$ and identified by the rotor angle δ in the fixed (a, b) reference frame attached to the stator, while (ψ_{sa}, ψ_{sb}) and (u_{sa}, u_{sb}) denote the (a, b)-components of the stator flux and the stator voltage vectors in the fixed (a, b) frame.

1.2 State Space Models

Let T_e be the electromagnetic torque produced by the motor and J the motor moment of inertia. Recall that T_L is the load torque while R_s and R_r are the stator and rotor resistances, respectively. Since in balanced operating conditions (1.5) $i_{s0} = 0$, $i_{r0} = 0$ and $u_{s0} = 0$, on the basis of (1.1), (1.6), and (1.7), for an induction motor with one pole pair we can write (the damping friction torque is usually negligible in induction motors and it is therefore set equal to zero)

$$\begin{aligned} \dot{\psi}_{sa} + R_s i_{sa} &= u_{sa} \\ \dot{\psi}_{sb} + R_s i_{sb} &= u_{sb} \\ \dot{\psi}_{rd'} + R_r i_{rd'} &= 0 \\ \dot{\psi}_{rq'} + R_r i_{rq'} &= 0 \\ \dot{\delta} &= \omega \\ J\dot{\omega} &= T_e - T_L \end{aligned} \tag{1.8}$$

in which

$$\begin{bmatrix} i_{rd'} \\ i_{rq'} \end{bmatrix} = \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} i_{ra} \\ i_{rb} \end{bmatrix}$$
$$\begin{bmatrix} \psi_{rd'} \\ \psi_{rq'} \end{bmatrix} = \begin{bmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{bmatrix} \begin{bmatrix} \psi_{ra} \\ \psi_{rb} \end{bmatrix}$$

with (ψ_{ra}, ψ_{rb}) and (i_{ra}, i_{rb}) denoting the (a,b)-components of the rotor flux and the rotor current in the fixed (a,b) frame. Since

$$\begin{aligned}\begin{bmatrix}\dot\psi_{ra}\\ \dot\psi_{rb}\end{bmatrix} &= \begin{bmatrix}\cos\delta & -\sin\delta\\ \sin\delta & \cos\delta\end{bmatrix}\begin{bmatrix}\dot\psi_{rd'}\\ \dot\psi_{rq'}\end{bmatrix} + \omega\begin{bmatrix}-\sin\delta & -\cos\delta\\ \cos\delta & -\sin\delta\end{bmatrix}\begin{bmatrix}\psi_{rd'}\\ \psi_{rq'}\end{bmatrix}\\ &= -\begin{bmatrix}R_r & 0\\ 0 & R_r\end{bmatrix}\begin{bmatrix}i_{ra}\\ i_{rb}\end{bmatrix} + \begin{bmatrix}-\omega\psi_{rb}\\ \omega\psi_{ra}\end{bmatrix}\end{aligned} \tag{1.9}$$

we can write in (a,b) coordinates

$$\begin{aligned}\begin{bmatrix}\dot\psi_{sa}\\ \dot\psi_{sb}\end{bmatrix} + \begin{bmatrix}R_s & 0\\ 0 & R_s\end{bmatrix}\begin{bmatrix}i_{sa}\\ i_{sb}\end{bmatrix} &= \begin{bmatrix}u_{sa}\\ u_{sb}\end{bmatrix}\\ \begin{bmatrix}\dot\psi_{ra}\\ \dot\psi_{rb}\end{bmatrix} + \begin{bmatrix}R_r & 0\\ 0 & R_r\end{bmatrix}\begin{bmatrix}i_{ra}\\ i_{rb}\end{bmatrix} + \begin{bmatrix}0 & \omega\\ -\omega & 0\end{bmatrix}\begin{bmatrix}\psi_{ra}\\ \psi_{rb}\end{bmatrix} &= \begin{bmatrix}0\\ 0\end{bmatrix}.\end{aligned} \tag{1.10}$$

From (1.2), (1.4), (1.6), and (1.7), the electromagnetic equations are

$$\begin{bmatrix}\psi_{sa}\\ \psi_{sb}\\ \psi_{ra}\\ \psi_{rb}\end{bmatrix} = \begin{bmatrix}L_s & 0 & M & 0\\ 0 & L_s & 0 & M\\ M & 0 & L_r & 0\\ 0 & M & 0 & L_r\end{bmatrix}\begin{bmatrix}i_{sa}\\ i_{sb}\\ i_{ra}\\ i_{rb}\end{bmatrix} \triangleq L\begin{bmatrix}i_{sa}\\ i_{sb}\\ i_{ra}\\ i_{rb}\end{bmatrix}. \tag{1.11}$$

The matrix L is positive definite, *i.e.* the quadratic form associated to L

$$\frac{1}{2}i^{\mathrm{T}}Li \tag{1.12}$$

is positive for any nonzero value of the current vector $i = [i_{sa}, i_{sb}, i_{ra}, i_{rb}]^{\mathrm{T}}$: (1.12) represents the magnetic energy. This implies that $L_sL_r > M^2$. Define

$$R \triangleq \begin{bmatrix}R_s & 0 & 0 & 0\\ 0 & R_s & 0 & 0\\ 0 & 0 & R_r & 0\\ 0 & 0 & 0 & R_r\end{bmatrix} \tag{1.13}$$

so that, if we eliminate $(\psi_{sa}, \psi_{sb}, \psi_{ra}, \psi_{rb})$ in (1.10) by using (1.11), we obtain the first state space model

$$\begin{aligned}L\begin{bmatrix}\frac{\mathrm{d}i_{sa}}{\mathrm{d}t}\\ \frac{\mathrm{d}i_{sb}}{\mathrm{d}t}\\ \frac{\mathrm{d}i_{ra}}{\mathrm{d}t}\\ \frac{\mathrm{d}i_{rb}}{\mathrm{d}t}\end{bmatrix} &= -R\begin{bmatrix}i_{sa}\\ i_{sb}\\ i_{ra}\\ i_{rb}\end{bmatrix} - \begin{bmatrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & \omega M & 0 & \omega L_r\\ -\omega M & 0 & -\omega L_r & 0\end{bmatrix}\begin{bmatrix}i_{sa}\\ i_{sb}\\ i_{ra}\\ i_{rb}\end{bmatrix} + \begin{bmatrix}u_{sa}\\ u_{sb}\\ 0\\ 0\end{bmatrix}\\ J\frac{\mathrm{d}\omega}{\mathrm{d}t} &= T_e - T_L\end{aligned} \tag{1.14}$$

in which the state variables are $(i_{sa}, i_{sb}, i_{ra}, i_{rb}, \omega)$ and the electromagnetic torque T_e is still to be determined as a function of the state variables. This choice of state variables is naturally linked to the energy stored in the motor given by

$$E = \frac{1}{2} i^{\mathrm{T}} L i + \frac{1}{2} J \omega^2 \tag{1.15}$$

which is the sum of the magnetic energy (1.12) and of the kinetic energy

$$\frac{1}{2} J \omega^2 \, . \tag{1.16}$$

The expression of T_e can be obtained from the energy balance

$$\begin{aligned} \frac{\mathrm{d}E}{\mathrm{d}t} &= [i_{sa}, i_{sb}][u_{sa}, u_{sb}]^{\mathrm{T}} - T_L \omega - i^{\mathrm{T}} R i \\ &\quad + \omega [i_{ra}, i_{rb}] \begin{bmatrix} 0 & -M \\ M & 0 \end{bmatrix} [i_{sa}, i_{sb}]^{\mathrm{T}} + T_e \omega \, . \end{aligned} \tag{1.17}$$

Let P_{in}, P_{out}, and P_{loss} denote the input power, the output power, and the power losses, respectively; since

$$\begin{aligned} \frac{\mathrm{d}E}{\mathrm{d}t} &= P_{in} - P_{out} - P_{loss} \\ &= [i_{sa}, i_{sb}][u_{sa}, u_{sb}]^{\mathrm{T}} - T_L \omega - i^{\mathrm{T}} R i \, , \end{aligned} \tag{1.18}$$

comparing (1.17) with (1.18) it follows that

$$T_e = [i_{ra}, i_{rb}] \begin{bmatrix} 0 & -M \\ M & 0 \end{bmatrix} [i_{sa}, i_{sb}]^{\mathrm{T}} = M(i_{ra} i_{sb} - i_{rb} i_{sa}) \, . \tag{1.19}$$

Note that the electromagnetic torque T_e produced by the motor is a nonlinear function of the state variables and constitutes the main nonlinear term in the induction motor model. Since from (1.11)

$$\begin{aligned} i_{ra} &= -\frac{M}{L_r} i_{sa} + \frac{1}{L_r} \psi_{ra} \\ i_{rb} &= -\frac{M}{L_r} i_{sb} + \frac{1}{L_r} \psi_{rb} \end{aligned} \tag{1.20}$$

the electromagnetic torque T_e can also be expressed as

$$T_e = \frac{M}{L_r} (\psi_{ra} i_{sb} - \psi_{rb} i_{sa}) \, . \tag{1.21}$$

From (1.14) and (1.19) we obtain the overall state space model in terms of the state variables $(i_{sa}, i_{sb}, i_{ra}, i_{rb}, \omega)$ and the input variables (u_{sa}, u_{sb}, T_L)

$$B \begin{bmatrix} \frac{\mathrm{d}i_{sa}}{\mathrm{d}t} \\ \frac{\mathrm{d}i_{sb}}{\mathrm{d}t} \\ \frac{\mathrm{d}i_{ra}}{\mathrm{d}t} \\ \frac{\mathrm{d}i_{rb}}{\mathrm{d}t} \\ \frac{\mathrm{d}\omega}{\mathrm{d}t} \end{bmatrix} + (K+C) \begin{bmatrix} i_{sa} \\ i_{sb} \\ i_{ra} \\ i_{rb} \\ \omega \end{bmatrix} = \begin{bmatrix} u_{sa} \\ u_{sb} \\ 0 \\ 0 \\ -T_L \end{bmatrix} \tag{1.22}$$

with

$$B = \begin{bmatrix} L_s & 0 & M & 0 & 0 \\ 0 & L_s & 0 & M & 0 \\ M & 0 & L_r & 0 & 0 \\ 0 & M & 0 & L_r & 0 \\ 0 & 0 & 0 & 0 & J \end{bmatrix}, \qquad K = \begin{bmatrix} R_s & 0 & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 & 0 \\ 0 & 0 & R_r & 0 & 0 \\ 0 & 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega L_r & M i_{sb} \\ 0 & 0 & -\omega L_r & 0 & -M i_{sa} \\ 0 & 0 & -M i_{sb} & M i_{sa} & 0 \end{bmatrix}. \tag{1.23}$$

Note that the matrix C is skew-symmetric, *i.e.* $C = -C^{\mathrm{T}}$. If we differentiate with respect to time the total energy (1.15), which can also be expressed as

$$E = \frac{1}{2}[i^{\mathrm{T}}, \omega] B \begin{bmatrix} i \\ \omega \end{bmatrix}$$

we reobtain (1.18)

$$\begin{aligned} \frac{\mathrm{d}E}{\mathrm{d}t} &= -[i^{\mathrm{T}}, \omega] K \begin{bmatrix} i \\ \omega \end{bmatrix} + i_{sa}u_{sa} + i_{sb}u_{sb} - \omega T_L \\ &= -P_{loss} + P_{in} - P_{out} \end{aligned}$$

since $C = -C^{\mathrm{T}}$. The model (1.22) is very advantageous to analyze the energy balance: for this reason the model (1.22) will be referred to as the energy model. In fact, integrating with respect to time the power balance (1.18), we obtain the total energy balance from an initial time t_0 to the time t:

$$\begin{aligned} & E(t) - E(t_0) + \int_{t_0}^{t} i(\tau)^{\mathrm{T}} R i(\tau) \mathrm{d}\tau \\ &= \frac{1}{2} i(t)^{\mathrm{T}} L i(t) - \frac{1}{2} i(t_0)^{\mathrm{T}} L i(t_0) + \frac{1}{2} J\omega^2(t) - \frac{1}{2} J\omega^2(t_0) + \int_{t_0}^{t} i(\tau)^{\mathrm{T}} R i(\tau) \mathrm{d}\tau \\ &= \int_{t_0}^{t} [i_{sa}(\tau)u_{sa}(\tau) + i_{sb}(\tau)u_{sb}(\tau)] \mathrm{d}\tau - \int_{t_0}^{t} T_L \omega(\tau) \mathrm{d}\tau \,. \end{aligned}$$

Eliminating $(\psi_{sa}, \psi_{sb}, i_{ra}, i_{rb})$ in (1.10) by using (1.11), namely (recall also (1.20))

$$\begin{bmatrix} i_{ra} \\ i_{rb} \end{bmatrix} = \begin{bmatrix} -\frac{M}{L_r} & 0 \\ 0 & -\frac{M}{L_r} \end{bmatrix} \begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} + \begin{bmatrix} \frac{1}{L_r} & 0 \\ 0 & \frac{1}{L_r} \end{bmatrix} \begin{bmatrix} \psi_{ra} \\ \psi_{rb} \end{bmatrix} \tag{1.24}$$

$$\begin{bmatrix} \psi_{sa} \\ \psi_{sb} \end{bmatrix} = \begin{bmatrix} L_s - \frac{M^2}{L_r} & 0 \\ 0 & L_s - \frac{M^2}{L_r} \end{bmatrix} \begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} + \begin{bmatrix} \frac{M}{L_r} & 0 \\ 0 & \frac{M}{L_r} \end{bmatrix} \begin{bmatrix} \psi_{ra} \\ \psi_{rb} \end{bmatrix} \tag{1.25}$$

we obtain a state space model in terms of the state variables $(\omega, \psi_{ra}, \psi_{rb}, i_{sa}, i_{sb})$ and the input variables (u_{sa}, u_{sb}, T_L)

$$\begin{aligned}
\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \mu\left(\psi_{ra} i_{sb} - \psi_{rb} i_{sa}\right) - \frac{T_L}{J} \\
\frac{\mathrm{d}\psi_{ra}}{\mathrm{d}t} &= -\alpha\psi_{ra} - \omega\psi_{rb} + \alpha M i_{sa} \\
\frac{\mathrm{d}\psi_{rb}}{\mathrm{d}t} &= -\alpha\psi_{rb} + \omega\psi_{ra} + \alpha M i_{sb} \\
\frac{\mathrm{d}i_{sa}}{\mathrm{d}t} &= -\gamma i_{sa} + \frac{u_{sa}}{\sigma} + \beta\alpha\psi_{ra} + \beta\omega\psi_{rb} \\
\frac{\mathrm{d}i_{sb}}{\mathrm{d}t} &= -\gamma i_{sb} + \frac{u_{sb}}{\sigma} + \beta\alpha\psi_{rb} - \beta\omega\psi_{ra}
\end{aligned} \tag{1.26}$$

in which the following reparameterization is used:

$$\begin{aligned}
\mu &= \frac{M}{JL_r} \\
\alpha &= \frac{R_r}{L_r} \\
\sigma &= L_s\left(1 - \frac{M^2}{L_s Lr}\right) \\
\beta &= \frac{M}{\sigma L_r} \\
\gamma &= \frac{R_s}{\sigma} + \beta\alpha M\,.
\end{aligned} \tag{1.27}$$

Note that since $\sigma > 0$, all the above parameters are greater than zero. From (1.26) it follows that

$$\begin{aligned}
\frac{\mathrm{d}i_{sa}}{\mathrm{d}t} &= -\frac{R_s}{\sigma} i_{sa} + \frac{u_{sa}}{\sigma} - \beta\frac{\mathrm{d}\psi_{ra}}{\mathrm{d}t} \\
\frac{\mathrm{d}i_{sb}}{\mathrm{d}t} &= -\frac{R_s}{\sigma} i_{sb} + \frac{u_{sb}}{\sigma} - \beta\frac{\mathrm{d}\psi_{rb}}{\mathrm{d}t}\,.
\end{aligned} \tag{1.28}$$

The state space model (1.26), which will be referred to as the fixed frame model, has some advantages from the control view point since it clarifies that the control inputs (u_{sa}, u_{sb}) directly affect the dynamics of the stator currents (i_{sa}, i_{sb}) which can be viewed as intermediate control variables since they control the rotor speed ω and the rotor flux modulus $\sqrt{\psi_{ra}^2 + \psi_{rb}^2}$ in the first three equations in (1.26). Note that the model (1.26) is highly nonlinear due to the expression of the produced electromagnetic torque $T_e = \mu(\psi_{ra} i_{sb} - \psi_{rb} i_{sa})$ and to the products $\omega\psi_{rb}$ and $\omega\psi_{ra}$ appearing in the last four equations in (1.26) which are originated by the rotation of the rotor at speed $\omega(t)$, according to (1.9).

Let us now introduce a time-varying (d,q) frame which rotates at an arbitrary speed $\omega_0(t)$ and is identified at each time t by the angle $\varepsilon_0(t)$ so that

$$\frac{\mathrm{d}\varepsilon_0}{\mathrm{d}t} = \omega_0 \tag{1.29}$$

with $\varepsilon_0(0)$ an arbitrary initial condition. Rotor fluxes (ψ_{ra}, ψ_{rb}), stator currents

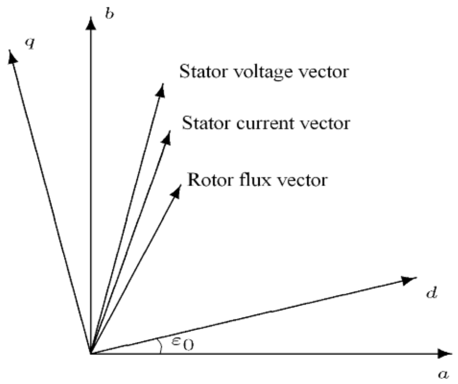

Fig. 1.2 (d,q) reference frame for the rotating frame model

(i_{sa}, i_{sb}), and stator voltages (u_{sa}, u_{sb}) are expressed with respect to the time-varying rotating (d,q) frame as (see Figure 1.2)

$$\begin{aligned}
\begin{bmatrix} \psi_{rd} \\ \psi_{rq} \end{bmatrix} &= \begin{bmatrix} \cos\varepsilon_0 & \sin\varepsilon_0 \\ -\sin\varepsilon_0 & \cos\varepsilon_0 \end{bmatrix} \begin{bmatrix} \psi_{ra} \\ \psi_{rb} \end{bmatrix} \\
\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} &= \begin{bmatrix} \cos\varepsilon_0 & \sin\varepsilon_0 \\ -\sin\varepsilon_0 & \cos\varepsilon_0 \end{bmatrix} \begin{bmatrix} i_{sa} \\ i_{sb} \end{bmatrix} \\
\begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix} &= \begin{bmatrix} \cos\varepsilon_0 & \sin\varepsilon_0 \\ -\sin\varepsilon_0 & \cos\varepsilon_0 \end{bmatrix} \begin{bmatrix} u_{sa} \\ u_{sb} \end{bmatrix} .
\end{aligned} \tag{1.30}$$

If the new state variables $(\omega, \psi_{rd}, \psi_{rq}, i_{sd}, i_{sq})$ and input variables (u_{sd}, u_{sq}, T_L) are used, in the new (d,q) rotating coordinates the induction motor model (1.26) becomes

$$\begin{aligned}
\frac{\mathrm{d}\omega}{\mathrm{d}t} &= \mu\left(\psi_{rd} i_{sq} - \psi_{rq} i_{sd}\right) - \frac{T_L}{J} \\
\frac{\mathrm{d}\psi_{rd}}{\mathrm{d}t} &= -\alpha\psi_{rd} + (\omega_0 - \omega)\,\psi_{rq} + \alpha M i_{sd}
\end{aligned}$$

$$\frac{d\psi_{rq}}{dt} = -\alpha\psi_{rq} - (\omega_0 - \omega)\,\psi_{rd} + \alpha M i_{sq}$$
$$\frac{di_{sd}}{dt} = -\gamma i_{sd} + \omega_0 i_{sq} + \beta\alpha\psi_{rd} + \beta\omega\psi_{rq} + \frac{u_{sd}}{\sigma}$$
$$\frac{di_{sq}}{dt} = -\gamma i_{sq} - \omega_0 i_{sd} + \beta\alpha\psi_{rq} - \beta\omega\psi_{rd} + \frac{u_{sq}}{\sigma} \tag{1.31}$$

which generalizes the fixed frame model (1.26) since (1.31) becomes (1.26) in the special case in which the speed ω_0 of the rotating coordinate frame is zero along with the initial angle $\varepsilon_0(0)$ (*i.e.* $\omega_0 = \varepsilon_0(0) = 0$ in (1.31)). The state space model (1.31) will be referred to as the rotating frame model.

In the third equation in (1.31) ω_0 can be freely chosen to our advantage. If we set, assuming $\psi_{rd} \neq 0$,

$$\omega_0 = \omega + \frac{\alpha M i_{sq}}{\psi_{rd}} \tag{1.32}$$

the third equation in (1.31) becomes

$$\frac{d\psi_{rq}}{dt} = -\alpha\psi_{rq} \tag{1.33}$$

which implies, since $\alpha > 0$, that $\psi_{rq}(t)$ tends exponentially to zero for any initial condition $\psi_{rq}(0)$, *i.e.*

$$\psi_{rq}(t) = e^{-\alpha t}\psi_{rq}(0)\,. \tag{1.34}$$

If $\psi_{rq}(0) = 0$ then $\psi_{rq}(t) = 0$ for every $t \geq 0$. Equations (1.29) and (1.32) give

$$\frac{d\varepsilon_0}{dt} = \omega + \frac{\alpha M i_{sq}}{\psi_{rd}}\,. \tag{1.35}$$

Substituting (1.32) in (1.31) we obtain

$$\frac{d\omega}{dt} = \mu\left(\psi_{rd} i_{sq} - \psi_{rq} i_{sd}\right) - \frac{T_L}{J}$$
$$\frac{d\psi_{rd}}{dt} = -\alpha\psi_{rd} + \frac{\alpha M i_{sq}}{\psi_{rd}}\psi_{rq} + \alpha M i_{sd}$$
$$\frac{d\psi_{rq}}{dt} = -\alpha\psi_{rq}$$
$$\frac{di_{sd}}{dt} = -\gamma i_{sd} + \omega i_{sq} + \frac{\alpha M i_{sq}^2}{\psi_{rd}} + \beta\alpha\psi_{rd} + \beta\omega\psi_{rq} + \frac{u_{sd}}{\sigma}$$
$$\frac{di_{sq}}{dt} = -\gamma i_{sq} - \omega i_{sd} - \frac{\alpha M i_{sq} i_{sd}}{\psi_{rd}} + \beta\alpha\psi_{rq} - \beta\omega\psi_{rd} + \frac{u_{sq}}{\sigma}\,. \tag{1.36}$$

The model (1.36) is a special case of the rotating frame model (1.31) in which ω_0 is chosen according to (1.32). If $\psi_{rq}(0) = 0$ and consequently, according to (1.34),

$\psi_{rq}(t) = 0$ for every $t \geq 0$, then the (d,q) frame rotates so that the direct axis coincides with the rotor flux vector and ε_0 coincides with the angle ρ between the flux vector and the a-axis (see Figure 1.3), that is

$$\begin{aligned} \psi_{ra} &= \psi_{rd} \cos\rho \\ \psi_{rb} &= \psi_{rd} \sin\rho \end{aligned} \tag{1.37}$$

with

$$\begin{aligned} \psi_{rd} &= \sqrt{\psi_{ra}^2 + \psi_{rb}^2} \\ \rho &= \arctan\left(\frac{\psi_{rb}}{\psi_{ra}}\right) . \end{aligned} \tag{1.38}$$

In this case, *i.e.* when $\psi_{rq}(0) = 0$ or equivalently $\varepsilon_0 = \rho$, equations (1.36) become

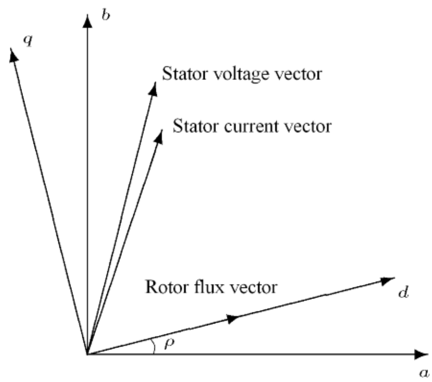

Fig. 1.3 (d,q) reference frame when $\varepsilon_0 = \rho$ for the field-oriented model

$$\begin{aligned} \frac{\mathrm{d}\omega}{\mathrm{d}t} &= \mu\psi_{rd}i_{sq} - \frac{T_L}{J} \\ \frac{\mathrm{d}\psi_{rd}}{\mathrm{d}t} &= -\alpha\psi_{rd} + \alpha M i_{sd} \\ \frac{\mathrm{d}\rho}{\mathrm{d}t} &= \omega + \frac{\alpha M i_{sq}}{\psi_{rd}} \\ \frac{\mathrm{d}i_{sd}}{\mathrm{d}t} &= -\gamma i_{sd} + \omega i_{sq} + \frac{\alpha M i_{sq}^2}{\psi_{rd}} + \beta\alpha\psi_{rd} + \frac{u_{sd}}{\sigma} \\ \frac{\mathrm{d}i_{sq}}{\mathrm{d}t} &= -\gamma i_{sq} - \omega i_{sd} - \frac{\alpha M i_{sq} i_{sd}}{\psi_{rd}} - \beta\omega\psi_{rd} + \frac{u_{sq}}{\sigma} \end{aligned} \tag{1.39}$$

which constitutes a motor state space model with state variables $(\omega, \psi_{rd}, \rho, i_{sd}, i_{sq})$ and input variables (u_{sd}, u_{sq}, T_L). The model (1.39) will be referred to as field-oriented model. Several important comments on the field-oriented model (1.39) are in order:

1. The difference between the rotor flux speed of rotation $\dot{\rho}$ and the rotor speed ω is equal to $\frac{\alpha M i_{sq}}{\psi_{rd}}$: it is usually called slip speed ω_s and is expressed as follows:

$$\begin{aligned}\omega_s = \dot{\rho} - \omega &= \frac{\alpha M i_{sq}}{\psi_{rd}} = \frac{\alpha M T_e}{\mu \psi_{rd}^2} \\ &= \frac{R_r M(\psi_{ra} i_{sb} - \psi_{rb} i_{sa})}{L_r(\psi_{ra}^2 + \psi_{rb}^2)}\ ;\end{aligned} \tag{1.40}$$

   it is proportional to the electromagnetic torque T_e and inversely proportional to the the flux modulus squared: the smaller the flux modulus, the larger the flux speed of rotation $\dot{\rho}$ while the larger the electromagnetic torque T_e, the larger the flux speed of rotation $\dot{\rho}$.
2. No matter how $\varepsilon_0(0)$, or equivalently $\psi_{rq}(0)$, is chosen, (1.39) describes the limiting behavior of (1.36) as t goes to infinity according to (1.34).
3. The field-oriented model (1.39) is an equivalent (except at $\psi_{ra} = \psi_{rb} = 0$) description of the fixed frame model (1.26) in the new state variables

$$\begin{aligned}\omega &= \omega \\ \psi_{rd} &= \sqrt{\psi_{ra}^2 + \psi_{rb}^2} \\ \rho &= \arctan\left(\frac{\psi_{rb}}{\psi_{ra}}\right) \\ i_{sd} &= \frac{\psi_{ra} i_{sa} + \psi_{rb} i_{sb}}{\sqrt{\psi_{ra}^2 + \psi_{rb}^2}} = i_{sa}\cos\rho + i_{sb}\sin\rho \\ i_{sq} &= \frac{\psi_{ra} i_{sb} - \psi_{rb} i_{sa}}{\sqrt{\psi_{ra}^2 + \psi_{rb}^2}} = -i_{sa}\sin\rho + i_{sb}\cos\rho\end{aligned} \tag{1.41}$$

   and new control input coordinates

$$\begin{aligned}u_{sd} &= \frac{\psi_{ra} u_{sa} + \psi_{rb} u_{sb}}{\sqrt{\psi_{ra}^2 + \psi_{rb}^2}} = u_{sa}\cos\rho + u_{sb}\sin\rho \\ u_{sq} &= \frac{\psi_{ra} u_{sb} - \psi_{rb} u_{sa}}{\sqrt{\psi_{ra}^2 + \psi_{rb}^2}} = -u_{sa}\sin\rho + u_{sb}\cos\rho\ .\end{aligned} \tag{1.42}$$

4. The field-oriented model (1.39) is the most advantageous from the control view point since the control inputs (u_{sd}, u_{sq}) directly affect the currents (i_{sd}, i_{sq}) dynamics only; the stator current vector (i_{sd}, i_{sq}) can be viewed as an intermediate control vector in the reduced order model