

## INDUSTRIAL STRUCTURE AND THE NATURE OF INNOVATIVE ACTIVITY\*

A substantial portion of the increase in the output of advanced industrial nations is widely judged to have been attributable to technical progress. There is also overwhelming evidence that this progress has not occurred merely in a random manner (see, e.g. Schmookler, 1962). Yet there is not much in the way of economic theory to explain either its rate or its direction, and particularly little that has a reasonably precise micro-economic foundation.<sup>1</sup> This lacuna is important, because a recognition of the importance of technical progress raises serious doubts about the adequacy with which traditional micro-economic models allow one to understand the functioning of modern market economies, and to develop policy prescriptions (e.g. with respect to anti-trust policy). The lacuna is also surprising, for it has become a commonplace, at least since the work of Schumpeter (1947), that the pace of inventive and innovative activity is related to market structure. Moreover, there must be many who share the Schumpeterian belief that perfect competition is inimical to inventive activity, and that the gains from such activity more than offset the welfare loss arising from production inefficiency associated with market power.<sup>2</sup>

However, a substantial part of the recent industrial organisation literature would appear to have interpreted tests of the Schumpeterian thesis in a somewhat different manner.<sup>3</sup> It is often argued now that industrial concentration and large size *spur* inventive activity. It is as though concentration is a *cause* of innovations.<sup>4</sup>

\* This is a revised and expanded version of the first part of an invited paper (Dasgupta and Stiglitz, 1977) presented at the World Congress of the *International Economic Association on Economic Growth and Resources*, held in Tokyo during 27 August-3 September 1977. This version was prepared whilst Dasgupta was a Visiting Professor at the School of International Studies, Jawaharlal Nehru University, and the Delhi School of Economics, Delhi, and Stiglitz was Oskar Morgenstern Distinguished Fellow at Mathematica, and Visiting Professor at the Institute of Advanced Studies, Princeton, during the Autumn of 1978. We have gained much from discussions with Hans Biswinger, Sukhamoy Chakravarty, A. K. Dasgupta, Paul David, Richard Gilbert, Sanford Grossman, Glen Loury, Edwin Mansfield, and most especially, from the comments of Ashok Guha.

<sup>1</sup> For this last, see, for example Arrow (1962), Barzel (1968), Stiglitz (1970), Kamien and Schwartz (1972), Evenson and Kieslev (1975), Nelson, Winter and Schuette (1976), Dasgupta and Stiglitz (1977), Loury (1977), Nelson and Winter (1977), Dasgupta and Stiglitz (1978), Levin (1978) and, of course, the pioneering work of Schumpeter (1947).

<sup>2</sup> 'A system . . . that at every given point of time fully utilises its possibilities to the best advantage may yet in the long run be inferior to a system that does so at no given point of time, because the latter's failure to do so may be a condition for the level or speed of long run performance' (Schumpeter (1947), p. 83). It should be noted that Schumpeter is here comparing industrial capitalism with perfect competition.

<sup>3</sup> The empirical findings suggest that whilst up to a point industrial concentration is positively correlated with innovative activity, it is negatively correlated when an industry is too concentrated. See Scherer (1970), and Kamien and Schwartz (1975) for a survey of the empirical literature.

<sup>4</sup> Thus, for example, after noting in a sentence that technological change has effects on market structure, the body of the chapter devoted to R & D in Scherer (1970), is ' . . . concerned with a possible causal flow in the opposite direction; from market structure to technological innovation' and, among others, with the question: 'Is monopoly power, e.g. as manifested in high market concentration, a favourable climate for innovation and technical progress?' Scherer (1970), p. 347.

In this paper we attempt to provide an analytical framework relating market structure to the nature of inventive activity. In doing so we have come to modify this neo-Schumpeterian view in a fundamental way. We shall argue that except in the short run *both* market structure and the nature of inventive activity are endogenous; that the degree of concentration in an industry ought not to be treated as given, as it recently has been in the industrial organisation literature; that they both depend on more basic ingredients, such as the technology of research, demand conditions, the nature of the capital market (i.e. market rates of interest, and the ability of firms to borrow to finance research and development (R & D)), and the legal structure (e.g. patent rights). We shall, to be sure, explore the relationship between the degree of concentration and the nature of innovative activity. But as they are both endogenous, their relationship, unlike the neo-Schumpeterian thesis, ought not to be regarded as a *causal* one.

A major objective of our study is to formulate models within which the efficiency of a market economy can be assessed and where the tradeoff between atemporal production efficiency and dynamic gains can be meaningfully discussed. Not surprisingly these questions, once properly posed, turn out to be far more complicated than they appear at first sight. It is not a case of a single firm making a single decision (e.g. the total volume of R & D expenditure), but rather a case in which several firms make a complex of decisions; and it is the consequences of these with which we are concerned. For example, each firm needs to decide both on how much to spend on R & D and also on which research strategies to pursue. Moreover, research strategies can differ not only with respect to the probability distribution of the dates at which success occurs, but also as regards how much is learnt even if the principal objectives of the research project fail, and to how similar, both in process and in objectives, they are to those being pursued by other firms. Each of these decisions has important consequences not only for the aggregate rate of technological progress, but also for industrial structure and the performance of a market economy. If firms tend to imitate each other's research strategies then much of R & D expenditure may be essentially duplicative, and consequently socially wasteful. If firms engage in excessively risky projects it may lead to too fast a rate of technical progress and high degrees of industrial concentration. This last in turn may imply large losses in production efficiency. What makes the analysis all the more difficult is that each decision on the part of a firm has to be made within an industrial structure which is itself endogenous.

Questions regarding the *consequences* of market structure on R & D are, of course, not new. That there is *underinvestment* in R & D both under competition and monopoly is probably a general presumption. This presumption is based partly on the fact that knowledge – the output of R & D – has the attributes of a public good; but partly also on an argument, due to Arrow (1962), that relies solely on a comparison of the magnitudes of the payoff to the successful firm under alternative market structures.

In Section I, we shall review this argument and see where it is in error. This review will also help set the stage for the formal analysis that follows. In fact a

central conclusion of this paper is that on balance there is no reason for supposing that a market economy sustains too low a level of investment in R & D. There may well be over-investment.

Sections II, III and IV contain analyses of various aspects of R & D activity and their relationship to market structure. The discussions are based on models that are natural extensions of a simple model of process innovation presented in Section I. In Section II, we explore the relationship between market structure and expenditure in R & D. The model we shall present, whilst very simple, is at once rich enough to illustrate some points that we have raised above and several that will be raised subsequently. However, it is as well to mention at the outset that a central feature that the model of Section II is designed to display is that while R & D expenditures on the part of firms are rather like fixed-costs in production, the levels of such 'fixed costs' are themselves choice variables for firms (firms can engage in less or more R & D expenditure). This results in a key difference between the analysis of competition in R & D and the more conventional analysis of product competition, as our subsequent discussion will display. Moreover, we shall note formally that there are some basic non-convexities that may be present in the production and use of knowledge, and since knowledge is the output of R & D effort, any analysis of R & D must take into account such possible non-convexities. To illustrate the matter simply, it should be noted that the indirect social benefit function of a commodity is a decreasing but *convex* function of the unit cost of production (see Fig. 4). An immediate implication of this is that as between two research strategies that yield the same mean reduction in production cost, a risk-averse society will prefer the *riskier* one if the expenditure on the two are more or less the same. Another implication of this, as we shall note in Section II, is that net social benefit of R & D expenditure is not necessarily a concave function even if one postulates diminishing returns in cost reduction due to increased R & D effort. The point is that the same piece of knowledge can in principle be applied at any scale of operation. Thus, the cost of information per unit of scale decreases as the scale increases; but the value of information per unit scale need not. Earlier writings, in the main, have concentrated on the failure of the price system to sustain an efficient production and utilisation of knowledge, not only because knowledge has the attributes of a public good, but also because of the impossibility of establishing a complete set of perfectly competitive contingent markets in the face of, say, the phenomenon of moral hazard. A part of our analysis will be directed at noting the natural non-convexities that arise in the production and use of knowledge. Our aim in general is to see how each of these features affects the structure of an industry and the scale and direction of innovative activity.

The example of Section II, being timeless and devoid of uncertainty, suffers from several drawbacks, two crucial ones being that one is unable to study the *degree of risk-taking* in research activity, and that one cannot identify the *speed of research* (the pace of inventive activity). Consequently, it is not possible to analyse how each of these characteristics is related to market structure. It is clear enough in advance, of course, that each is in turn related not only to the structure of the

product market but also to the degree of competition in R & D activity.<sup>1</sup> The models that follow will capture this feature sharply. The example discussed in Section III is similar to that of Section II, but it is now supposed that research activities have uncertain payoffs. In this paper we are concerned with *process* innovation (i.e. R & D designed to reduce cost of production). This means that attention is drawn to the distribution of extreme values. This in turn implies, as was noted by Evenson and Kieslev (1975), that even risk-averse firms might wish to engage in randomisation. The example to be discussed in Section III will bear this out. But the desire for randomisation will, in our model, be reinforced by the feature which we have noted earlier.

In Section IV time is introduced into the analysis. It will be supposed that the first firm to invent captures all benefits that are to be had among firms (i.e. we suppose that the winner takes all). No doubt this is a simplification. But it should be transparent in which direction the model needs to be modified, were one to recognise the fact that firms usually are able to invent around patents and that as a result the first firm to make a breakthrough is not necessarily the most advantaged.

A critical assumption of the construction in Section IV is that all firms are obliged to follow the *same* research strategy; i.e. they all face the same decision tree. This has important consequences. In a sequel to this paper we explore the opposite extreme and suppose that firms face uncertainties about the date of success that are independent of one another.

In Section V, we gather together what we regard as the basic morals emerging from our analysis. The Appendices explore the constructions in Sections II and III in detail, verifying some of the claims made in the text. In particular, Appendix 1 contains a theorem regarding the existence of an equilibrium with free entry which may be of wider interest.

#### I. MARKET BIAS IN PROCESS INNOVATION

In what follows we suppose an absence of income effects. Let  $Q$  denote the quantity of a given commodity. The gross social benefit of consuming  $Q$  is  $u(Q)$  with  $u'(Q) > 0$  and  $u''(Q) < 0$ . Market demand is given by

$$p = p(Q) = u'(Q). \quad (1)$$

Write  $R(Q) = p(Q)Q$  as gross revenue to a monopolist. Assume for the moment that marginal revenue is decreasing in output.

Suppose that the current best-practice technique for producing the commodity involves  $c$  as the unit cost of production. Suppose also that a particular process innovation reduces the unit cost of production to  $c^*$ . In a pioneering paper Arrow (1962) explored the gain to the innovator under three forms of the product market. The first, the socially managed market, is one where the

<sup>1</sup> Among other things it is this last that distinguishes the analyses undertaken in this paper from those undertaken by Arrow (1962), in that Arrow concentrated exclusively on differing structures of the product market and supposed no competition in R & D activity when analysing the incentives to innovate under different market structures. We shall comment on this at a greater length in Section I.

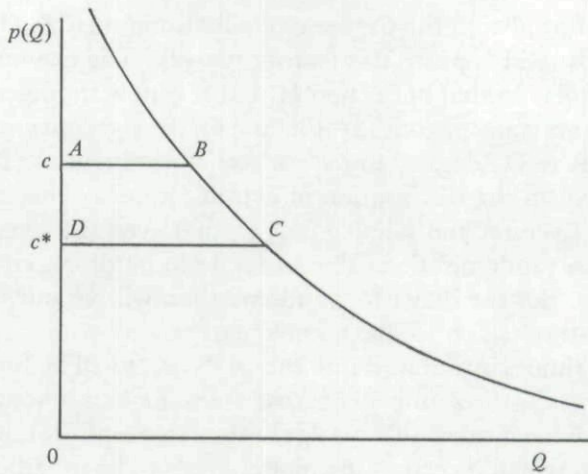


Fig. 1

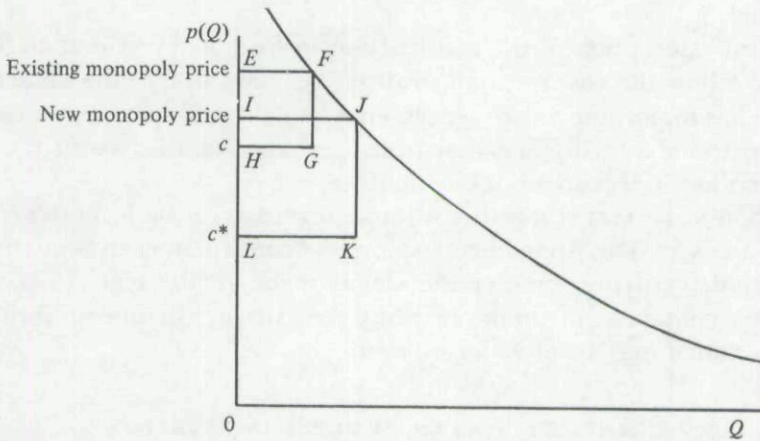


Fig. 2

market price is the cost of production. For this the per period pay-off due to the innovation is the gain in net social surplus, which is represented by the area  $ABCD$  in Fig. 1. Let  $\pi_s$  denote this gain. The second is the case of the pure monopolist (i.e. where there are barriers to entry). Let  $\pi_m$  represent the increase in per period monopoly profit if the monopolist undertakes the innovation. In Fig. 2,  $\pi_m$  is given by the difference between the areas  $IJKL$  and  $EFGH$ . The third is the competitive economy. Assume then that the  $c$ -technology is competitively exploited and that the innovator acquires a patent on the  $c^*$ -technology. Let  $\pi_c$  denote the per-period monopoly profit accruing to the innovator during the life of the patent. There are clearly two cases to be considered. In Fig. 3 the case where the monopoly price exceeds  $c$  is shown. Thus there is a limit price phenomenon here, and the innovator will supply the entire market and charge  $c$ . For this case  $\pi_c$  is given by the area  $ABC'D$ .

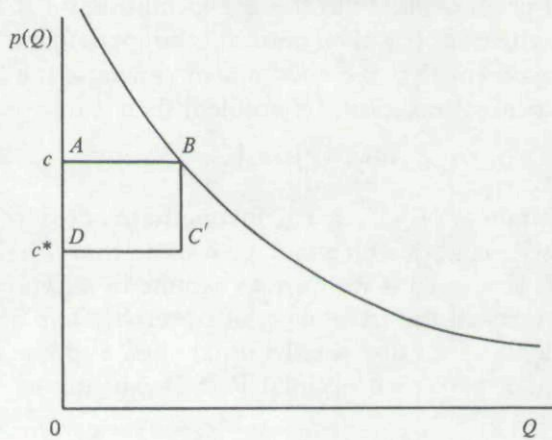


Fig. 3

Now it is possible to show that

$$\pi_s > \pi_c > \pi_m.^1 \quad (2)$$

From this observation Arrow concluded, 'the incentive to invent is less under monopolistic than under competitive conditions, but even in the latter case it will be less than is socially desirable' (Arrow (1962), p. 152). There would appear to be at least two weaknesses in Arrow's contention. First, the analysis is restricted to differences in the supply side of the *product* market, and does not consider alternative environments in which R & D is undertaken. If this latter is considered the *net* gains from the invention may not satisfy inequality (2). In particular, his modelling of the competitive economy appears to suppose that there is no competition in R & D activity and that there is a single firm engaged in it (see footnote 1, p. 269). Secondly, the model hypothesises the feasibility of discrete choices only. More generally, one would like to hypothesise the existence of margins of choice: research strategies aimed at a slight lowering of costs, a slight increase in risk, and a slight increase in speed. In the following three sections we explore these three features of process innovation in turn.

## II. MARKET STRUCTURE AND THE VOLUME OF R & D EXPENDITURE

### (1) *The Socially Managed Industry*

Imagine first that the market for the commodity in question is socially managed. If  $c$  is the (constant) unit cost of production then the net social benefit of consuming  $Q$  is  $u(Q) - cQ$ . Define

$$V(c) = \max_Q [u(Q) - cQ] \quad (3)$$

as the indirect utility function. It is well known that  $V(c)$  is a declining convex function of  $c$  (see Fig. 4). We suppose that R & D expenditure is designed to

<sup>1</sup> For a demonstration, see Arrow (1962).

reduce the cost of production. Thus if  $x$  is expenditure on R & D then  $c(x)$  is the unit cost of production. It is then natural to suppose that  $c'(x) < 0$ .<sup>1</sup>

Assume for the moment that the government can raise the funds for R & D expenditure from general taxation. Its problem then is to

$$\max_{x \geq 0} \{V[c(x)] - x\}. \quad (4)$$

Suppose that a solution to (4) exists. The first point to note is that unless further structure is imposed on  $c(x)$  we cannot guarantee that  $V[c(x)]$  is a concave function of  $x$ . This is so even if we were to assume in addition that  $c''(x) > 0$  (i.e. decreasing returns in the technology of research). The problems that the possible non-concavity of  $V$  raises are the usual ones. Suppose that the solution to (4) dictates that  $x > 0$ , then optimal R & D expenditure must satisfy the condition

$$\frac{dV}{dc} c'(x) = 1,$$

but

$$\frac{dV}{dc} = -Q,$$

hence

$$-c'(x)Q = 1. \quad (5)$$

The interpretation of equation (5) is clear enough. The LHS is the marginal social benefit of increasing R & D expenditure, which in fact is the reduction in production cost when output is optimal. The RHS is, of course, the marginal cost of increasing R & D expenditure. If  $V$  is concave in  $x$  there is no need for the government to calculate consumer surplus in locating the optimum. But if  $V$  is not concave in  $x$  the government needs in general to conduct global cost-benefit analysis to locate the optimum. The need for computing consumer surplus in such circumstances is obvious.

In order to bring these points out more clearly it will prove helpful to specify the functions somewhat. Suppose that

$$u(Q) = \frac{\sigma Q^{1-\epsilon}}{(1-\epsilon)} \quad (\sigma, \epsilon > 0). \quad (6)$$

Thus if we write by  $p(Q)$  the market demand curve,

$$p(Q) = \sigma Q^{-\epsilon}, \quad (7)$$

where  $\epsilon^{-1}$  is the elasticity of demand. Therefore, (7) supposes that market demand is iso-elastic.

It is then simple to confirm that

$$V[c(x)] = \left(\frac{\epsilon}{1-\epsilon}\right) (\sigma)^{1/\epsilon} / [c(x)]^{(1-\epsilon)/\epsilon}. \quad (8)$$

Suppose in addition that it is a new product which is under consideration.

<sup>1</sup> It is as though Mother Nature has a patent on all techniques of production with unit cost  $c(x)$ , ( $x > 0$ ) and that society has to pay  $x$  to purchase the right to use the technique of production with unit cost  $c(x)$ .

In particular assume

$$c(x) = \beta x^{-\alpha} \quad (\alpha, \beta > 0). \quad (9)$$

In this case

$$V[c(x)] = \left( \frac{\epsilon}{1-\epsilon} \right) \frac{\sigma^{1/\epsilon}}{\beta^{(1-\epsilon)/\epsilon}} x^{\alpha(1-\epsilon)/\epsilon}.$$

We may now note that  $V[c(x)]$  is strictly concave in  $x$  if  $\epsilon > \alpha(1-\epsilon)$ . If the reverse inequality were to hold,  $V[c(x)]$  would be strictly convex in  $x$ , and in particular, (4) will not possess a finite solution. Social benefits will continually increase with increasing R & D expenditure.<sup>1</sup>

The foregoing condition for ensuring that  $V[c(x)]$  is strictly concave is really rather transparent. It says that  $1 + 1/\alpha > 1/\epsilon$ . Since  $1/\epsilon$  is the elasticity of demand, and since  $1/\alpha$  is the production cost elasticity of R & D investment, the condition says that this latter elasticity must be sufficiently large to entertain continual diminishing (utility) returns on R & D expenditure. It should be noted that the condition is readily satisfied if demand is inelastic (i.e.  $\epsilon > 1$ ). It is satisfied even if demand is elastic (i.e.  $\epsilon < 1$ ) so long, of course, it is not too elastic. The condition makes precise the range on  $\epsilon$  that yields a well defined planning problem on the assumption that income effects are nil.<sup>2</sup>

Suppose then that  $\epsilon > \alpha(1-\epsilon)$ . Let  $x_s$  and  $Q_s$  denote the socially optimal levels of R & D expenditure and total output, respectively. If we now use (7) and (9) in the optimality conditions it is simple to verify that

$$x_s = (\alpha^\epsilon \sigma \beta^{\epsilon-1})^{1/[\epsilon-\alpha(1-\epsilon)]} \quad (10)$$

and

$$Q_s = (\alpha\beta)^{-1} (\sigma\alpha^\epsilon \beta^{\epsilon-1})^{(1+\alpha)/[\epsilon-\alpha(1-\epsilon)]}. \quad (11)$$

It follows immediately from equations (10) and (11) that both optimum output and optimum R & D expenditure are greater in industries that face larger markets (i.e. larger value of  $\sigma$ ). Thus the larger is the size of the market the greater ought (unit) cost reduction be. It is also worth noting that total output ought to be less in industries characterised by more costly R & D technology (i.e. a larger value of  $\beta$ ). One expects these features of the optimum. However, equation (10) implies that optimum R & D expenditure is *less* in industries characterised by more costly R & D technology if demand is *elastic* ( $\epsilon < 1$ ), and is *greater* if demand is *inelastic* ( $\epsilon > 1$ ). On reflection, this too is possibly in line with intuition.

<sup>1</sup> This is, of course, absurd since we would not be justified in supposing the marginal utility of income to remain constant.

<sup>2</sup> Our purpose so far has been solely to draw attention to the *existence* of non-concavities in the payoff function in the context of process innovation under certain circumstances; nothing more. As we noted in footnote 1 it is rather silly to explore non-concavities while assuming away income effects. The obvious question to ask is how to reformulate the model if in fact  $\epsilon < \alpha(1-\epsilon)$ . One obviously needs to assume that the marginal utility of income is not constant. Suppose, for example, that in the obvious notation  $u(Q, x) = \sigma [Q^{(1-\epsilon)/(1-\epsilon)}] [M-x-c(x)Q]^\delta$ , where  $1 > \epsilon$ ,  $\delta > 0$ , and  $M$  is total income. Let  $W(x) = \max_Q u(Q, x)$ . It is simple to verify that if  $c(x)$  satisfies (9) then  $W(x)$  is strictly convex for small  $x$  if  $1 < \alpha(1-\epsilon)$ . Nevertheless,  $W(x)$  has a unique maximum.



(2) *Oligopoly with Free-Entry*

It remains to explore a market economy. Suppose now that R & D activity is undertaken exclusively in the private sector. For simplicity of exposition we consider a market for a new product and suppose, also for simplicity, that all firms face the same R & D technology,  $c(x)$ . Thus if  $x_i (\geq 0)$  is the R & D expenditure of firm  $i$ ,  $c(x_i)$  is the unit cost of production that it faces for the product. In pursuing this formulation we are supposing that knowledge is monopolised by a firm when it pays for it.<sup>1</sup> Firms are profit maximising, behave non-cooperatively, and our task is to describe an equilibrium for this market. Towards this we assume that firms behave in a Cournot fashion (i.e. each firm chooses its own R & D expenditure level and its own output level) and that they all entertain Cournot conjectures (i.e. each firm supposes that none of the remaining firms will deviate from its course of action if the given firm deviates).

Begin by supposing that there is free entry into the industry; i.e. assume that market structure is endogenous. Let  $n$  be the number of firms in the industry and let  $Q_i (\geq 0)$  denote the output of the  $i$ th firm ( $i = 1, \dots, n$ ). It follows from our assumptions that

$$[n^*, (Q_1^*, x_1^*), (Q_2^*, x_2^*), \dots, (Q_i^*, x_i^*), \dots, (Q_{n^*}^*, x_{n^*}^*)]$$

is an equilibrium with free entry if, for  $i = 1, 2, \dots, n^*$ ,

$$[p(\sum_{j \neq i} Q_j^* + Q_i^*) - c(x_i^*)] Q_i^* - x_i^* \geq [p(\sum_{j \neq i} Q_j^* + Q) - c(x_i)] Q_i - x_i,$$

$$\text{for all } x_i, Q_i \geq 0, \quad (12)$$

and

$$\left[ p \left( \sum_{i=1}^{n^*} Q_i^* + Q \right) - c(x) \right] Q - x \leq 0 \quad \text{for all } x, Q \geq 0. \quad (13)$$

Condition (12) is clear enough. It says that  $(Q_i^*, x_i^*)$  is profit maximising for firm  $i$  ( $i = 1, \dots, n^*$ ) if it assumes that each of the remaining firms in the industry,  $j$  ( $j \neq i$ ), chooses output level  $Q_j^*$ , and that those firms outside the industry remain outside. Condition (13) must be satisfied if there are no barriers to entry. It says that there is no way for a firm not in the industry to enter and make positive profits if the  $i$ th firm in the industry ( $i = 1, \dots, n^*$ ) chooses to produce at the level  $Q_i^*$ . We have supposed, therefore, that each firm, whether in the industry or not, entertains Cournot conjectures regarding all other firms.

We are here concerned not only with the question of whether a free-entry market equilibrium exists but also with the characteristics of such equilibria in those circumstances where they do exist. This latter task is greatly simplified if we restrict our attention to *symmetric* equilibria; i.e. equilibria in which firms in the industry behave identically. In Appendix 1, we shall locate circumstances in which symmetric equilibria can be shown to exist. For the moment we suppose that such equilibria exist and we proceed to characterise them.

<sup>1</sup> Since the product is new,  $c_i(0) = \infty$ , by assumption. We leave it to the reader to analyse the case where the existing competitive product price is  $c$  and firms spend resources to cut costs (i.e.  $c_i(0) = c$ ). An alternative interpretation of this model would be to suppose that preferences are defined over the *characteristics* of commodities, and that firms in the industry compete to produce *different* commodities but which possess the *same* characteristic. Hence the commodities are perfect substitutes in consumption.

Let  $n^*$  be the equilibrium number of firms in the industry (to be determined, of course). Let  $\hat{Q}_i$  denote the total output of all firms other than the  $i$ th. Thus  $Q = Q_i + \hat{Q}_i$ . Firm  $i$  chooses  $x_i$  and  $Q_i$  with a view to

$$\text{maximising } \{[p(Q_i + \hat{Q}_i) - c(x_i)]Q_i - x_i\}. \quad (14)$$

On the assumption that profit maximising  $Q_i$  and  $x_i$  are both positive, the first-order conditions are

$$p(Q)[1 - \epsilon(Q)Q_i/Q] = c(x_i) \quad (15)$$

and

$$-Q_i c'(x_i) = 1, \quad (16)$$

where  $\epsilon(Q) \equiv -Qp'(Q)/p(Q)$ , is the inverse of the elasticity of demand. Since we are exploring symmetric equilibria,  $Q_i$  and  $x_i$  must be independent of  $i$  for  $i = 1, \dots, n^*$ . Therefore, if  $n^*$ ,  $Q^*$ ,  $x^*$  characterise a symmetric equilibrium with free entry they must satisfy conditions (15) and (16), which reduce to

$$p(Q^*)[1 - \epsilon(Q^*)/n^*] = c(x^*) \quad (17)$$

and

$$-c'(x^*) Q^*/n^* = 1. \quad (18)$$

(Note that  $Q^*$  denotes total output in equilibrium and therefore  $Q^*/n^*$  is output per firm in the industry.)

Moreover, the free entry condition (13) reduces to

$$[p(Q^* + Q) - c(x)]Q - x \leq 0 \quad \text{for all } x, Q \geq 0. \quad (19)$$

Finally, note that since  $(x^*, Q^*/n^*)$  is the profit maximising pair of choices for the representative firm in the industry in equilibrium, it must yield non-negative profits. Thus

$$[p(Q^*) - c(x^*)] Q^* \geq n^*x^*. \quad (20)$$

It is clear that the analysis would be greatly eased if free-entry were to result in firms earning negligible profits. Suppose then that  $\{[p(Q^*) - c(x^*)]Q^* - n^*x^*\}/n^*x^*$  is small enough to be neglected. We can then, instead of (20), work with the zero-profit condition:

$$[p(Q^*) - c(x^*)]Q^* = n^*x^*. \quad (21)$$

If (21) is satisfied, condition (19) is most certainly satisfied. But one can locate conditions in which (19) is satisfied even though (20) is a strict inequality. We shall explore these issues in detail in Appendix 1, where we shall also enquire into the circumstances in which (21) is a good approximation. In fact one expects that (21) is a reasonable approximation in those circumstances where  $n^*$  is 'large' and that one can identify parametric conditions for which the number of firms in equilibrium is 'large'. In Appendix 1 we shall note by way of an important class of cases that this is so, but that  $n^*$  need not necessarily be large in those circumstances in which the use of (21) as an equilibrium condition is justifiable. For the moment we assume that it is a good approximation. It follows that  $n^*$ ,  $Q^*$ ,  $x^*$  are obtained from equations (17), (18) and (21).

A glance at these conditions shows that a market equilibrium is not character-

ised by an optimal allocation of resources. For one thing, producers exercise a degree of monopoly power at a market equilibrium which sustains the level of R & D expenditure they incur. For another the private firm's marginal benefit of its R & D expenditure is calculated on the basis of its contribution to *its* scale of output (condition (18)) and not for the entire market (condition (5)).

We now proceed to analyse the market equilibrium conditions in detail. On using equations (17) and (21) we note that

$$1/n^* = Z^*/\epsilon(Q^*) \quad (\text{where } Z^* = n^*x^*/p(Q^*)Q^*), \quad (22)$$

(22) is the fundamental equation of this section. As we are analysing symmetric equilibria we cannot compare the degrees of concentration in different markets by indices such as the concentration ratio. For our model it would seem natural to regard  $1/n^*$  as the index of the *degree of concentration*.  $Z^*$ , being the fraction of industry sales that is spent on R & D, is an obvious index for research intensity in the industry. Equation (22) says that if  $\epsilon$  is constant they are proportional to each other. Thus, in a cross-section study of different industries with the same demand elasticity in equilibrium, but varying by way of the size of the market and R & D technologies they face, one would observe a linear relationship between research intensity and concentration. *But there is no causality to be imputed to this relationship: industrial concentration and research intensity are simultaneously determined.*<sup>1</sup>

It will have been noted that in arriving at equation (22) no use was made of equation (18). Thus (22) obtains under more general circumstances than we have allowed, in that it does not depend on firms choosing their R & D strategies with a view to profit maximisation. We therefore proceed to obtain a somewhat sharper characterisation. Let  $\alpha(x) \equiv -xc'(x)/c(x)$  denote the elasticity of unit cost of production with respect to R & D expenditure. It then follows from equations (18) and (22) that

$$Z^* \equiv n^*x^*/p(Q^*)Q^* = \alpha(x^*)/[1 + \alpha(x^*)]. \quad (23)$$

Therefore, in a cross-section study of industries facing different demand conditions but the same elasticity of the unit cost function ( $\alpha$ ) in equilibrium, one would observe that the index of research intensity,  $Z^*$ , is the same. But these industries would be characterised by different degrees of concentration; for on using (23) in (22) one obtains the equilibrium number of firms as

$$n^* = \epsilon(Q^*) [1 + \alpha(x^*)]/\alpha(x^*). \quad (24)$$

Therefore, the greater is the elasticity of demand ( $1/\epsilon(Q^*)$ ) the smaller is the number of firms that one will observe in such a cross-section study.

<sup>1</sup> The relationship between the degree of concentration and R & D expenditure established here is not inconsistent with the empirical findings mentioned in footnote 3, p. 266 above, for note that equation (22) has been obtained on the assumption that (21) is a good approximation. In Appendix 1 we shall note that (21) is indeed a good approximation for certain ranges of the parameters underlying the model and, in particular, for those ranges for which  $n^*$  is large (i.e. the degree of concentration is small). To get a feel for orders of magnitude it may be noted that for firms in the United States in 1961 employing more than 5,000 persons,  $Z$  was on average 5.2%, for those employing 1,000-5,000 persons it was 2.2% and for those employing less than 1,000 persons it was 2% (see Nelson *et al.* (1967), p. 67).

Notice, however, that equations (22) and (24) tell us that industries with smaller demand elasticities will, with any given number of firms, be characterised by a higher value of the index of research intensity. The lower demand elasticity leads, with a given degree of concentration, to higher markups; equilibrium is maintained not by entry, but by firms spending enough on R & D to forestall entry (essentially by spending all of their profits on R & D).

These foregoing propositions have an unmistakable Schumpeterian flavour to them. Since R & D involves fixed costs we cannot expect an industry that engages in it to be characterised by perfect competition. Nevertheless, *effective* competition is maintained by firms entering the market. Restrictive practices in our model, such as market price exceeding the unit cost of production (equation (17)), must be understood in the context of what Schumpeter called a 'perennial gale of creative destruction'. Our analysis has shown that the number of firms in an industry is no measure of the extent of this effective competition. Nor does the size of the market directly influence the number of firms in an industry for, as equation (24) makes clear, the equilibrium number of firms depends solely on the elasticities of the demand and innovation functions. If these elasticities are constant, as in (7) and (9), the equilibrium number of firms can be calculated directly, and is independent of the size of the market. However, the size of the market does influence the extent to which process innovation occurs in a market economy. To see this sharply suppose that market demand and innovation functions satisfy (7) and (9). If we now use these functional forms in the equilibrium conditions (17), (18) and (21), routine calculations yield their solution as:

$$n^* = \epsilon(1 + \alpha) / \alpha, \tag{25}$$

$$Q^* = \frac{\epsilon(1 + \alpha)}{\alpha^2 \beta} [\sigma \alpha^{2\epsilon} \beta^{\epsilon-1} \epsilon^{-\epsilon} (1 + \alpha)^{-(1+\epsilon)}]^{(1+\alpha)/[\epsilon - \alpha(1-\epsilon)]} \tag{26}$$

and

$$x^* = [\sigma \alpha^{2\epsilon} \beta^{\epsilon-1} \epsilon^{-\epsilon} (1 + \alpha)^{-(1+\epsilon)}]^{1/[\epsilon - \alpha(1-\epsilon)]}. \tag{27}$$

Now it will be recalled that (17) and (18) are merely the first-order conditions for each firm's profit maximising exercise. In Appendix 1 we shall note that for (25)-(27) to represent a market equilibrium we shall need to suppose that  $\epsilon > \alpha(1 - \epsilon)$ . Otherwise an equilibrium does not exist. Assume then that  $\epsilon > \alpha(1 - \epsilon)$ . (This, as we noted earlier, is trivially true if market demand is inelastic (i.e.  $\epsilon > 1$ )). It is then immediate from (27) that the greater is the size of the market (i.e. the greater is  $\sigma$ ), the greater is R & D expenditure per firm and, therefore, the greater is (unit) cost reduction. And so is industry output in equilibrium the greater.<sup>1</sup> Likewise, the costlier is R & D technology (i.e. the greater is  $\beta$ ), the smaller is industry output. This is precisely what intuition suggests. However, note that the costlier is R & D technology the *smaller* is equilibrium R & D expenditure per firm (and hence industry-wide R & D expenditure) if demand is *elastic*; but it is *greater* if demand is *inelastic*. It is this last which is not intuitively immediate.

<sup>1</sup> The late Jacob Schmookler, in a series of writings, stressed the importance of the growth in demand for a product in stimulating R & D activity designed towards cost reduction and quality improvement. See, for example Schmookler (1962).

It will be recalled that equations (25)–(27) have been arrived at on the supposition that firms in equilibrium earn negligible profits. It is then clear that if  $\epsilon(1+\alpha)/\alpha$  is an integer, a free-entry symmetric equilibrium exists, where firms in equilibrium earn precisely zero profits and where the number of firms in equilibrium is given by (25).<sup>1</sup> Now, while the number of firms must be an integer,  $\epsilon(1+\alpha)/\alpha$  will not be, except by fluke. However, if  $\epsilon/\alpha$  is ‘large’, the largest integer less than  $\epsilon(1+\alpha)/\alpha$  is ‘large’. Then condition (25) suggests that if we set  $n^* = [\epsilon(1+\alpha)/\alpha]$  (i.e. the largest integer less than  $\epsilon(1+\alpha)/\alpha$ ), then such a number of firms can sustain an equilibrium, in the sense that if each chooses the pair  $(x^*, Q^*/n^*)$  which satisfies conditions (26) and (27), each will have maximised its profits given the choice of others, and its maximised profit level will be so low that condition (19) will also be satisfied, thus deterring further entry into the industry. In Appendix 1, we shall note that this is in fact the case. By construction we shall note that a free-entry symmetric equilibrium exists if  $\epsilon/\alpha$  is ‘large’, and that by choosing either  $\alpha$  to be sufficiently small or  $\epsilon$  to be sufficiently large we can force the equilibrium level of profit per firm to be as small as we care to make, so as to allow the zero profit condition (21) to be as good an approximation as we want.<sup>2</sup> However, it should be noted that the two limiting values of  $\alpha$  and  $\epsilon$  lead to different characteristics of industry equilibrium. To see this use equation (24) in equation (17) to obtain

$$p(Q^*)/c(x^*) = 1 + \alpha(x^*). \quad (28)$$

Assume that  $\alpha$  and  $\epsilon$  are both constants. Notice now that  $\epsilon/n^* \rightarrow 0$  as  $\alpha \rightarrow 0$ , and from (28) we conclude that  $p(Q^*) \rightarrow c(x^*) = \beta$  as  $\alpha \rightarrow 0$ . In the limit, as  $\alpha \rightarrow 0$ , the present model reduces to the conventional model of a perfectly competitive industry. However,  $\epsilon/n^* \rightarrow \alpha/(1+\alpha)$  as  $\epsilon \rightarrow \infty$  and in particular, equation (28) implies that if industry demand is highly inelastic market equilibrium sustains a great many firms. Nevertheless the ratio of market price to marginal cost of production can be much greater than unity.

Earlier we noted that it was natural to regard  $n^{-1}$  as the index of industrial concentration for the model at hand. Indeed, unguided intuition might suggest that if the equilibrium number of firms is large the industry resembles the conventional competitive model. The foregoing result suggests that this is wrong. In fact several authors (e.g. Kalecki (1954)) have used instead the ratio of product price to ‘prime cost’ (i.e.  $p(Q)/c(x)$  here) as a measure of the *degree of monopoly* in an industry. Now, a glance at equation (28) tells us that an industry could at the same time be characterised by a low degree of concentration (large  $n^*$ ) and a high degree of monopoly (large  $p(Q^*)/c(x^*)$ ). This would be so if both  $\epsilon$  and  $\alpha$  are ‘large’, so that  $n^* \equiv [\epsilon(1+\alpha)/\alpha] \simeq \epsilon$ , and therefore,  $n^*$  is ‘large’. Indeed, presently we shall note that for the model at hand the degree of monopoly is a much better index of market imperfection than the degree of concentration.

Now, in a cross-section study of industries differing from one another in terms of the elasticity of the unit cost function, we would note that those facing a

<sup>1</sup> Since we have already supposed that  $\epsilon > \alpha(1-\epsilon)$ , it follows that  $\epsilon(1+\alpha)/\alpha > 1$ .

<sup>2</sup> For equation (17) to make sense  $n^*$  must exceed  $\epsilon(Q^*)$ . But if  $\epsilon$  and  $\alpha$  are both constants, then if either  $\alpha$  is ‘small’ or  $\epsilon$  is ‘large’,  $n^* = [\epsilon(1+\alpha)/\alpha] > \epsilon$ . Here, and in what follows, bold square brackets round a number will denote the largest integer not exceeding the number.

larger elasticity are characterised by a greater degree of monopoly. If these industries face the same demand elasticity, equation (17) tells us that those characterised by a greater degree of monopoly contain a smaller number of firms. But then equation (22) tells us that these in turn are associated with a higher index of research intensity,  $Z^*$ . The question arises whether R & D expenditure per firm is higher. This in general is hard to tell. However, suppose  $\alpha$  and  $\epsilon$  are both constant, so that equilibrium is characterised by (25)–(27). It should now be noted from (27) that if  $\alpha$  is 'small' and demand is inelastic (i.e.  $\epsilon > 1$ ),  $\partial x^*/\partial \alpha > 0$ , so that the greater is  $\alpha$  the greater is R & D expenditure per firm in oligopoly equilibrium. But this in turn means that cost reduction is greater in such industries. *Somewhat paradoxically, then, we would observe greater cost reduction in those industries that are characterised by a higher degree of monopoly power.*

It remains for us to compare the performance of the free entry oligopoly equilibrium with that of the socially managed industry. Continue to assume that  $p(Q)$  and  $c(x)$  satisfy (7) and (9) respectively. We then note from (11) and (26) that  $Q_s > Q^*$ . Market equilibrium output is less than the socially optimal output, a result which is not immediately obvious since one may have thought that the pressure of competition (free-entry) would drive the market price down to a level below what is socially optimal, by forcing each firm to invest more in R & D than is desirable for society. The question then arises whether cost reduction is greater or less in the oligopoly market than in the socially optimal one. On comparing (10) and (27) one notes that  $x^* \geq x_s$  as  $(n^*)^{-\epsilon} \geq (1 + \alpha)$ . It follows that  $x^* < x_s$ , and hence that there is insufficient cost reduction in the oligopoly industry. But industry-wide R & D expenditure in the market economy is  $n^*x^*$  and the question arises how this compares with  $x_s$ . On using (10), (25) and (27) it is now easy to confirm that  $n^*x^* \geq x_s$  as  $(n^*)^{\alpha(\epsilon-1)} \geq (1 + \alpha)$ . This implies that if  $\epsilon$  is 'large' (i.e. demand is highly inelastic), then  $n^*x^* > x_s$  and, therefore, that total R & D expenditure in the market economy exceeds the socially optimal level. *Thus the market economy may be characterised both by excessive expenditure on R & D ( $n^*x^*$ ) and too low a rate of technical progress ( $x^*$ ) as compared to the socially managed one.* The point of course is that the market encourages too much duplication here. To see this sharply we may note from (10) that  $x_s \rightarrow (\alpha\beta)^{1/(1+\alpha)}$  as  $\epsilon \rightarrow \infty$ . However, from equation (27) it is immediate that  $x^* \rightarrow 0$  as  $\epsilon \rightarrow \infty$ , but  $n^*x^* \rightarrow \infty$ . *Thus welfare loss due to competition is unbounded as  $\epsilon \rightarrow \infty$ , even though each firm serves an infinitesimal fraction of a finite demand as  $\epsilon \rightarrow \infty$ .*

Matters are quite different though for the other limit economy. On comparing equations (10) and (27) it is immediate that  $x^* \rightarrow 0$  and  $x_s \rightarrow 0$  as  $\alpha \rightarrow 0$ . Note as well that as  $\alpha \rightarrow 0$  we have  $n^*x^* \rightarrow 0$ , even though  $n^* \rightarrow \infty$ . Therefore, in the limit, as  $\alpha \rightarrow 0$ , welfare loss due to the industry being privately managed is nil. As we noted earlier, the industry in this case is the conventional competitive one, characterised by a lack of distortion. Welfare loss due to competition in this limit economy is nil.

### (3) Oligopoly with Barriers to Entry

It has been our purpose so far to explore the implications of an endogenous market structure on the amount of innovative activity. We have captured this

in our model via the free-entry condition (13). Entry sustains effective competition. This, as we noted in the introduction, implies that the pace of innovative activity must be traced neither to the degree of concentration, nor to the degree of monopoly in the industry in question, but to more basic ingredients such as demand conditions, R & D technology, the nature of the capital market, etc. Nevertheless, it is the case that several authors have claimed a slowing down in the intensity of innovations in advanced capitalist economies, and have traced it to an increasingly monopolistic character of capitalism.<sup>1</sup>

Now, we have already noted on several occasions that the degree of monopoly,  $p(Q)/c(x)$ , is not an explanatory variable. Nor, if there is free-entry, is the number of active firms an explanatory variable. Indeed, we have noted that with free entry into the industry it can readily happen that industries characterised by a greater degree of monopoly are *more* innovative, *not less*. It is then worthwhile inquiring into the relationship between the degree of monopoly and the reduction in the unit cost of production in an environment where there are *barriers to entry*. We can then see whether allowing new firms to enter would result in greater cost reduction in equilibrium. Thus let the number of firms be exogenously given, say  $n$ . Formally,  $[(Q_1^*, x_1^*), \dots, (Q_i^*, x_i^*), \dots, (Q_n^*, x_n^*)]$  is an equilibrium if for  $i = 1, \dots, n$

$$\left. \begin{aligned} & [p(\sum_{j \neq i} Q_j^* + Q_i^*) - c(x_i^*)] Q_i^* - x_i^* \\ & \geq [p(\sum_{j \neq i} Q_j^* + Q_i) - c(x_i)] Q_i - x_i \quad \text{for all } x_i, Q_i \geq 0. \end{aligned} \right\} \quad (29)$$

Since we are interested only in symmetric equilibria, the foregoing definition of an equilibrium implies that conditions (17) and (18) must be satisfied, which we re-write here as

$$p(Q^*)[1 - \epsilon(Q^*)/n] = c(x^*) \quad (30)$$

and

$$-c'(x^*) Q^*/n = 1. \quad (31)$$

Since  $n$  is given, there are only two unknowns,  $Q^*$  and  $x^*$ , to be determined from (30) and (31). Suppose, to get explicit solutions, that  $p(Q)$  and  $c(x)$  satisfy (7) and (9) respectively. From (30) it is clear that since  $n$  is exogenously given in the present analysis, we must assume  $n > \epsilon$  (otherwise (30) and (31) will not possess a solution). It must also be supposed that  $n \leq \epsilon(1 + \alpha)/\alpha$ , since otherwise the solution of equation (30) and (31) will result in firms making negative profits and so they would not represent equilibrium conditions. Finally, we shall need to suppose that  $\epsilon > \alpha(1 - \epsilon)$ , a condition we required earlier (for details see Appendix 1). Routine calculations now enable one to solve equations (30) and (31), and they yield

$$x^* = [\sigma(\alpha/n)^\epsilon \beta^{\epsilon-1} (1 - \epsilon/n)]^{1/[\epsilon - \alpha(1 - \epsilon)]} \quad (32)$$

and

$$Q^* = (n/\alpha\beta) [\sigma(\alpha/n)^\epsilon \beta^{\epsilon-1} (1 - \epsilon/n)]^{(1 + \alpha)/[\epsilon - \alpha(1 - \epsilon)]}. \quad (33)$$

<sup>1</sup> 'Another (reason for the slowing down of the growth of advanced capitalist economies) is the hampering of application of new inventions which results from the increasingly monopolistic character of capitalism' (Kalecki (1954), p. 159).

It will be noticed immediately from equation (33) that in the admissible range ( $\epsilon, \epsilon(1+\alpha)/\alpha$ ],  $Q^*$  is an increasing function of  $n$ . *Industry output therefore increases with the number of firms.* Consequently product price decreases. This is the advantage of greater competition. Moreover, from equation (30) it is clear that the degree of monopoly declines as the number of firms in the industry increases. The question that we began with is whether an increase in the number of firms results in greater innovation (i.e. greater cost reduction). From equation (32) it is immediate that the answer is 'no'. *If the number of firms is increased, each firm in equilibrium spends less on R & D, and so unit cost of production in equilibrium is higher.* However, it is easy to confirm from equation (32) that total R & D expenditure,  $nx^*$ , increases with the number of firms. The point is that while the industry spends more on R & D as a consequence of increased competition, each firm spends less. The extra expenditure is essentially wasted in duplication. In fact a comparison of equations (10) and (32) shows that  $x^* < x_s$ . We conclude that for the model at hand (unit) cost reduction is insufficient in a market economy whether or not there are barriers to entry; and consequently, market price for the product is higher than is socially desirable.

This is brought out most forcefully if we consider a monopolist protected by entry barriers. In this case there is, of course, no duplication in R & D. Nevertheless, as we have seen, the monopolist engages in less R & D activity than is socially desirable. This remains true even if the government in a socially managed industry is forced to raise its revenue for R & D expenditure through benefit taxation.

However, the *speed* with which firms carry out their R & D work and, consequently, the *rapidity* with which technological innovations take place in a market economy would appear to be greater if there is free-entry into R & D activity, than if active firms were protected by entry barriers into the research sector. In Section IV we shall study the implication of free-entry into the research sector on the speed with which R & D is undertaken. For a more complete discussion of the issues see Dasgupta and Stiglitz (1980).

### III. NON-CONCAVITIES IN THE VALUE OF PROCESS INNOVATION AND THE DEGREE OF RISK-TAKING

It has already been noted that the indirect social utility function  $V(c)$  in equation (3) is a declining convex function (see Fig. 4). We revert to the basic model of Section I and suppose that  $c$  is the unit cost of production associated with the existing best-practice technique. Our aim is to explore the implications of the convexity of  $V(c)$  on the choice among risky research projects. To begin with, suppose there are two research strategies to choose from, one of which reduces the cost of production to  $c^*$  with certainty, whilst the other, if successful, reduces it to  $\tilde{c}$  (with  $\tilde{c} < c^* < c$ ). If unsuccessful, the cost of production remains at  $c$ . But suppose the *expected* cost reduction associated with the risky research project is  $c - c^*$ . If society's welfare criteria are derived with a view to maximising expected social utility then if the costs of the two research projects are the



same, it would prefer the risky project to the riskless one. This is depicted in Fig. 4.<sup>1</sup> Let us now generalise this example.

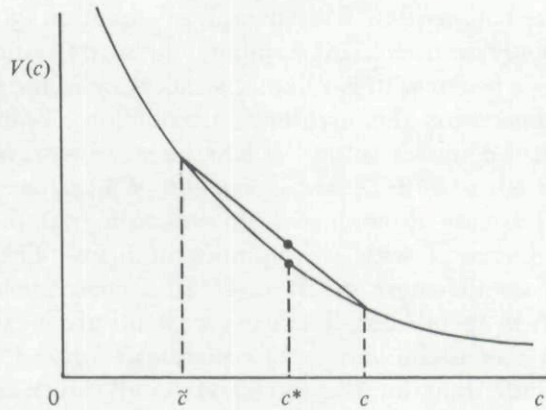


Fig. 4

Suppose that research projects are indexed by  $\alpha$  ( $\bar{\alpha} \geq \alpha \geq 0$ ). Project- $\alpha$  has probability  $h(\alpha)$  of success. If successful, the unit cost of production reduces to  $c(\alpha)$ . If unsuccessful, the cost of production remains at  $c$ . The projects are so labelled that we suppose, without loss of generality, that  $c'(\alpha) < 0$ . For simplicity, continue to assume that this family of research projects has the same mean outcome; i.e.

$$\left. \begin{aligned} E[c(\alpha)] &= c^* = h(\alpha)c(\alpha) + [1 - h(\alpha)]c, \\ \bar{\alpha} \geq \alpha \geq 0 \quad \text{and} \quad h(0) &= 1. \end{aligned} \right\} \quad (34)$$

(34) implies that  $c(0) = c^*$  and hence  $\alpha = 0$  is the riskless project. If we differentiate (34) we obtain

$$c'(\alpha)h(\alpha) = [c - c(\alpha)]h'(\alpha). \quad (35)$$

Consequently  $h'(\alpha) < 0$ . Therefore a higher value of  $\alpha$  is associated with a more risky project.

R & D expenditure associated with project- $\alpha$  is  $R(\alpha)$ . Quite obviously  $R(\alpha) > 0$ ,  $\alpha \geq 0$ . If  $R'(\alpha) \leq 0$ , nothing of interest remains to be said. Since  $\bar{\alpha}$  is the riskiest project available (the highest value of  $\alpha$  in the given family of available research projects) the planner ought to choose  $\bar{\alpha}$  (or undertake no research at all). Consequently we consider the other special case; where  $R'(\alpha) > 0$ . For obvious reasons, suppose in addition that  $R''(\alpha) > 0$ .

In what follows, we take it that  $R(0)$  is 'small', so that the optimum involves some R & D activity. Now if the planner were to choose project- $\alpha$ , expected net social benefits,  $E(\alpha)$ , would be

$$E(\alpha) \equiv h(\alpha)V[c(\alpha)] + [1 - h(\alpha)]V(c) - R(\alpha). \quad (36)$$

<sup>1</sup> If R & D cost is too high society would, presumably, not wish to engage in any research at all, and continue to rely on the  $c$ -technology.

Notice that unless we impose further structure on the functions  $h(\cdot)$  and  $V(\cdot)$ , one cannot ensure that  $E(\cdot)$  is concave. If it is not, then in general the government will need to conduct global cost-benefit analysis of the 'random' research projects.

Having indicated the problem let us simplify and suppose that (36) is strictly concave. Assuming optimal  $\alpha$  lies strictly between 0 and  $\bar{\alpha}$  we note that on using condition (35), the necessary and sufficient condition for optimality is:

$$-h(\alpha)c'(\alpha) \left( -V'[c(\alpha)] - \frac{\{V[c(\alpha)] - V(c)\}}{[c - c(\alpha)]} \right) = R'(\alpha). \quad (37)$$

The social benefit-cost criterion (37) has an unusual simplicity about it.  $-h(\alpha)c'(\alpha)$  is the expected marginal reduction in production cost - when a marginally more risky research project is chosen. What the LHS of condition (37) says is that the marginal benefit of stepping up R & D expenditure is  $-h(\alpha)c'(\alpha)$  times the difference between the *marginal* social benefit from cost reduction  $\{-V'[c(\alpha)]\}$  and the *average* social benefit from cost reduction,  $\{V[c(\alpha)] - V(c)\}/[c - c(\alpha)]$ .

To look at the matter another way, we know in advance that

$$-V'[c(\alpha)] > \{V[c(\alpha)] - V(c)\}/[c - c(\alpha)]$$

(since  $V(c)$  is convex and declining). What (37) says is that at an optimum the difference between marginal and average returns ought to be  $-R'(\alpha)/h(\alpha)c'(\alpha)$ .

We have established that a social planner will wish to engage in risky research projects. But then so will a pure monopolist. To see this suppose that marginal revenue from sales is a declining function of sales. Then it is simple to confirm that the monopolist's maximised profit from sales is a convex and declining function of his unit cost of production. Consequently, if the monopolist is concerned with maximising expected profits, an argument identical to the one we have presented above comes into play. The monopolist too favours risk.

The natural question to ask is whether the monopolist is inclined to engage in the optimum degree of risk and, if not, whether one can establish a bias in his behaviour towards risk. In Appendix 2, we demonstrate that if the family of research projects is restricted to the class we have discussed above, the monopolist undertakes insufficient risk in his R & D effort and invests too little in research and development. But then clearly this would be true even if the family of available research projects were characterised by a slight lowering of expected cost reduction,  $(c - c^*)$ , with increasing risk (i.e. increase in  $\alpha$ ). In this case one would observe greater cost reduction *on average* if the industry is controlled by a private monopolist, than if it were socially controlled. The point about this observation is that while cost reduction is a 'good thing' the monopolist, paradoxically, would on average be reducing costs too much, even though his R & D expenditure would be less than that which is socially desirable.

These pure economic environments are the easiest to analyse. With free entry the analysis appears to be unusually complex. But in an oligopolistic environment the market power of a producer increases as his cost advantage over his

rivals increases. There is then an *a-priori* presumption that competition may encourage excessive risk-taking on the part of producers engaged in process innovation. We have been unable to settle the issue when there is free entry into an industry.

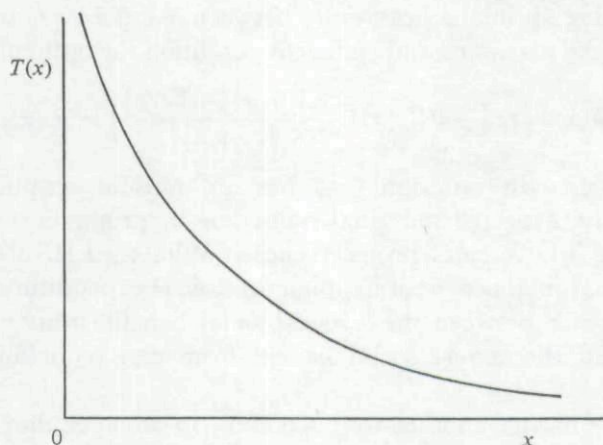


Fig. 5

#### IV. MARKET STRUCTURE AND THE SPEED OF RESEARCH

We are here concerned with the speed of research, and we continue to illustrate matters in the context of process innovation. Revert to the basic model of Section I and suppose  $c$  to be the unit cost of production associated with the existing best practice technique. As in Sections I and II, we simplify and hypothesise highly goal orientated research. Research now is taken to consist of solving a sequence of problems that will enable the commodity to be produced at a unit cost  $c^*$  ( $c^* < c$ ). There is no uncertainty.<sup>1</sup> If a research unit invests  $x$  at  $t = 0$  it solves the entire set of problems at date  $T(x)$ , where  $T'(x) < 0$  and  $T''(x) > 0$  with  $T(x) \rightarrow 0$  as  $x \rightarrow \infty$  and  $T(x) \rightarrow \infty$  at  $x \rightarrow 0$  (see Fig. 5).<sup>2</sup>

Consider first competitive conditions. We suppose that currently the market price for the commodity is  $c$ . It is understood that the first firm to make the invention is awarded a patent of fixed duration. If there are several winners they share the patent. We need not specify whether in this latter case the winners propose to collude by forming a cartel, or whether they play a Cournot game, as in Section II. All that we need to postulate is that each winner enjoys a positive flow of profits while the patent lasts. It follows that firms may well have an incentive to engage in R & D activity. We take it that each firm knows the R & D strategy of the other firms.

Firms are assumed to be able to borrow freely to finance their R & D activity

<sup>1</sup> Models incorporating uncertainty in the date of invention have been discussed in Stiglitz (1970), Dasgupta, Heal and Majumdar (1977), Kamien and Schwartz (1978), Loury (1977) and Dasgupta and Stiglitz (1978).

<sup>2</sup> I.e. all potential research units are of identical ability. This assumption is made solely for expositional ease, and can be relaxed in the obvious manner.

at a competitive rate of interest  $r (> 0)$ . Since by hypothesis  $T'(x) < 0$ , each firm engaged in R & D will establish only one research unit. In order to explore game equilibria with free entry we shall, as earlier, suppose that firms entertain Cournot conjectures regarding those that are engaged in R & D activity. However, we shall wish to capture the fact that firms engaged in R & D fear the threat of entry by potential entrants. Therefore, it will be supposed that active firms entertain Stackelberg conjectures regarding potential entrants; i.e. they work on the reaction functions of potential entrants. This latter assumption implies that under free entry into R & D activity an equilibrium is characterised by each firm receiving zero present-value of profits, and the former implies that only future winners are engaged in R & D.

Now suppose that more than one firm is active at a potential equilibrium. They all then invest the same amount. But this cannot be an equilibrium. For, with Cournot conjectures about each other, any one of these firms can increase its R & D expenditure marginally, thereby ensuring that it is the sole winner, and so increase the present value of its profits by a discrete amount. *We conclude that with free entry, at most one firm will be engaged in R & D activity at an equilibrium, and its net present value of profits will be nil.*

What this result is telling one is that the fact that a single firm is engaged in R & D activity is not in itself an evidence that there is little competition in this field. For the model at hand competition is intense. This single firm raises its R & D expenditure and so speeds up its research to a level high enough to forestall entry.

It remains to calculate the equilibrium level of expenditure. Let  $T^* (> 0)$  be the length of the patent. We now recall the notation introduced in Section I. If  $x^*$  is the equilibrium level of investment undertaken by the single firm, it satisfies the zero-profit condition

$$(\pi_c/r) (1 - e^{-rT^*}) e^{-rT(x)} = x. \quad (38)$$

The date of invention is  $T(x^*)$ . One notes first that  $x = 0$  is a solution of equation (38). One notes as well that the LHS of equation (38) is not necessarily a concave function of  $x$ . In Fig. 6 we have drawn both the LHS and the RHS of equation (38). As the figure makes clear, the largest solution of (38) is the equilibrium level of R & D expenditure,  $x^*$ . An equilibrium therefore exists and it is unique. A glance at Fig. 6 also shows that in a cross-section study of industries with the same R & D technology those characterised by greater demand (i.e. greater  $\pi_c$ ) will sustain a greater speed of research.<sup>1</sup>

We now compare the competitive outcome with the socially optimal speed of research. We take it that  $r$  is regarded as appropriate to use for discounting

<sup>1</sup> To obtain a feel for orders of magnitude it may be noted that in large corporations R & D projects are often expected to be completed in 4 to 5 years and that many such projects involve not undue risk, in many cases the estimated probability of success exceeding 0.8 (see Mansfield (1967)). Notable examples of risky R & D projects would appear to be in the pharmaceutical industry (see Schwartzman (1977)). For a theoretical exploration of the relationship between market structure, risk-taking in R & D, and the speed of research, see Dasgupta and Stiglitz (1977, 1980).

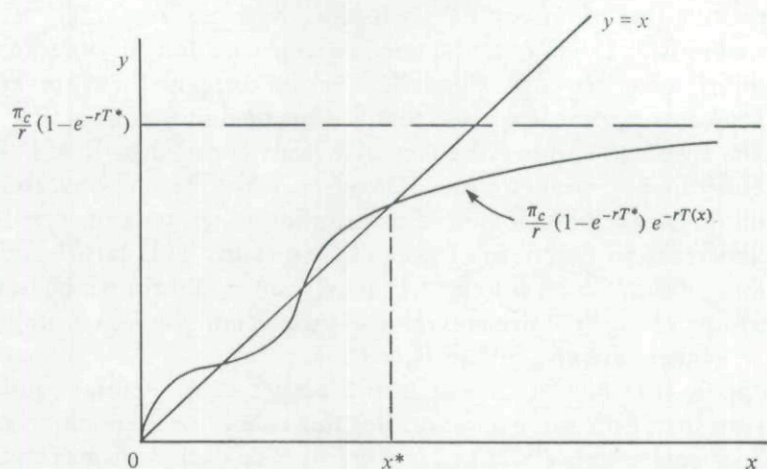


Fig. 6

social benefits and costs. The social planner is then concerned with choosing  $x$  so as to maximise

$$\left[ \frac{\pi_s}{r} e^{-rT(x)} - x \right]. \quad (39)$$

Notice again that (39) is not in general a concave function of  $x$ . However, given the assumptions that we have made about  $T(x)$  it is immediate that (39) has a maximum. Let  $x_s$  be the optimum R & D expenditure. Then provided it is positive it satisfies the cost-benefit rule

$$-T'(x) \pi_s e^{-rT(x)} = 1. \quad (40)$$

It will be recalled that  $\pi_s > \pi_c$  (inequality (2)). But a glance at (38) and (40) tells us that one can easily have  $x^* > x_s$ . When this is so the single firm engaged in R & D activity is forced to incur excessive R & D expenditure because of the *threat* of competition. Market forces encourage too high a speed of research.

The central result of this Section, that under free-entry, a Cournot-Nash equilibrium for the model considered here sustains at most one firm, survives if we introduce uncertainty in the special form where all firms are obliged to follow the *same* steps in solving the problems that are necessary to solve in order to make the  $c^*$ -technology viable (i.e. if all research units face the same decision tree). This means that given the pace of R & D activity of the remaining firms a given firm can guarantee that it is the first to invent by choosing a sufficiently high pace of research, even though it is still unable to say at which date the sequence of tasks will be completed by its research unit. With this form of uncertainty it is immediate that at most one firm will incur R & D expenditure at an equilibrium.

It will be granted that the foregoing postulates undue interdependence of research strategies amongst firms. In the sequel to this paper (Dasgupta and Stiglitz, 1978) we move to the opposite extreme and suppose complete independence. In such a case it is clear enough in advance that an equilibrium

may well sustain several firms competing in R & D; for no firm can guarantee itself to be the winner.

The point then is this. A key element in the determination of the number of firms competing in R & D activity at an equilibrium is the degree of correlation in the probabilities of success. Typically one would expect the number to be small when correlation is great. But the number of firms engaged in R & D is not an appropriate index for measuring the degree of competition. As we have seen, competition may result in a single firm being so engaged; and engaged in excessive expenditure, leading to excessive speed in research effort.

#### V. CONCLUDING REMARKS

There is now a fairly large empirical literature investigating various aspects of R & D activity and relating them to the structure of industries in which such activity is undertaken. It is therefore particularly surprising to note the paucity of theoretical explorations in this area. Moreover, so far as we are aware the majority of such explorations have been directed at the behaviour of a single firm engaged in R & D in the midst of an exogenously given environment. In order to make contact with some of the empirical findings and to obtain a less incomplete understanding of these matters it is necessary to move beyond the analysis of a single firm and to consider a set of interacting firms and, if one is interested in the long run as well, to postulate an endogenous market structure. What results is a game environment, and in this paper we have attempted to come to grips with a few of the traditional issues in the theory of industrial organisation in the context of a set of simple constructions. In investigating R & D activity in a market economy we have throughout made use of the concept of a game equilibrium. It is, of course, well known that both the existence and characteristics of such equilibria depend on what types of action agents are postulated to choose and on the conjectures that are entertained by agents regarding the responses by other agents to their choice. In this paper we have supposed that firms choose quantities (*viz.* output and R & D expenditure levels). As regards conjectures the simplest by far to analyse are Cournot ones, namely that other agents do not respond to the given agent's choice. We have, for the most part, supposed such conjectures. However, while plausible in certain circumstances (e.g. in the model of Section II), such conjectures are not plausible in many others. For example, the reader can readily check that an equilibrium does not exist in the model of Section IV, if all firms entertain Cournot conjectures about *all* other firms. We have consequently resorted to a set of hybrid conjectures on the part of firms in the analysis of Section IV; conjectures that are *a-priori*, not implausible. There is a clear need, though, of a study of the characteristics of oligopoly equilibria under more sophisticated conjectures on the part of firms and to see how these characteristics vary as conjectures are varied.<sup>1</sup>

One of the most oft discussed issues in the industrial organisation literature

<sup>1</sup> For an important analysis of oligopoly equilibrium with barriers to entry, where the focus of attention is on alternative specifications of the conjectures entertained by firms, see Marschak and Selton (1974).

has been the relationship between the size of firms and the pace of R & D activity. We have discussed some aspects of this issue in our introduction and in Section II. Now if firms are forced to finance their R & D expenditure from internal funds there is a clear presumption that industrial concentration is positively correlated with R & D activity. The model of Section II, being timeless, is consistent with the internal finance hypothesis. However, in Section IV, we supposed a perfect capital market with a view to demonstrating that one does not need to resort to the internal finance hypothesis if one wants to argue that only a limited number of firms will typically be engaged in competing R & D activities. Nor in fact, as we have shown, does one need to postulate non-classical goals on the part of firms in order to make contact with some of the empirical findings. We have throughout formalised the market economy as one in which firms are engaged in games with complete information. In such contexts it hardly makes sense to postulate anything other than profits as a firm's goals. Matters are different if incomplete information is postulated for firms; and a particularly interesting avenue that has recently been explored (see, e.g. Winter (1971), and Nelson, Winter and Schuette (1976)) consists in supposing in addition that agents have only limited ability to solve complicated maximisation problems. Endowed only with 'bounded rationality', firms are postulated to follow some 'satisficing' course of action, and to search locally for improvements if existing courses of action cease to produce satisfactory results.

Quite apart from any details that may be found interesting, the central conclusions of our analysis would appear to be:

(1) Even when one regards industrial structure to be endogenous, theoretical considerations are consistent with the empirical finding that when the degree of concentration in industries is small, industry-wide R & D effort is positively correlated with concentration (Section II).

(2) High degrees of concentration are by themselves not an evidence of lack of effective competition (Section II).

(3) When the degree of concentration in industries with free-entry is small, R & D effort per firm (and therefore cost reduction) is often positively correlated with concentration. Moreover, the degree of concentration is positively correlated with the degree of monopoly (Section II).

(4) Both optimal R & D expenditure and R & D expenditure per firm in a market economy increase with the size of the market. They decrease with increasing costs associated with R & D technology if demand is elastic and increase with increasing costs if demand is inelastic (Section II).

(5) If there are barriers to entry an increase in the number of firms would result in a decrease in R & D expenditure per firm in a market economy, although industry output would increase, and therefore, the degree of monopoly would decrease (Section II).

(6) There is some presumption that cost-reducing in an industry in a market economy, even when there is free entry, is less than the socially optimal level (Section II). However,

(7) If demand is highly inelastic, *total* R & D expenditure in an industry with free entry exceeds the socially optimal level (Section II). (6) and (7) imply

(8) There may be excessive duplication of research effort in a market economy in the sense that industry-wide R & D expenditure exceeds the socially optimal level even though cost-reduction is lower. In particular, an industry may be characterised by a very low degree of concentration (i.e. a large number of firms) and at the same time engage in a great deal of social waste.

(9) In the case of process innovation a good case can be made for encouraging investment in risky research projects, even if society is risk-averse (Section III).

(10) A pure monopolist (i.e. one protected by entry barriers) appears to have insufficient incentive (a) to undertake R & D expenditure (Section II) and (b) to engage in risky research ventures (Section III and Appendix 2).

(11) Since the market power of a firm increases as its cost advantage over its rivals increases there is a presumption that competitive markets encourage firms to engage in overly risky research projects (Section III).

(12) If the first firm to succeed is awarded most of the reward for invention, then to the extent the risks that firms undertake are positively correlated, pressure of competition will ensure that only a few firms engage in R & D activity; in extreme cases at most one firm will be engaged in research (Section IV).

(13) But the observation that only a few firms are engaged in R & D is not in itself an evidence that a market economy sustains too little R & D activity (Section IV). In particular

(14) Pressure of competition may result in excessive speed in research (Section IV), and in general

(15) There is no presumption that a market economy has a tendency to generate insufficient information.

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*Date of receipt of final typescript: September 1979*

#### APPENDIX I

We shall by construction locate conditions under which a free-entry symmetric equilibrium exists for the model of section II, when  $p(Q)$  and  $c(x)$  satisfy (7) and (9) in the text. In doing this we reverse the order followed in the text and suppose first that there are barriers to entry, so that  $n$  is exogenously given.

Consider the solution (32) and (33) of the two first-order conditions (30) and (31). For it to be real-valued one must suppose that  $n > \epsilon$ . Let us do so. It will have been noted that the pair  $(x^*, Q^*/n)$  in (32) and (33) is the unique solution of the (interior) first-order conditions for the representative profit maximising firm when it assumes that each of the remaining firms chooses  $Q^*/n$  as its output level. We must now locate conditions under which the representative firm earns non-negative profits when it chooses  $(x^*, Q^*/n)$ .

Let  $\pi(x^*, Q^*/n)$  denote the level of its profit. Then  $\pi(x^*, Q^*/n) = [p(Q^*) - c(x^*)] Q^*/n - x^*$ . For this to be non-negative it is necessary and sufficient that

$$[p(Q^*) - c(x^*)] Q^*/nx^* \geq 1. \quad (\text{A } 1)$$



If we now appeal to the functional forms (7) and (9) and use (32) and (33), routine calculations yield

$$[p(Q^*) - c(x^*)] Q^*/nx^* = \epsilon/\alpha(n - \epsilon). \quad (\text{A } 2)$$

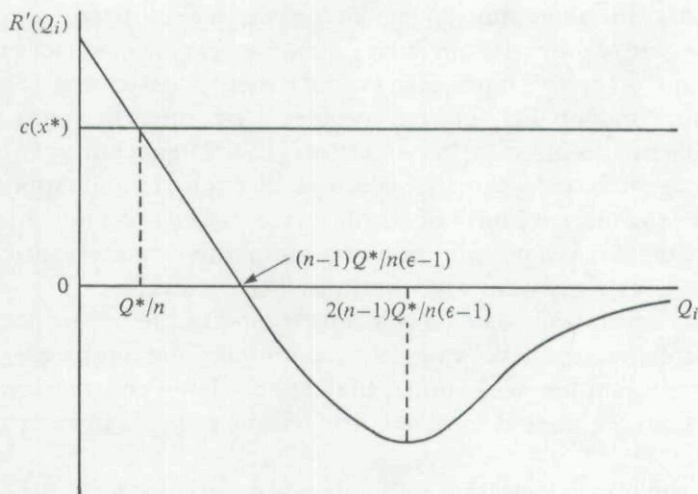


Fig. 7. Marginal revenue schedule,  $R'(Q_i)$ , of firm  $i$ , when gross revenue,

$$R(Q_i) = \sigma[Q_i + (n-1)Q^*/n]^{-\epsilon}Q_i,$$

where  $Q^*$  is equilibrium industry output,  $\epsilon > 1$  and  $n > \epsilon$ .  $x^*$  is equilibrium R & D expenditure per firm.

It will be noted that for  $n > \epsilon$ , the RHS of (A 2) is a decreasing function of  $n$  and is equal to unity at  $n = \epsilon(1 + \alpha)/\alpha$ . Combining (A 1) and (A 2) we therefore conclude that

$$\pi(x^*, Q^*/n) \geq 0 \quad \text{if } \epsilon < n \leq \epsilon(1 + \alpha)/\alpha. \quad (\text{A } 3)$$

(A 3) therefore instructs us to suppose that  $\epsilon < n \leq \epsilon(1 + \alpha)/\alpha$ . Let us do so.

It remains to find conditions under which the profit function,  $\{p[(n-1)Q^*/n + Q_i] - c(x_i)\} Q_i - x_i$ , for firm  $i$  is concave in the neighbourhood of the point  $x_i = x^*$ ,  $Q_i = Q^*/n$ . In fact it can be readily checked that if  $p(Q)$  and  $c(x)$  have the functional forms (7) and (9) and  $x^*$  and  $Q^*/n$  are given by (32) and (33), the profit function of the representative firm satisfies the second-order conditions at  $x^*$  and  $Q^*/n$  if  $\epsilon > \alpha(1 - \epsilon)$  (i.e. if  $\epsilon(1 + \alpha)/\alpha > 1$ ). Thus,  $(x^*, Q^*/n)$  is the global profit maximising choice for the representative firm if each of the other firms chooses  $(x^*, Q^*/n)$  (see Fig. 7). We have therefore proved:

*Theorem 1.* If  $p(Q)$  and  $c(x)$  satisfy (7) and (9) and if  $n$  is a positive integer, then a symmetric Cournot equilibrium amongst  $n$  firms exists if

- (i)  $\epsilon < n \leq \epsilon(1 + \alpha)/\alpha$ , and (ii)  $\epsilon(1 + \alpha)/\alpha > 1$ .

Now clearly  $\epsilon(1 + \alpha)/\alpha > 1$  if  $\epsilon > 1$ . What we have therefore shown is that if  $\epsilon < n \leq \epsilon(1 + \alpha)/\alpha$ , then a symmetric Cournot equilibrium with  $n$  firms exists even if market demand is throughout *inelastic*.

We turn now to locating conditions under which a *free-entry* symmetric Cournot equilibrium exists. Suppose there are  $n$  firms in the industry, and suppose for the moment barriers to entry. Then let  $(x^*, Q^*/n)$  be the chosen course of action for the representative firm at a symmetric Cournot equilibrium. Then market price for the product is  $p(Q^*)$ . Now define by  $\hat{x}$  the solution of the equation

$$p(Q^*) = c(x). \quad (\text{A } 4)$$

$\hat{x}$  is like an entry cost. It is clear that  $\exists m > 0$ , independent of  $n$ , such that if  $\pi(x^*, Q^*/n) \leq m\hat{x}$ , then no additional firm will find it profitable to enter the market even if there were no barriers to entry. Thus, in fact if  $n$  has such a value that  $\pi(x^*, Q^*/n) \leq m\hat{x}$ , condition (19) in the text is satisfied. We now proceed to locate conditions under which  $\pi(x^*, Q^*/n) \leq m\hat{x}$ . If  $p(Q)$  and  $c(x)$  satisfy (7) and (9) and if the value of  $Q^*$  given in equation (33) is used in (A 4), routine calculations show that

$$\hat{x} = (\beta/\sigma)^{1/\alpha} (n/\alpha\beta)^{\epsilon/\alpha} [\sigma(\alpha/n)^{\epsilon} \beta^{\epsilon-1} (1-\epsilon/n)]^{\epsilon(1+\alpha)/\alpha[\epsilon-\alpha(1-\epsilon)]}. \quad (\text{A } 5)$$

Moreover,

$$\begin{aligned} \pi(x^*, Q^*/n) &= [p(Q^*) - c(x^*)]Q^*/n - x^* \\ &= (n-\epsilon)^{-1} [\sigma(\alpha/n)^{\epsilon} \beta^{\epsilon-1} (1-\epsilon/n)]^{1/[\epsilon-\alpha(1-\epsilon)]} [\epsilon(1+\alpha)/\alpha - n]. \end{aligned} \quad (\text{A } 6)$$

Using (A 5) and (A 6) one notes that

$$\pi(x^*, Q^*/n) \leq \hat{x} \quad \text{if and only if} \quad \epsilon(1+\alpha)/\alpha - n \leq mn(1-\epsilon/n)^{(1+\alpha)/\alpha}. \quad (\text{A } 7)$$

As we are concerned with locating circumstances under which a free entry symmetric Cournot equilibrium *exists*, we are entitled to choose  $n$ . In what follows we shall always choose  $n$  to be the largest positive integer not exceeding  $\epsilon(1+\alpha)/\alpha$ . Call this  $n^*$ . Then  $n^* = [\epsilon(1+\alpha)/\alpha]$ . On appealing to Theorem 1, we may now note:

*Theorem 2.*<sup>1</sup> If  $p(Q)$  and  $c(x)$  satisfy (7) and (9) respectively, a free-entry symmetric Cournot equilibrium exists if (i)  $\epsilon(1+\alpha)/\alpha > 1$ , (ii)  $n^* = [\epsilon(1+\alpha)/\alpha] > \epsilon$ , and (iii)  $\epsilon(1+\alpha)/\alpha - n^* \leq mn^*(1-\epsilon/n^*)^{(1+\alpha)/\alpha}$ .

The foregoing theorem makes explicit the relationship between the parameters of the economy that are sufficient to guarantee the existence of a free-entry symmetric Cournot equilibrium. It should be noted in particular that the conditions in *Theorem 2* are satisfied if for any given value of  $\epsilon$ ,  $\alpha$  is chosen small enough and also, for any given value of  $\alpha$ , if  $\epsilon$  is chosen large enough. That is, an equilibrium exists if  $\epsilon/\alpha$  is sufficiently large. This confirms the claims made in the text.

It remains to find conditions under which the use of the zero-profit condition (21) is a good approximation. In fact we have already located such conditions. What we are looking for are conditions under which  $\pi(x^*, Q^*/n^*)/x^*$  is 'small'. (A 2) then tells us that for the specification given by (7) and (9) in the text

<sup>1</sup> Novshek (1977) has independently proved a result similar to this in a somewhat different context. We are grateful to Morton Kamien for drawing our attention to Novshek's work.

this is the same as finding conditions under which  $\epsilon/\alpha(n^* - \epsilon) - 1$  is negligible. Since by construction  $n^* = [\epsilon(1+\alpha)/\alpha]$ , one can locate ranges for the pair  $(\alpha, \epsilon)$  for which the zero profit condition is a good approximation. In particular, one notes that it is a good approximation if  $\epsilon/\alpha$  is 'large'. In this case,  $n^*$  is, of course 'large'.

## APPENDIX 2

In this appendix we compare the degree of risk-taking on the part of the pure monopolist with the socially optimum degree of risk-bearing. Towards this suppose the family of research projects is characterised by (34) and suppose  $R(\alpha) > 0$  for  $\alpha \geq 0$ ,  $R'(\alpha) > 0$  and  $R''(\alpha) < 0$ . Let the market demand function be generated by the social utility function (6). Thus the demand function is

$$p(Q) = \sigma Q^{-\epsilon} \quad (1 > \epsilon > 0). \quad (\text{A } 8)$$

Let  $V_m(c)$  denote the monopolist's maximum profit from sales when  $c$  is the unit cost of production. It is simple to confirm that

$$V_m(c) = \epsilon \sigma^{1/\epsilon} (1 - \epsilon)^{(1-\epsilon)/\epsilon} c^{-(1-\epsilon)/\epsilon}. \quad (\text{A } 9)$$

If the monopolist selects project- $\alpha$  his net expected profit is

$$h(\alpha)V_m[c(\alpha)] + [1 - h(\alpha)]V_m(c) - R(\alpha), \quad (\text{A } 10)$$

where  $V_m(c)$  is given by (A 9). Write

$$A \equiv \epsilon^{1/\epsilon} (1 - \epsilon)^{(1-\epsilon)/\epsilon}, \quad g(\alpha) \equiv \{h(\alpha)c(\alpha)^{-(1-\epsilon)/\epsilon} + [1 - h(\alpha)]c^{-(1-\epsilon)/\epsilon}\}.$$

Then (A 10) can be expressed as

$$Ag(\alpha) - R(\alpha). \quad (\text{A } 11)$$

We now suppose for analytical simplicity that the family of research projects is such that  $g(\alpha)$  is concave in  $\alpha$ . Then provided the (expected) profit maximising R & D project lies strictly between  $\alpha = 0$  and  $\alpha = \bar{\alpha}$ , the monopolist will choose that  $\alpha$  which is the solution of

$$Ag'(\alpha) = R'(\alpha). \quad (\text{A } 12)$$

Let  $\alpha_m$  be the (unique) solution of (A 12).

We turn now to the social planner. He is concerned with social benefit (36). Since social utility has the form (6),  $V(c)$  satisfies (8). Write  $B \equiv \epsilon \sigma^{1/\epsilon} (1 - \epsilon)^{-1}$ . Then (36) can be expressed as

$$Bg(\alpha) - R(\alpha). \quad (\text{A } 13)$$

Since by hypothesis (A 13) is strictly concave in  $\alpha$  the planner will also wish to engage in an elementary research project. Let  $\alpha_s$  ( $0 < \alpha_s < \bar{\alpha}$ ) be the socially optimal research project. Then  $\alpha_s$  must be the solution of

$$Bg'(\alpha) = R'(\alpha). \quad (\text{A } 14)$$

But  $B > A$ . On comparing (A 12) and (A 14) it follows that  $\alpha_s > \alpha_m$ . This establishes the claim made in the text.

## REFERENCES

- Arrow, K. J. (1962). 'Economic welfare and the allocation of resources for invention.' In *The Rate and Direction of Inventive Activity: Economic and Social Factors* (ed. R. Nelson). (NBER) Princeton University Press.
- Barzel, Y. (1968). 'Optimal timing of innovation.' *Review of Economics and Statistics* (August), pp. 348-55.
- Dasgupta, P., Heal, G. M. and Majumdar, M. (1977). 'Resource depletion and research and development.' In *Frontiers of Quantitative Economics*, vol. III B (ed. M. Intriligator). Amsterdam: North Holland.
- and Stiglitz, J. E. (1977). 'Market structure and research and development.' London School of Economics (mimeo).
- (1980). 'Uncertainty, market structure, and the speed of research.' *Bell Journal of Economics* Spring.
- Evenson, R. and Kieslev, Y. (1975). 'A model of technological research.' *Journal of Political Economy*.
- Kalecki, M. (1954). *Theory of Economic Dynamics*. London: George Allen and Unwin.
- Kamien, M. and Schwartz, N. (1972). 'Timing of innovations under rivalry.' *Econometrica*, 40, no. 1, pp. 43-60.
- (1975). 'Market structure and innovation: a survey.' *Journal of Economic Literature*, vol. 13, no. 1, pp. 1-37.
- (1978). 'Optimal exhaustible resource depletion with endogenous technical change.' *Review of Economic Studies*, vol. 45, pp. 179-96.
- Levin, R. C. (1978). 'Technical change, barriers to entry and market structure.' *Economica*, vol. 45, no. 180, pp. 347-62.
- Loury, G. (1977). 'Market structure and innovation.' NorthWestern University, Center for Mathematical Studies in Economics and Management Science, Discussion Paper, no. 256.
- Mansfield, E. (1967). *Econometric Studies of Industrial Research and Technological Innovation*. New York: W. W. Norton & Co.
- Marschak, T. and Selton, R. (1974). *General Equilibrium with Price-Making Firms*. Berlin: Springer-Verlag.
- Nelson, R. R., Peck, M. J. and Kalachek, E. D. (1967). *Technology, Economic Growth and Public Policy*. Washington, D.C.: Brookings Institution.
- and Winter, S. (1977). 'Forces generating and limiting concentration under Schumpeterian competition.' *Bell Journal of Economics*, vol. 9, no. 2 (Autumn), pp. 524-48.
- and Schuette, H. L. (1976). 'Technical change in an evolutionary model.' *Quarterly Journal of Economics*, vol. 90, no. 1, pp. 90-118.
- Novshek, W. (1977). 'Nash-Cournot equilibrium with entry.' Mimeo. Princeton University.
- Scherer, F. M. (1970). *Industrial Market Structure and Economic Performance*. Chicago: Rand McNally.
- Schmookler, J. (1962). 'Economic sources of inventive activity.' *Journal of Economic History*, vol. 22 (March).
- Schumpeter, J. (1947). *Capitalism, Socialism and Democracy* (2nd edn.). London: Allen and Unwin.
- Schwartzman, D. (1977). *Innovation in the Pharmaceutical Industry*. Baltimore: John Hopkins University Press.
- Stiglitz, J. E. (1970). 'Perfect and imperfect capital markets.' Yale University, mimeo. Paper presented to the Winter meeting of the Econometric Society (1970).
- Winter, S. (1971). 'Satisficing, selection, and the innovating remnant.' *Quarterly Journal of Economics*.

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