

# Inefficient star formation through turbulence, magnetic fields and feedback

Christoph Federrath<sup>★</sup>

*Research School of Astronomy and Astrophysics, The Australian National University, Canberra, ACT 2611, Australia*

Accepted 2015 April 24. Received 2015 April 24; in original form 2015 March 10

## ABSTRACT

Star formation is inefficient. Only a few per cent of the available gas in molecular clouds forms stars, leading to the observed low star formation rate (SFR). The same holds when averaged over many molecular clouds, such that the SFR of whole galaxies is again surprisingly low. Indeed, considering the low temperatures, molecular clouds should be highly gravitationally unstable and collapse on their global mean freefall time-scale. And yet, they are observed to live about 10–100 times longer, i.e. the SFR per freefall time ( $\text{SFR}_{\text{ff}}$ ) is only a few per cent. Thus, other physical mechanisms must counteract the quick global collapse. Turbulence, magnetic fields and stellar feedback have been proposed as regulating agents, but it is still unclear which of these processes is the most important and what their relative contributions are. Here, we run high-resolution simulations including gravity, turbulence, magnetic fields and jet/outflow feedback. We confirm that clouds collapse on a mean freefall time, if only gravity is considered, producing stars at an unrealistic rate. In contrast, if turbulence, magnetic fields and feedback are included step-by-step, the SFR is reduced by a factor of 2–3 with each additional physical ingredient. When they all act in concert, we find a constant  $\text{SFR}_{\text{ff}} = 0.04$ , currently the closest match to observations, but still about a factor of 2–4 higher than the average. A detailed comparison with other simulations and with observations leads us to conclude that only models with turbulence producing large virial parameters, and including magnetic fields and feedback can produce realistic SFRs.

**Key words:** MHD – turbulence – stars: formation – ISM: clouds – ISM: jets and outflows – ISM: kinematics and dynamics.

## 1 INTRODUCTION

Why is star formation so inefficient? For example, our entire home galaxy – the Milky Way – only produces about  $2 M_{\odot} \text{ yr}^{-1}$  (Chomiuk & Povich 2011), while there is plenty of gas available to form hundreds of stars per year (Zuckerman & Evans 1974; Zuckerman & Palmer 1974). Based on the range of typical molecular cloud masses of  $\sim 10^2$ – $10^7 M_{\odot}$  and sizes of  $\sim 1$ – $100$  pc, the average cloud mass density  $\rho$  is a few times  $10^{-21}$ – $10^{-19} \text{ g cm}^{-3}$ , corresponding to an average particle number density of about  $10^3$ – $10^5 \text{ cm}^{-3}$  for standard molecular composition. This would render the clouds highly unstable and immediately leads to a very short freefall time,  $t_{\text{ff}} = \sqrt{3\pi/(32G\rho)} = 0.1$ – $1$  Myr, which is at least one order of magnitude shorter than the typical lifetime of molecular clouds. Thus, other mechanisms than thermal pressure must be at work to prevent molecular clouds from collapsing globally and from forming stars at a 10–100 times higher rate than observed.

For a long time, the standard picture of star formation was that magnetic fields provide the primary source of stabilizing pressure and tension, which acts against the quick global collapse. Only after the neutral species had slowly diffused through the charged particles over an ‘ambipolar-diffusion’ time-scale of about 10 Myr, would star formation proceed in the central regions of magnetized clouds (Mestel & Spitzer 1956; Mouschovias 1976; Shu 1983). This standard theory requires that clouds start their lives dominated by a strong magnetic field, rendering them initially subcritical to collapse. After about one ambipolar-diffusion time-scale, magnetic flux was left behind in the cloud envelope, while the mass has increased in the cloud core such that some stars could form in centre. Thus, star formation regulated by ambipolar diffusion predicts a higher mass-to-flux ratio in the cores than in the envelopes of the clouds, which is – however – typically not observed (Crutcher, Hakobian & Troland 2009; Lunttila et al. 2009; Mouschovias & Tassis 2009; Santos-Lima et al. 2010; Bertram et al. 2012; Lazarian, Esquivel & Crutcher 2012).

An alternative scenario is that clouds are in fact collapsing globally, thus initially forming stars at a very high rate, but that the

<sup>★</sup> E-mail: [christoph.federrath@gmail.com](mailto:christoph.federrath@gmail.com)

ionization feedback from massive stars eventually terminates the global collapse and disperses the clouds (Vázquez-Semadeni 2015).

A third alternative is that the observed supersonic random motions (Zuckerman & Evans 1974; Zuckerman & Palmer 1974; Larson 1981; Solomon et al. 1987; Falgarone, Puget & Perault 1992; Ossenkopf & Mac Low 2002; Heyer & Brunt 2004; Roman-Duval et al. 2011; Schneider et al. 2011) regulate star formation. In this modern picture, turbulence plays a crucial dual role. On the one hand, the turbulent kinetic energy stabilizes the clouds on large scales and prevents global collapse, on the other hand, it induces local compressions in shocks, because the turbulence is supersonic (Elmegreen & Scalo 2004; Mac Low & Klessen 2004; McKee & Ostriker 2007). This generates the initial conditions for star formation, because the local compressions typically produce filaments and dense cores at the intersections of filaments (Schneider et al. 2012).

So which scenario is right or wrong? As it turns out, neither of the three is entirely wrong, nor do they explain everything by themselves. In the most recent years, we have come to a deeper understanding of the relative importance of magnetic fields and turbulence through numerical simulations. While the role of turbulence cannot be denied, because it naturally explains most of the observed velocity and density structure of molecular clouds, the role of magnetic fields remained less clear. Simulation work by e.g. Price & Bate (2007) and Hennebelle & Teyssier (2008) suggested that magnetic fields can suppress fragmentation on small scales and hence influence the shape of the initial mass function of stars. Moreover, Wang et al. (2010), Padoan & Nordlund (2011), Federrath & Klessen (2012) and Myers et al. (2014) showed that magnetic fields reduce the SFR by a factor of 2–3 compared to the non-magnetized case. Importantly however, magnetic fields also launch fast, powerful, mass-loaded jets and outflows from the protostellar accretion disc. These jets and outflows can drive turbulence and alter the cloud structure and dynamics so profoundly that the SFR and the initial mass function might be even more affected by this jet/outflow feedback mechanism (Federrath et al. 2014; Krumholz et al. 2014).

The aim of this study is to determine the physical processes that make star formation inefficient. We measure which physical mechanisms are relevant for bringing the SFR in agreement with observations and we determine their relative importance, i.e. by what amount they reduce the SFR individually and when acting all in concert.

Section 2 summarizes our numerical methods and simulations from which we measure the SFR. Our results are presented in Section 3, where we find that purely self-gravitating molecular clouds are indeed highly unstable, while the step-by-step inclusion of turbulence, magnetic fields and jet/outflow feedback brings the SFR down by one to two orders of magnitude when all of these physical mechanisms act together. In Section 4, we review the existing literature and compare our simulations and those of other groups with observational data for the SFR. We summarize our findings and conclusions in Section 5.

## 2 NUMERICAL SIMULATION TECHNIQUES

We use the multi-physics, adaptive mesh refinement (AMR; Berger & Colella 1989) code FLASH (Fryxell et al. 2000; Dubey et al. 2008) in its latest version (v4), to solve the compressible magnetohydrodynamical (MHD) equations on three-dimensional (3D) periodic grids of fixed side length  $L$ , including turbulence, magnetic fields, self-gravity and outflow feedback. The positive-definite HLL5R Riemann solver (Waagan, Federrath & Klingenberg 2011) is used

to guarantee stability and accuracy of the numerical solution of the MHD equations.

### 2.1 Turbulence driving

Turbulence is a key for star formation (Elmegreen & Scalo 2004; Mac Low & Klessen 2004; McKee & Ostriker 2007; Krumholz 2014; Padoan et al. 2014), so most of our simulations include a turbulence driving module that produces turbulence similar to what is observed in real molecular clouds, i.e. driving on the largest scales (Heyer, Williams & Brunt 2006; Brunt, Heyer & Mac Low 2009) and with a power spectrum,  $E(k) \sim k^{-2}$ , consistent with supersonic, compressible turbulence (Larson 1981; Heyer & Brunt 2004; Roman-Duval et al. 2011) and confirmed by simulations (Kritsuk et al. 2007; Federrath et al. 2010a; Federrath 2013a). We drive turbulence by applying a stochastic Ornstein–Uhlenbeck process (Eswaran & Pope 1988; Schmidt, Hillebrandt & Niemeyer 2006) to construct an acceleration field  $F_{\text{stir}}$ , which serves as a momentum and energy source term in the momentum equation of MHD. As suggested by observations,  $F_{\text{stir}}$  only contains large-scale modes,  $1 < |k| L/2\pi < 3$ , where most of the power is injected at the  $k_{\text{inj}} = 2$  mode in Fourier space, i.e. on half of the box size. The turbulence on smaller scales is not directly affected by the driving and develops self-consistently. The turbulence forcing module used here excites a natural mixture of solenoidal and compressible modes, corresponding to a turbulent driving parameter  $b = 0.4$  (Federrath et al. 2010a), although some cloud-to-cloud variations in this parameter from  $b \sim 1/3$  (purely solenoidal driving) to  $b \sim 1$  (purely compressive driving) are expected for real clouds (Price, Federrath & Brunt 2011; Kainulainen, Federrath & Henning 2013).

### 2.2 Sink particles

In order to measure the SFR, we use the sink particle method developed by Federrath et al. (2010b). Sink particles form dynamically in our simulations when a local region in the simulation domain undergoes gravitational collapse and forms stars. This is technically achieved by first flagging each computational cell that exceeds the Jeans resolution density,

$$\rho_{\text{sink}} = \frac{\pi c_s^2}{G \lambda_J^2} = \frac{\pi c_s^2}{4 G r_{\text{sink}}^2}, \quad (1)$$

with the sound speed  $c_s$ , the gravitational constant  $G$  and the local Jeans length  $\lambda_J$ . Thus, the sink particle accretion radius is given by  $r_{\text{sink}} = \lambda_J/2$  and set to 2.5 grid cell lengths in order to capture star formation and to avoid artificial fragmentation on the highest level of AMR (Truelove et al. 1997). If the gas density in a cell exceeds  $\rho_{\text{sink}}$ , a spherical control volume with radius  $r_{\text{sink}}$  is constructed around that cell and it is checked that all the gas within the control volume is Jeans-unstable, gravitationally bound and collapsing towards the central cell. A sink particle is only formed in the central cell of the control volume, if all of these checks are passed. This avoids spurious formation of sink particles and guarantees that only bound and collapsing gas forms stars (Federrath et al. 2010b), which is important for accurately measuring the SFR.

On all the lower levels of AMR (except the highest level, where sink particles form), we use an adaptive grid refinement criterion based on the local Jeans length, such that  $\lambda_J$  is always resolved with at least 32 grid cell lengths in each of the three spatial directions of our 3D domain. This resolution criterion is very conservative and computationally costly, but guarantees that we resolve turbulence on the Jeans scale (Federrath et al. 2011), potential dynamo amplification of the magnetic field in the dense cores (Sur et al. 2010),

**Table 1.** Key simulation parameters.

Model name (1)	Turbulence (2)	$\sigma_v$ (km s <sup>-1</sup> ) (3)	$\mathcal{M}$ (4)	$B$ ( $\mu$ G) (5)	$\beta$ (6)	$\mathcal{M}_A$ (7)	Jets/Outflows (8)	$N_{\text{res}}^3$ (9)	SFR( $M_{\odot}$ yr <sup>-1</sup> ) (10)	$\Sigma_{\text{SFR}}$ ( $M_{\odot}$ pc <sup>-2</sup> Myr <sup>-1</sup> ) (11)	SFR <sub>ff</sub> (12)
G	None	0	0	0	$\infty$	$\infty$	No	1024 <sup>3</sup>	$1.6 \times 10^{-4}$	39	0.47
GvsT	Mix	1.0	5.0	0	$\infty$	$\infty$	No	1024 <sup>3</sup>	$8.3 \times 10^{-5}$	21	0.25
GvsTM	Mix	1.0	5.0	10	0.33	2.0	No	1024 <sup>3</sup>	$2.8 \times 10^{-5}$	6.9	0.083
GvsTMJ	Mix	1.0	5.0	10	0.33	2.0	Yes	1024 <sup>3</sup>	$1.4 \times 10^{-5}$	3.4	0.041
GvsTMJ512	Mix	1.0	5.0	10	0.33	2.0	Yes	512 <sup>3</sup>	$1.3 \times 10^{-5}$	3.2	0.039
GvsTMJ256	Mix	1.0	5.0	10	0.33	2.0	Yes	256 <sup>3</sup>	$8.9 \times 10^{-6}$	2.2	0.027

*Notes.* Column 1: simulation name. Columns 2–4: the type of turbulence driving (Federrath et al. 2010a), turbulent velocity dispersion and turbulent rms sonic Mach number. Columns 5–7: magnetic field strength, the ratio of thermal to magnetic pressure (plasma  $\beta$ ) and the Alfvén Mach number. Column 8: whether jet and outflow feedback was included or not. Column 9: maximum effective grid resolution (note that refinement is based on the Jeans length with a minimum of 32 cells per Jeans length). Columns 10–12: the absolute SFR, the SFR column density and the SFR per mean global freefall time. The standard four simulation models are in the first four rows. Two additional simulations with lower resolution, but otherwise identical to GvsTMJ, are listed in the last two rows, in order to check convergence of our results for the SFR.

and capture the basic structure of accretion discs forming on the smallest scales (Federrath et al. 2014). If a cell within the accretion radius of an existing sink particle exceeds  $\rho_{\text{sink}}$  during the further evolution, is bound to the sink particle and is moving towards it, then we accrete the excess mass above  $\rho_{\text{sink}}$  on to the sink particle, conserving mass, momentum and angular momentum. We compute all contributions to the gravitational interactions between the gas on the grid (with the iterative multigrid solver by Ricker 2008) and the sink particles (by direct summation over all sink particles and grid cells). A second-order leapfrog integrator is used to advance the sink particles on a timestep that allows us to resolve close and highly eccentric orbits (for details, see the tests in Federrath et al. 2010b).

### 2.3 Outflow/Jet feedback

Powerful jets and outflows are launched from the protostellar accretion discs around newborn stars. These outflows carry enough mass, linear and angular momentum to transform the structure of their parent molecular cloud and to potentially control star formation itself. In order to take this most important mechanical feedback effect (Krumholz et al. 2014) into account, we recently extended the sink particle approach such that sink particles can launch fast collimated jets together with a wide-angle, lower speed outflow component, to reproduce the global features of observed jets and outflows, as well as to be consistent with high-resolution simulations of the jet launching process and with theoretical predictions (Federrath et al. 2014). The most important feature of our jet/outflow feedback model is that it converges and produces the large-scale effects of jets and outflows already with relatively low resolution, such as with sink particle radii  $r_{\text{sink}} \sim 1000$  au used in our star cluster simulations here. Our module has been carefully tested and compared to previous implementations of jet/outflow feedback such as the models implemented in Wang et al. (2010) and Cunningham et al. (2011). The most important difference to any previous implementation is that our feedback model includes angular momentum transfer, reproduces the fast collimated jet component and demonstrated convergence (for details, see Federrath et al. 2014).

### 2.4 Simulation parameters

All our simulations share the same global properties: a cloud size  $L = 2$  pc, a total cloud mass  $M = 388 M_{\odot}$  and a mean density  $\rho_0 = 3.28 \times 10^{-21}$  g cm<sup>-3</sup>, resulting in a global mean freefall time

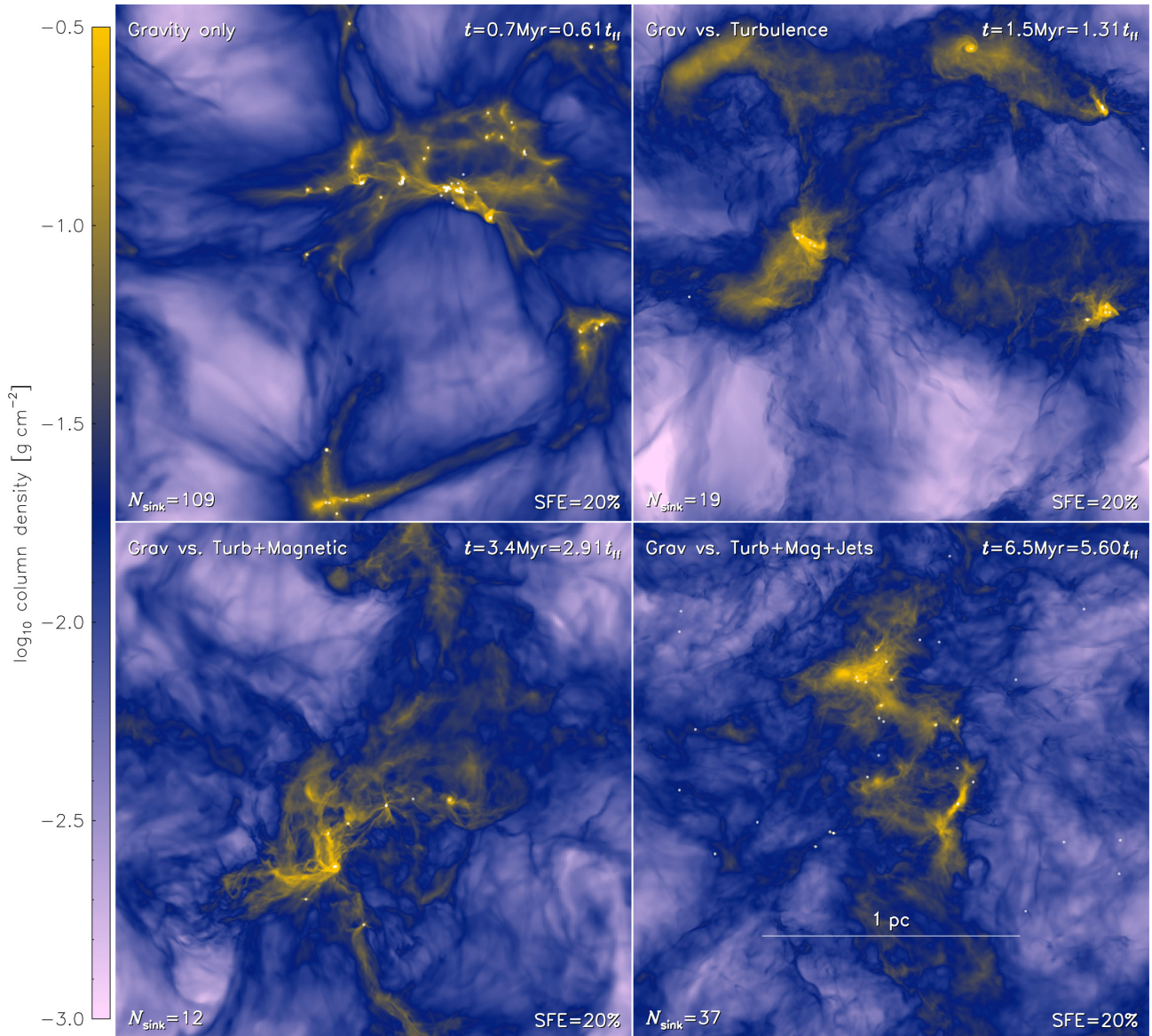
$t_{\text{ff}} = 1.16$  Myr. Models including turbulence have a velocity dispersion  $\sigma_v = 1$  km s<sup>-1</sup> and an rms Mach number of  $\mathcal{M} = 5$ , given the sound speed  $c_s = 0.2$  km s<sup>-1</sup>, appropriate for molecular gas with temperature  $T = 10$  K over the wide range of densities that lead to dense core and eventually star formation (Omukai et al. 2005). Finally, models including a magnetic field start with a uniform initial field of  $B = 10$   $\mu$ G, which is subsequently compressed, tangled and twisted by the turbulence, similar to how it would be structured in real molecular clouds (Planck Collaboration XXXV 2015). The magnetic field strength, the turbulent velocity dispersion and the mean density all follow typical values derived from observations of clouds with the given physical properties (Falgarone et al. 1992; Crutcher et al. 2010). This leads to the dimensionless virial ratio  $\alpha_{\text{vir}} = 1.0$  (also typical for molecular clouds in the Milky Way; see Kauffmann, Pillai & Goldsmith 2013) and to a plasma beta parameter (ratio of thermal to magnetic pressure)  $\beta = 0.33$  or an Alfvén Mach number  $\mathcal{M}_A = 2.0$ . Falgarone et al. (2008) find an average Alfvén Mach number of about  $\mathcal{M}_A = 1.5$  in 14 different star-forming regions in the Milky Way. Thus, the assumed magnetic field in our simulation models is very close to the values typically observed in molecular clouds and cloud cores.

We run four basic models, which – step-by-step – include more physics. In the first simulation, we only include self-gravity with a given initial density distribution resembling molecular cloud structure, but we do not include any turbulent velocities or magnetic field. In the second model, we include a typical level and mixture of molecular cloud turbulence (see Section 2.1). The third model is identical to the second model, but adds a standard magnetic field for the given cloud size and mass. Finally, the fourth model is identical to the third model, but additionally includes jet and outflow feedback (see Section 2.3). These four basic models were all run with a maximum effective grid resolution of 1024<sup>3</sup> cells. Their key parameters are listed in Table 1. We also run two additional models, which are identical to the fourth model (with jet/outflow feedback), but have a lower maximum effective resolution of 512<sup>3</sup> and 256<sup>3</sup> cells, respectively, in order to check numerical convergence of our results for the SFR. Those models are listed in the bottom two rows of Table 1.

## 3 RESULTS

### 3.1 Cloud structure and stellar distribution

Fig. 1 shows column density projections of our four basic models from Table 1. The simulation that only includes self-gravity



**Figure 1.** Column density projections at the end of each simulation model: Gravity only (top left), Gravity versus Turbulence (top right), Gravity versus Turbulence + Magnetic Fields (bottom left), and Gravity versus Turbulence + Magnetic Fields + Jet/Outflow Feedback (bottom right). The time to reach a realistic SFE = 20 per cent with small star clusters having formed, is shown in the top right corner of each panel. The time increases significantly for each model that adds physical processes opposing gravitational collapse. This demonstrates that only the combination of turbulence, magnetic fields and feedback can produce realistic SFRs, which is quantified below and summarized in Table 1. An animation of these still frames is available in the online version.

(no turbulence, no magnetic fields and no feedback) is shown in the top left-hand panel. The gas structures we see in the figure resemble the typical distribution found in gravity-only simulations such as simulations of pure dark matter in the cosmological case. Gas from the voids continuously falls on to dense filaments. Stars form inside the filaments and in particular where filaments intersect. This seems to be true also for real molecular clouds to some degree (Schneider et al. 2012), but the interfilament gas is much more disturbed and turbulent in real molecular clouds compared to the gravity-only case. We stop the simulation when 20 per cent of the gas is wound up in stars, i.e. the star formation efficiency  $SFE \equiv M_{\text{stars}}/(M_{\text{stars}} + M_{\text{gas}}) = 20$  per cent, which is already reached within a fraction of a global mean freefall time,  $t = 0.61 t_{\text{ff}}$ , for the gravity-only simulation. Thus, the whole cloud forms stars in roughly a global freefall time, which is the ex-

pected – and unrealistic – outcome if no other physics but gravity is considered.

When we add a realistic level of turbulence to the cloud, shown in the top right-hand panel of Fig. 1, we find that the time to reach SFE = 20 per cent increases to  $t = 1.31 t_{\text{ff}}$  or 1.5 Myr. The overall structure is now much closer to what we see in real molecular clouds, especially the turbulent velocity dispersion measured in observations is approximately reproduced in this model. However, the SFR is still an order of magnitude too large compared to typical observations and there is no magnetic field present, contrary to what is observed (Crutcher et al. 2010).

Thus, we add magnetic fields in the simulation shown in the bottom left-hand panel of Fig. 1 and see that the end time is shifted by slightly more than a factor of 2,  $t = 2.91 t_{\text{ff}} = 3.4$  Myr, compared to non-MHD case. Magnetic fields thus reduce the SFR by

about a factor of 2–3, as found in previous simulations (Wang et al. 2010; Padoan & Nordlund 2011; Federrath & Klessen 2012; Padoan, Haugbølle & Nordlund 2012). Although we added a considerable set of physics (gravity, turbulence and magnetic fields) in this simulation model, the SFR is still much higher than inferred from observations, but many spatial and morphological features seen in real star-forming molecular clouds are well reproduced with this set of physics.

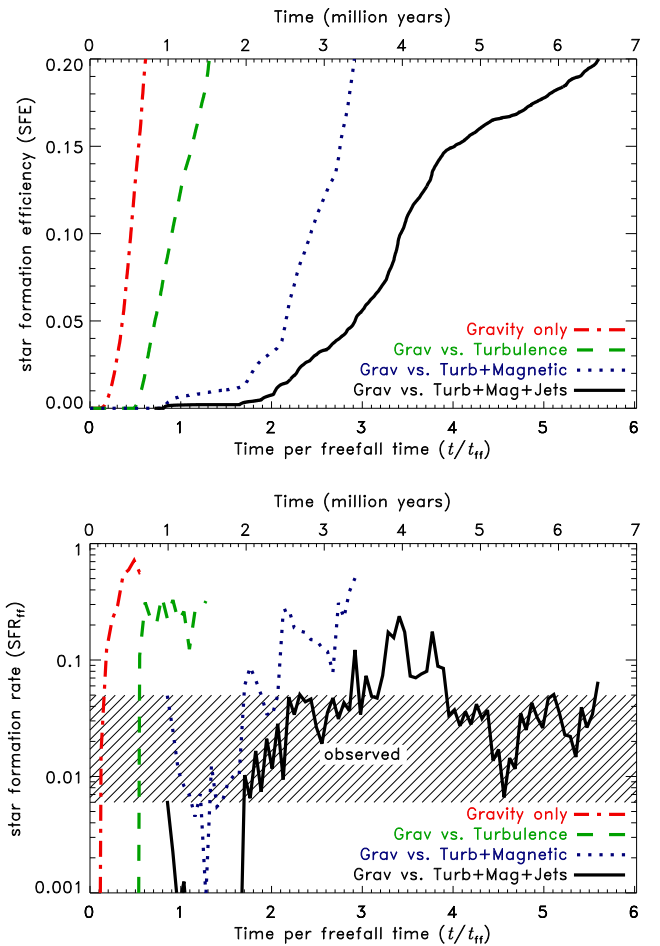
Finally, we add jet and outflow feedback in the simulation shown in the bottom right-hand panel of Fig. 1. The overall cloud structure is similar to the turbulent MHD case without feedback, but the SFR is further reduced by about another factor of 2. Now it takes  $6.5 t_{\text{ff}} = 5.6 \text{ Myr}$  to reach a reasonable SFE = 20 per cent in this small cluster-forming region of a molecular cloud. The most interesting morphological difference compared to all the no-feedback cases is that the oldest stars have wandered away from their formation sites and are now distributed over a much wider volume. Newborn stars are still always located on or very close to high-density peaks. A total of 37 stars have formed in the whole field, while some of those stars might actually represent close binaries or small multiple systems given the limited resolution available in these calculations. While the absolute fragmentation is not fully converged in these simulations, we emphasize that the SFR and the amount of mechanical feedback is well captured and is converged with our numerical resolution. This is shown in Table 1, where in the last three columns, we compare this feedback model (GvsTMJ) with two lower resolution equivalents (GvsTMJ512 and GvsTMJ256), demonstrating convergence of the SFR to within 5 per cent.

### 3.2 Star formation rate

Now we concentrate on the quantitative determination of the SFR. Fig. 2 shows the time evolution of the SFE (top panel) and the SFR (bottom panel) in our four fiducial simulation models. While the gravity-only model (dot-dashed line) turns all the gas into stars in about a global mean freefall time, the SFR per freefall time defined as  $\text{SFR}_{\text{ff}} \equiv d(\text{SFE})/d(t/t_{\text{ff}})$  (Krumholz & McKee 2005; Federrath & Klessen 2012) is reduced by about a factor of 2 when turbulence is included (dashed line). The  $\text{SFR}_{\text{ff}}$  is reduced by at least another factor of 2 when turbulence and magnetic fields are present (dotted line), and by another factor of 2 when jet and outflow feedback is included (solid line).

The hatched region in the bottom panel of Fig. 2 shows the observed range of  $\text{SFR}_{\text{ff}}$  based on the observational data compiled in Krumholz & Tan (2007). Our feedback model eventually settles into that observed range.

A very interesting and noteworthy feature of our feedback model is the evolution of the SFR with time, in particular the fact that the SFR slowly but steadily increases until about  $t \sim 4.0 \text{ Myr}$  and then drops significantly by almost an order of magnitude until  $t \sim 5.3 \text{ Myr}$ , followed by a period of nearly constant SFR. This is a manifestation of self-regulation by feedback. As more material is accreted and the SFR increases, the amount of gas being re-injected into the interstellar medium in the form of fast mass-loaded jets and outflows increases proportionally. Eventually, the feedback enhances the turbulence and the amount of kinetic energy in the system, thereby increasing the virial parameter,  $\alpha_{\text{vir}} = 2E_{\text{kin}}/E_{\text{grav}}$ , such that star formation is rapidly quenched. This quenching of the SFR in turn reduces the amount of feedback such that the system eventually settles into a self-regulated state of star formation in which any intermittent increase in accretion triggers feedback that regulates the SFR down to a nearly constant level.



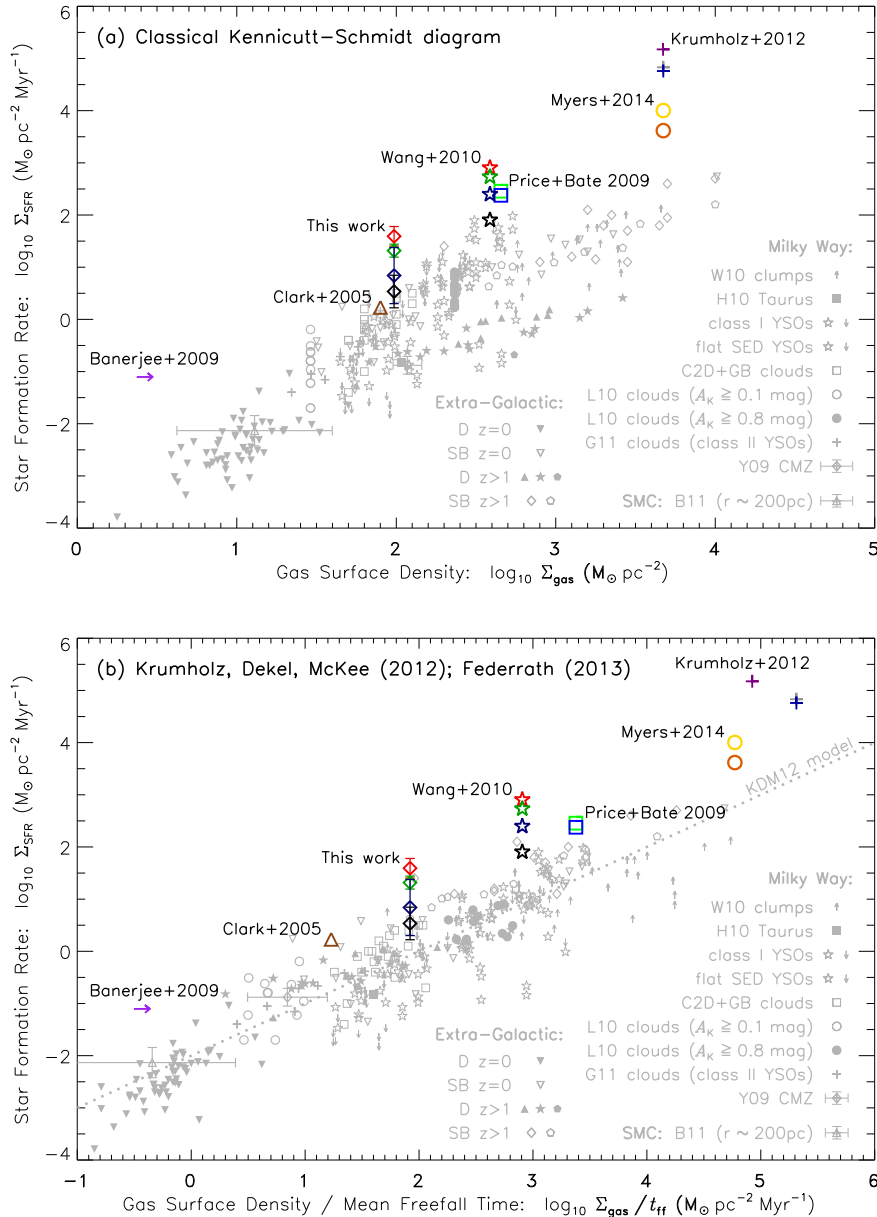
**Figure 2.** Time evolution of the SFE (top panel) and the SFR per freefall time,  $\text{SFR}_{\text{ff}} = d(\text{SFE})/d(t/t_{\text{ff}})$  (bottom panel) for each of our four standard models from Table 1. We immediately see the drastic reduction in the SFR when turbulence, magnetic fields and jet/outflow feedback together counteract gravity, in which case the SFR is reduced to values consistent with the observed range of SFRs, shown as the hatched region.

The last three columns in Table 1 list the time-averaged SFRs in each simulation model. For the gravity-only model, we find unrealistically high  $\text{SFR}_{\text{ff}} = 0.47$ , while the feedback model including turbulence and magnetic fields has a time-averaged  $\text{SFR}_{\text{ff}} = 0.041$ . Thus, the combination of turbulence, magnetic fields and jet/outflow feedback reduces the SFR by more than an order of magnitude and brings the SFR into the observed range.

## 4 COMPARISON WITH OBSERVATIONS

In Fig. 3, we show how our simulations and those by other groups compare to observations of the SFR. The observational data are a collection of star-forming Milky Way clouds, molecular clumps and young stellar objects from Heiderman et al. (2010), Lada, Lombardi & Alves (2010) and Gutermuth et al. (2011), as well as the Central Molecular Zone (CMZ) from Yusef-Zadeh et al. (2009). The extragalactic data of disc (D) and starburst (SB) galaxies at low and high redshift were compiled in Krumholz et al. (2012a) and Krumholz, Dekel & McKee (2013), and summarized and extended by the Small Magellanic Cloud (SMC) in Federrath (2013b).

The top panel of Fig. 3 shows the classical Kennicutt–Schmidt diagram, i.e. a plot of the SFR surface density ( $\Sigma_{\text{SFR}}$ ) versus the



**Figure 3.** SFR column density ( $\Sigma_{\text{SFR}}$ ) as a function of the gas surface density,  $\Sigma_{\text{gas}}$  (classical Kennicutt–Schmidt diagram, top panel) and  $\Sigma_{\text{SFR}}$  as a function of the gas surface density divided by the mean freefall time,  $\Sigma_{\text{gas}}/t_{\text{ff}}$  (Krumholz, Dekel & McKee 2012a; Federrath 2013b, bottom panel). The grey data points are from observations in the Milky Way, the SMC, and in disc and starburst galaxies at low and high redshift summarized in Federrath (2013b). The coloured and labelled symbols are from numerical simulations: Krumholz, Klein & McKee (2012b, crosses), Myers et al. (2014, circles), Price & Bate (2009, squares), Wang et al. (2010, stars), Clark et al. (2005, triangle), Banerjee et al. (2009, arrow) and from this work (diamonds). The simulations use a wide range of different setups, initial conditions, physics and numerical methods. The general trend is that most simulations produce SFRs about one to two orders of magnitude too high compared to the typical values found in the observations. Only simulations that include turbulence, magnetic fields and jet/outflow feedback or have very strong turbulence (producing very large virial ratios  $\alpha_{\text{vir}} = 2E_{\text{kin}}/E_{\text{grav}} \gtrsim 4$ ) are consistent with the observations.

gas surface density ( $\Sigma_{\text{gas}}$ ). The bottom panel shows a more recent representation of a star formation law introduced in Krumholz et al. (2012a), where  $\Sigma_{\text{SFR}}$  is plotted as a function of  $\Sigma_{\text{gas}}/t_{\text{ff}}$ , i.e.  $\Sigma_{\text{gas}}$  divided by the global mean freefall time  $t_{\text{ff}}$ , which exhibits a better correlation than the classical Kennicutt–Schmidt relation. The remaining scatter in this new relation is discussed and explained in Federrath (2013b), by variations in the turbulent properties – primarily in the turbulent Mach number – of the molecular clouds forming stars in the observations.

In both panels of Fig. 3, we superimpose simulation data on to the observational data points. The crosses are simulations from

Krumholz et al. (2012b) with the highest to the lowest  $\Sigma_{\text{SFR}}$  produced in their simulations without turbulence and isolated gravity boundaries (top cross), including turbulence and periodic gravity boundaries (middle cross), and finally adding outflow feedback (bottom cross). The circles show simulations from Myers et al. (2014). The top circle is a purely hydrodynamical simulation and the bottom circle included a magnetic field, yielding a reduction of the SFR by about a factor of 2–3, consistent with Wang et al. (2010), Padoan & Nordlund (2011), Padoan et al. (2012), Federrath & Klessen (2012) and this work. Both the Krumholz et al. (2012b) and Myers et al. (2014) simulations probe a regime of extremely

high column density and extremely high  $\Sigma_{\text{gas}}/t_{\text{ff}}$ . The squares in Fig. 3 are simulations by Price & Bate (2009) with a strong magnetic field. The square with slightly lower  $\Sigma_{\text{SFR}}$  is from their simulation model that included radiative heating, while the other one did not. From this, we can conclude that radiative heating – at least from low-mass stars – only reduces the SFR by  $\lesssim 20$  per cent.

The star symbols in Fig. 3 show a series of simulations by Wang et al. (2010), similar to ours, but at higher mean column density. The stars from high to low  $\Sigma_{\text{SFR}}$  correspond to gravity-only, plus turbulence, plus magnetic fields and plus outflow feedback, respectively. Our set of basic models is shown as diamonds with error bars based on the time-variations of the SFR shown in Fig. 2. When we compare the Wang et al. (2010) series of simulation models with the ones in this work, we find the same basic trend. Adding turbulence, magnetic fields and outflow feedback reduce the SFR step-by-step. However, the minimum  $\text{SFR}_{\text{ff}}$  obtained in Wang et al. (2010) was  $\text{SFR}_{\text{ff}} = 0.10$ , which is significantly larger than our best model, which has  $\text{SFR}_{\text{ff}} = 0.041^{+0.043}_{-0.021}$ . Given the higher virial parameter of  $\alpha_{\text{vir}} = 2E_{\text{kin}}/E_{\text{grav}} = 1.5$  compared to our value ( $\alpha_{\text{vir}} = 1$ ) and their very strong magnetic field (100  $\mu\text{G}$  compared to our 10  $\mu\text{G}$ ), we would have expected that the Wang et al. (2010) simulation would form stars at a lower rate, because of the stronger support from the turbulence and the stronger support from the magnetic field than in our case. The most likely reason for this discrepancy is that Wang et al. (2010) use a centrally concentrated density profile with  $\rho \sim r^{-2}$  as an initial condition such that star formation in the centre of the cloud is much more violent than in the outskirts (Girichidis et al. 2011).

A colliding flow simulation by Banerjee et al. (2009) with magnetic field is also shown in Fig. 3 as an arrow. In order to compute  $\Sigma_{\text{SFR}}$ ,  $\Sigma_{\text{gas}}$  and  $t_{\text{ff}}$  for their simulation model, we used the average  $\text{SFR} = 5 \times 10^{-4} M_{\odot} \text{yr}^{-1}$  quoted in Banerjee et al. (2009), the average cloud mass of  $1.5 \times 10^4 M_{\odot}$ , the average cloud area of  $\pi(45 \text{ pc})^2$  and the average cloud thickness of 2 pc. This leads to  $\Sigma_{\text{SFR}} = 0.079 M_{\odot} \text{pc}^{-2} \text{Myr}^{-1}$ ,  $\Sigma_{\text{gas}} = 2.4 M_{\odot} \text{pc}^{-2}$ ,  $t_{\text{ff}} = 7.5 \text{ Myr}$  and thus  $\text{SFR}_{\text{ff}} = 0.25$ .

Finally, the triangle in Fig. 3 shows the simulation from Clark et al. (2005), which reaches low SFRs despite the fact that neither magnetic fields nor feedback were included, but just turbulence. The explanation for this behaviour is that the virial parameter is  $\alpha_{\text{vir}} = 4$  in Clark et al. (2005). Thus, their  $\alpha_{\text{vir}}$  is very high, such that large fractions of the cloud are essentially unbound and cannot form stars. This is in line with the theoretical prediction that the SFR drops exponentially with increasing  $\alpha_{\text{vir}}$  (Krumholz & McKee 2005; Federrath & Klessen 2012; Padoan et al. 2012). However, only few clouds have such high virial parameters and the SFR per freefall time is still quite high in Clark et al. (2005) with  $\text{SFR}_{\text{ff}} \sim 0.10$ , despite the large  $\alpha_{\text{vir}}$ . Thus, magnetic fields and feedback seem to be essential physical ingredients to reach realistic SFRs.

## 5 SUMMARY AND CONCLUSIONS

We presented high-resolution hydrodynamical simulations of star cluster formation including turbulence, magnetic fields and jet/outflow feedback. Although our simulation models with turbulence, magnetic fields and jet/outflow feedback produce  $\text{SFR}_{\text{ff}} \sim 0.04^{+0.04}_{-0.02}$ , which is currently the closest available match to observations, they still overproduce stars by a factor of 2–4 compared to the observed average  $\text{SFR}_{\text{ff}} \sim 0.01$ . This discrepancy might be resolved by considering other types of feedback in addition to jet/outflow feedback, such as radiation pressure, which seems to be capable of reducing the SFR further by another factor of 2 (MacLachlan

et al. 2015, note however that radiative heating from low- and intermediate-mass stars does not affect the SFR significantly; cf. Section 4). Our results also indicate that the typical level of turbulence and magnetic fields in molecular clouds may be higher than previously thought. We conclude that only the combination of strong turbulence with virial parameters above unity, strong magnetic fields and mechanical plus radiative feedback can produce realistic SFRs consistent with observations.

## ACKNOWLEDGEMENTS

We thank Enrique Vázquez-Semadeni and the anonymous referee for their useful comments on the manuscript. CF acknowledges funding provided by the Australian Research Council’s Discovery Projects (grants DP130102078 and DP150104329). The author gratefully acknowledges the Jülich Supercomputing Centre (grant hhd20), the Leibniz Rechenzentrum and the Gauss Centre for Supercomputing (grant pr32lo), the Partnership for Advanced Computing in Europe (PRACE grant pr89mu), and the National Computational Infrastructure (grant ek9), supported by the Australian Government. This work was further supported by resources provided by the Pawsey Supercomputing Centre with funding from the Australian Government and the Government of Western Australia. The software used in this work was in part developed by the DOE-supported Flash Center for Computational Science at the University of Chicago.

## REFERENCES

- Banerjee R., Vázquez-Semadeni E., Hennebelle P., Klessen R. S., 2009, *MNRAS*, 398, 1082
- Berger M. J., Colella P., 1989, *J. Comput. Phys.*, 82, 64
- Bertram E., Federrath C., Banerjee R., Klessen R. S., 2012, *MNRAS*, 420, 3163
- Brunt C. M., Heyer M. H., Mac Low M., 2009, *A&A*, 504, 883
- Chomiuk L., Povich M. S., 2011, *AJ*, 142, 197
- Clark P. C., Bonnell I. A., Zinnecker H., Bate M. R., 2005, *MNRAS*, 359, 809
- Crutcher R. M., Hakobian N., Troland T. H., 2009, *ApJ*, 692, 844
- Crutcher R. M., Wandelt B., Heiles C., Falgarone E., Troland T. H., 2010, *ApJ*, 725, 466
- Cunningham A. J., Klein R. I., Krumholz M. R., McKee C. F., 2011, *ApJ*, 740, 107
- Dubey A. et al., 2008, in Pogorelov N. V., Audit E., Zank G. P., eds, *ASP Conf. Ser. Vol. 385, Numerical Modelling of Space Plasma Flows: Astronom-2007*. Astron. Soc. Pac., San Francisco, p. 145
- Elmegreen B. G., Scalo J., 2004, *ARA&A*, 42, 211
- Eswaran V., Pope S. B., 1988, *Comput. Fluids*, 16, 257
- Falgarone E., Puget J.-L., Perault M., 1992, *A&A*, 257, 715
- Falgarone E., Troland T. H., Crutcher R. M., Paubert G., 2008, *A&A*, 487, 247
- Federrath C., 2013a, *MNRAS*, 436, 1245
- Federrath C., 2013b, *MNRAS*, 436, 3167
- Federrath C., Klessen R. S., 2012, *ApJ*, 761, 156
- Federrath C., Roman-Duval J., Klessen R. S., Schmidt W., Mac Low M., 2010a, *A&A*, 512, A81
- Federrath C., Banerjee R., Clark P. C., Klessen R. S., 2010b, *ApJ*, 713, 269
- Federrath C., Sur S., Schleicher D. R. G., Banerjee R., Klessen R. S., 2011, *ApJ*, 731, 62
- Federrath C., Schrön M., Banerjee R., Klessen R. S., 2014, *ApJ*, 790, 128
- Fryxell B. et al., 2000, *ApJ*, 131, 273
- Girichidis P., Federrath C., Banerjee R., Klessen R. S., 2011, *MNRAS*, 413, 2741
- Gutermuth R. A., Pipher J. L., Megeath S. T., Myers P. C., Allen L. E., Allen T. S., 2011, *ApJ*, 739, 84

- Heiderman A., Evans N. J., II, Allen L. E., Huard T., Heyer M., 2010, *ApJ*, 723, 1019
- Hennebelle P., Teyssier R., 2008, *A&A*, 477, 25
- Heyer M. H., Brunt C. M., 2004, *ApJ*, 615, L45
- Heyer M. H., Williams J. P., Brunt C. M., 2006, *ApJ*, 643, 956
- Kainulainen J., Federrath C., Henning T., 2013, *A&A*, 553, L8
- Kauffmann J., Pillai T., Goldsmith P. F., 2013, *ApJ*, 779, 185
- Kritsuk A. G., Norman M. L., Padoan P., Wagner R., 2007, *ApJ*, 665, 416
- Krumholz M. R., 2014, *Phys. Rep.*, preprint ([arXiv:1402.0867](https://arxiv.org/abs/1402.0867))
- Krumholz M. R., McKee C. F., 2005, *ApJ*, 630, 250
- Krumholz M. R., Tan J. C., 2007, *ApJ*, 654, 304
- Krumholz M. R., Dekel A., McKee C. F., 2012a, *ApJ*, 745, 69
- Krumholz M. R., Klein R. I., McKee C. F., 2012b, *ApJ*, 754, 71
- Krumholz M. R., Dekel A., McKee C. F., 2013, *ApJ*, 779, 89
- Krumholz M. R. et al., 2014, in Beuther H., Klessen R. S., Dullemond C. P., Henning T., eds, *Protostars and Planets VI*. Univ. Arizona Press, Tuscan, AZ, p. 243
- Lada C. J., Lombardi M., Alves J. F., 2010, *ApJ*, 724, 687
- Larson R. B., 1981, *MNRAS*, 194, 809
- Lazarian A., Esquivel A., Crutcher R., 2012, *ApJ*, 757, 154
- Lunttila T., Padoan P., Juvela M., Nordlund Å., 2009, *ApJ*, 702, L37
- Mac Low M.-M., Klessen R. S., 2004, *Rev. Mod. Phys.*, 76, 125
- McKee C. F., Ostriker E. C., 2007, *ARA&A*, 45, 565
- MacLachlan J. M., Bonnell I. A., Wood K., Dale J. E., 2015, *A&A*, 573, A112
- Mestel L., Spitzer L., Jr, 1956, *MNRAS*, 116, 503
- Mouschovias T. C., 1976, *ApJ*, 207, 141
- Mouschovias T. C., Tassis K., 2009, *MNRAS*, 400, L15
- Myers A. T., Klein R. I., Krumholz M. R., McKee C. F., 2014, *MNRAS*, 439, 3420
- Omukai K., Tsuribe T., Schneider R., Ferrara A., 2005, *ApJ*, 626, 627
- Ossenkopf V., Mac Low M.-M., 2002, *A&A*, 390, 307
- Padoan P., Nordlund Å., 2011, *ApJ*, 730, 40
- Padoan P., Haugbølle T., Nordlund Å., 2012, *ApJ*, 759, L27
- Padoan P., Federrath C., Chabrier G., Evans N. J., II, Johnstone D., Jørgensen J. K., McKee C. F., Nordlund Å., 2014, in Beuther H., Klessen R. S., Dullemond C. P., Henning T., eds, *Protostars and Planets VI*. Univ. Arizona Press, Tuscan, AZ, p. 77
- Planck Collaboration XXXV, 2015, preprint ([arXiv:1502.04123](https://arxiv.org/abs/1502.04123))
- Price D. J., Bate M. R., 2007, *MNRAS*, 377, 77
- Price D. J., Bate M. R., 2009, *MNRAS*, 398, 33
- Price D. J., Federrath C., Brunt C. M., 2011, *ApJ*, 727, L21
- Ricker P. M., 2008, *ApJ*, 176, 293
- Roman-Duval J., Federrath C., Brunt C., Heyer M., Jackson J., Klessen R. S., 2011, *ApJ*, 740, 120
- Santos-Lima R., Lazarian A., de Gouveia Dal Pino E. M., Cho J., 2010, *ApJ*, 714, 442
- Schmidt W., Hillebrandt W., Niemeyer J. C., 2006, *Comput. Fluids*, 35, 353
- Schneider N. et al., 2011, *A&A*, 529, A1
- Schneider N. et al., 2012, *A&A*, 540, L11
- Shu F. H., 1983, *ApJ*, 273, 202
- Solomon P. M., Rivolo A. R., Barrett J., Yahil A., 1987, *ApJ*, 319, 730
- Sur S., Schleicher D. R. G., Banerjee R., Federrath C., Klessen R. S., 2010, *ApJ*, 721, L134
- Truelove J. K., Klein R. I., McKee C. F., Holliman J. H., II, Howell L. H., Greenough J. A., 1997, *ApJ*, 489, L179
- Vázquez-Semadeni E., 2015, in Lazarian A., de Gouveia Dal Pino E. M., Melioli C., eds, *Astrophysics and Space Science Library*, Vol. 407, *Magnetic Fields in Diffuse Media*. Springer-Verlag, Berlin, p. 401
- Waagan K., Federrath C., Klingenberg C., 2011, *J. Comput. Phys.*, 230, 3331
- Wang P., Li Z.-Y., Abel T., Nakamura F., 2010, *ApJ*, 709, 27
- Yusef-Zadeh F. et al., 2009, *ApJ*, 702, 178
- Zuckerman B., Evans N. J., II, 1974, *ApJ*, 192, L149
- Zuckerman B., Palmer P., 1974, *ARA&A*, 12, 279

## SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article:

### **Federrath\_inefficient\_star\_formation\_lowres.mp4**

(<http://mnras.oxfordjournals.org/lookup/suppl/doi:10.1093/mnras/stv941/-/DC1>).

Please note: Oxford University Press are not responsible for the content or functionality of any supporting materials supplied by the authors. Any queries (other than missing material) should be directed to the corresponding author for the article.

This paper has been typeset from a  $\text{\TeX}/\text{\LaTeX}$  file prepared by the author.