Inequality adjusted income growth*

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Abstract

We introduce and characterize a new measure of aggregate income growth that allows to give more weight to individuals with lower individual income growth. Our measure includes several important measures of directional mobility encountered in the literature. The empirical application compares the measure of income growth between the United States and Germany, and finds that giving more weight to individuals with lower income growth reverses the ranking.

Keywords: income growth, income mobility, rank dependency.

JEL classification: D31, D63

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1 Introduction

This paper provides an axiomatic characterization of a new measure of aggregate income growth. Our measure differs from other measures of income growth in two respects. First, the unit of analysis in our measure is given by the growth rates of the individual incomes within the population. In this respect, we disagree with the common view that aggregate income growth should be defined as the change of an aggregate income statistic, like, for example, the growth of mean or median income. Second, our growth measure takes into account the inequalities among individual income growth rates. Combining these two ideas, we characterize a rank dependent growth measure that gives more weight to individuals that experience lower individual income growth. Our empirical application demonstrates that the sensitivity with respect to inequality in individual income growth, is crucial to rank the inequality adjusted growth rates of the United States and Germany.

This research challenges the conventional viewpoint that income growth in a society should be evaluated on the basis of a change in some aggregate income variable, such as mean or median income. Indeed, from a micro perspective, the unit of analysis should be the individual (or household) and not a representative aggregate of the whole population. In order to understand our perspective, it is necessary to make a distinction between aggregate income growth, which measures income growth for a country, population or society as a whole, and individual income growth, which is the growth experienced by a single individual. By definition, individual income growth depends only on the individual's income in the initial and final period. Having established this distinction, we believe that it is, for example, more informative to look at the average of individual income growth than to look at the growth of average income.[1] The example in table 1 motivates our point of view. The table presents initial and final incomes for a 3 person society in four hypothetical situations. The initial income distribution is determined by the vector $\mathbf{x}$. We assume that this vector is the same in every situation. The final income distribution in situation $k = 1, \ldots, 4$ is labeled by $\mathbf{y}^k$. Finally, the table provides information on the ratios of final to initial incomes $\mathbf{g}^k$ in each of the four cases, representing the relevant measure of individuals' income growth. The bottom row gives the same statistics for the means.

Table 1 around here

Let us start by comparing the two processes $\mathbf{x} \to \mathbf{y}^1$ and $\mathbf{x} \to \mathbf{y}^2$. The bottom row shows that the growth of mean income is almost two times larger in the latter than in the former transition. However, if we look at the individual income ratios, we see that there is no difference between the two situations, except for a re-ranking of the individuals. Provided that we are impartial concerning who encounters which income growth within the population, we should judge the two situations as having equal growth.

[1]Interestingly, it can be shown that it is possible to decompose the growth of mean income as a weighted sum of the individual growth rates, with base period income as the weights (see, for example, Fell and Green, 1983; Ahluwalia and Chenery, 1974). In this respect, measuring income growth by the growth rate of mean income gives more weight to individuals at the top of the initial income distribution.

Now, consider the changes $\mathbf{x} \to \mathbf{y}^3$ versus $\mathbf{x} \to \mathbf{y}^4$. In this case, we have the opposite phenomenon: the growth rate in mean income is the same in both situations, but the distribution of individual income ratios looks very different. In particular, for the situation $\mathbf{x} \to \mathbf{y}^4$ we see that there are two individuals who double their incomes, while one individual experiences a status-quo. On the other hand for the transition $\mathbf{x} \to \mathbf{y}^3$ we see the reverse: two individuals have no change in income while one individual doubles his income. From this perspective, it is difficult to argue that the two processes represent an equal amount of income growth.

The idea to focus on individual income growth rather than the growth in average incomes has been around some time (see, e.g. Klasen, 1994). Ahluwalia and Chenery (1974) observed that looking at the growth in average incomes implies that every unit increase in income gets the same value, irrespective whether the income accrues to a poor or rich individual : to have one more unit of y_j, one is always willing to give up one unit of y_i, irrespective of i and j's first period incomes. They proposed to look at the unweighted average of individual income growth instead, such that every percentage increase in incomes has the same value.[2] As a result, to have one more unit of y_j, one is willing to give up x_i/x_j units of y_i. As such, a unit increase in income for an initially poor individual is worth more than a unit increase in income for an initially richer individual.

Our approach goes further. In addition to the focus on individual income growth, our measure embodies the idea that in the aggregation of the individual growth measures to an aggregate measure, one should give more weight to the individuals that experience the lowest income growth. In other words, our measure satisfies the criterion that aggregate growth increases if growth is redistributed from an individual with high individual growth towards an individual with low income growth. Similarly to the idea that income equality is maximal if total income is distributed equally among all individuals, our approach supports the idea that aggregate growth will be maximal if total growth is distributed equally among all individuals.

An alternative motivation for our approach can be obtained from recent happiness research.[3] Clark, Frijters, and Shields (2008) cite a lot of evidence showing that an individual's happiness depends on her income relative to her income in the past (habituation). Hence, individual income growth can be interpreted as a measure of individual happiness. Sensitivity to the bottom of the individual mobility distribution then translates to the requirement that our measure of aggregate 'happiness' should give priority to people that experience lower individual happiness.

Our measure of income growth bears a lot of similarity with measures from the literature on income mobility. Measures of income mobility evaluate the change from an initial to a final income distribution. In particular, our measure of income growth can be seen

[2] As an alternative, they propose to use a weighted average of individual income growths, giving a larger weight to the initially poor. This has recently been taken up by part of the pro-poor growth literature -see, e.g. Jenkins and Van Kerm (2006); Grimm (2007); Van Kerm (2009); Bourguignon (2011) and Jenkins and Van Kerm (2011).

[3] We thank an anonymous member of the editorial team for this motivation.

as a directional measure of income mobility which takes into account inequality in the distribution of individual mobilities (measured by individual income growth rates) within a population.[4] A measure is directional if it distinguishes between upward mobility (income increase) and downward mobility (income decrease). Moreover, given that we want to express aversion with respect to inequality in the distribution of individual income growth, we can learn from the literature on income inequality measurement which provides several alternative ways to do so (see for example Ebert, 1988; Lambert, 2001). For our measure, we axiomatize aggregate growth as a rank dependent mean of individual income growth to express inequality aversion with respect to differences in individual growth: larger weights are given to individuals with lower income growth and these weights depend on the rank order in the distribution of the individual growth measures only. An alternative approach is to take a concave transformation of the individual growth measures before aggregating. For this case, the weight attached to each individual growth measure depends on the size of this growth relative to the other individuals' growth (and not on the rank order in the distribution). At the end of section 2 we show that, taking a concave transformation exhibiting constant relative risk aversion, as in Atkinson (1970), results in a ranking that is formally similar to the mobility measure derived by Schluter and Van de gaer (2010). Alternatively, one can look for stochastic dominance relationships between the distributions of individual growth rates, a suggestion formulated by Fields (2000) and Fields, Leary, and Ok (2002). We illustrate this approach in the empirical application.

As a first step for our characterization we impose Weak Decomposability (axiom WD). This condition states that aggregate income growth should be measured solely in terms of individual income growth measures. The axiom is frequently used in the axiomatic treatment of income mobility and has important implications. Foremost, it rules out the use of all information other than the individual growth measures. In terms of the illustrative example, it means that our judgement has to be exclusively based on the vectors $\mathbf{g}^k$.[5] The imposition of WD implies that our measure ignores information on, for example, the position of the individuals in the initial and final income distribution. Observe, however, that a unit increase (decrease) in income has a larger positive (negative) effect on the growth rate of the initially poor. Therefore, a social planner whose actions are guided by our measure of aggregate income growth will tend to favor the initially poor.

Once WD is accepted, the specific form of the aggregate growth measure can be established by solving two remaining issues. The first concerns the particular functional form for

[4] See, for example Schluter and Van de gaer (2010) and Fields and Ok (1999b) for other measures of directional mobility. The literature on mobility measurement can and has been looked at from many different perspectives including the following ones: mobility as income movement (Cowell, 1985; Fields and Ok, 1999b,a; D'Agostino and Dardanoni, 2009b), mobility as change of positions in the income distribution (D'Agostino and Dardanoni, 2009a; Cowell and Flachaire, 2011), mobility as a process that equalizes lifetime income (Shorrocks, 1978; Maasoumi and Zandvakili, 1986; Chakravarty et al., 1985; Fields, 2010)) and mobility as a process that should be mostly in favor of the initially poor (Van Kerm, 2009). We refer to Fields (2000) and Fields (2007) for a more complete overview of this literature.

[5] As such, it excludes the following perspectives on growth (or mobility): positional mobility, the equalization of lifetime incomes, favorable treatment of the initially poor and macro growth (e.g. growth of mean income).

the individual growth measures. As this form depends only on two variables —initial and final income— it can be quickly characterized by combining certain elementary structural axioms. We characterize two directional measures of individual growth. Both measures are scale invariant: they are unchanged when the individual's initial and final income are multiplied by the same positive scalar (axiom SI). They are directional: they are increasing in final and decreasing in initial income (axiom M). Finally they are path independent: one measure satisfies multiplicative path independence (axiom MPI), the other additive path independence (API). The former (latter) means that when moving first from an initial to an intermediate and then to a final income level, the growth in moving from the initial to the final income level can be written as a multiplicative (additive) function of the growth in the transition from the initial to the intermediate and the growth of the intermediate to the final income level.

Next, we move to a more important contribution of this paper: the characterization of the rank dependent aggregation procedure. We use a framework that is similar in spirit to Bossert (1990)'s characterization of the S-Gini index. To apply it to our framework, we use the requirement of Weak Decomposability (axiom WD), explained above. Bossert's first structural assumption applied to the growth context becomes then that the aggregate growth measure must be both relative and absolute with respect to the individual growth measures (axioms RI and TI). His second structural assumption becomes that the aggregate growth measure has to satisfy a decomposability requirement: for a population of size n, the income growth of the population depends on the aggregate growth of the group of $n-1$ members with the highest individual growth in the society and the income growth of the individual with the lowest individual growth (axiom D-HG). Perhaps these requirements are less intuitive in our context than in the context of inequality measurement, but many mobility measures proposed in the literature satisfy them. Beside these structural axioms we also impose two normative axioms. The first axiom, population invariance, implies that any k-fold reproduction of the society should leave aggregate growth unchanged (axiom PI). The last axiom, priority for lower growth, expresses our aversion towards inequality in individual growth rates. It states that aggregate growth increases more when additional income growth is allocated to individuals with lower individual growth than when it is allocated to individuals with higher individual growth (axiom PLG).

The combination of these axioms leads, in the Donaldson and Weymark (1980) terminology, to a family of single-series Gini indices defined over two possible measures of individual income growth. The first yields a generalization of the directional measure of income mobility proposed by Schluter and Van de gaer (2010), the second a generalization of the directional measure of income mobility developed by Fields and Ok (1999b). We derive the asymptotic distribution of our new measure and we illustrate our findings by comparing income growth between the United States and (west) Germany. Our findings show that the ranking between the two countries depends crucially on the value of a parameter which controls the degree to which more weight is given to individuals with lower individual income growth.

In section 2, we provide our characterization and section 3 gives the empirical illustration. Section 4 concludes. All proofs are in the appendix.

2 Characterization

Consider a set of individuals $\{1, \dots, n\}$ and two income distributions $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n_{++}$. Individual i's initial income is given by x_i while his final income is given by y_i. We measure aggregate income growth of going from the initial situation $\mathbf{x}$ to the final income distribution $\mathbf{y}$ by a real valued and continuous function $G^n(\mathbf{x}, \mathbf{y})$. In particular, given the income distribution vectors $\mathbf{x}, \mathbf{x}', \mathbf{y}$ and $\mathbf{y}'$, we have that $G^n(\mathbf{x}, \mathbf{y}) \geq G^n(\mathbf{x}', \mathbf{y}')$ if the process $\mathbf{x} \to \mathbf{y}$ has more income growth than the process $\mathbf{x}' \to \mathbf{y}'$. For a scalar $x \in \mathbb{R}_+$ we write $x \cdot \mathbf{1}$ to indicate the n-dimensional vector $(x, x, \dots, x)$. We begin by imposing some properties on the measure of individual income growth, G^1. Our first, rather uncontroversial, property states that individual income growth should be increasing in final period income and decreasing in initial period income.

Axiom: Monotonicity [M]: *$x, y, x', y' \in \mathbb{R}_{++}$, if $x \leq x'$ and $y \geq y'$, then:*

$$G^1(x, y) \geq G^1(x', y) \text{ and } G^1(x, y) \geq G^1(x, y').$$

Our second condition says that the measure of individual income growth should remain invariant when both initial and final income are scaled up or down with a common factor. This condition is important when comparing income growth between countries that use different currencies, as will be the case in our empirical application.

Axiom: Scale Invariance [SI]: *$x, y \in \mathbb{R}_{++}$ and $\lambda > 0$:*

$$G^1(x, y) = G^1(\lambda x, \lambda y).$$

The condition of scale invariance allows us to define a real valued function f such that, $G^1(x, y) = G^1(1, y/x) = f(y/x)$. In order to pinpoint the functional form of f, we need an additional condition. We choose to impose a path independency property. Consider three periods. An individual's first period income x changes to y in the second period and to z in the third. Our path independence axiom states that the individual's measure of income growth, from the first to the third period (x to z) can be written as a function of the two single period measures (from x to y and from y to z). In its most general form, it requires the existence of a function H such that $H(G^1(x, y), G^1(y, z)) = G^1(x, z)$. We choose two particular forms for this function H.[6]

Axiom: Multiplicative Path Independence [MPI]: *For all $x, y, z \in \mathbb{R}_{++}$:*

$$G^1(x, z) = G^1(x, y).G^1(y, z).$$

Axiom: Additive Path Independence [API]: *For all $x, y, z \in \mathbb{R}_{++}$:*

$$G^1(x, z) = G^1(x, y) + G^1(y, z).$$

[6]Observe that the mere existence of H does not impose any additional requirements on f. Indeed, one can always choose $H(a, b) = h^{-1}(h(a) + h(b))$ with $h = \ln \circ f^{-1}$.

One can easily verify that MPI requires the function f to satisfy Cauchy's fourth functional equation while API requires f to satisfy Cauchy's third functional equation. This gives the following partial result (see, for example Aczél, 1966, p.39):

Lemma 1.

- *$G^1(x, y)$ satisfies M, SI and MPI if and only if there exists a number $r > 0$ such that $G^1(x, y) = (y/x)^r$.*
- *$G^1(x, y)$ satisfies M, SI and API if and only if there exists a number $r > 0$ such that $G^1(x, y) = r \ln(y/x)$.*

In this lemma, the parameter r is a sensitivity parameter: higher values of r lead to larger differences in the measure of individual income growth. Our next axiom states that total growth should only depend on the values of the individual growth measures. Its interpretation was already given in the introduction.

Axiom: Weak Decomposability [WD]: *For all $n \in \mathbb{N}$ and all $\mathbf{x}, \mathbf{y}, \mathbf{x}'$ and $\mathbf{y}' \in \mathbb{R}^n_{++}$, if for all $i \in \{1, \ldots, n\}$ and $i \neq j$,*

$$G^1(x_i, y_i) = G^1(x'_i, y'_i),$$

then:

$$G^n(\mathbf{x}, \mathbf{y}) \geq G^n(\mathbf{x}', \mathbf{y}') \text{ if and only if } G^1(x_j, y_j) \geq G^1(x'_j, y'_j).$$

Given two income distributions $\mathbf{x}$ and $\mathbf{y}$ and an individual growth measure G^1, we may construct the vector of individual income growth indices $\mathbf{g}$, determined by its elements $g_i = G^1(x_i, y_i)$. The following is an immediate consequence of axiom WD.

Lemma 2. *G^n satisfies WD if and only if there exist strictly increasing and continuous functions W^n such that for all $\mathbf{x}$ and $\mathbf{y} \in \mathbb{R}^n$:*

$$G^n(\mathbf{x}, \mathbf{y}) = W^n(\mathbf{g}).$$

Lemma 2 shows that we may restrict ourselves to the ranking of all vectors composed of the individual growth measures $\mathbf{g}$. In the sequel, our axioms will be formulated in terms of the function $W^n(\mathbf{g})$. In this perspective, one should take into account that the domain of W^n differs depending on the functional form for the individual growth measure. If $G^1(x, y) = (y/x)^r$, its domain equals $\mathbb{R}^n_{++}$. On the other hand, if $G^1(x, y) = r \ln(y/x)$, then the domain equals $\mathbb{R}^n$. In the remaining part of this paper, we will denote the relevant domain by $\mathcal{D}^n$, where $\mathcal{D}^n = \mathbb{R}^n_{++}$ or $\mathcal{D}^n = \mathbb{R}^n$ depending on the underlying individual growth measure.

Our following axiom states that comparisons between income growth measures remain invariant under a common multiplication of the individual growth measures. In other words, the ranking derived from the aggregate growth index should remain invariant when individual growths are multiplied by the same constant.

Axiom: Relative Invariance [RI]: *For all* $\mathbf{r}, \mathbf{s} \in \mathcal{D}^n$ *and* $\lambda > 0$,

$$\textit{if } W^n(\mathbf{r}) = W^n(\mathbf{s}) \textit{ then } W^n(\lambda\mathbf{r}) = W^n(\lambda\mathbf{s}).$$

The next axiom states that comparisons between growth measures remain invariant under a common translation of the individual growth measures. In other words, the ranking derived from the aggregate growth indices should not depend on the particular origin that is chosen for the measurement of the individual indices.

Axiom: Translation Invariance [TI]: *For all* $\mathbf{r}, \mathbf{s} \in \mathcal{D}^n$ *and* $\lambda > 0$,

$$\textit{if } W^n(\mathbf{r}) = W^n(\mathbf{s}) \textit{ then } W^n(\mathbf{r} + \lambda \cdot \mathbf{1}) = W^n(\mathbf{s} + \lambda \cdot \mathbf{1}).$$

For any vector $\mathbf{g} \in \mathbb{R}^n$, let $\tilde{\mathbf{g}}$ be a permutation of $\mathbf{g}$ such that $g_1 \geq g_2 \ldots \geq g_n$. Our next axiom states that aggregate growth only depends on the individual growth measures with the lowest level of individual growth and on the aggregate growth level of the $n-1$ other individuals. Although it may seem like a strong restriction, we should note that it is much weaker than most of the decomposability axioms in the mobility measurement literature. See, for example, Fields and Ok (1996) axiom 2.4, Fields and Ok (1999b) subgroup decomposability, Mitra and Ok (1998) axiom PC, Schluter and Van de gaer (2010) subgroup consistency, D'Agostino and Dardanoni (2009b) subvector consistency.

Axiom: Decomposability with respect to Highest Growth [D-HG]: *For all* $n \in \mathbb{N}$ *and all* $\mathbf{g}, \mathbf{g}' \in \mathcal{D}^n$,

$$\textit{if } W^{n-1}(\tilde{g}_1, \ldots, \tilde{g}_{n-1}) = W^{n-1}(\tilde{g}'_1, \ldots, \tilde{g}'_{n-1}) \textit{ and } \tilde{g}_n = \tilde{g}'_n,$$
$$\textit{then } W^n(\mathbf{g}) = W^n(\mathbf{g}').$$

Our next axiom is known as population invariance or replication invariance. Replication invariance says that a k-fold replication of the population does not change aggregate growth. The axiom allows us to compare aggregate income growth between populations of different sizes.

Axiom: Population Invariance [PI]: *For all* $n, k \in \mathbb{N}$ *and all* $\mathbf{g} \in \mathcal{D}^n$,

$$W^n(\mathbf{g}) = W^{kn}(\underbrace{\mathbf{g}, \mathbf{g}, \ldots, \mathbf{g}}_{k times}).$$

Finally, we introduce one additional axiom. This axiom states that an allocation of additional growth increases aggregate growth more if it is allocated to an individual with lower individual growth: it expresses inequality aversion with respect to the distribution of individual growth. The motivation for this axiom was given in the introduction.

Axiom: Priority for Lower Growth [PLG]: *For all* $n \in \mathbb{N}$, $\mathbf{g} \in \mathbb{R}^n$ *and* $\sigma > 0$, *if* $g_i < g_j$, *then:*

$$W^n(g_1, \ldots, g_i + \sigma, \ldots, g_j, \ldots, g_n) \geq W^n(g_1, \ldots, g_i, \ldots, g_j + \sigma, \ldots, g_n).$$

The following proposition is proven in appendix A:

Proposition 1. *For all $n \in \mathbb{N}$, G^n satisfies M, SI, MPI, WD, RI, TI, D-HG, PI and PLG if and only if there exist a number $\delta \geq 1$ and a number $r > 0$, such that:*

$$G^n(\mathbf{x}, \mathbf{y}) = \frac{1}{n^\delta} \sum_{i=1}^{n} (i^\delta - (i-1)^\delta)\ \tilde{g}_i.$$

with $\tilde{g}_i$ the individual growth measure g_i ranked increasingly, and

$$g_i = G^1(x_i, y_i) = (y_i/x_i)^r.$$

If API is satisfied instead of MPI, then,

$$g_i = G^1(x_i, y_i) = r \ln(y_i/x_i).$$

The first measure of Proposition 1, for $\delta = 1$, reduces to the directional mobility measure proposed in Schluter and Van de gaer (2010),

$$G^n_{SV}(\mathbf{x}, \mathbf{y}) = \frac{1}{n} \sum_{i=1}^{n} (y_i/x_i)^r\,,$$

where the parameter $r > 0$ is a sensitivity parameter: higher values of r lead to larger differences in individual growths without changing their ranking. Especially, the relative size of high values of y_i/x_i increases rapidly as r increases. The second measure of Proposition 1, for $\delta = 1$, reduces to the directional mobility measure of Fields and Ok (1999b):

$$G^n_{FO}(\mathbf{x}, \mathbf{y}) = \frac{1}{n} \sum_{i=1}^{n} r \ln\left(y_i/x_i\right).$$

The value of r does not change the ranking when comparing aggregate growth in two situations, and so, in this case r can be put equal to 1. These two measures compute aggregate growth (or mobility) by taking the unweighed sum of all individual growth measures. In the literature on mobility measurement the notion of exchange mobility takes an important place. It requires that covariance decreasing income swaps in either the initial or the final period increase mobility.[7] The measure G^n_{SV} satisfies this notion, while measure G^n_{FO} is insensitive to covariance decreasing income swaps.

Since $\delta \geq 1$ in proposition 1, our measure generalizes the measures G^n_{SV} and G^n_{FO}. For $\delta > 1$ it gives more weight to individuals with lower income growth, for $\delta = 2$ we have the traditional Gini weights and if δ converges to ∞, only the comparison between lowest income growths matters. Naturally, this increased generality comes at a cost, as the generalizations of both G^n_{SV} and G^n_{FO} are sensitive to covariance decreasing income swaps, but in a non-trivial way: covariance decreasing income swaps may actually decrease the value of these measures. In addition, contrary to measures G^n_{SV} and G^n_{FO}, our measure is no longer additively decomposable in the income growth measures of subgroups of the

[7] See Tsui (2009) for various equivalent ways to define exchange mobility.

population. In the next section, we will see that the possibility to attach more weight to individual income growth at the bottom of the individual income growth distribution can reverse the ranking between countries.

An obvious alternative approach to account for inequality of individual income growth would be to take an Atkinson-type constant relative risk aversion evaluation function of the individual growth measures,[8] resulting in

$$G_A^n(\mathbf{x}, \mathbf{y}) = \frac{1}{n}\frac{1}{1-e}\sum_{i=1}^{n}(g_i)^{1-e},$$

with $e > 0$ an inequality aversion parameter. Combined with individual growth measure $G^1(x, y) = (y/x)^r$ with $r > 0$, it is clear that the mobility ordering defined by G_A^n ranks distributions of mobility on the basis of

$$G_A^n(\mathbf{x}, \mathbf{y}) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{x_i}\right)^{r(1-e)} \text{ if } 0 < e < 1 \text{ and } G_A^n(\mathbf{x}, \mathbf{y}) = -\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{x_i}\right)^{r(1-e)} \text{ if } e > 1.$$

Hence, for $e < 1$, the ranking coincides with the Schluter and Van de gaer (2010) measure with parameter $\widehat{r} = r(1-e) > 0$. When $e > 1$ the criterion to rank the mobility numbers is formally identical (with $\widehat{r} = r(1-e) < 0$) , but opposite in sign. This way of aggregating individual mobilities has the advantage that the ranking is additively decomposable in subgroups. On the other hand, the measure does not satisfy TI and it does not allow us to distinguish between the effects of the sensitivity parameter r and the degree of inequality aversion (measured by e). One possible solution to this issue is to fix the parameter r, for example $r = 1$. In this way, the parameter $\widehat{r}$ only measures the degree of inequality aversion. Fixing r to one can easily be done by introducing an axiom that fixes the scale of the individual mobilities, for example: for all $\lambda > 0, G^1(x, \lambda y) = \lambda G^1(x, y)$. In the empirical application we compare our results for the rank dependent growth measure $G^n(\mathbf{x}, \mathbf{y})$ with those for $G_A^n(\mathbf{x}, \mathbf{y})$.

There is one final note that we want to stress before we begin at the empirical application. Our measure of economic growth is increasing in both the mean level of individual growth and in the degree of equality in individual growth. Now, let $\mu^n(\mathbf{x}, \mathbf{y})$ be the level of mean individual growth, i.e.

$$\mu^n(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^{n} G^1(x_i, y_i)$$

In other words, μ^n coincides with our measure of income growth $G^n(\mathbf{x}, \mathbf{y})$ with $\delta = 1$. Then, it is easy to see that:

$$G^n(\mathbf{x}, \mathbf{y}) = (1 - I^n(\mathbf{x}, \mathbf{y}))\, \mu^n(\mathbf{x}, \mathbf{y})$$
$$I^n(\mathbf{x}, \mathbf{y}) = \left(1 - \frac{G^n(\mathbf{x}, \mathbf{y})}{\mu^n(\mathbf{x}, \mathbf{y})}\right)$$

[8] It is possible to obtain a characterization of this aggregation function. See, e.g., Ebert (1988, theorem 14)

The above expression for $G^n(\mathbf{x},\mathbf{y})$ makes explicit why our measure is a measure of inequality adjusted income growth. It decomposes the aggregate growth measure $G^n(\mathbf{x},\mathbf{y})$ in a mean growth effect $\mu^n(\mathbf{x},\mathbf{y})$ and a measure of individual growth equality $(1-I^n(\mathbf{x},\mathbf{y}))$. In other words, the higher $I^n(\mathbf{x},\mathbf{y})$, the more unequal the distribution of individual income growth within the population and the lower our measure for aggregate income growth. We need to make one cautionary remark with respect to this decomposition. The decomposition should not be used for the individual mobility measures $r\ln(y/x)$. Indeed, for this functional specification, it is possible that the average growth is zero (or negative) which would render the computation and interpretation of I^n difficult or impossible. We illustrate this decomposition (for the measure $(y/x)^r$) in the next section.

3 Application

3.1 Statistical analysis

For the statistical analysis, we rely on large sample theory. The primitive for the statistical inference is the individual growth measure $G^1(x,y)$. We assume that this growth measure has a differentiable cumulative distribution function, denoted by $F(.)$ with finite mean such that we have the following proposition.

Proposition 2.

(i) $G^n(\mathbf{x},\mathbf{y}) \xrightarrow{p} G^\infty(F)$, *where:*

$$G^\infty(F) = \delta \int_0^1 g(1-F(g))^{\delta-1} dF(g).$$

(ii) $\sqrt{n}\,(G^n(\mathbf{x},\mathbf{y}) - G^\infty(F)) \overset{a}{\sim} N(0,\sigma^2(F))$, *where:*

$$\sigma^2(F) = \delta^2 \int_0^1 \int_0^1 (F(min(m,n)) - F(m)F(n))(1-F(m))^{\delta-1}(1-F(n))^{\delta-1} dndm,$$

which can be consistently estimated by the estimator

$$s^2(\mathbf{x},\mathbf{y}) = \sum_{i=1}^{n-1}\sum_{j=1}^{n-1} \phi(i,j)(\tilde{m}_{n-i} - \tilde{m}_{n-i+1})(\tilde{m}_{n-j} - \tilde{m}_{n-j+1}),$$
$$\phi(i,j) = \delta^2 \left(min(i/n, j/n) - \frac{i}{n}\frac{j}{n} \right)(1-i/n)^{\delta-1}(1-j/n)^{\delta-1}.$$

(iii) Consider two independent joint income distributions $(\mathbf{x},\mathbf{y})$ *and* $(\mathbf{x}',\mathbf{y}')$ *and mobility measures* $G^n(\mathbf{x},\mathbf{y})$ *and* $G^{n'}(\mathbf{x}',\mathbf{y}')$ *with* $\sqrt{n}\,(G^n(\mathbf{x},\mathbf{y}) - G^\infty(F)) \overset{a}{\sim} N(0,\sigma^2(F))$ *and*

$\sqrt{n'}\left(G^{n'}(\mathbf{x}',\mathbf{y}') - G^{\infty}(F')\right) \overset{a}{\sim} N(0,\sigma^2(F'))$. *Then, under the null hypothesis that* $G^{\infty}(F) = G^{\infty}(F')$, *we have that,*

$$\frac{G^n(\mathbf{x},\mathbf{y}) - G^{n'}(\mathbf{x}',\mathbf{y}')}{[s^2(\mathbf{x},\mathbf{y})/n + s^2(\mathbf{x}',\mathbf{y}')/n']^{0.5}} \overset{a}{\sim} N(0,1).$$

.

The first point, which follows from Zitikis and Gastwirth (2002, theorem 1), states that our growth measure converges to the infinite sample analogue. The second result shows that our measure is asymptotically normally distributed. The asymptotic distribution follows from the theory on L-Statistics (see Shorack and Wellner, 1986, theorem 5 (i)) and the fact that $s^2(\mathbf{x},\mathbf{y})$ is a consistent estimator of $\sigma^2(F)$ follows from Zitikis and Gastwirth (2002, theorem 3). Finally, given two independent samples the third result gives a statistical test to verify whether the respective populations have different inequality adjusted income growth. This result follows immediately from the independence of the two samples. This will be the test statistic upon which we base our empirical analysis.

The asymptotic inference for the measure G_A^n follows from Schluter and Van de gaer (2010, lemma 2) (for $\hat{r} = r(1-e)$):

(i) Let $G_A^{\infty}(F) = E\left((y/x)^{\hat{r}}\right)$ and $\sigma_A^2(F) = E\left[\left((y/x)^{\hat{r}} - E\left[(y/x)^{\hat{r}}\right]\right)^2\right]$. Then,

$$\sqrt{n}\left(G_A^n(\mathbf{x},\mathbf{y}) - G_A^{\infty}(F)\right) \sim^a N\left(0,\sigma^2(F)\right).$$

Both G_A^{∞} and σ_A^2 can be consistently estimated by their sample analogues, $G_A^n(\mathbf{x},\mathbf{y})$ and $s_A^2(\mathbf{x},\mathbf{y})$.

(ii) Consider two independent joint income distributions and growth measures $G_A^n(\mathbf{x},\mathbf{y})$ and $G_A^{n'}(\mathbf{x}',\mathbf{y}')$ with $\sqrt{n}\left(G_A^n(\mathbf{x},\mathbf{y}) - G_A^{\infty}(F)\right) \sim^a N\left(0,\sigma_A^2(F)\right)$ and $\sqrt{n'}\left(G_A^{n'}(\mathbf{x}',\mathbf{y}') - G_A^{\infty}(F')\right) \sim^a N\left(0,\sigma_A^2(F')\right)$. Under the null hypothesis that $G_A^{\infty}(F) = G_A^{\infty}(F')$ we have that

$$\frac{G_A^n(\mathbf{x},\mathbf{y}) - G_A^{n'}(\mathbf{x}',\mathbf{y}')}{[(s_A^2(F)/n) + (s_A^2(F')/n')]^{0.5}} \overset{a}{\sim} N(0,1).$$

It should be noted that the proposed inference procedures are only valid under the assumption that the samples of individual mobilities are independent. This assumption may be violated for many datasets because of complex sampling design features.[9] Unfortunately, such issues are beyond the scope of this paper.

[9] See, for example Howes and Lanjouw (1998) for a discussion of the implications of sampling issues for the measurement of poverty indices.

3.2 Empirical application

In our empirical application, we compare the inequality adjusted income growth of (West) Germany and the United States. For Germany, our data are obtained from the German Socio-Economic Panel (GSOEP) survey while for the United States, we retrieved our data from the Panel Study of Income Dynamics (PSID). Both data sets are widely used in studies of income mobility.[10] In order to obtain comparable income variables we used the "Cross National Equivalent data files"[11] version of the PSID and GSOEP data sets. The unit of analysis is the individual and the income concept is the post-tax, post-benefit household income in 1996 prices. We corrected for differences in household size by using the OECD equivalence scale (equal to the square root of the household size). As customary in the literature (e.g. Schluter and Trede (2003) and Schluter and Van de gaer (2010)) we trimmed each sample at the 1 and 99 percent quantiles. We further restrict our analysis to the years 1984/85 and 1996/97 but the results for other years are similar. The sample 1984/85 gives us 17,727 observations for the US and 13,022 observations for Germany. The sample sizes for 1996/97 are 21,290 for the US and 15,860 for Germany. Table 2 provides the result of the statistical test whether inequality adjusted income growth in the US is larger than in Germany or not.[12]

Table 2 around here

Observe that for $\delta = 1$, the US has always a higher growth than Germany. This is the case where our measure coincides with the measures of Schluter and Van de gaer (2010) and Fields and Ok (1999b) discussed in the previous section. However, when δ increases, inequality adjusted growth in Germany becomes higher than in the US. For the individual growth measure $(y_i/x_i)^r$, how much δ has to increase for the ranking to reverse depends on the value of r. Larger values for r do not change the ranking of individual growth, but increases the difference between high and low individual growth measures, such that in the comparison between the US and Germany a greater weight has to be given to individuals with low income growth before the ranking reverses.

Clearly, for our measures, it makes a big difference whether individual income growth is weighted or not before aggregation. Our procedure (with $\delta > 1$) gives more weight to the individuals with low income growth. Hence, our result indicates that those at the bottom end of the income growth distribution in the US face a larger percentage decrease in their incomes than those that are at the same percentile of the distribution in Germany. This is confirmed by the plot of the cumulative distribution function of (y_i/x_i) for 1996/1997 in figure 1 below (the figure looks similar for all years), which was already observed by Chen (2009), who considers 5 year income movements for a set of countries including the

[10]See, for example Burkhauser and Poupore (1997), Trede (1998), Maasoumi and Trede (2001), Schluter and Trede (2003) and Schluter and Van de gaer (2010).

[11]See http://www.human.cornell.edu/pam/research/centers-programs/german-panel/cnef-data-files.cfm

[12]The inequality adjusted growth rates themselves are not of primary importance, as we are only interested here in the ranking of the two countries.

US and Germany (see figure 4 p.85 and the discussion following it). Since the cumulative distribution functions cross, there is no first order dominance of one distribution over the other. Hence, the judgment depends on the particular way individual income growth measures are aggregated. Our inequality adjusted growth measure is the first to make this explicit. It allows us to conclude that, even though average individual income growth is higher in the US, a moderate concern with the distribution of income growth across the population, leads to the conclusion that Germany has a higher inequality adjusted aggregate growth than the US.

Figure 1 around here.

Table 3 present a comparison between the growth of the US versus Germany in terms of the measure G_A^n presented at the end of section 2. As mentioned before, for this measure, it is no longer possible to separate the sensitivity parameter in the individual growth measure r from the parameter that determines the degree of inequality aversion (i.e. e). However, as can be seen from the table, if their combination $\hat{r} = r(1-e)$ is small enough, we again obtain a reversal in the ranking of the two countries in terms of inequality adjusted income growth.

Table 3 around here

Finally, table 4 presents the decomposition of $G^n(\mathbf{x}, \mathbf{y})$ in terms of the mean growth rate $\mu^n(\mathbf{x}, \mathbf{y})$ and the inequality measure $I^n(\mathbf{x}, \mathbf{y})$ as derived at the end of the previous section. The table confirms our discussion above. In all cases, the US has a higher mean growth rate than Germany while Germany has a lower inequality of individual growth. As r increases, differences in the mean values of the measures of individual growth increase, favoring the US distribution of individual growth rates. As δ increases, the contribution of differences in inequalities of individual growth becomes more important, favoring the German distribution of individual growth rates. Hence, depending on the level of δ and r, either the larger mean individual growth rate of the US or the smaller inequality in the distribution of individual growth in Germany dominates in the ranking of the two countries' inequality adjusted growth rate.

Table 4 around here.

4 Conclusion

We argue that the standard practice of expressing income growth as change in aggregate income can be questioned. Furthermore, simply adding individual growth measures to obtain an aggregate growth number can also be questioned in contexts where individual income growth varies within the population. In these contexts, keeping average individual income growth constant, a more equal distribution of the growth rates within the population is to be preferred. This leads us to axiomatize rank dependent measures of income growth, such that more weight can be given to individuals with lower income growth. The result is a generalization of the unweighed measures of upward structural mobility proposed by Fields and Ok (1999b) and Schluter and Van de gaer (2010).

Our empirical application shows that the issue is relevant in the comparison of income growth between the US and Germany: while unweighed individual growth in the US is higher than Germany, the ranking reverses when more concern is given to individuals with lower individual growth. We therefore conclude that the issue raised in this paper is of importance when comparing growth rates between countries.

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A Proof of proposition 1

For each $n \in \mathbb{N}$, consider the function $\varepsilon^n : \mathcal{D}^n \to \mathbb{R}$ such that:

$$W^n(\mathbf{g}) = W^n(\varepsilon^n(\mathbf{g}) \cdot \mathbf{1}).$$

The function ε^n is similar to the equally distributed equivalent income that is well known from the literature on inequality measurement (see Atkinson (1970)). It is the amount of individual growth, which if distributed equally to everyone would render aggregate growth equal to the case where the individual growth vector is equal to $\mathbf{g}$. The 'greater than or equal to' ordering implied by ε^n coincides with the ordering derived from W^n. This follows from axiom M, which implies monotonicity of the function W^n, such that

$$\begin{aligned} &W^n(\mathbf{g}) \geq W^n(\mathbf{g}') \\ \iff &W^n(\varepsilon^n(\mathbf{g}) \cdot \mathbf{1}) \geq W^n(\varepsilon^n(\mathbf{g}') \cdot \mathbf{1}) \\ \iff &\varepsilon^n(\mathbf{g}) \geq \varepsilon^n(\mathbf{g}'). \end{aligned}$$

We proceed by deriving the implications of the axioms for the function ε^n. Observe that for all $\mathbf{g} \in \mathbb{R}$:

$$W^n(g \cdot \mathbf{1}) = W^n(\varepsilon^n(g \cdot \mathbf{1}) \cdot \mathbf{1}).$$

This implies that $\varepsilon^n(g \cdot \mathbf{1}) = g$ for all values of g and n.

The implication of axiom RI is that the function ε^n becomes homogeneous of degree one. Indeed, from the definition of ε^n, we have that $W^n(\mathbf{g}) = W^n(\varepsilon^n(\mathbf{g}) \cdot \mathbf{1})$ such that, by RI

$$W^n(\lambda\mathbf{g}) = W^n(\lambda\varepsilon^n(\mathbf{g}) \cdot \mathbf{1}).$$

From the definition of ε^n we also have that,

$$W^n(\lambda\mathbf{g}) = W^n(\varepsilon^n(\lambda\mathbf{g}) \cdot \mathbf{1}).$$

Combining the last two equalities, we get that $W^n(\lambda\varepsilon^n(\mathbf{g}) \cdot \mathbf{1}) = W^n(\varepsilon^n(\lambda\mathbf{g}) \cdot \mathbf{1})$, from which since W^n is monotonic

$$\lambda\varepsilon^n(\mathbf{g}) = \varepsilon^n(\lambda\mathbf{g}). \tag{ARI}$$

Axiom TI imposes that ε^n is independent of origin. From the definition of ε^n, we have $W^n(\mathbf{g}) = W^n(\varepsilon^n(\mathbf{g}) \cdot \mathbf{1})$, such that, by TI

$$W^n(\mathbf{g} + \lambda \cdot \mathbf{1}) = W^n(\varepsilon^n(\mathbf{g} \cdot \mathbf{1} + \lambda \cdot \mathbf{1}) = W^n((\varepsilon^n(\mathbf{g}) + \lambda) \cdot \mathbf{1}).$$

At the same time, from the definition of ε^n,

$$W^n(\mathbf{g} + \lambda \cdot \mathbf{1}) = W^n\left(\varepsilon^n(\mathbf{g} + \lambda \cdot \mathbf{1}) \cdot \mathbf{1}\right),$$

such that the combination of the last two equations yields, because of the monotonicity of W^n,

$$\varepsilon^n(\mathbf{g} + \lambda \cdot \mathbf{1}) = \varepsilon^n(\mathbf{g}) + \lambda. \tag{ATI}$$

Together, axioms WD, RI and TI impose a very specific functional for $\varepsilon^2(g_1, g_2)$: for populations of two individuals, aggregate growth can be written as a weighted sum of individual growth. Formally:

Lemma 3. *The function W^2 satisfies WD, RI and TI, if and only if there exist numbers γ_1^2 and $\gamma_2^2 \in [0,1]$, such that:*

$$\gamma_1^2 + \gamma_2^2 = 1 \text{ and } \varepsilon^2(g_1, g_2) = \gamma_1^2 g_1 + \gamma_2^2 g_2.$$

Proof. Consider $\mathbf{g} = (g_1, g_2)$ and assume wlog that $g_1 \geq g_2$ then, using first (ATI) and then (ARI),

$$\begin{aligned}\varepsilon^2(g_1, g_2) &= \varepsilon^2(g_1 - g_2, 0) + g_2 \\ &= \varepsilon^2(1,0)(g_1 - g_2) + g_2 \\ &= \varepsilon^2(1,0)g_1 + (1 - \varepsilon^2(1,0))g_2.\end{aligned}$$

Now, let $\gamma_1^2 = \varepsilon^2(1,0)$ and set $\gamma_2^2 = (1 - \varepsilon^2(1,0))$. By WD:

$$0 = \varepsilon^2(0,0) \leq \varepsilon^2(1,0) \leq \varepsilon^2(1,1) = 1.$$

Hence, both γ_1^2 and γ_2^2 are positive. □

Using axiom $D - HG$ together with WD, RI and TI, we can derive the following partial result:

Lemma 4. *The function G^n satisfies WD, RI, TI and D-HG, if and only if there exist positive numbers $\gamma_1^n, \ldots, \gamma_n^n$ summing to one, such that:*

$$\varepsilon^n(\mathbf{g}) = \sum_{i=1}^{n} \gamma_i^n \tilde{g}_i.$$

Proof. Observe that Axiom D-HG allows the existence of a two placed function L^n such that:

$$\varepsilon^n(\tilde{g}_1, \ldots, \tilde{g}_n) = L^n(\varepsilon^{n-1}(\tilde{g}_1, \ldots, \tilde{g}_{n-1}), \tilde{g}_n). \tag{AD-HM}$$

The proof of the lemma is by induction. Lemma 3 gives the proof for $n = 2$. Now, assume that it holds up to $n - 1$ and let us show that the result holds for n. Then:

$$\begin{aligned}\varepsilon^n(\tilde{g}_1, \ldots, \tilde{g}_n) =& L^n(\varepsilon^{n-1}(\tilde{g}_1, \ldots, \tilde{g}_{n-1}), \tilde{g}_n) && \text{(by AD-HM)} \\ =& L^n(\varepsilon^{n-1}(\tilde{g}_1 - \tilde{g}_n, \ldots, \tilde{g}_{n-1} - \tilde{g}_n), 0) + \tilde{g}_n && \text{(by ATI)} \\ =& L^n(\varepsilon^{n-1}(\mathbf{1} \cdot \varepsilon^{n-1}(\tilde{g}_1 - \tilde{g}_n, \ldots, \tilde{g}_{n-1} - \tilde{g}_n)), 0) + \tilde{g}_n && \\ =& L^n(\varepsilon^{n-1}(\mathbf{1}, 0) \cdot \varepsilon^{n-1}(\tilde{g}_1 - \tilde{g}_n, \ldots \tilde{g}_{n-1} - \tilde{g}_n)) + \tilde{g}_n && \text{(by ARI)} \\ =& L^n(1,0) \cdot (\varepsilon^{n-1}(\tilde{g}_1, \ldots, \tilde{g}_{n-1}) - \tilde{g}_n) + \tilde{g}_n && \text{(by ATI)} \\ =& L^n(1,0)\varepsilon^{n-1}(\tilde{g}_1, \ldots, \tilde{g}_{n-1}) + (1 - L^n(1,0))\tilde{g}_n. &&\end{aligned}$$

Now, substituting $\varepsilon^{n-1}(\tilde{g}_1, \ldots, \tilde{g}_{n-1}) = \sum_{i=1}^{n-1} \gamma_i^{n-1} \tilde{g}_i$ and defining for $i < n$,

$$\gamma_i^n = \gamma_i^{n-1} L^n(1,0),$$

and for $i = n$,

$$\gamma_n^n = (1 - L^n(1,0)),$$

we derive the expression:

$$\varepsilon^n(\widetilde{\mathbf{g}}) = \sum_{i=1}^{n} \gamma_i^n \tilde{g}_i.$$

It is easy to see that $\sum_{i=1}^{n} \gamma_i^n = 1$ and that all terms are positive. □

Axiom *PI* allows us to determine the functional form of the coefficients γ_i. Indeed, theorems 1 and 2 of Donaldson and Weymark (1980) show that PI imposes that there exist a $\delta \in \mathbb{R}_{++}$ such that for all $i \in \mathbb{N}$,

$$\gamma_i^n = (i^\delta - (i-1)^\delta)/n^\delta.$$

Hence, the function G^n satisfies WD, TI, RI , D-HG and PI if and only if there exists a number δ such that:

$$\varepsilon^n(\mathbf{g}) = \frac{\sum_{i=1}^{n}(i^\delta - (i-1)^\delta)\tilde{g}_i}{n^\delta}.$$

Combining this expression with lemma 1 completes the proof of the proposition.

B Figures and Tables

Table 1: Illustrative example

individual	$\mathbf{x}$	$\mathbf{y}^1$	$\mathbf{y}^2$	$\mathbf{y}^3$	$\mathbf{y}^4$	$\mathbf{g}^1$	$\mathbf{g}^2$	$\mathbf{g}^3$	$\mathbf{g}^4$
1	10	20	5	10	20	2	0.5	1	2
2	20	20	20	20	40	1	1	1	2
3	30	15	60	60	30	0.5	2	2	1
mean	20	16.3	28.3	30	30	0.81	1.42	1.5	1.5

Table 2: Comparison of growth between Germany and US for the hypothesis: $G^{\infty}_{PSID} > G^{\infty}_{GSOEP}$

		r					
1984/85 δ	log	0.2	0.4	0.7	1	1.5	2
1	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
2	FALSE^{ns}	FALSE^{ns}	TRUE^{ns}	TRUE^{ns}	TRUE	TRUE	TRUE
4	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE^{ns}
6	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
8	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

		r					
1996/97 δ	log	0.2	0.4	0.7	1	1.5	2
1	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE	TRUE
2	FALSE	FALSE	FALSE	FALSE	FALSE^{ns}	TRUE	TRUE
4	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
6	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE
8	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE	FALSE

NOTE: All signs except the ones with ns as superscript are significant at the 95% level.

Figure 1: Empirical CDF of individual growth ratios 1996/1997

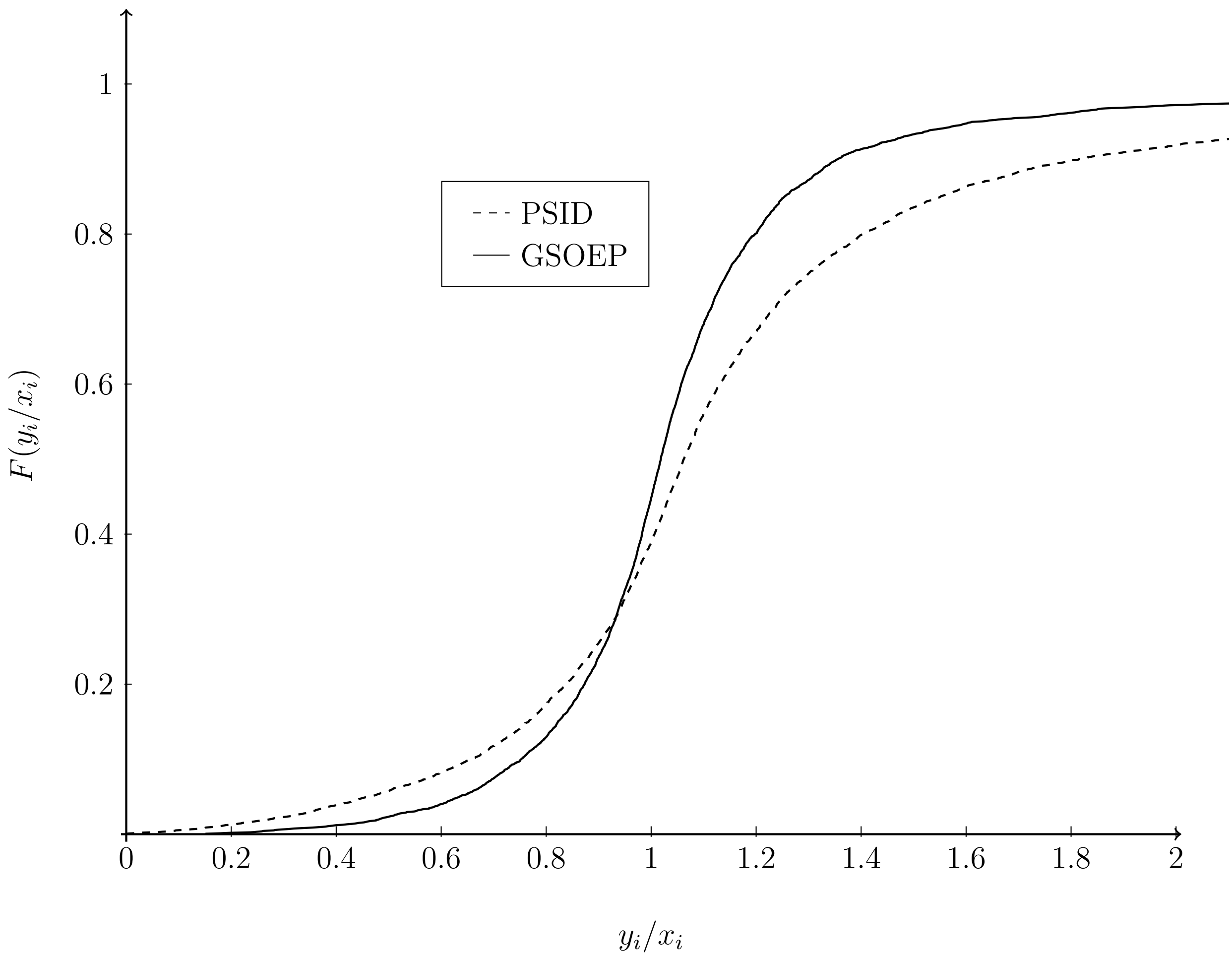

Table 3: Comparison of growth between the US and Germany for the hypothesis: $G^{\infty}_{A,PSID} > G^{\infty}_{A,GSOEP}$

	$\hat{r} = r(1-e)$							
year	-2	-1.5	-1	-0.5	0.5	1	1.5	2
1984/1985	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE	TRUE
1996/1997	FALSE	FALSE	FALSE	FALSE	TRUE	TRUE	TRUE	TRUE

NOTE: All signs are significant at the 95% level.

Table 4: Decomposition into growth inequality and mean growth

1984/85		r					
		0.2	0.4	0.7	1	1.5	2
δ	data set						
				$I^n(\mathbf{x},\mathbf{y})$			
2	PSID	0.0431	0.0871	0.1571	0.2364	0.4161	0.6888
	GSOEP	0.0315	0.0629	0.11	0.158	0.2415	0.333
4	PSID	0.0774	0.1505	0.2554	0.3593	0.5505	0.7803
	GSOEP	0.059	0.1149	0.1938	0.2678	0.3834	0.4926
6	PSID	0.0981	0.1875	0.3099	0.4238	0.6147	0.8202
	GSOEP	0.0758	0.1459	0.2419	0.3286	0.457	0.5703
8	PSID	0.1136	0.2147	0.3487	0.4685	0.6573	0.8455
	GSOEP	0.0882	0.1686	0.2763	0.3712	0.5068	0.621
				$\mu^n(\mathbf{x},\mathbf{y})$			
	PSID	1.0109	1.0301	1.0778	1.1593	1.4764	2.7442
	GSOEP	0.9996	1.0031	1.0157	1.0379	1.1011	1.2107

1996/97		r					
		0.2	0.4	0.7	1	1.5	2
δ	data set						
				$I^n(\mathbf{x},\mathbf{y})$			
2	PSID	0.0559	0.1143	0.2141	0.3403	0.6303	0.8915
	GSOEP	0.032	0.0648	0.1165	0.1731	0.2848	0.4269
4	PSID	0.0985	0.1901	0.3248	0.4669	0.7312	0.9302
	GSOEP	0.0575	0.1128	0.1931	0.2725	0.4081	0.554
6	PSID	0.1253	0.2355	0.3864	0.5322	0.7766	0.9449
	GSOEP	0.0734	0.1419	0.2379	0.3282	0.4728	0.6161
8	PSID	0.146	0.2696	0.431	0.578	0.8068	0.9541
	GSOEP	0.0854	0.1638	0.2708	0.3683	0.5178	0.658
				$\mu^n(\mathbf{x},\mathbf{y})$			
	PSID	1.0224	1.0613	1.1637	1.3657	2.4397	8.6488
	GSOEP	1.0068	1.0185	1.0465	1.0908	1.222	1.4943