Inequality: Methods and Tools

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1. Introduction

Inequality means different things to different people: whether inequality should encapsulate ethical concepts such as the desirability of a particular system of rewards or simply mean differences in income is the subject of much debate[1]. Here we will conceptualise inequality as the dispersion of a distribution, whether that be income, consumption or some other welfare indicator or attribute of a population.

We begin with some notation. Define a vector **y** of incomes, $y_1, y_2 \ldots . y_i \ldots . y_n$; $y_i \in \Re$, where n represents the number of units in the population (such as households, families, individuals or earners for example). Let F(y) be the cumulative distribution function of y, and I(y) an estimate of inequality.

Inequality is often studied as part of broader analyses covering poverty and welfare, although these three concepts are distinct. Inequality is a broader concept than poverty in that it is defined over the whole distribution, not only the censored distribution of individuals or households below a certain poverty line, y^p. Incomes at the top and in the middle of the distribution may be just as important to us in perceiving and measuring inequality as those at the bottom, and indeed some measures of inequality are driven largely by incomes in the upper tail (see the discussion in section 2 below). Inequality is also a much narrower concept than welfare. Although both of these capture the whole distribution of a given indicator, inequality is independent of the mean of the distribution (or at least this is a desirable property of an inequality measure, as is discussed below in section 2) and instead solely concerned with the second moment, the dispersion, of the distribution. However these three concepts are closely related and are sometimes used in composite measures. Some poverty indices incorporate inequality in their definition: for example Sen's poverty measure contains the Gini coefficient among the poor (Sen, 1976) and the Foster-Greer-Thorbecke measure with parameter $\alpha \geq 2$ weights income gaps from the poverty line in a convex manner, thus taking account of the distribution of incomes below the poverty line (Foster et al, 1984). Inequality may also appear as an argument in social welfare functions of the form $W=W(\mu(y), I(y))$: this topic is discussed more fully below under the subject of stochastic dominance.

2. Measuring Inequality

There are many ways of measuring inequality, all of which have some intuitive or mathematical appeal[2]. However, many apparently sensible measures behave in perverse fashions. For example, the variance, which must be one of the simplest measures of inequality, is not

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[1] See Atkinson (1983) for a brief summary.
[2] Cowell (1995) contains details of at least 12 summary measures of inequality.

independent of the income scale: simply doubling all incomes would register a quadrupling of the estimate of income inequality. Most people would argue that this is not a desirable property of an inequality measure and so it seems appropriate to confine the discussion to those that conform to a set of axioms. Even this however may result in some measures ranking distributions in different ways and so a complementary approach is to use stochastic dominance. We begin with the axiomatic approach and outline five key axioms which we usually require inequality measures to meet[3].

2.1. The Axiomatic approach.

The Pigou-Dalton Transfer Principle (Dalton, 1920, Pigou, 1912). This axiom requires the inequality measure to rise (or at least not fall) in response to a mean-preserving spread: an income transfer from a poorer person to a richer person should register as a rise (or at least not as a fall) in inequality and an income transfer from a richer to a poorer person should register as a fall (or at least not as an increase) in inequality (see Atkinson, 1970, 1983, Cowell, 1985, Sen, 1973). Consider the vector **y'** which is a transformation of the vector **y** obtained by a transfer δ from y_j to y_i , where $y_i > y_j$, and $y_i + \delta > y_j - \delta$, then the transfer principle is satisfied iff $I(\mathbf{y'}) \geq I(\mathbf{y})$. Most measures in the literature, including the Generalized Entropy class, the Atkinson class and the Gini coefficient, satisfy this principle, with the main exception of the logarithmic variance and the variance of logarithms (see Cowell, 1995).

Income Scale Independence. This requires the inequality measure to be invariant to uniform proportional changes: if each individual's income changes by the same proportion (as happens say when changing currency unit) then inequality should not change. Hence for any scalar $\lambda > 0$, $I(\mathbf{y}) = I(\lambda\mathbf{y})$. Again most standard measures pass this test except the variance since $var(\lambda\mathbf{y}) = \lambda^2 var(\mathbf{y})$. A stronger version of this axiom may also be applied to uniform absolute changes in income and combinations of the form $\lambda_1\mathbf{y} + \lambda_2\mathbf{1}$ (see Cowell, 1999).

Principle of Population (Dalton, 1920). The population principle requires inequality measures to be invariant to replications of the population: merging two identical distributions should not alter inequality. For any scalar $\lambda > 0$, $I(\mathbf{y}) = I(\mathbf{y}[\lambda])$, where $\mathbf{y}[\lambda]$ is a concatenation of the vector **y**, λ times.

Anonymity. This axiom – sometimes also referred to as '*Symmetry*' - requires that the inequality measure be independent of any characteristic of individuals other than their income (or the welfare indicator whose distribution is being measured). Hence for any permutation **y**' of **y**, $I(\mathbf{y}) = I(\mathbf{y}')$.

Decomposability. This requires overall inequality to be related consistently to constituent parts of the distribution, such as population sub-groups. For example if inequality is seen to rise amongst each sub-group of the population then we would expect inequality overall to also increase. Some measures, such as the Generalised Entropy class of measures, are easily decomposed and into intuitively appealingly components of within-group inequality and between-group inequality: I_{total} =

[3] See Cowell (1985) on the axiomatic approach. Alternative axioms to those listed below are possible and the appropriateness of these axioms has been questioned. See Amiel (1998), Amiel and Cowell (1998), Harrison and Seidl (1994a, 1994b) amongst others for questionnaire experimental tests of the desirability of these axioms, and Cowell (1999), for an introduction to alternative approaches to inequality.

$I_{within} + I_{between}$. Other measures, such as the Atkinson set of inequality measures, can be decomposed but the two components of within- and between-group inequality do not sum to total inequality. The Gini coefficient is only decomposable if the partitions are non-overlapping, that is the sub-groups of the population do not overlap in the vector of incomes. See section 3 for full details of decomposition techniques.

Cowell (1995) shows that any measure I(y) that satisfies all of these axioms is a member of the Generalized Entropy (GE) class of inequality measures, hence we focus our attention on this reduced set. We do however also present formula for the Atkinson class of inequality measures, which are ordinally equivalent to the GE class, and the popular Gini coefficient.

2.1.1. Inequality Measures

Members of the Generalised Entropy class of measures have the general formula as follows:

$$GE(\alpha) = \frac{1}{\alpha^2 - \alpha}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i}{\bar{y}}\right)^{\alpha} - 1\right]$$

where n is the number of individuals in the sample, y_i is the income of individual i, $i \in (1, 2,...,n)$, and $\bar{y} = (1/n) \sum y_i$, the arithmetic mean income. The value of GE ranges from 0 to ∞, with zero representing an equal distribution (all incomes identical) and higher values representing higher levels of inequality[4]. The parameter α in the GE class represents the weight given to distances between incomes at different parts of the income distribution, and can take any real value. For lower values of α GE is more sensitive to changes in the lower tail of the distribution, and for higher values GE is more sensitive to changes that affect the upper tail. The commonest values of α used are 0,1 and 2: hence a value of α=0 gives more weight to distances between incomes in the lower tail, α=1 applies equal weights across the distribution, while a value of α=2 gives proportionately more weight to gaps in the upper tail. The GE measures with parameters 0 and 1 become, with l'Hopital's rule, two of Theil's measures of inequality (Theil, 1967), the mean log deviation and the Theil index respectively, as follows:

$$GE(0) = \frac{1}{n}\sum_{i=1}^{n}\log\frac{\bar{y}}{y_i}$$

$$GE(1) = \frac{1}{n}\sum_{i=1}^{n}\frac{y_i}{\bar{y}}\log\frac{y_i}{\bar{y}}$$

With α=2 the GE measure becomes 1/2 the squared coefficient of variation, CV:

$$CV = \frac{1}{\bar{y}}\left[\frac{1}{n}\sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2\right]^{1/2}$$

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[4] In the presence of any zero income values GE(0) will always attain its maximum, ∞. Negative incomes restrict the choice of α to values greater than 1.

The Atkinson class of measures has the general formula:

$$A_\varepsilon = 1 - \left[\frac{1}{n} \sum_{i=1}^{n} \left[\frac{y_i}{\bar{y}} \right]^{1-\varepsilon} \right]^{1/(1-\varepsilon)}$$

where ε is an inequality aversion parameter, $0<\varepsilon<\infty$: the higher the value of ε the more society is concerned about inequality (Atkinson, 1970). The Atkinson class of measures range from 0 to 1, with zero representing no inequality[5]. Setting $\alpha=1-\varepsilon$, the GE class becomes ordinally equivalent to the Atkinson class, for values of $\alpha<1$ (Cowell, 1995).

The Gini coefficient satisfies axioms 1-4 above, but will fail the decomposability axiom if the sub-vectors of income overlap. There are ways of decomposing the Gini but the component terms of total inequality are not always intuitively or mathematically appealing (see for example Fei et al, 1978, and an attempt at a decomposition with a more intuitive residual term by Yitzhaki and Lerman, 1991). However the Gini's popularity merits it a mention here. It is defined as follows (Gini, 1912):

$$Gini = \frac{1}{2n^2\bar{y}} \sum_{i=1}^{n} \sum_{j=1}^{n} \left| y_i - y_j \right|$$

The Gini coefficient takes on values between 0 and 1 with zero interpreted as no inequality[6].

2.2. An Alternative Approach: Stochastic Dominance.

Although the measures discussed above generally meet the set of desirable axioms it is possible that they will rank the same set of distributions in different ways, simply because of their differing sensitivity to incomes in different parts of the distributions. When rankings are ambiguous, the alternative method of stochastic dominance can be applied. We discuss three types of stochastic dominance below. The first two are sensitive to the mean of the distribution, and are therefore not applicable to establishing inequality rankings. As the discussion below suggests, first- and second- order stochastic dominance are fundamentally of use in comparisons of social welfare. They are presented first, however, because they are logically prior to the dominance category which is associated with unambiguous comparisons of inequality across distributions: mean-normalised second-order dominance, or Lorenz dominance. [7]

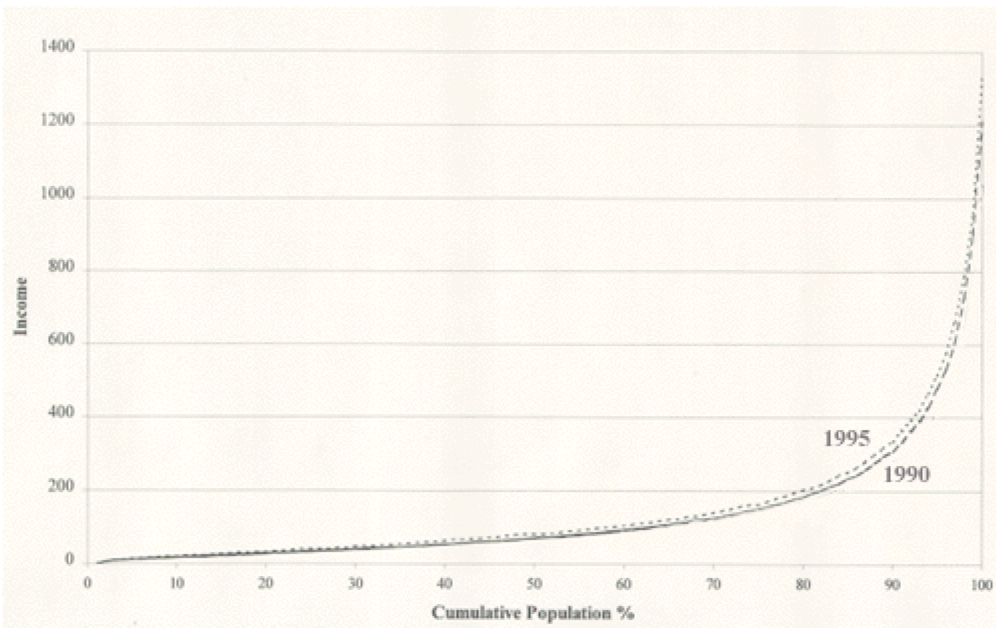
First order stochastic dominance. Consider two income distributions $\mathbf{y}_1$ and $\mathbf{y}_2$ with cumulative distribution functions (CDFs) $F(\mathbf{y}_1)$ and $F(\mathbf{y}_2)$. If $F(\mathbf{y}_1)$ lies nowhere above and at least somewhere below $F(\mathbf{y}_2)$ then distribution $\mathbf{y}_1$ displays first order stochastic dominance over distribution $\mathbf{y}_2$: $F(\mathbf{y}_1)\leq F(\mathbf{y}_2)$ for all y. Hence in distribution $\mathbf{y}_1$ there are no more individuals with income less than a given income level than in distribution $\mathbf{y}_2$, for all levels of income. We can

[5] In the presence of any zero incomes A(1) always attains its maximum value, 1.
[6] Zero incomes pose no problem for the Gini. However it may take a negative value if mean income is negative or a value greater than 1 if there are some very large negative incomes (see Scott and Litchfield, 1994).

[7] See Ferreira and Litchfield (1996) for an application of stochastic dominance analysis to Brazil.

express this in an alternative way using the inverse function $\mathbf{y}=F^{-1}(\mathbf{p})$ where p is the share of the population with income less than a given income level: first order dominance is attained if $F_1^{-1}(\mathbf{p}) \geq F_2^{-1}(\mathbf{p})$ for all p. The inverse function $F^{-1}(\mathbf{p})$ is known as a Pen's Parade (Pen, 1974) which simply plots incomes against cumulative population, usually using ranked income quantiles[8]. The dominant distribution is that whose Parade lies nowhere below and at least somewhere above the other. First order stochastic dominance of distribution $\mathbf{y}_1$ over $\mathbf{y}_2$ implies that any social welfare function that is increasing in income, will record higher levels of welfare in distribution $\mathbf{y}_1$ than in distribution $\mathbf{y}_2$ (Saposnik, 1981, 1983).

Figure 1: First Order Stochastic Dominance
Brazil 1981-1995: Pen's Parades



Source: Ferreira and Litchfield, 1999, "Inequality, Poverty and Welfare, Brazil 1981-1995". London School of Economics Mimeograph.

Second order stochastic dominance. Consider now the deficit functions (the integral of the CDF) of distributions $\mathbf{y}_1$ and $\mathbf{y}_2$: $G(y_{i,k}) = \int_0^{y_k} F(y_i)dy$ i=1,2. If the deficit function of distribution $\mathbf{y}_1$ lies nowhere above and somewhere below that of distribution $\mathbf{y}_2$, then distribution $\mathbf{y}_1$ displays second order stochastic dominance over distribution $\mathbf{y}_2$: $G(\mathbf{y}_{1,k}) \leq G(\mathbf{y}_{2,k})$ for all y_k. The dual of the deficit

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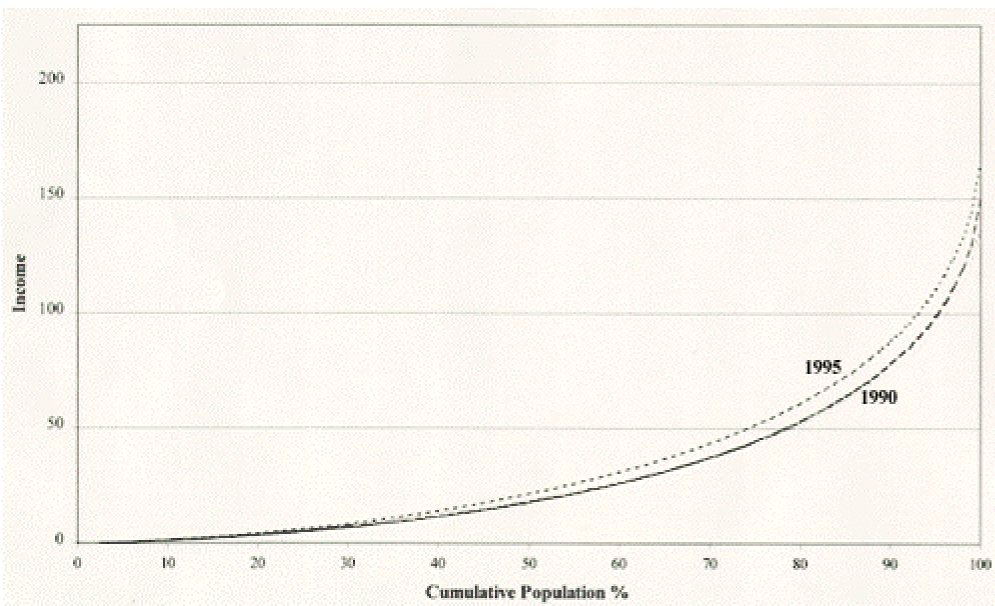
[8] The original Pen's Parades of Jan Pen were conceptualised by comparing the incomes of every individual in a population. The example that Pen gave was of lining up individuals in ascending order of income and re-scaling their heights to represent their income level. If these individuals were to be paraded past an observer she would typically see a large number of dwarves (poor people), eventually followed by individuals of average height (income) and finally followed by a small number of giants (very rich people). In practice comparing incomes at every income level proves too laborious, hence some degree of aggregation is usually employed and quantiles are compared.

curve is the Generalised Lorenz curve (Shorrocks, 1983) defined as $GL(p) = \int_0^{y_k} y dF(y)$, which plots cumulative income shares scaled by the mean of the distribution against cumulative population, where the height of the curve at p is given by the mean of the distribution below p. As Atkinson and Bourguignon (1989) and Howes (1993) have shown, second order dominance established by comparisons of the deficit curves for complete, uncensored distributions implies and is implied by Generalised Lorenz dominance: $GL_1(\mathbf{p}) \geq GL_2(\mathbf{p})$ for all p. Second order dominance of distribution $\mathbf{y}_1$ over distribution $\mathbf{y}_2$ implies that any social welfare function that is increasing and concave in income will record higher levels of welfare in $\mathbf{y}_1$ than in $\mathbf{y}_2$ (Shorrocks, 1983). It should now be apparent that second order stochastic dominance is therefore implied by first order stochastic dominance, although the reverse is not true.

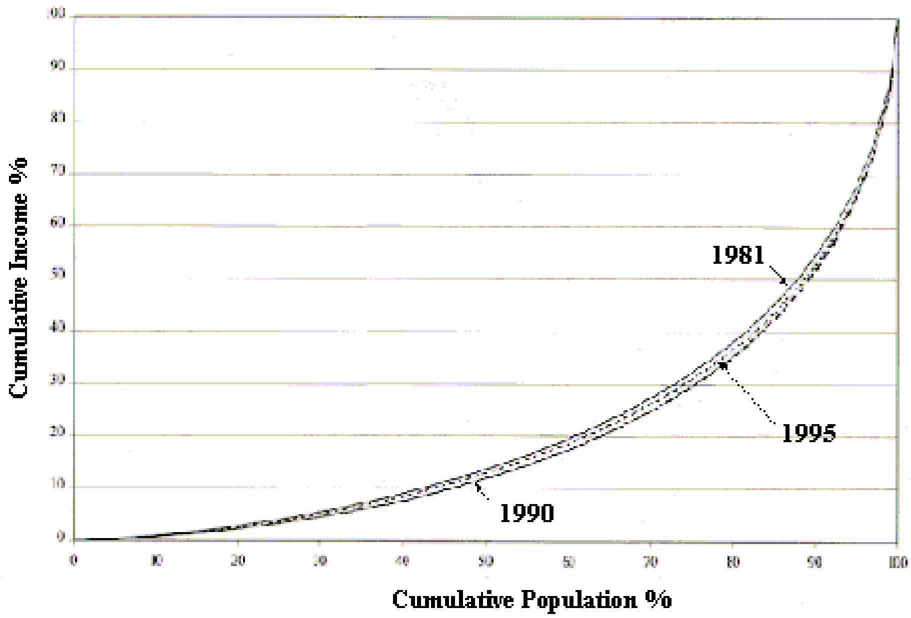
Figure 2: Second Order Stochastic Dominance
Brazil 1981-1995: Generalised Lorenz Curves



Source: Ferreira and Litchfield, 1999, "Inequality, Poverty and Welfare, Brazil 1981-1995". London School of Economics Mimeograph.

Mean-normalised second order stochastic dominance. In order to rank distributions in terms of inequality alone, rather than welfare, a third concept (also known as Lorenz dominance) is applied. If the Lorenz curve, the plot of cumulative income shares against cumulative population shares, of distribution $\mathbf{y}_1$ lies nowhere below and at least somewhere above the Lorenz curve of distribution $\mathbf{y}_2$ then $\mathbf{y}_1$ Lorenz dominates $\mathbf{y}_2$. Any inequality measure which satisfies anonymity and the Pigou-Dalton transfer principle will rank the two distributions in the same way as the Lorenz curves (Atkinson, 1970).

Figure 3: Mean-normalized second order stochastic dominance
Brazil 1981-1995: Lorenz curves



Note: 1995 Lorenz dominates 1990, but 1981 Lorenz dominates 1990 and dominates 1995.
Source: Ferreira and Litchfield, 1999, "Inequality, Poverty and Welfare, Brazil 1981-1995". London School of Economics Mimeograph.

2.3. Statistical Inference and Sampling Variance.

In order to make meaningful comparisons between estimates of inequality of different distributions and of the rankings implied by stochastic dominance we need to examine the statistical significance of the results. In the case of summary inequality measures we need to examine the standard errors of the estimates. There are a number of ways this can be done, depending on the degree of sophistication one wishes to apply and the measure of inequality in question.

Cowell (1995) lists some rough approximations for a number of inequality measures if the sample size is large and if one is prepared to make assumptions about the underlying distribution from which the sample is taken. For example the coefficient of variation CV (which is a simple transformation of GE(2) – see above) has standard error $CV\sqrt{([1+2CV^2]/2n)}$ if we assume the underlying distribution is normal. The Gini coefficient, again assuming a normal distribution, has standard error $0.8086CV/\sqrt{n}$.

However this may be too rough for some purposes in which case a more accurate method can be applied. Cowell (1989) shows that many inequality measures can be expressed in terms of their sample moments about zero. For example the Atkinson class can be written as

$$A_\varepsilon = 1 - \frac{[\mu'_r]^{1/r}}{\mu'_1}$$

where $\mu_r{}'$ is the r^{th} moment about zero, r=1-ε and $\mu_1{}'$= the mean of the distribution, and the GE class as:

$$GE(\alpha) = \frac{1}{\alpha^2 - \alpha}\left[\mu_{1\alpha}[\mu_{11}]^{-\alpha}[\mu_{10}]^{\alpha-1} - 1\right], \alpha \neq 0,1$$

where $\mu_{v\alpha}$ are the moments about zero defined as:

$$\mu_{v\alpha} = \iint z^v y^\alpha dF(z,y)$$

where z is household size (or some other weighting variable), v=1,2 and $-\infty<\alpha<\infty$. The sample moments $m_{v\alpha}$ can be expressed as $m_{v\alpha}=1/n\sum z_i{}^v y_i{}^\alpha$. If both mean household size and mean income are known, then Var(GE(α)) is relatively easy to derive:

$$\hat{V} = \frac{1}{[n-1][\alpha^2-\alpha]^2}[m_{11}]^{-2\alpha}[m_{10}]^{2\alpha-2}[m_{2,2\alpha} - m_{1\alpha}^2]$$

For the full details of the method and for the cases where α=0 and 1, and where the population mean income and household size are not known see Cowell (1989, 1995).

It is also possible to test the statistical significance of any stochastic dominance results. Howes (1993) describes one test based on a simple test of sample mean differences:

$$z_i = \frac{\hat{\xi}_i - \hat{\xi}_i^*}{\left(\frac{\hat{C}_{ii}}{N} + \frac{\hat{C}_{ii}^*}{N^*}\right)^{1/2}}$$

where $\xi=(\xi_1, \ldots \xi_w)$ is a vector of heights of curves (Pen's Parades, Lorenz or Generalized Lorenz curves), C_{ii} is the relevant element in the diagonal of the variance-covariance matrix associated with ξ and N is the sample size. Z_i is asymptotically normally distributed. See Howes (1993) for fuller details and an empirical application to China, and Ferreira and Litchfield (1996) for an application to Brazil.

3. The Determinants of Inequality

The preceding discussion of measurement and comparisons of inequality should have sufficed to establish what a complex and multifaceted phenomenon it is. Because it is influenced by the welfare of any individual or household in a society, and because welfare itself is affected by so many factors, and determined in general equilibrium, the study of causation or determination of inequality is a perilous field. Authors aware of the muddy waters in which they wade qualify their every statement with cautionary remarks about how results are merely 'indicative' or 'suggestive'.

They are often at pains to point out that decomposition results are descriptive, and inferences of causation are merely suggestive. The analytical techniques are in their infancy, and such caution is both warranted and necessary. Nevertheless, once the appropriate caution of interpretation is internalised, some techniques do allow us to glimpse interesting patterns. In the absence of more definitive inference methods, some of these decomposition and regression analyses are often worthwhile exercises. We begin with decomposition techniques.

3.1. Decomposition techniques

Decomposability is desirable for both arithmetic and analytic reasons. Economists and policy analysts may wish to assess the contribution to overall inequality of inequality with and between different sub-groups of the population, for example within and between workers in agricultural and industrial sectors, or urban and rural sectors. Decompositions of inequality measures can shed light on both its structure and dynamics. Inequality decomposition is a standard technique for examining the contribution to inequality of particular characteristics and can be used to assess income recipient characteristics and income package influences. The field was pioneered by Bourguignon (1979), Cowell (1980), and Shorrocks (1982a, 1982b, 1984). For more details on the methodologies, see Deaton (1997), and Jenkins (1995). Fields (1980) provides summaries of applications to developing countries.

Decomposition by population sub-group.

The point of this decomposition is to separate total inequality in the distribution into a component of inequality between the chosen groups (I_b), and the remaining within-group inequality (I_w). Two types of decomposition are of interest: firstly the decomposition of the level of inequality in any one year, i.e a static decomposition, and secondly a decomposition of the change in inequality over a period of time, i.e. a dynamic decomposition.

The static decomposition. When total inequality, I, is decomposed by population subgroups, the Generalised Entropy class can be expressed as the sum of within-group inequality, I_w, and between group inequality, I_b. Within-group inequality I_w is defined as:

$$I_w = \sum_{j=1}^{k} w_j GE(\alpha)_j$$

$$w_j = v_j^{\alpha} f_j^{1-\alpha}$$

where f_j is the population share and v_j the income share of each partition j, j=1,2,..k. In practical terms the inequality of income within each sub-group is calculated and then these are summed, using weights of population share, relative incomes or a combination of these two, depending on the particular measure used. Between-group inequality, I_b, is measured by assigning the mean income of each partition j, $\overline{y_j}$, to each member of the partition and calculating:

$$I_b = \frac{1}{\alpha^2 - \alpha}\left[\sum_{j=1}^{k} f_j \left(\frac{\overline{y_j}}{\overline{y}}\right)^{\alpha} - 1\right]$$

Cowell and Jenkins (1995) show that the within- and between-group components of inequality, defined as above, can be related to overall inequality in the simplest possible way: $I_b + I_w = I$. They then suggest an intuitive summary measure, R_b , of the amount of inequality explained by differences between groups with a particular characteristic or set of characteristics, $Rb = I_b / I$. Hence we can conclude that x% of total inequality is "explained" by between group inequalities, and (100-x)% is accounted for by inequalities within groups. By increasing the number of partitions we can account for the effect of a wider range of structural factors[9].

The dynamic decomposition. Accounting for changes in the level of inequality by means of a partition of the distribution into sub-groups must entail at least two components of the change: one caused by a change in inequality between the groups and one by a change in inequality within the groups. The second one is the "pure inequality" effect, but the first one can be further disaggregated into an effect due to changes in relative mean incomes between the subgroups - an "income effect" - and one due to changes in the size of the subgroups - an "allocation effect". Hence we can decompose the change in total inequality into three components: an allocation effect arising from changes in the number of people within different partitions, an income effect arising from changes in relative incomes between partitions, and finally a pure inequality effect arising from changes in inequality within partitions (Mookerjee and Shorrocks, 1982). The arithmetic becomes complicated for some measures, so this is usually only applied to *GE(0)*, as follows:

$$\Delta GE(0) = \begin{bmatrix} \sum_{j=1}^{k} \overline{f}_j \Delta GE(0)_j \\ + \sum_{j=1}^{k} \overline{GE(0)}_j \Delta f \ \ + \sum_{j=1}^{k} \left[\overline{\lambda}_j - \overline{\log(\lambda_j)} \right] \Delta f_j \\ + \sum_{j=1}^{k} \left(\overline{v}_j - \overline{f}_j \right) \Delta \log(\mu(y)_j \end{bmatrix}$$

where y is income, Δ is the difference operator, λ_j is the mean income of group j relative to the overall mean, i.e. $\mu(y_j)/\mu(y)$ and the over-bar represents a simple average. The first term captures the pure inequality effect, the second and third term capture the allocation effect and the final term the income effect. By dividing both sides through by the initial value of *GE(0)* proportionate changes in inequality can be compared to proportionate changes in the individual effects[10].

Decomposition by income source

Total income is usually made up of more than one source: labour earnings, income from capital, private and public transfers, etc; and so it is useful to express total inequality *I* as the sum of factor contributions, where each contribution depends on the incomes from a given factor source, *f*, i.e.

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[9] The Atkinson measure can also be decomposed by population sub-group – see Cowell and Jenkins (1995).
[10] This is actually an approximation of the true decomposition, but both Mookherjee and Shorrocks (1982) and, later, Jenkins (1995) argue that for computational purposes this approximation is sufficient.

$$I = \sum_f S_f$$

where S_f depends on incomes from source *f*. Factor income source *f* provides a disequalising effect if S_f >0, and an equalising effect if S_f <0. Now define

$$s_f \equiv \frac{S_f}{I}$$

so $\sum_f s_f = 1$. S_f is the absolute contribution of factor *f* to overall inequality, while s_f is the proportional factor contribution. The exact decomposition procedure depends on the measure of inequality used, but whichever measure is used must naturally be decomposable and, given the large number of income sources, it must be defined for zero incomes. In practice the easiest measure to decompose in this way is GE(2). In that case:

$$S_f = s_f GE(2) = \rho_f \chi_f \sqrt{GE(2).GE(2)_f}$$

where ρ_f is the correlation between component *f* and total income and $\chi_f = \mu_f / \mu$ is *f*'s factor share. A large value of S_f suggests that factor f is an important source of total inequality.

For the dynamic decomposition we can write

$$\Delta GE(2) = GE(2)_{t+1} - GE(2)_t = \sum_f \Delta S_f = \sum_f \Delta\left[\rho_f \chi_f \sqrt{GE(2).E(2)_f}\right]$$

and proportionate inequality changes as

$$\%\Delta GE(2) = \Delta GE(2) / GE(2)_t = \sum_f s_f \%\Delta S_f$$

A large value of $s_f \%\Delta S_f$ suggests that changes in factor *f* have a large influence in changes in total inequality. See Jenkins (1995) for the complete methodology and an application to the UK, and Theil (1979) [11].

3.2. Regression analyses

The decomposition techniques described above are very suitable for assessing the contribution of a set of factors (household-specific attributes or income sources) to inequality. However one drawback is that the importance of a particular attribute will vary depending on the measure of inequality that is decomposed. Fields (1997) proposes an alternative decomposition technique, which allows one to assess the importance of household specific attributes in explaining the level

[11] STATA routines for decompositions by population sub-group and by income source are available at the Boston College Department of Economics. See above for details and conditions of use.

of inequality, where the amount explained by each factor is independent of the inequality measures used[12]. The method involves running a standard set of regressions of the form:

$$\ln(y_{ij}) = \alpha_j + \beta_j X + \varepsilon_j$$

where the subscript i refers to the individual, j refers to the population sub-group and X is a vector of explanatory variables. Then the relative contributions, s_j , of each factor can then be estimated as:

$$s_j = \mathrm{cov}[a_j Z_j, Y] / \sigma^2(Y) = a_j * \sigma(Z_j) * cor[Z_j, Y] / \sigma(Y)$$

where a is the vector of coefficients (α, β_i), Z is the vector of explanatory variables plus a constant (1, x_i), and Y is log income. The change in inequality over time can also be decomposed using the s_j's estimated above, although the estimates are sensitive to the inequality measure used. See Fields (1997) for full details and an application to data from Bolivia and Korea.

An alternative approach is the *quantile regression* methodology, where instead of estimating the mean of a dependent variable conditional on the values of the independent variables, one estimates the median: minimising the sum of the absolute residuals rather than the sum of squares of the residuals as in ordinary regressions. It is possible to estimate different percentiles of the dependent variables, and so to obtain estimates for different parts of the income distribution. Furthermore, it is possible to use different independent variables for different quantiles, reflecting the view that data may be heteroskedastic with different factors affecting the rich and poor. See Deaton (1997) for an introduction to quantile regressions and some applications to developing country data.

Regression techniques are also applied when we want to model the effect of aggregate factors rather than specific attributes of a household. One method is to regress the level of inequality in each year (or each country, population sub-group etc.) on a set of explanatory variables, such as the rates of unemployment (UE) and inflation (INF), as follows:

$$I(y)_t = \alpha + \beta_1 UE_t + \beta_2 INF_t + u_t$$

A second method that is often applied in this macro-economic context is to regress a set of income shares on the independent variables, as follows:

$$s_{it} = \alpha_i + \beta_{1i} UE_t + \beta_{2i} INF_t + u_{it}$$

where s_{it} denotes the income share of the ith quantile group in year t[13]. The i quantile share regressions are a set of seemingly unrelated regressions (see Zellner, 1962) but since the right hand side variables are the same in each equation, the SURE estimation technique suggested by Zellner is equivalent to a set of simple OLS regressions. See Blinder and Esaki (1978) for an

[12] The result holds for all inequality measures that are symmetric and continuous, e.g. all members of the Generalised Entropy class, the Atkinson class and the Gini.

[13] See Blinder and Esaki (1978) for the original specifications.

application to the USA, Buse (1982) on Canadian inequality, Ferreira and Litchfield (1999) on Brazil and Nolan (1987) on UK inequality.

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