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INERTIA CALCULATION PROCEDURE FOR PRELIMINARY DESIGN.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report explains the methods involved in estimating aircraft moments of inertia for preliminary design purposes. Assumptions that were made for this procedure and the derivation of equations that evolved from these assumptions are included. An example using the method on the C-5A aircraft is shown. This procedure requires a knowledge of the major aircraft group weights, the location of major components (landing gear, avionics bay, etc.), geometry information, and inertias of some major subsystem items. Using

this data, the moments of inertia about the roll, pitch, and yaw axes are calculated as well as the roll-yaw cross-product of inertia.

FOREWORD

The purpose of this work was to establish a method to predict aircraft inertia suitable to preliminary design. It must be applicable to all types of military aircraft and be usable with the level of information normally available during preliminary design.

The material in this report was compiled as a part of the continuing methods development effort under project AFSD00710000N, Flying Qualities Methodology and Development. The effort was accomplished within ASD/XRHI by Charles Lanham while a cooperative student under the direction of Wayne M. O'Connor.

TABLE OF CONTENTS

LIST OF SYMBOLS

TERM	DEFINITION
b_1	span of surface panel from root chord to tip*
b_2	span of surface panel from root chord to break*
b_3	span of surface panel from break chord to tip
b_4	span of wing fuel tank
c_r	length of surface root chord*
$CREW_{cg}$	perpendicular distance from YZ plane of the remote axes to the crew center of gravity.
c_2	length of surface chord at break
c_3	length of most inboard chord for wing fuel tank
d_p	average diameter of payload
I	moment of inertia of a group or component about the remote axes
I_{cg}	moment of inertia of total aircraft about its own center of gravity
I_o	moment of inertia of a group or component about its own center of gravity
I_1	moment of inertia of a surface about the leading edge of the root chord or break chord*
I_4	moment of inertia of a fuel tank about the leading edge of the most inboard tank chord
l_c	length of fuselage center section
l_e	length of nacelle (for buried engines just length of engine)
l_f	longitudinal length of fuselage tank
l_n	length of fuselage nose cone
l_p	length of fuselage tail cone
l_v	length of item used as a volume in fuselage

LIST OF SYMBOLS (cont'd)

TERM	DEFINITION
R	average fuselage radius ($\frac{Smax}{\pi}$)
R_e	average nacelle radius (for buried engines use radius of engine)
R_v	average radius of item used as a volume in fuselage
S_c	wetted area of fuselage center section
S_1	external store or tank length
S_n	wetted area of fuselage nose cone
S_r	average radius of external tank or store
S_t	wetted area of fuselage tail cone
t_b	thickness of surface at break chord ($\frac{t}{c} \cdot c$) *
t_f	thickness of wing fuel tank at most inboard chord
t_{f_o}	thickness of wing fuel tank at most outboard chord
t_r	thickness of surface at root chord ($\frac{t}{c} \cdot c$)
t_t	thickness of surface at tip chord ($\frac{t}{c} \cdot c$)
W_c	weight of fuselage center section (structure only)
W_{d_c}	total weight of contents to be distributed throughout the fuselage
W_e	total propulsion group weight divided by the number of engines
W_{f_f}	weight of fuel in the fuselage
W_{f_w}	weight of fuel in both wing fuel tanks
W_h	weight of total horizontal tail group
W_i	weight of both surface inboard of break
W_n	weight of fuselage nose cone (structure only)

TERM	DEFINITION
W_o	weight of both surfaces outboard of break
W_p	weight of one point mass
W_{p_c}	total weight of point masses in the fuselage center section
W_{pnc}	total weight of point masses in nose and tail cones
W_s	weight of fuselage structure
W_{s_t}	weight of external fuel tank or store
W_t	weight of fuselage tail cone (structure only)
W_v	weight of total vertical tail group
W_{v_o}	weight of one volume of mass
W_w	weight of total wing group
XF1	distance from the wing fuel tank leading edge at most inboard tank chord to the longitudinal tank center of gravity
XF2	perpendicular distance from YZ plane of the remote axes to leading edge of wing fuel tank most inboard chord
XP	perpendicular distance from YZ plane of the remote axes to engine center of gravity
XS1	distance from the surface leading edge at root chord to the longitudinal surface center of gravity
XS2	(surface with leading and/or trailing edge break) distance from the surface leading edge of root chord to the longitudinal center of gravity for the surface section inboard of the break. *
XS3	(surface with leading and/or trailing edge break) distance from the surface leading edge break chord to the longitudinal center of gravity for the surface section outboard of the break.
XS4	perpendicular distance from YZ plane of the remote axes to leading edge of surface root chord
XS5	perpendicular distance from YZ plane of the remote axes to leading edge of surface break chord

LIST OF SYMBOLS (cont'd)

TERM	DEFINITION
$\bar{X}$	perpendicular distance from Z axis to aircraft center of gravity
$\bar{\bar{X}}$	distance from a defined reference point to the surface longitudinal center of gravity
YF1	distance from the wing fuel tank most inboard chord to the spanwise tank center of gravity
$Y\dot{F}1$	perpendicular distance from the XZ plane of wing fuel tanks most inboard chord to the spanwise tank center of gravity. ($Y\dot{F}1$ cos θ)
YF2	perpendicular distance from XZ plane of the remote axes to most inboard chord of wing fuel tank
YP	perpendicular distance from XZ plane of the remote axes to engine center of gravity
YS1	distance along span of surface from root chord to center of gravity
$Y\dot{S}1$	perpendicular distance from XZ plane of surface root chord to spanwise center of gravity (YS1 cos θ)
YS2	distance along span of surface from root chord to center of gravity for inboard surface
$Y\dot{S}2$	perpendicular distance from XA plane of surface root chord to spanwise center of gravity for inboard surfaces. (YS2 cos θ)
YS3	distance along span of surface from break chord to center of gravity for outboard surface
$Y\dot{S}3$	perpendicular distance from XA plane if surface break chord to spanwise center of gravity for outboard surfaces (YS3 cos θ)
YS4	perpendicular distance from XZ plane of the remote axes to the surface root chord
$\overset{*}{Y}$	distance from some reference point a surface spanwise center of gravity
Z_b	perpendicular distance from XY plane of the remote axes to fuselage centerline

TERM	DEFINITION
ZF	perpendicular distance from XY plane of the remote axes to wing fuel at most inboard chord
ZF2	(YF1 sin θ) (needed only for wing internal tanks with anhedral or dihedral) perpendicular distance from the XY plane of the surface root chord to the vertical center of gravity
ZP	perpendicular distance from XY plane of the remote axes to engine center of gravity
ZS1	perpendicular distance from XY plane of the remote axes to root chord of surface
ZS3	(YS1 sin θ) perpendicular distance from the XY plane of the surface root chord to the vertical surface center of gravity. *
ZS4	(YS2 sin θ) perpendicular distance from the XY plane of the surface root chord to the vertical center of gravity of the surface panel inboard of the break. *
ZS5	(YS3 sin θ) perpendicular distance from the XY plane of the surface break chord to the vertical center of gravity of surface panel outboard of the break
$\bar{z}$	perpendicular distance from X axes to aircraft center of gravity
$\bar{\bar{z}}$	distance from some reference point to a vertical surface center of gravity

*

LIST OF SYMBOLS (cont'd)

TERM	DEFINITION
ZF	perpendicular distance from XY plane of the remote axes to wing fuel at most inboard chord
ZF2	(YF1 sin θ) (needed only for wing internal tanks with anhedral or dihedral) perpendicular distance from the XY plane of the surface root chord to the vertical center of gravity
ZP	perpendicular distance from XY plane of the remote axes to engine center of gravity
ZS1	perpendicular distance from XY plane of the remote axes to root chord of surface
ZS3	(YS1 sin θ) perpendicular distance from the XY plane of the surface root chord to the vertical surface center of gravity. *
ZS4	(YS2 sin θ) perpendicular distance from the XY plane of the surface root chord to the vertical center of gravity of the surface panel inboard of the break. *
ZS5	(YS3 sin θ) perpendicular distance from the XY plane of the surface break chord to the vertical center of gravity of surface panel outboard of the break
$\bar{Z}$	perpendicular distance from X axes to aircraft center of gravity
$\bar{\bar{Z}}$	distance from some reference point to a vertical surface center of gravity

LIST OF SYMBOLS (con't)

TERM	DEFINITION
Λ_{L1}	sweep of surface leading edge at root
Λ_{La}	sweep of surface leading edge outboard of break
Λ_{T1}	sweep of surface trailing edge at root *
Λ_{T2}	sweep of surface trailing edge outboard of break
Λ_{L3}	sweep of wing fuel tank leading edge
Λ_{T3}	sweep of wing fuel tank trailing edge
ρ	density of fuel
θ	angle in degrees between plane of surface and XY plane of remote axes (positive for dihedral, negative for anhedral)
*	root chord can be defined as either theoretical or exposed

(see section II A)

SECTION I

INTRODUCTION

The purpose of this procedure is to determine the inertias of an aircraft at the preliminary design level so that the dynamic performance (flying qualities) can be examined. The purpose of analyzing dynamic performance at the preliminary design level is to insure adequate control surface sizing, develop control surface sizing rules for parametric design studies, and to determine the complexity of the flight control system necessary to adequately perform all required maneuvers. To do this, the procedure must be able to provide reasonably accurate estimates for different fuel and loading states.

The method described in this report has been incorporated into the ASD/XR Interactive Computer Design (ICAD) system. The flying qualities analysis portion of this system is described in Reference 1.

SECTION II

BACKGROUND

A. Basic Moment-of-Inertia Theory

Moment of inertia is the measure of resistance to angular acceleration, as mass is the measure of resistance to linear acceleration. Moment of inertia may be mathematically derived as follows.

If torque is expressed as the product of force and radius ($T = Fr$) and the following substitutions are made: $F = ma$ and $a = \alpha r$ then $T = mar$ or $T = mr^2 \alpha$ where a is the linear acceleration, α is the angular acceleration, and m is the mass.

The term mr^2 is defined as the moment of inertia (I) and this equation may be written $T = I\alpha$.

If a body of mass m is caused to rotate about a remote axis y the following relationship exists: $I_y = mr^2 = m(x^2 + z^2)$.

However, since mass m not only offers resistance to rotation about the y axis but also offers resistance to rotation about its own centroidal axis, the total inertia of m about y is $I_y = mr^2 + I_{oy}$ where I_{oy} is the inertia of m about its own centroidal axis.

When the full angular momentum equations are developed, there are nine I_o terms given in general by: $I_{o_{x_i x_j}} = \int x_i x_j \, dm$ where x_i, x_j, can be x, y, or z. Since the symmetric terms are equal, e.g., $I_{o_{xy}} = I_{o_{yx}}$, there are actually six independent moments of inertia.

For most aircraft problems, the vehicle is symmetric about the XZ plane. Although there are asymmetries in equipment locations which give rise to some non-zero values, it can be assumed for preliminary design purposes that I_{xy} and I_{yz} are zero. There are some configurations where this assumption obviously is not correct, such as skewed wings. For these aircraft the additional terms should be calculated. This method is limited to predicting the four remaining moments of inertia, I_x, I_y, I_z, and I_{xz}.

B. Method Description

There are three steps involved in obtaining these moments of inertia:

1) Allocate the total aircraft weight to six seperate groups:

a. wing group
b. horizontal tail group
c. vertical tail group
d. fuselage group
e. propulsion group
f. additional items

The level of detail of the weight breakdown given in Table 9 of Section II is adequate for determining the inertias. This allocation primarily involves distributing the subsystems throughout the aircraft without identifying the actual location of each wire, cable, line, or component. Since this is done on an "historical" or "accepted design practice" basis, adjustments may be needed for designs with unusual concepts or distributions.

2) Calculate the moment of inertia of each group about its own centroid and then transfer these inertias to a set of remote axes.

3) Locate the aircraft center of gravity, sum the inertias, and translate them back to the aircraft center of gravity to obtain the desired moments of inertia. The last two steps are described in detail in Section III.

C. Weight Allocation

Allocation of the total aircraft weight to the major groups is accomplished by a apckage of rules extracted from a structural weight estimation program (SWEEP) written by Rockwell International and from statistical data. The aircraft items are distributed as shown in Table 1.

TABLE 1

WEIGHT ALLOCATION

Item	Fraction in Fuselage	Fraction in Wing	Fraction Horiz Tail	Fraction Vert Tail	Fraction With Engine Package	Fraction With Items
Horiz tail structure	-	-	1.0	-	-	-
Vertical tail structure	-	-	-	1.0	-	-
Fuselage structure	1.0	-	-	-	-	-
Main gear		D	D	-	-	-
Nose gear	1.0	-	-	-	-	-
Engine Nacelle & Pylons	-	-	-	-	1.0	-
Other structure	1.0	-	-	-	-	-
Engine	-	-	-	-	1.0	-
Aux gearboxes	-	-	-	-	1.0	-
Exhaust system	-	-	-	-	1.0	-
Cooling & drains	-	-	-	-	1.0	-
Lubricating sys	-	-	-	-	1.0	-
Engine controls	-	-	-	-	1.0	-
Starting sys	-	-	-	-	1.0	-
Auxiliar power unit	1.0	-	-	-	-	-
Instruments	1.0	-	-	-	-	-
Hydraulics	0.67	-	-	-	0.33	-
Electrical	0.75	-	-	-	0.25	-
Electronics	1.0	-	-	-	-	-
Armament	1.0	-	-	-	-	-
Air conditioning	1.0	-	-	-	-	-
Photographic	1.0	-	-	-	-	-
Auxiliary gear	1.0	-	-	-	-	-
Other equipment	1.0	-	-	-	-	-
Crew	1.0	-	-	-	-	-
Oil	-	-	-	-	1.0	-
Liquid Nitrogen	1.0	-	-	-	-	-
Miscellaneous	1.0					
Payload	-	-	-	-	-	1.0
Guns	-	-	-	-	-	1.0

Table 1 Cont'd

Wing Pylons	-	-	-	-	-	1.0
Ext Wing tanks	-	-	-	-	-	1.0
fuselage pylons	-	-	-	-	-	1.0
Ext Fus tanks	-	-	-	-	-	1.0
Fuel	-	-	-	-	-	1.0

Note D = dependent on input location definition.

Items which need further discussion:

a. Fuel System - Distribute between the fuselage and wing group according to the fraction of fuel weight contained in each group.

b. Surface Controls: Summarized in table:

Table 2

SURFACE CONTROL WEIGHT ALLOCATION

	Fraction of Total Surface Control Weight				
Configuration Code W, H, V*	Wing	Horizontal Tail	Vertical Tail	Fuselage Cockpit	Fuselage Distributed
0, 0, 0	0.532	0.128	0.124	0.038	0.178
0, 0, 1	0.457	0.110	0.247	0.033	0.153
0, 1, 0	0.464	0.239	0.108	0.034	0.155
0, 1, 1	0.406	0.209	0.220	0.029	0.136
1, 0, 0	0.608	0.108	0.103	0.032	0.149
1, 1, 0	0.541	0.205	0.092	0.029	0.133
1, 0, 1	0.534	0.094	0.213	0.028	0.131
1, 1, 1	0.482	0.182	0.192	0.026	0.118

*W, wing — 0 = fixed — 1 = variable sweep
H, horizontal — 0 = elevator type — 1 = all moveable type
V, vertical tail — 0 = rudder type — 1 = all moveable type

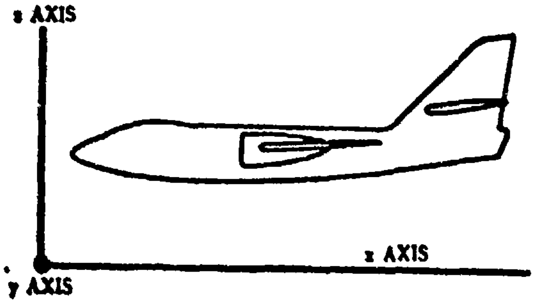
c. Trapped fuel - Distribute between fuselage and wing group according to fraction of fuel weight contained in each group.

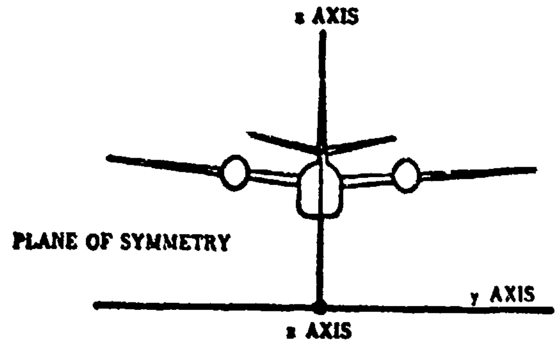
d. Air induction system - Add weight to fuselage group if engines are buried. Add to engine group if engines are podded.

e. Wing structure - If there is a wing carry-through structure, the weight should be added to the wing group - Otherwise only the exposed wing structure is in the wing group.

SECTION III
GROUP INERTIAS

Before the centroidal inertias (I_o's) of each group can be calculated and then translated to the remote set of axes, a certain amount of component location and geometry information must be known. Everything should be referenced in accordance with the chosen set of remote axes (see sketch). The exact position of these axes can be varied, but to make the calculations easiest, the Z axis should be located at the nose of the aircraft.





Below is a list of additional components whose X, Y, and Z locations have to be determined if they are to be included. All geometry information that is needed is included with the discussion of each major group.

1) Main and nose landing gear
2) Auxiliary power unit
3) Air conditioning
4) Auxiliary gear
5) Gun
6) Crew
7) Weapons
8) Fuel system (Centroid of fuselage fuel tank)
9) Avionics bays
10) Radar

11) Furnishings & Equipment (centroid of total group or centroids of major items)

12) Photographic equipment

13) Other equipment

14) Liquid nitrogen

15) Miscellaneous items

16) Fuselage store and tank pylons

17) Fuselage external stores and tanks

18) Wing store and tank pylons

19) Wing external stores and tanks

20) Internal Payload

Using this information we can now proceed to calculate the moments of inertia of the separate groups about their own centroidal axes and translate these to the remote axes.

A. <u>Surfaces</u>

Wing, horizontal tail, and vertical tail groups are all common surfaces. To define the shape of the surface the normal planview (one side of wing and horizontal since they're symmetrical) is used. The equations are derived for a trapezoidal panel with the thickness varying linearly from root to tip. If a surface has edge or thickness breaks, it should be separated into inner and outer trapezoidal panels with the inertia of each calculated separately. The thickness is assumed constant as you go from leading to trailing edge and equal to the maximum for that section.

$$I_{1x} = \frac{Wb^3}{V}\left\{\left[(t_r - t_t)\left(\frac{c}{4} + \frac{b\tan\Lambda_T}{5} - \frac{b\tan\Lambda_L}{5}\right)\right] + \left[t_r\left(\frac{c}{3} + \frac{b\tan\Lambda_T}{4} - \frac{b\tan\Lambda_L}{4}\right)\right]\right\} \quad (1)$$

$$I_{1y} = \frac{Wb}{V}\left\{\left[t_r\left(\frac{c^3}{3} + bc\tan\Lambda_T\left(\frac{c}{2} + \frac{b\tan\Lambda_T}{3}\right) + \frac{b^3}{12}\left(\tan^3\Lambda_T - \tan^3\Lambda_L\right)\right)\right] - \left[(t_r - t_t)\left(\frac{c^3}{6} + bc\tan\Lambda_T\left(\frac{c}{3} + \frac{b\tan\Lambda_T}{4}\right) + \frac{b^3}{15}\left(\tan^3\Lambda_T - \tan^3\Lambda_L\right)\right)\right]\right\} \quad (2)$$

$$I_{1z} = I_{1x} + I_{1y} \quad (3)$$

$$V = b\left\{t_r\left[c + \frac{b}{2}\left(\tan\Lambda_T - \tan\Lambda_L\right)\right] - (t_r - t_t)\left[\frac{c}{2} + \frac{b}{3}\left(\tan\Lambda_T - \tan\Lambda_L\right)\right]\right\} \quad (4)$$

The inertia equations for this volumetric shape are derived (see Appendix Section 1) with the assumption that all surfaces lie in planes parallel to the XY plane of the remote axes.

To take into account the fact that surfaces don't always lie in planes parallel to the XY plane but usually have some anhedral or dihedral:

$$\text{True } I_{1y} = (I_{1y}\cos\theta + I_{1z}\sin\theta) \quad (5)$$

$$\text{True } I_{1z} = (I_{1y}\sin\theta + I_{1z}\cos\theta) \quad (6)$$

I_{1x} is not affected by dihedral.

The product of inertia I_{1xz} is non-zero only if there is some dihedral or anhedral.

$$I_{1xz} = \frac{W}{V} t_r \sin\Theta \left[\frac{c_r^2 b^2}{4} + \frac{c_r b^3}{3} \tan\Lambda_T + \frac{b^4}{8} (\tan^2\Lambda_T - \tan^2\Lambda_L) \right] - \frac{W}{V} (t_r - t_t) \sin\Theta \left[\frac{c_r^2 b^2}{6} + \frac{c_r b^3}{4} \tan\Lambda_T + \frac{b^4}{10} (\tan^2\Lambda_T - \tan^2\Lambda_L) \right] \quad (7)$$

These equations calculate the inertias for the entire wing, horizontal, or vertical tail as long as the total group is used.

If a wing does not have a carry-through structure, the exposed wing should be used and the symbols should be defined accordingly. Otherwise, a theoretical wing should be used. All horizontal and vertical tails should be defined with exposed parameters. The equations shown here calculate the inertias for the entire inboard or out-board surfaces (left and right) as long as the total weight for each was used and the symbols were defined correctly.

Table 3 shows how to define the general symbols used in all equations dealing with surfaces, for each separate surface.

Table 3. Surface Symbols

Before I_1 can be translated back to obtain the surface I_o, the centroids of the surfaces must be known. All longitudinal surface centroids can be found by a method from DATCOM (See Appendix Section 2) as long as the parameters c, b, and Λ_L are again properly defined for each surface.

Table 3. Surface Symbols

GENERAL SYMBOL	DEFINED SYMBOLS			
	WING (NO BREAK)	INBOARD SURFACE	OUTBOARD SURFACE	HORIZ. & VERT.
Λ_L	Λ_{L1}	Λ_{L1}	Λ_{L2}	Λ_{L1}
Λ_T	Λ_{T1}	Λ_{T1}	Λ_{T2}	Λ_{T2}
b	b_1	b_2	b_3	b_1
c	c_r	c_r	c_2	c_1
t_r	t_r	t_r	t_b	t_r
t_t	t_t	t_b	t_t	t_t
W	W_w	W_i	W_o	W_h, W_v
$\bar{\bar{X}}$	XS1	XS2	XS3	XS1
$\bar{\bar{Y}}$	YS1*	YS2*	YS3	YS1**
$\bar{\bar{Z}}$	ZS3***	ZS4***	ZS5***	ZS3***

* If YS4 = 0 and $\Theta = 0$ Set these = 0.

** If YS4 = 0 and $\Theta = 0$ Set these = 0.

*** Not needed if $\Theta = 0$.

Before I_1 can be translated back to obtain the surface I_o, the centroids of the surfaces must be known. All longitudinal surface centroids can be found by a method from DATCOM (See Appendix, Section 2) as long as the parameters c, b, and Λ_L are again properly defined for each surface.

$$\text{XS1, XS2, XS3} = \frac{(-C_a^2 + C_b^2 + C_c C_b + C_c^2)}{3\,(C_b + C_c - C_a)} \sqrt{(K_o)} \qquad (8)$$

where C_a is the smallest of the following values: c, b $\tan\Lambda_L$, b $\tan\Lambda_L$ + c C_b is the intermediate value; C_c is the largest value

K_o = .703 for a wing

K_o = .771 for horizontal or vertical tail

All spanwise surface centroids are assumed to be at the spanwise surface center of volume. These centroids are needed only for exposed wing surfaces, outboard surfaces and for vertical tails since these all have inertias that need to be translated (see Appendix, Section 2).

$$YS1,\ YS2,\ YS3 = \frac{b^2}{V}\left[\left(t_r\left(\frac{c}{2} + \frac{b}{3}\right.\right.\right.(\tan\Lambda_T - \tan\Lambda_L)) - (t_r - t_t)\left(\frac{c}{3} + \frac{b}{4}\right.(\tan\Lambda_T - \tan\Lambda_L))\Big] \quad (9)$$

All vertical center of gravity distances for surfaces with $\Theta = 0$ are negligible because of the small thickness of surfaces compared with their length and span. For surfaces with either anhedral or dihedral the vertical center of gravity ($\bar{\bar{Z}}$) no longer lies in the XY plane of the root or break chord and can be calculated by:

$$ZS3 = YS1 \sin\theta \quad (10)$$

$$ZS4 = YS2 \sin\theta \quad (11)$$

$$ZS5 = YS3 \sin\theta \quad (12)$$

With the surface centroid location known, I_1 can be translated to the centroid and then to the remote axes. For wings (no break), horizontal, and vertical surfaces, the I values are calculated by:

$$I_x = I_{1x} - W(YS1)^2 - W(ZS3)^2 + W(YS1 + YS4)^2 + W(ZS3 + ZS1)^2 \quad (13)$$

$$I_y = I_{1y} - W(XS1)^2 - W(ZS3)^2 + W(XS1 + XS4)^2 + W(ZS3 + ZS1)^2 \quad (14)$$

$$I_z = I_{1z} - W(XS1^2 + YS1^2) + W(XS1 + XS4)^2 + W(YS1 + YS4)^2 \quad (15)$$

$$I_{xz} = I_{1xz} - W(XS1)(ZS3) + W(XS1 + XS4)(ZS3 + ZS1) \quad (16)$$

For inboard surfaces:

$$I_x = I_{1x} - W(YS2^2 + ZS4^2) + W(ZS4 + ZS1)^2 + W(YS2 + YS4)^2 \quad (17)$$

$$I_y = I_{1y} - W(XS2^2 + ZS4^2) + W(XS2 + XS4)^2 + W(ZS1 + ZS4)^2 \quad (18)$$

$$I_z = I_{1z} - W(XS2^2 + YS2^2) + W(XS2 + XS4)^2 + W(YS2 + YS4)^2 \quad (19)$$

$$I_{xz} = I_{IXZ} - W\,(XS2)\,(ZS4) + W\,(XS2 + XS4)\,(ZS4 + ZS1) \tag{20}$$

For outboard surfaces:

$$I_x = I_{ox} + W\,(YS3 + b_2\cos\theta + YS4)^2 + W\,(ZS1 + b_2\sin\theta + ZS5)^2 \tag{21}$$

$$I_y = I_{oy} = W\,(XS5 + XS3)^2 + W(ZS1 + b_2\sin\theta + ZS5)^2 \tag{22}$$

$$I_z = I_{oz} + W\,(YS3 + b_2\cos\Theta + YS4)^2 + W\,(XS5 + XS3)^2 \tag{23}$$

$$I_{xz} = I_{oxz} - W\,(XS3)\,(ZS5) + W\,(XS3 + XS4)\,(ZS5 + ZS1) \tag{24}$$

B. FUSELAGE

The fuselage data needed for inertia calculations is:

l_n, l_c, l_t, R, Z_b, W_s, W_{pc}, XS4, W_{vo}, R_v, l_v, W_{pc}, W_{pnc}

Fuselage weight is divided into four areas:

1. Structure
2. Distributed contents
3. Volumes of mass
4. Point masses

1. Structure. Fuselage structural weight includes wing carry-through structure (if it was added to the fuselage group) and air induction system weight (if you have buried engine installations). This weight is assumed to be distributed between a conical nose shell, open-ended right-cylindrical shell, and a conical tail shell. For buried engine installations, the conical tail shell is neglected. Fuselage structure is distributed to each geometric shape in proportion to the surface area. ($\frac{\text{weight}}{\text{area}}$ = constant). Air induction

system weight should be added to the open ended right-circular shell. Moments of inertia of fuselage sturcture about the remote axes (see Appendix, Section 3) are given by:

$$I_x = \frac{R^2}{2}(W_n + 2\ W_c + W_t) + W_s\ (Z_b)^2 \tag{25}$$

$$I_y = \frac{R^2}{4}(W_n + 2\ W_c + W_t) + l_n^{\ 2}\ (\frac{W_n}{2} + W_c + W_t) + l_c^{\ 2}\ (\frac{W_c}{3} + W_t) + \frac{l_t^{\ 2} W_t}{6}$$

$$+ l_c l_n\ (W_c + 2W_t) + 2/3\ l_t l_c W_t + 2/3\ l_t l_n W_t + W_s\ (Z_b)^2 \tag{26}$$

$$I_z = I_y - W_s\ (Z_b)^2 \tag{27}$$

$$I_{xz} = W_n\ (3/4\ l_n\ Z_b) + W_c Z_b\ (ln + \frac{lc}{2}) + W_t Z_b\ (l_n + l_c + \frac{lt}{4}) \tag{28}$$

2) <u>Distributed Contents</u>. This consists of four main items: electrical system, instruments and navigation, hydraulics, and surface controls. They are assumed to be randomly spread throughout the fuselage from the cockpit to the leading edge of the horizontal tail in the shape of an open ended right-cylindrical shell. Moments of inertia of distributed contents about the remote axes (see Appendix, Section 4) are given by:

$$I_x = W_{dc} R^2 + W_{dc}\ (Z_b)^2 \tag{29}$$

$$I_y = \frac{W_{dc}}{2}\ (R^2 + 1/6\ (XS4\text{-}CREW_{cg}))^2) +$$

$$W_{dc}\ (\frac{XS4\text{-}CREW_{cg}}{2} + CREW_{cg})^2 + W_{dc}\ Z_b^{\ 2} \tag{30}$$

$$I_z = I_y - W_{dc}\ (Z_b)^2 \tag{31}$$

$$I_{xz} = \frac{W_{dc}}{2}\ (XS4 + CREW_{cg})\ (Z_b) \tag{32}$$

Here W_{dc} is defined as the weight of surface controls allocated to the fuselage + weight of electrical system allocated to the fuselage + weight of hydraulic system allocated to the fuselage + 30% of weight of instruments and navigation allocated to the fuselage.

3) Volumes of Mass consist of items such as the fuel system in the fuselage, the avionics bay, and furnishings. It is left to the user to decide whether to use these as volumes or point masses because of the variability of the items. Either a cylindrical shell or a solid rectangular shape can be used. Moments of inertia of these volumes about the remote axes (see Appendix, Section 5) are given by:

Cylindrical shell - (33)

$$I_x = W_{vo}R_v^2 + W_{vo}Z^2$$

$$I_y = \frac{W_{vo}}{2} (R_v^2 + \frac{l_v^2}{6} + W_{vo} (X^2 + Z^2) \quad (34)$$

$$I_z = \frac{W_{vo}}{2} (R_v^2 + \frac{l_v^2}{6}) + W_{vo}X^2 \quad (35)$$

$$I_{xz} = W_{vo}XZ \quad (36)$$

Rectangular solid -

$$I_x = \frac{W_{vd}}{12} (2R_v^2 + 2R_v^2) + W_{vo}Z^2 \quad (37)$$

$$I_y = \frac{W_{vo}}{12} (l_v^2 + 2R_v^2) + W_{vo} (X^2 + Z^2) \quad (38)$$

$$I_z = \frac{W_{vo}}{12} (l_v^2 + 2R_v^2) + W_{vo}X^2 \quad (39)$$

$$I_{xz} = W_{vo}XZ \quad (40)$$

4. Point Masses. Each point mass is generally considered separately for calculating inertias. Aggregate small items, such as troop provisions in cargo aircraft are handled differently. For roll (I_x) inertia the point mass total weight for aggregate items is distributed between a solid cone and a solid right circular cylinder. All aggregate point masses located in the nose or tail cone of the fuselage are put in the solid cone and all point masses in the center section are put in the solid right circular cylinder. For I_y and I_z they are lumped at some average location. The moment of inertia of point masses about the remote axes (see Appendix, Section 6) are given by:

$$I_x = \Sigma W_p \, (Y^2 + Z^2)$$

or

$$I_x = \frac{Wpc}{2} R^2 + \frac{3}{10} Wpnc \, R^2 + (Wpc + Wpnc) \, (Z_b)^2 \tag{41}$$

$$I_y = \Sigma W_p \, (X^2 + Z^2) \tag{42}$$

$$I_z = \Sigma Wp \, (X^2 + Y^2) \tag{43}$$

$$I_{xz} = \Sigma W_p XZ \tag{44}$$

Items usually considered as point masses:

Main and nose landing gear

Auxiliary power unit

Air Conditioning

Auxiliary gear

Gun

Crew

Armament

Surface controls assigned to cockpit

Radar

Photographic

70% of instruments and navigation weight (locate at cockpit)

Other equipment

Liquid nitrogen

Miscellaneous items

C. Propulsion

The propulsion data needed for inertia calculations is: W_e, R_e, l_e, XP, YP, ZP, I_o of engines.* The total group weight is divided by the number of engines; this is the weight of each engine and accessories. If the I_o's of the engines are not known, they can be approximated by using a solid cylinder (see Appendix, section 9). The moments of inertia of each engine about the remote axes are given by:

I_o approximated:

$$I_x = \frac{W_e R_e^2}{2} + W_e (YP^2 + ZP^2) \quad (45)$$

$$I_y = \frac{W_e}{12} (3R_e^2 + l_e^2) + W_e (XP^2 + ZP^2) \quad (46)$$

$$I_z = \frac{W_e}{12} (3R_e^2 + l_e^2) + W_e (XP^2 + YP^2) \quad (47)$$

$$I_{xz} = W_e (XP)(ZP) \quad (48)$$

I_o input:

$$I_x = I_{ox} + W_e (YP^2 + ZP^2) \quad (49)$$

$$I_y = I_{oy} + W_e (XP^2 + ZP^2) \quad (50)$$

$$I_z = I_{oz} + W_e (XP^2 + YP^2) \quad (51)$$

$$I_{xz} = I_{oxz} + W_e (XP)(ZP) \quad (52)$$

---

*R_e and l_e are not needed if inertias are given.

D. Internal Fuel

1. Wing fuel tanks

Internal wing fuel is defined in the same manner as surfaces because of the wing fuel tank shape being similar to a surface. We assume the wing tank is full of fuel and has a constant density. The I_1 equations for surfaces (see Appendix, Section 2) can now be used as long as we substitute ρ (density of fuel) for $\frac{W}{V}$.

$$I_{4X} = 2b^3\rho \left[-(t_r-t_t) \left(\frac{c}{4} + \frac{b}{5} (\tan \Lambda_T - \tan \Lambda_L)\right) + t_r \left(\frac{c}{3} + \frac{b}{4} (\tan \Lambda_T - \tan \Lambda_L)\right) \right] \quad (53)$$

$$I_{4y} = 2b\rho \left\{ t_r \left[\frac{c^3}{3} + bc\tan \Lambda_T \left(\frac{c}{2} + \frac{b\tan \Lambda_T}{3}\right) + \frac{b^3}{12} (\tan^3\Lambda_T - \tan^3 \Lambda_L)\right] - [(t_r-t_t) \left(\frac{c^3}{6} + bc\tan \Lambda_T \left(\frac{c}{3} + \frac{b\tan \Lambda_T}{4}\right) + \frac{b^3}{15} (\tan^3\Lambda_T - \tan^3\Lambda_L)\right)] \right\} \quad (54)$$

$$I_{4z} = I_{4x} + I_{4y} \quad (55)$$

Again realizing wing fuel tanks may be at some dihedral angle:

$$\text{TRUE } I_{4y} = (I_{4y} \cos \Theta + I_{4z} \sin \Theta) \quad (56)$$

$$\text{TRUE } I_{4z} = (I_{4y} \sin \Theta + I_{4z} \cos \Theta) \quad (57)$$

I_{4x} is not affected by dihedral. If there is dihedral, the product of inertia of the fuel is given by:

$$I_{4xz} = 2\rho \sin \Theta \left\{ \left[t_r \left(\frac{c^2b^2}{12} + \frac{cb^3}{12} \tan \Lambda_T + \frac{b^4}{40} (\tan^2 \Lambda_T - \tan^2 \Lambda_L)\right) \right] + t_t \left[\frac{c^2b^2}{6} + \frac{cb^3}{4} \tan \Lambda_T + \frac{b^4}{10} (\tan^2 \Lambda_T - \tan^2 \Lambda_L) \right] \right\} \quad (58)$$

These equations calculate the total I_4 for total wing fuel (right and left wing fuel tanks).

TABLE 4

WING INTERNAL FUEL TANK SYMBOLS

General Symbol	Defined Symbol
Λ_L	Λ_{L3}
Λ_T	Λ_{T3}
b	b_4
c	c_3
t_r	t_f
t_t	t_{f_o}
ρ	JP4 = .02814 $\frac{lb}{in^3}$
$\bar{\bar{X}}$	XF1
$\bar{\bar{Y}}$	$\dot{Y}$F1
$\bar{\bar{Z}}$	ZF2

Again, as in the case for surfaces, the centroid of the fuel tank must be calculated. The centroid is assumed to be located at the center of volume of the tank. (See Appendix, Section 7.)

$$XF1 = \frac{b}{v} \left\{ \left[(t_r \left(\frac{c^2}{2} + \frac{bc\tan\Lambda_T}{2} + \frac{b^2}{6} (\tan^2 \Lambda_T - \tan^2 \Lambda_L)\right)\right] - \left[(t_r - t_t) \left(\frac{c^2}{4} + \frac{cb\tan\Lambda_T}{3} + \frac{b^2}{8} (\tan^2 \Lambda_T - \tan^2 \Lambda_L) \right)\right] \right\} \quad (59)$$

$$YF1 = \frac{b^2}{v} \left[t_r \left(\frac{c}{2} + \frac{b}{3} (\tan\Lambda_T - \tan\Lambda_L)\right)\right] - \left[(t_r - t_t) \left(\frac{c}{3} + \frac{b}{4} (\tan\Lambda_T - \tan\Lambda_L)\right)\right] \quad (60)$$

The vertical fuel tank centroid is zero unless the wing has anhedral or dihedral, in which case:

$$ZF2 = YF1 \sin \Theta$$

The fuel tank I_4 can be translated to obtain I_o and then I by:

$$I_x = I_{4x} - W_{fw} \dot{Y}F1^2 - W_{fw} (ZF2)^2 + W_{fw} (\dot{Y}F1 + YF2)^2 + W_{fw} (ZF^2 + ZF)^2 \quad (61)$$

$$I_y = I_{4y} - W_{fw} XF1^2 - W_{fw} ZF2^2 + W_{fw} (XF1 + XF2)^2 + W_{fw} (ZF2 + ZF)^2 \quad (62)$$

$$I_z = I_{4z} - W_{fw} (XF1^2 + \dot{Y}F1^2) + W_{fw} (XF1 + XF2)^2 + W_{fw} (\dot{Y}F1 + YF2)^2 \quad (63)$$

$$I_{xz} = I_{4xz} - W_{fw} (XF1)(ZF2) + W_{fw} (XF1 + XF2)(ZF2 + ZF) \quad (64)$$

If there is more than one internal wing fuel tank, this total procedure can be used for each subsequent tank in the same manner.

2. Fuselage Fuel Tanks

Fuselage internal fuel is assumed to be in the shape of a solid right cylinder. These inertia calculations are to be used in aircraft flying qualities studies; only short period rolling motions will be examined, and the fuel will not attain any appreciable rotational motion during these maneuvers. The rolling inertia of the fuel about its own axis is therefore assumed to be zero. The moments of inertia of fuselage internal fuel about the remote axes (see Appendix, Section 8) is given by:

$$I_{ox} = 0 \quad (65)$$

$$I_{oy} = \frac{W_{ff}}{12}\left(\frac{W_{ff}}{\Pi \rho l_f} + l_f^2\right) + W_{ff} (X^2 + Z^2) \quad (66)$$

$$I_{oz} = \frac{W_{ff}}{12}\left(\frac{W_{ff}}{\Pi \rho l_f} + l_f^2\right) + W_{ff} X^2 \quad (67)$$

$$I_{oxz} = W_{ff} XZ \quad (68)$$

E. Payload

1. Transport Payload

Payload inertia is estimated by using a solid rectangular mass or series of masses as were the volumes of mass in the fuselage. Moments of inertia for payload about the remote axes (see Appendix 5, Section 9) are given by:

$$I_x = \frac{W}{12} (d_p^2 + d_p^2) + WZ^2 \quad (69)$$

$$I_y = \frac{W}{12} (l_p^{\ 2} + d_p^{\ 2}) + W (X^2 + Z^2) \quad (70)$$

$$I_z = \frac{W}{12} (l_p^{\ 2} + d_p^{\ 2}) + WX^2 \quad (71)$$

$$I_{xz} = WXZ \quad (72)$$

2. Internal Weapons

It is assumed that the inertia and locations of these items are given.

F. Additional Items

1. External Stores and Tanks

Wing and fuselage store and tank pylons are to be used as point masses to calculate I_x, I_y, and I_z. External wing and fuselage tanks and stores can be approximated by shells and solid right cylinders (s e Appendix, Section 9) depending on whether the tanks are full or empty.

TANKS:

$$I_z = \frac{W_{st}}{2} \left(SR^2 + \frac{SL^2}{6}\right) + W_{st} (Y^2 + X^2) \quad (73)$$

$$I_y = \frac{W_{st}}{2} \left(SR^2 + \frac{SL^2}{6}\right) + W_{st} (X^2 + Z^2) \quad (74)$$

$$I_x = W_{st}\, SR^2 + W_{st} (Y^2 + Z^2) \quad (75)$$

$$I_{xz} = W_{st} XZ \quad (76)$$

STORES:

$$I_x = \frac{W_{st}}{2} SR^2 + W_{st} (Y^2 + Z^2) \quad (77)$$

$$I_y = \frac{W_{st}}{12} (3\ SR^2 + SL^2) + W_{st} (X^2 + Z^2) \quad (78)$$

$$I_z = \frac{W_{st}}{12} (3\ SR^2 + SL^2) + W_{st} (X^2 + Y^2) \quad (79)$$

$$I_{xz} = W_{st}\ XZ \quad (80)$$

G. Total Aircraft Inertias

The total inertia about the remote axes from all groups are now summed to achieved a complete inertia for the total aircraft. For this to be translated back to the center of gravity of the total vehicle, the center of gravity location must first be calculated. By definition:

$$\bar{X} = \frac{\Sigma WX}{\Sigma W} \quad \bar{Z} = \frac{\Sigma WZ}{\Sigma W} \quad \bar{Y} = 0 \qquad (81)$$

where X and Z are distances to the item or group centroid. All item and group weights and distances to the remote axes are already known, except for the fuselage structure longitudinal distances. These are given by:

$$WX \text{ nose cone} = W_n \,(2/3\ l_n) \qquad (82)$$

$$WX \text{ center} = W_c \,(l_n + 1/2\ l_c) \qquad (83)$$

$$WX \text{ tail cone} = W_t \,(l_n + l_c + 1/3\ l_t) \qquad (84)$$

W should equal the total aircraft weight. $\bar{Y}$ is zero because of the already assumed symmetry of the aircraft. The translation of the total inertias to the aircraft center of gravity is then:

$$I_{cgx} = I_x - W\bar{Z}^2 \qquad (85)$$

$$I_{cgy} = I_y - W(\bar{Z}^2 + \bar{X}^2) \qquad (86)$$

$$I_{cgz} = I_z - W\bar{X}^2 \qquad (87)$$

$$I_{cgxz} = I_{xz} - W\overline{XZ} \qquad (88)$$

Results from the use of this method on various types of aircraft is given in Table 5. Data on these aircraft were obtained from References 3 - 6.

(Moments of Inertia X10 - $^{-6}$)
(lb - in^2)

Table 5. Summary Comparison

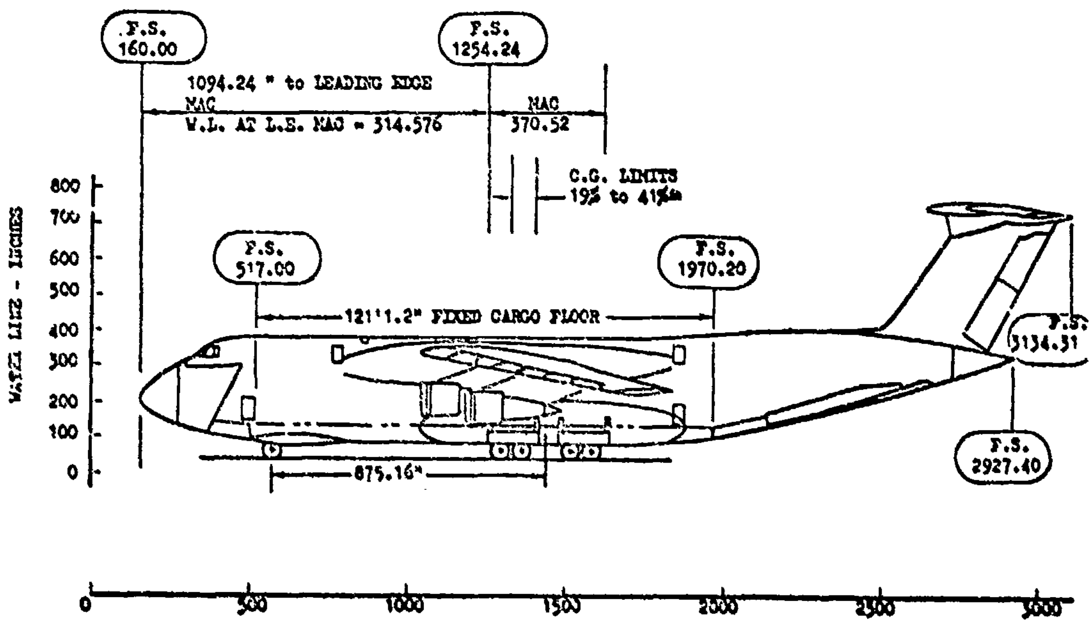
Configuration	Roll		Pitch		Yaw	
	Actual	Calc.	Actual	Calc.	Actual	Calc.
F-15A						
Operating Weight Empty	97.8	129.0	747.2	762.2	822.3	835.6
Air Superiority Takeoff Weight	166.5	190.8	824.0	829.2	946.0	951.8
C-5A						
Operating Weight Empty	57909	54246	101486	98853	146944	140694
Basic Flight Design Max Fuel	170867	158941	124744	116564	279748	266755
A-10						
Weight Empty	168	203	413	356	580	543
Ferry Mission Gross Weight	293	279	604	608	817	891
B-52G						
Weight Empty	26011	23270	22551	19380	48562	42216
Design Gross Weight	69163	64142	39520	37350	108683	92696
Average Error (%)		11.4		5.7		6.8

Average percent error of actual versus calculated values is 8.6%

(Moments of Inertia X10^{-6})
(lb - in^2)

III. Sample Problem: C-5A

Moments of inertia are first calculated for operating weight empty and then for basic flight design weight with maximum fuel. All units are pounds and inches. Basic geometry and weight data are given in Figures 1 and 2 and Tables 6 and 7. This data was taken from Reference 3.



FUSELAGE STATION - INCHES

FIGURE 1

PROFILE SKETCH

Figure 2. C-5A Three-View

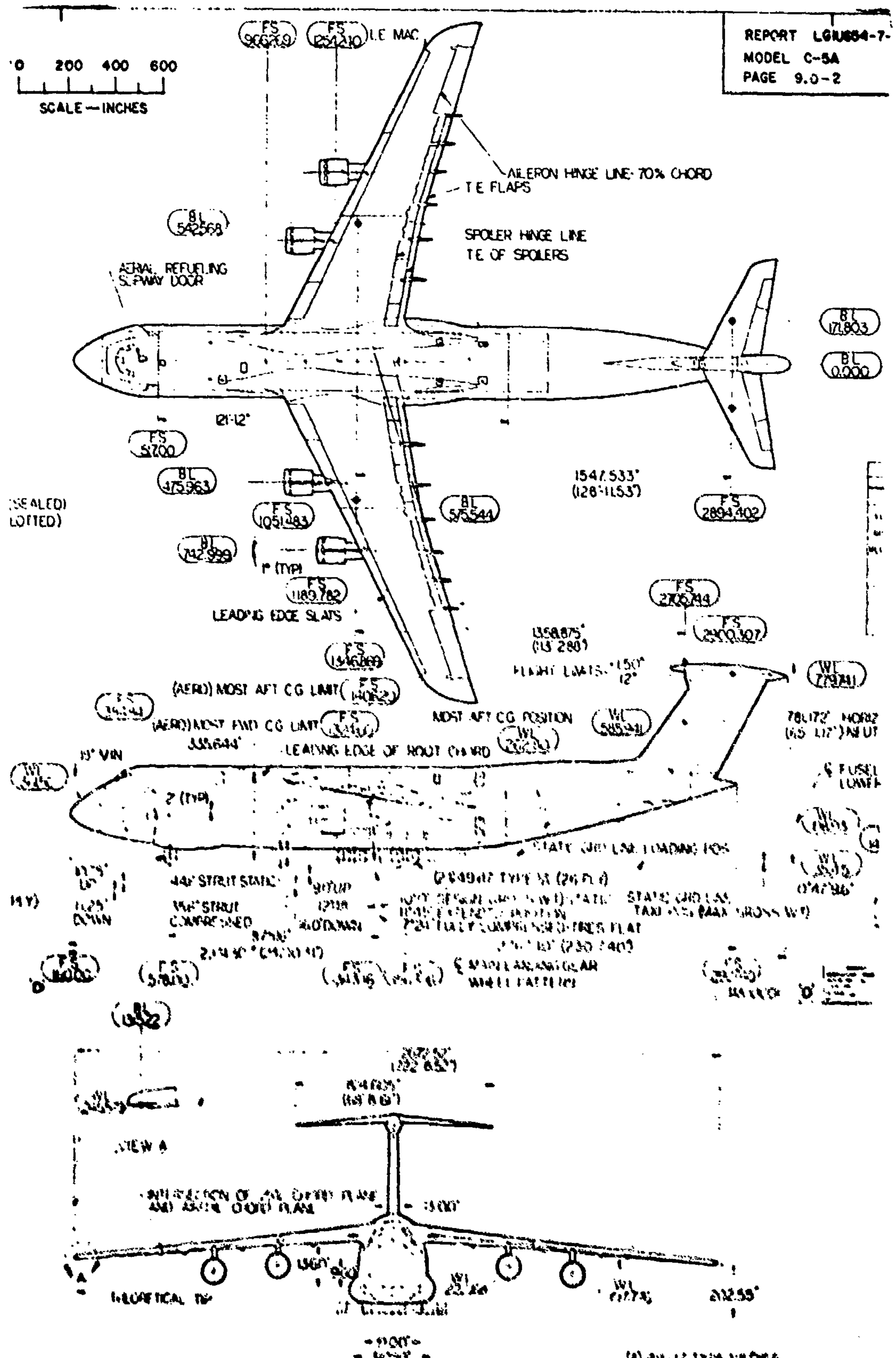


Table 6. C-5A Weight Statement

WING GROUP						82044.8
BASIC STRUCTURE - CENTER SECTION					8621.2	
- INTERMEDIATE PANEL					30638.4	
- OUTER PANEL					13160.0	
SECONDARY STRUCTURE					3729.6	
AILERONS (INCLUDING 916. LB BALANCE WEIGHT)					2327.0	
FLAPS - TRAILING EDGE					10736.8	
SLATS - LEADING EDGE					4955.0	
SPOILERS					2474.8	
TAIL GROUP						12395.9
STABILIZER - BASIC STRUCTURE					4711.9	
- SECONDARY STRUCTURE					829.9	
- ELEVATOR (INCL. 241.0 LB BALANCE WEIGHT)					1251.2	
FIN - BASIC STRUCTURE					4577.5	
- SECONDARY STRUCTURE					346.7	
- RUDDER	.0				678.7	
BODY GROUP (INCLUDING MANUFACTURING VARIATION OF -27.2)						116047.5
FUSELAGE - BASIC STRUCTURE					61394.8	
GEAR PODS (INCLUDING MLG AND NLG DOORS)					3661.2	
CARGO FLOOR (INCLUDING TIEDOWN RINGS + RECEPTACLES)					17074.0	
RAILS AND ROLLERS (RAIL SYSTEM)					2590.2	
FUSELAGE - SECONDARY STRUCTURE					6308.2	
FORWARD LOADING DOORS (VISOR DOOR)					5322.5	
FORWARD RAMP (INCLUDING RAMP EXTENSION + TOE)					4375.6	
AFT LOADING DOORS (SIDE + CENTER)					2236.1	
AFT RAMP (INCLUDING PRESSURE DOOR + TOE)					6483.1	
ALIGHTING GEAR GROUP						38087.9
		ROLLING				
	LOCATION	STOCK	STRUCT	CONTROLS	TOTALS	
	MAIN	9130.8	18127.9	6412.2	33680.9	
	NOSE	1066.[illegible]	2073.2	1361.6	4407.0	
SURFACE CONTROLS GROUP						7134.6
COCKPIT CONTROLS					116.0	
AUTOMATIC PILOT					151.3	
SYSTEM CONTROLS					6866.7	
ENGINE SECTION OR NACELLE GROUP						9586.4
INBOARD - POD STRUCTURE					1169.0	
- PYLON					2969.4	
OUTBOARD - POD STRUCTURE					1166.2	
- PYLON					3210.6	
COWLS, PANELS, AND MISCELLANEOUS					1071.2	
TOTAL (TO BE BROUGHT FORWARD)						265297.1

Table 6-continued

PROPULSION GROUP						36261.7
ENGINE INSTALLATION					29017.2	
AFTERBURNERS (IF FURNISHED SEPARATELY)						
ACCESSORY GEAR BOXES AND DRIVES (CSD)					.0	
SUPERCHARGERS (FOR TURBO TYPES)					.0	
AIR INDUCTION SYSTEM					.0	
EXHAUST SYSTEM					276.4	
COOLING SYSTEM					174.0	
LUBRICATION SYSTEM					10.4	
TANKS						
COOLING SYSTEM						
DUCTS, PLUMBING, ETC.						
FUEL SYSTEM					2457.8	
TANKS - WING				580.2		
- BODY						
PLUMBING, ETC.				1877.6		
WATER INJECTION SYSTEM					.0	
ENGINE CONTROLS					178.2	
STARTING SYSTEM					137.5	
THRUST REVERSERS					3943.4	
AUXILIARY POWER PLANT GROUP						332.5
INSTRUMENTS AND NAVIGATIONAL EQUIPMENT GROUP						936.2
HYDRAULIC GROUP						3976.5
ELECTRICAL GROUP						5450.8
ELECTRONICS GROUP						3893.5
EQUIPMENT					2571.5	
INSTALLATION					1318.1	
ARMAMENT GROUP						
FURNISHINGS AND EQUIPMENT GROUP						6835.5
ACCOMMODATIONS FOR PERSONNEL					2150.4	
MISCELLANEOUS EQUIPMENT					982.4	
FURNISHINGS					3232.9	
EMERGENCY EQUIPMENT					469.8	
AIR CONDITIONING AND ANTI-ICING GROUP						3640.0
AIR CONDITIONING					3411.1	
ANTI-ICING					228.9	
AUXILIARY GEAR GROUP						38.9
HANDLING GEAR					38.8	
ARRESTING GEAR					.0	
CATAPULTING GEAR					.1	
MANUFACTURING VARIATION (INCLUDED IN BODY GROUP)						
TOTAL FROM PRECEDING PAGE						265297.1
WEIGHT EMPTY						325265.0

Table 6-continued

LOAD CONDITION			BASIC WEIGHT	OPERAT. WEIGHT	O.W. + ATC	FLT DES MAX CAR
CREW (NO. 6) (INCLUDING BAGGAGE)			0	1,290	1,290	1,290
FUEL	TYPE	GAL.				
UNUSABLE	JP-4	87	562	562	562	562
INTERNAL	JP-4	0	0	0	0	
INTERNAL	JP-4	28,322				184,096
INTERNAL	JP-4					
INTERNAL	JP-4					
OIL						
TRAPPED			58	58	58	58
ENGINE			0	206	206	206
BAGGAGE - CREW (INCLUDED IN CREW WEIGHT)						
TIEDOWN DEVICES			1,750	1,750	1,750	1,750
PAYLOAD			(7,581)	(0)	(7,581)	(214,45[illegible]
DELIVERABLE CARGO						
AUXILIARY CREW						
BAGGAGE						
PASSENGERS/TROOPS						
PALLETS (300 LB EACH)						6,600
PALLET NETS (54 LB EACH)						1,188
PALLET LOAD						193,405
VEHICLES						
CHAINS (IN EXCESS OF 1750 LB)						
WRM KIT						680
REMOVABLE TROOP PROVISIONS (DRY)			7,581		7,581	7,581
AFT TOE RAMPS						
EQUIPMENT						
PYROTECHNICS						
PHOTOGRAPHIC						
LIFE RAFTS - CREW (INCL. EQUIPMENT)			200	200	200	200
OXYGEN - CREW (25 LITERS)			63	63	63	63
FOOD - CREW			17	17	17	17
WATER - CREW			43	43	43	43
USEFUL LOAD			10,274	4,189	11,770	402,737
WEIGHT EMPTY			325,263	325,263	325,263	[illegible]
GROSS WEIGHT			[illegible]	[illegible]	[illegible]	[illegible]

Table 6-continued

LOAD CONDITION	TYPE	GAL.	FLT DES MAX FUEL	MAX DES MAX CAR	MAX DES MAX FUEL	FERRY MISSION
CREW (NO. 6) (INCLUDING BAGGAGE)			1,290	1,290	1,290	1,290
FUEL						
UNUSABLE	JP-4	87	562	562	562	562
INTERNAL	JP-4	49,000	318,500		318,500	318,500
INTERNAL	JP-4	27,707		180,096		
INTERNAL	JP-4					
INTERNAL	JP-4					
OIL						
TRAPPED			58	58	58	58
ENGINE			206	206	206	206
BAGGAGE - CREW (INCLUDED IN CREW WEIGHT)						
TIEDOWN DEVICES			1,750	1,750	1,750	1,750
PAYLOAD			(80,048)	(259,452)	(121,048)	(7,581)
DELIVERABLE CARGO						
AUXILIARY CREW						
BAGGAGE						
PASSENGERS/TROOPS						
PALLETS (300 LB EACH)			2,400	7,800	3,600	
PALLET NETS (54 LB-EACH)			432	1,404	648	
PALLET LOAD			68,955	241,987	108,539	
VEHICLES						
CHAINS (IN EXCESS OF 1750 LB)						
WRM KIT			680	680	680	
REMOVABLE TROOP PROVISIONS (DRY)			7,581	7,581	7,581	7,581
AFT TOE RAMPS						
EQUIPMENT						
PYROTECHNICS						
PHOTOGRAPHIC						
LIFE RAFTS - CREW (INCL. EQUIPMENT)			200	200	200	200
OXYGEN - CREW (25 LITERS)			63	63	63	63
FOOD - CREW			17	17	17	17
WATER - CREW			43	43	43	43
USEFUL LOAD			402,737	443,737	443,737	330,270
WEIGHT EMPTY			325,263	325,263	325,263	325,263
GROSS WEIGHT			[illegible]	[illegible]	[illegible]	[illegible]

Table 7. C-5A Geometric Data

No.	Item	Loadable Fwd Ramp	Loadable Aft Ramp	Loadable Cargo Floor	Cargo Compt	Fuse.	Nacelles Inbd	Outbd
1	LENGTH - OVERALL (FT) 247.8[illegible] HEIGHT - OVERALL (FT) 65.1							
2		LOADABLE	LOADABLE	LOADABLE				
3		FWD	AFT	CARGO	CARGO		NACELLES	7SIDE
4		RAMP	RAMP	FLOOR	COMPT	FUSE.	INBD	OUTBD
5	LENGTH - MAX (FT)	9.8	13.4	121.1	144.8	230.6	26.0	26.0
6	DEPTH - MAX (FT)				13.5	28.9	8.5	8.5
7	WIDTH - MAX (FT)	19.0	19.0	19.0	19.0	23.9	8.5	8.5
8	WETTED AREA (SQ FT)					16645.7	469.[illegible]	469.[illegible]
9	FLAT SURF (SQ FT)	196.1	254.6	2300.9	2741.6			
10	FUSE VOLUME (CU FT)			PRESSURIZED		65631.7		
11				TOTAL		96610.1		
12								
13						WING	H. TAIL	V. TAIL
14	GROSS AREA (SQ. FT)					6200.0	965.6	961.1
15	EXPOSED PLANFORM AREA (SQ FT)					5377.5	873.9	862.7
16	SPAN (FT)					222.7	68.7	34.5
17	EXPOSED WETTED AREA (SQ FT)					10[illegible]9.2	1843.6	1759.[illegible]
18	STRUCTURAL BOX WIDTH AT BODY JUNCTION (IN)					240.4	117.4	196.5
19	SWEEPBACK AT 25 % CHORD (DEGREES) - C.L. TO BREAK					24.0	24.6	34.9
20	- BREAK TO TIP					25.0	24.6	34.9
21	THEOR. ROOT CHORD (IN.) - INBD L.E. AND T.E. EXTD.					545.5	250.2	371.2
22	- MAX THICK (INT EXTD TO CL)					71.5	26.5	48.3
23	CHORD AT PLANFORM BREAK (IN.) - LENGTH					336.4		
24	- MAX THICKNESS					37.5		
25	THEORETICAL TIP CHORD (IN.) - LENGTH					194.0	92.6	237.0
26	- MAX THICKNESS					19.6	9.7	35.6
27	MEAN AERODYNAMIC CHORD (IN.) (PROJECTED)					370.5	193.4	335.5
28	L.E. OF M.A.C. (F.S.)					1254.2	2848.5	2621.5
29	TAIL LENGTH - 25 % MAC WING TO 25 % MAC H. TAIL (FT)							129.0
30	- 25 % MAC WING TO 25 % MAC V. TAIL (FT)							113.2
31								
32	AREAS (SQ FT)		LE SLATS	648.6 SPOILERS		410.7 ELEVATRS		256.7
33			TE FLAPS	991.7 AILERONS		252.8 RUDDERS		226.7
34								
35	ALIGHTING GEAR						MAIN	NOSE
36	LENGTH - OLEO EXTENDED - C.L. AXLE TO C.L. TRUNNION (IN)						81.8	55.1
37	OLEO TRAVEL - FULL EXTENDED TO FULL COLLAPSED (IN)						25.0	22.5
38	NUMBER OF WHEELS						24	4
39	NUMBER OF TIRES						24	4
40	NUMBER OF STRUTS						4	1
41	HYDRAULIC SYSTEM CAPACITY (GALLONS)				282.0			
42	FUEL AND LUBE SYSTEM						NO. TKS.	GAL.UNPT
43	FUEL - INTERNAL						12	49000
44	OIL						4	43
45	HORIZONTAL REFERENCE DATUM TO F.S. ZERO (IN)				0			
46	VERTICAL REFERENCE DATUM TO W.L. ZERO (IN)				0			
47	STRUCTURAL DATA - CONDITION					FUEL AT	STRESS GW	ULT LF
48	FLIGHT					164096	728000	3.75
49	LANDING					46346	635850	3.00
50	MAXIMUM DESIGN GROSS WEIGHT					180096	769000	3.75
51	MAXIMUM GROSS WEIGHT WITH ZERO WING FUEL					0	525904	3.38
52	MINIMUM FLYING WEIGHT					9311	371468	3.75
53	LIMIT LANDING SINKING SPEED (FT/SEC)				9	159250	635850	3.00
54	WING LIFT ASSUMED FOR LANDING DESIGN CONDITION (PER CENT WT)							111
55	STALL SPEED - LANDING CONFIGURATION - POWER OFF (KNOTS)							102
56	CABIN AND CARGO COMPT. ULT. DESIGN PRESS DIFFERENTIAL-FLT (PSI)							12.45
57	AIRFRAME WEIGHT (AS DEFINED IN [illegible])							280967.1

TABLE 8

C-5A GROSS WEIGHT BALANCE AND INERTIA SUMMARY

GROSS WEIGHT CONDITION (LANDING GEAR DOWN)	WEIGHT (LB)	F.S.	% MAC	B.L.	W.L.	INCHES BELOW LEMAC	I_x (Lb-In^2 x 10^{-6})	I_y (Lb-In^2 x 10^{-6})	I_z (Lb-In^2 x 10^{-6})	I_{xz} (Lb-In^2 x 10^{-6})	* δ
Weight Empty	325,263	1400	39.3	1	252	63	57,478.5	99,560.9	145,183.5	10,721.4	6.87
Basic Weight Plus Troop Kit	335,537	1405	40.7	1	254	61	57,863.4	101,742.1	147,586.2	11,043.8	6.91
Operating Weight	329,452	1393	37.5	1	252	63	57,909.0	101,485.9	146,943.8	10,697.5	6.76
Operating Weight & Troop Kit	337,033	1402	39.9	1	254	61	57,991.9	102,999.2	148,411.1	10,951.2	6.81
Basic Flight Design-Max.Cargo	728,000	1357	27.7	0	250	65	150,214.2	150,202.6	283,950.9	12,051.6	5.11
Basic Flight Design-Max. Fuel	728,000	1379	33.7	0	276	39	170,866.5	124,743.5	279,748.2	10,618.1	5.52
Maximum Design-Maximum Cargo	769,000	1350	25.8	0	246	69	149,599.9	158,894.7	291,557.3	12,322.0	4.92
Maximum Design-Maximum Fuel	769,000	1371	31.5	0	271	44	171,415.7	133,034.9	287,845.4	11,064.1	5.38
Ferry Mission (Zero Cargo)	655,533	1393	37.5	0	285	30	169,830.5	109,899.6	265,343.4	9,741.7	5.76
Landplane Landing (Max. Cargo)	635,850	1341	23.4	0	232	83	91,010.0	154,948.7	230,269.2	12,272.7	5.00
Personnel/Payload	728,000	1370	31.2	0	253	62	150,180.0	151,885.0	285,509.5	11,974.0	5.02
Typical Vehicle	728,000	1383	34.8	0	249	66	149,575.0	151,391.6	285,070.4	10,991.5	4.61
MAC Passenger	728,000	1369	31.0	0	266	49	166,248.6	151,574.2	301,287.4	12,191.9	5.12
** Most Forward C.G.	713,904	1339	22.9	0	249	66	139,966.0	152,015.6	275,469.4	12,351.7	5.17
*** Most Aft C.G. (Gear Up)	683,904	1406	41.0	0	246	69	135,513.4	145,574.8	265,053.0	10,261.4	4.50

L.E.M.A.C. = F.S. 1254.24
M.A.C. = 370.52 Inches

* Angle of Inclination of the principal axis (nose down) with respect to the air vehicle "x" axis.

** Represents the most critical forward center of gravity condition with respect to the allowable center of gravity limits for the 2.5g maximum cargo mission.

*** Represents the most critical aft center of gravity condition with respect to the allowable center of gravity limits for the 2.5g maximum cargo mission.

The weights have been reallocated for inertia calculation as shown in Table 9, and pertinent geometry items are given in Table 10.

Table 9

Reallocated Weights

WING GROUP	WEIGHT	X,Y,Z(when needed)
Structure	82045	
Surface Controls	3796	
Fuel System	2458	
Anti-Ice	229	
Trapped Fuel	562	
Total	89090	
FUSELAGE GROUP		
Structure	116048	
Distributed:		
Surface Control	1270	
Inst. & Navi.	281	
Hydraulic	2666	
Electrical	2761	
Total Dist.	6978	
POINT MASSES:		
Main Landing Gear	33681	1292,264,81
Nose Landing Gear	4407	418,0,86
Auxiliary Power Unit	933	1485,264,141
Air Conditioning	3411	964,0,294
Auxiliary Gear	39	2025,0,308
Crew	1290	318,0,332

Table 9 (cont'd)

	WEIGHT	X,Y,Z
Radar	376	80,0,260
Surface Controls	271	290,0,332
Instruments & Navigation	657	290,0,332
Other Equipment	0	
Tiedown Devices	1750	694,0,165
Life Rafts	200	698,0,334
Food	17	690,0,335
Water	43	691,0,365
Liquid 0_2	63	1280,0,153
Total Pt. Mass	47138	
Volumes:		
Avionics	3514	707,0,316
Furnishings	6836	763,0,281
Total Vol	10350	
HORIZONTAL TAIL GROUP		
Structure	6793	
Surface Controls	913	
Total	7706	
VERTICAL TAIL GROUP		
Structure	5603	
Surface Control	885	
Total	6488	
PROPULSION GROUP		
Engines & System	33804	
Hydraulic	1313	

Table 9. Continued

Electrical	690
Oil	264
Nacelles & Pylons	9586
Total	45657
Aircraft WE	

Table 10. C-5A Geometry Definitions for Inertia

GEOMETRY DATA

WING		HORIZ	VERT
Λ_{L1} -	28^o	30^o	37^o
Λ_T -	14^o	9^o	30^o
b -	1336	412	405
c -	525	250	371
t_r -	72	26	48
t_t -	20	10	39
XS4 -	806	2605	2425
ZS1 -	370	780	365
YS4 -	0	0	0
Θ -	-5^o	-5^o	90^o

FUSELAGE		PROPULSION			
l_h -	440	R_e -	80		
l_c -	1300	l_e -	312		
l_t -	1027	XP_1	-1020	XP_2 -	1165
R -	138	YP_1 -	476	YP_2 -	743
Z_b -	260	ZP_1 -	222	ZP_2 -	198

ADDITIONAL ITEMS

Λ_{L3} -	27^o	t_{fo} -	14	l_p -	1600

A. Centroids

WING

$C_a = 525$ $C_b = 710$ $C_c = 858$

USING Eq (8)*, XS1 = 422

USING Eq (9) , $YS^1 = 441$ $\dot{YS1} = YS1 \cos(-5^o) = 440$

$ZS3 = YS1 \sin(-5^o) = -38$

HORIZONTAL

$C_a = 238$ $C_b = 250$ $C_c = 315$

USING Eq (8), XS1 = 165

USING Eq (9), YS1 = 144 $\dot{YS1} = YS1 \cos(-5^o) = 143.5$

$ZS3 = YS1 \sin(-5^o) = -13$

VERTICAL

$C_a = 305$ $C_b = 371$ $C_c = 605$

USING Eq (8), XS1 = 277

USING Eq (9), YS1 = 188 $\dot{YS1} = YS1 \cos 90^o = 0$

$ZS3 = YS1 \sin 90^o = 188$

WING FUEL TANK

USING Eq (59) XF1 = 254

USING Eq (60) YF1 = 323 $\dot{YF1} = YF1 \cos(-5) = 321.6$

$ZF2 = YF1 \sin(-5) = -28$

B. Wing Group Inertia

USING Eq (1), $I_{1x} = 2.7028033 \times 10^{10}$

USING Eq (2), $I_{1y} = 1.9574734 \times 10^{10}$

USING Eq (3), $I_{1z} = 4.6602767 \times 10^{10}$

TRUE $I_{1y} = I_{1y} \cos(-5) + I_{1z} \sin(-5) = 1.5438548 \times 10^{10}$

TRUE $I_{1z} = I_{1y} \sin(-5) + I_{1z} \cos(-5) = 4.471938 \times 10^{10}$

USING (7), $I_{1xz} = -1.819266687 \times 10_9$

$$I_x = I_{1x} - 89090\,(440)^2 - 89090\,(-38)^2 + 89090\,(440 + 0)^2 + 89090\,(-38 + 370)^2 = 3.67192431 \times 10^{10}$$

$$I_y = I_{1y} - 89090\,(422)^2 - 89090\,(-38)^2 + 89090\,(422 + 806)^2 + 89090\,(-38 + 370)^2 = 1.4361055 \times 10^{11}$$

$$I_z = I_{1z} - 89090\,[(422)^2 + (440)^2] + 89090\,(422 + 806)^2 + 89090\,(440 + 0)^2 = 1.63200171 \times 10^{11}$$

$$I_{xz} = I_{1xz} - 89090\,(422)(-38) + 89090(422 + 806)(-38 + 370) = 3.5931017 \times 10^{10}$$

C. HORIZONTAL TAIL GROUP INERTIA

USING (1), $I_{1x} = 2.45844912 \times 10^8$

USING (2), $I_{1y} = 2.80151979 \times 10^8$

USING (3), $I_{1z} = 5.2599689 \times 10^8$

TRUE $I_{1y} = I_{1y} \cos(-5) + I_{1z} \sin(-5) = 2.3324227 \times 10^8$

TRUE $I_{1z} = I_{1y} \sin(-5) + I_{1z} \cos(-5) = 4.9957846 \times 10^8$

USING (7), $I_{1xz} = -1.941127 \times 10^7$

$$I_x = I_{1x} - 7706\,(143.5)^2 - 7706\,(-13)^2 + 7706\,(143.5 + 0)^2 + 7706\,(-13 + 780)^2 = 4.77784763 \times 10^9$$

$$I_y = I_{1y} - 7706\,(165)^2 - 7706\,(-13)^2 + 7706\,(165 + 2605)^2 + 7706\,(-13 + 780)^2 = 6.3682867 \times 10^{10}$$

$$I_z = I_{1z} - 7706\,(165^2 + 143.5^2) + 7706\,(165 + 2605)^2 + 7706\,(143.5 + 0)^2 = 5.82546306 \times 10^{10}$$

$$I_{xz} = I_{xz} - 7706(165)\,(-13) + 7706(165 + 2605)\,(-13 + 780) = 1.636920864 \times 10^{10}$$

D. VERTICAL TAIL GROUP INERTIA

USING (1), $I_{1x} = 3.17905191 \times 10^8$

USING (2), $I_{1y} = 7.2770685 \times 10^8$

USING (3), $I_{1z} = 1.045612041 \times 10^9$

TRUE $I_{1y} \approx I_{1y} \cos 90 + I_{1z} \sin 90 = 1.045612041 \times 10^9$

TRUE $I_{1z} = I_{1y} \sin 90 + I_{1z} \cos 90 = 7.2770685 \times 10^8$

USING (7), $I_{1xz} = 2.58157 \times 10^8$

$$I_x = I_{1x} - 6488\ (0)^2 - 6488\ (188)^2 + 6488\ (0 + 0)^2 + 6488\ (188 + 365)^2$$
$$= 2.07268211 \times 10^9$$

$$I_y = I_{1y} - 6488\ (277)^2 - 6488\ (188)^2 + 6488\ (277 + 2425)^2 + 6488\ (188 + 365)^2$$
$$= 4.967018756 \times 10^{10}$$

$$I_z = I_{1z} - 6488\ [(277)^2 + 0^2] + 6488\ (277 + 2425)^2 + 6488\ (0 + 0)^2$$
$$= 4.75975055 \times 10^{10}$$

$$I_{xz} = I_{1xz} - 6488\ (277)(188) + 6488(277 + 2425)(188 + 365) = 9.69440853 \times 10^9$$

E. FUSELAGE GROUP INERTIA

STRUCTURE:

$$S_n = \pi(138) \sqrt{138^2 + 440^2} = 199,920$$

$$S_c = 2\ \pi(138)(1300) = 1,127,203$$

$$S_t = \pi\ (138) \sqrt{138^2 + 1027} = \underline{449,247}$$

$$S_f = 1,776,370$$

$$W_n = \frac{199,920}{1,776,370}\ (116,048) = 13,061$$

$$W_c = \frac{1,127,203}{1,776,370}\ (116,048) = 73,639$$

$$W_t = \frac{449,247}{1,776,370}\ (116,048) = 29,349$$

USING (25), $I_x = 9.651054 \times 10^9$

USING (26), $I_y = 2.36853866 \times 10^{11}$

USING (27), $I_z = 2.290090211 \times 10^{11}$

USING (28), $I_{xz} = 3.722851418 \times 10^{10}$

DISTRIBUTED CONTENTS:

USING (29), $I_x = 6.04602 \times 10^8$

USING (30), $I_y = 1.79106443 \times 10^{10}$

USING (31), $I_z = 1.743893 \times 10^{10}$

USING (32), $I_{xz} = 2.6135712 \times 10^9$

VOLUMES:

AVIONICS: RECTANGULAR SOLID

USING (37), $I_x = \frac{3514}{12} (250^2 + 250^2) + 3514\ (316)^2$

$= 3.8749815 \times 10^8$

USING (38), $I_y = \frac{3514}{12} (250^2 + 1315^2) + 3514\ (707^2 + 316^2)$

$= 2.6320402 \times 10^9$

USING (39), $I_z = \frac{3514}{12} (1315^2 + 250^2) + 3514\ (707)^2$

$= 2.2811462 \times 10^9$

USING (40), $I_{xz} = 3514\ (707)(316) = 7.850698 \times 10^8$

FURNISHINGS: RECTANGULAR SOLID

USING (37), $I_x = \frac{6836}{12} (250^2 + 250^2) + 6836\ (281^2)$

USING (38), $I_y = \frac{6836}{12} (250^2 + 1100^2) + 6836\ (763^2 + 281^2)$

USING (39), $I_z = \frac{6836}{12} (1100^2 + 250^2) + 6836\ (763^2)$

USING (40), $I_{xz} = 6836\ (763)(281) = 1.4656589 \times 10^9$

POINT MASSES: for I_y,

$33681\ (1292^2 + 81^2)$

$4407\ (418^2 + 86^2)$

$933\ (1485^2 + 141^2)$

$3411\ (964^2 + 294^2)$

$39\ (2025^2 + 308^2)$

$1290\ (318^2 + 332^2)$

$376\ (80^2 + 260^2)$

$271\ (290^2 + 332^2)$

$657\ (290^2 + 332^2)$

$1750\ (694^2 + 165^2)$

$200\ (698^2 + 334^2)$

$17\ (690^2 + 335^2)$

$43\ (651^2 + 365^2)$

$63\ (1280^2 + 153^2)$

USING (42), $I_y = 6.45800764 \times 10^{10}$

$W_{pnc} = 0$

$W_{pc} = 1989$

USING (41), $I_x = \frac{1989\ (138)^2}{2} + \frac{3(0)\ (138)^2}{10} + 1989\ (260)^2 + \Sigma W\ (y^2 + z^2)$

$= 3.41181608 \times 10^9$

USING (43), $I_z = 6.36604587 \times 10^{10}$

USING (44), $I_{xz} = 5.376484824 \times 10^9$

F. PROPULSION GROUP INERTIA

USING (45), $I_x = 2\ [\frac{11414.3}{2}\ (80)^2 + 11,414.3\ (476^2 + 222^2)]$

$+ 2\ [\frac{11414.3}{2}\ (80^2) + 11,414.3\ (743^2 + 198^2)]$

$= 1.994107887 \times 10^{10}$

USING (46), $I_y = 2\ [\ \frac{11,414.3}{12}\ (3(80)^2 + 312^2) + 11,414.3\ (1020^2 + 222^2)]$

$+ 2\ [\ \frac{11,414.3}{12}\ (3(80)^2 + 312) + 11,414.3\ (1165^2 + 198^2)]$

$= 5.719790196 \times 10^{10}$

USING (47), $I_z = 2\ [\ \frac{11,414.3}{12}\ (3(80)^2 + 312^2) + 11,414.3\ (1020^2 + 476^2)]$

$+ 2\ [\frac{11,414.3}{12}\ (3(80)^2 + 312^2) + 11,414.3\ (1165^2 + 743^2)]$

$= 7.29527635 \times 10^{10}$

USING (48), $I_{xz} = 22829(1020)(222) + 22829(1165)(198) = 1.043536419 \times 10^{10}$

G. SUMMATION OF GROUP $I_{x,y,z}$

Total $I_x = 7.817685768 \times 10^{10}$

Total $I_y = 6.5382519 \times 10^{11}$

Total $I_z = 6.59099235 \times 10^{11}$

Total $I_{xz} = 1.198992973 \times 10^{11}$

H. AIRCRAFT CENTER OF GRAVITY

$\bar{X}$: W (X)

Wing

89,090 (1228)

Fuselage

13,061 (293)

73,639 (1090)

29,349 (2082)

6978 (1423)

Horizontal Tail

7706 (2770)

Vertical Tail

6488 (2702)

Propulsion

22,829 (1020)

22,829 (1165)

Point Masses

33681 (1292)

4407 (418)

933 (1485)

3411 (964)

39 (2025)

1290 (318)

376 (80)

271 (290)

657 (290)

1750 (694)

200 (698)

17 (690)

43 (651)

63 (1280)

Volumes

6836 (763)

3514 (707)

$WX = 4.13282587 \times 10^8$

$W = 329{,}456$

$$\bar{X} = \frac{WX}{W} = 1254.4$$

Fuselage

116,048 (260)

6.978 (260)

Wing

89,090 (332)

Horizontal Tail

7706 (767)

Vertical Tail

6488 (553)

Propulsion

22,829 (222)

22,829 (198)

Point Masses

33,681 (81)

4,407 (86)

933 (141)

3411 (294)

39 (308)

1290 (332)

376 (260)

271 (332)

657 (332)

1750 (165)

200 (334)

17 (335)

43 (365)

63 (153)

Volumes

3514 (316)

6836 (281)

$\Sigma\ WZ = 8.9156803 \times 10^7$

$\Sigma\ W = 329,456$

$\bar{Z} = \frac{\Sigma WZ}{\Sigma W} = 270.6$

I. MOMENTS OF INERTIA ABOUT AIRCRAFT CENTER OF GRAVITY (OPERATING WEIGHT EMPTY)

$I_{cgx} = I_x - (329,456)\ (270.6)^2$

$= 5.40461473 \times 10^{10}$

$I_{cgy} = I_y - 329{,}456\ (270.6^2 + 1254.4)^2$

$= 9.88529;998 \times 10^{10}$

$I_{cgz} = I_z - 329{,}456\ (1254.4)^2$

$= 1.406938403 \times 10^{11}$

$I_{cgyz} = 8.06854177 \times 10^{9}$

J. <u>CALCULATIONS FOR BASIC FLIGHT DESIGN WEIGHT WITH MAX FUEL:</u>

<u>INTERNAL WING FUEL:</u>

USING (53), $I_{4x} = 5.24631042 \times 10^{10}$

USING (54), $I_{4y} = 2.467113514 \times 10^{10}$

USING (55), $I_{4z} = 7.71342393 \times 10^{10}$

USING (58), $I_{4xz} = -2.96088621 \times 10^{9}$

TRUE $I_{4y} = I_{4y} \cos(-5) + I_{4z} \sin(-5) = 1.7854562 \times 10^{10}$

TRUE $I_{4z} = I_{4z} \sin(-5) + I_{4z} \cos(-5) = 7.4690489 \times 10^{10}$

$I_x = I_{4x} - 318500\ (321.6)^2 - 318500\ (-28)^2 + 318500\ (321.6 + 190)^2$

$+ 318500\ (-28 + 360)^2 = 1.3774084 \times 10^{11}$

$I_y = I_{4y} - 318500\ (254)^2 - 318500\ (-28)^2 + 318500\ (254 + 941)^2$

$= 4.869888185 \times 10^{11}$

$I_z = I_{4z} - 318500\ (254^2 + 321.6^2) + 381500\ (254 + 941)^2 + 318500\ (321.6 + 190)^2$

$= 5.618329538 \times 10^{11}$

$I_{xz} = I_{4xz} - 318500\ (254)\ (-28) + 318500\ (254 + 941)\ (-28 + 360)$

$= 1.25665746 \times 10^{11}$

PAYLOAD

USING (69), $I_x = \frac{71787}{(12)}(170^2 + 170^2) + 71787\ (192)^2 = 2.99213002 \times 10^9$

USING (70), $I_y = \frac{71787}{12}(1600^2 + 170^2) + 71787\ (1084^2 + 192^2)$

$= 1.0248755$

USING (71), $I_z = \frac{71787}{12}\ (1600^2 + 170^2) + 71787\ (1084)^2 = 9.98411921 \times 10^{10}$

USING (72), $I_{xz} = 71787\ (1084)\ (192) = 1.494088474 \times 10^{10}$

POINT MASSES:

$680\ (440^2 + 319^2)$

$7581\ (1619^2 + 341^2)$

$I_y = 2.09534 \times 10^{10}$

$I_z = 2.000267 \times 10^{10}$

$W_{pc} = 8261$

$I_x = \frac{8261\ (180)^2}{2} + 8261\ (335)^2 = 1.0609189 \times 10^9$

$I_{xz} = 4.2807557 \times 10^9$

Total $I_x = 2.199674956 \times 10^{11}$

Total $I_y = 1.25181229 \times 10^{12}$

Total $I_z = 1.340776051 \times 10^{12}$

Total $I_{xz} = 2.647866836 \times 10^{11}$

NEW CENTER OF GRAVITY DUE TO ADDITIONAL WEIGHT:

$\bar{x}$: 318500 (1195)

71787 (1084)

680 (440)

7581 (1619)

ΣWX (INCLUDING OPERATING WEIGHT) = 8.84280034×10^8

ΣW (INCLUDING OPERATING WEIGHT) = 728,025

$\bar{x} = 1214.6$

$\bar{Z}$: 318500 (332)

71787 (192)

680 (319)

7581 (341)

ΣWZ (INCLUDING OPERATING WEIGHT)= 2.114839×10^8

ΣW (INCLUDING OPERATING WEIGHT) = 728,025

$\bar{Z} = 290$

MOMENTS OF INERTIA ABOUT AIRCRAFT CENTER OF GRAVITY:

$I_{cgx} = I_x - 728025\ (290)^2 = 1.5874059 \times 10^{11}$

$I_{cgy} = I_y - 728025\ (290.^2 + 1214.6^2) = 1.16564203 \times 10^{11}$

$I_{cgz} = I_z - 728025\ (1214.6)^2 = 2.66754869 \times 10^{11}$

$I_{cgxz} = I_{xz} - 728025\ (290)\ (1214.6) = 8.3515259 \times 10^9$

SUMMARY OF ACTUAL VERSUS CALCULATED INERTIAS FOR C-5A

(MOMENTS OF INERTIA x 10^{-6})

	I_x (ROLL)			I_y (PITCH)		
	ACTUAL	CALCULATED	% ERROR	ACTUAL	CALCULATED	% ERROR
OPERATING WEIGHT EMPTY	57,909	54,246	6.3	101,486	98,853	2.6
WITH MAX FUEL	170,867	158,941	7.0	124,744	116,564	6.6

	I_z (YAW)			I_{xz}		
	ACTUAL	CALCULATED	% ERROR	ACTUAL	CALCULATED	% ERROR
OPERATING WEIGHT EMPTY	146,944	140,694	4.3	10,968	8068	26.4
WITH MAX FUEL	279,748	266,755	4.6	10,618	8351	21.4

References

1. USAF Stability and Control Datcom, Douglas Aircraft Division, October 1960.

1. Interactive Computer-Aided Design Aircraft Flying Qualities Program, ASD/XR 74-17, August 1974.

3. C-5A Actual Weight and Balance Report, Lockheed

4. F-15A Actual Weight Report, McDonnell Douglas MDC A3154, January 1975.

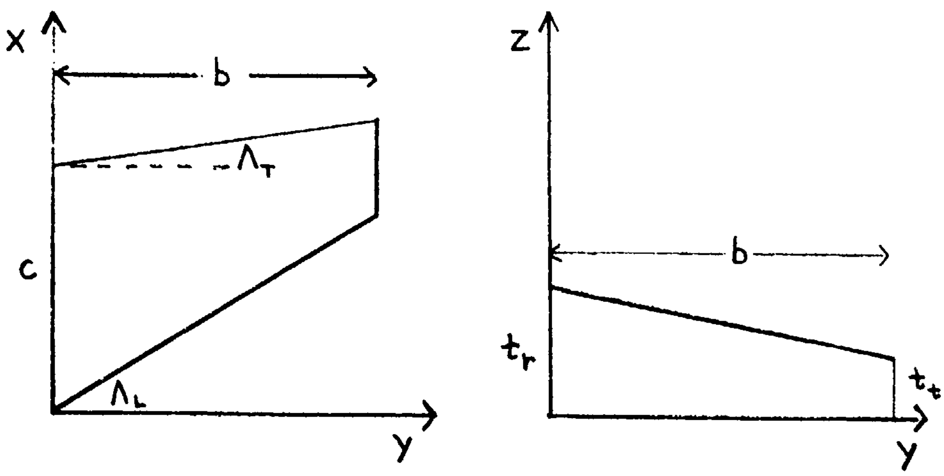
5. A-10A Actual Weight and Balance Report, Fairchild SW160 R0070

6. B-52G Actual Weight and Balance Report, Boeing D3-4586, August 1962

Appendix - Derivation of Equations

1. Surface Inertia and Volume

(Surface Diagram)



$$I = \int r^2\, dm, \quad dm = \rho\, dV, \quad I = \rho \int r^2\, dV, \quad dV = t\, dx\, dy$$

$$I_{1x}\ (\text{ROLL}) = \rho \int_0^b \int_{y\tan\Lambda_L}^{c + y\tan\Lambda_T} y^2 \left(t_r - \frac{t_r - t_t}{b}\, y\right) dx\, dy$$

$$= \rho \int_0^b \left(y^2 c + y^3 \tan\Lambda_T - y^3 \tan\Lambda_L\right)\left(t_r - \frac{t_r - t_t}{b}\, y\right) dy$$

$$= \frac{Wb^3}{V}\left\{\left[(t_r - t_t)\left(\frac{c}{4} + \frac{b\tan\Lambda_T}{5} - \frac{b\tan\Lambda_L}{5}\right)\right] + \left[t_r\left(\frac{c}{3} + \frac{b\tan\Lambda_T}{4} - \frac{b\tan\Lambda_L}{4}\right)\right]\right\}$$

Calculating the volume (V):

$$V = \iiint dx\, dy\, dz = \int_0^b \int_{y\tan\Lambda_L}^{c + y\tan\Lambda_T} \left(t_r - \frac{t_r - t_t}{b}\, y\right) dx\, dy$$

$$= \int_0^b \left[c + y\tan\Lambda_T - y\tan\Lambda_L\right]\left[t_r - \frac{t_r - t_t}{b} y\right] dy$$

$$= b\left\{t_r\left[c + \frac{b}{2}(\tan\Lambda_T - \tan\Lambda_L)\right] - (t_r - t_t)\left[\frac{c}{2} + \frac{b}{3}(\tan\Lambda_T - \tan\Lambda_L)\right]\right\}$$

$$I_{1y}\ (\text{PITCH}) = \rho \int_0^b \int_{y\tan\Lambda_L}^{c + y\tan\Lambda_T} x^2 \left(t_r - \frac{t_t - t_r}{b} y\right) dx\, dy$$

$$= \rho \int_0^b \left\{ \frac{(c + y\tan\Lambda_T)^3}{3}\left(t_r - \frac{t_r - t_t}{b} y\right) - \frac{y^3\tan^3\Lambda_L}{3}\left(t_r - \frac{t_r - t_t}{b} y\right)\right\} dy$$

$$= \rho \int_0^b \left\{\left[\left(-\frac{t_r - t_t}{b}\right)\left(\frac{c^3 y}{3} + c^2 y^2 \tan\Lambda_T + c y^3 \tan^2\Lambda_T + \frac{y^4\tan^3\Lambda_T}{3} - \frac{y^4\tan^3\Lambda_L}{3}\right)\right] + \left[t_r\left(\frac{c^3}{3} + c^2 y\tan\Lambda_T + c y^2 \tan^2\Lambda_T + \frac{y^3\tan^3\Lambda_T}{3} - \frac{y^3\tan^3\Lambda_L}{3}\right)\right]\right\} dy$$

$$= \frac{Wb}{V}\left\{\left[t_r\left(\frac{c^3}{3} + bc\tan\Lambda_T\left(\frac{c}{2} + \frac{b\tan\Lambda_T}{3}\right) + \frac{b^3}{12}(\tan^3\Lambda_T - \tan^3\Lambda_L)\right)\right] - \left[(t_r - t_t)\left(\frac{c^3}{6} + bc\tan\Lambda_T\left(\frac{c}{3} + \frac{b\tan\Lambda_T}{4}\right) + \frac{b^3}{15}(\tan^3\Lambda_T - \tan^3\Lambda_L)\right)\right]\right\}$$

$$I_{1z}\ (\text{YAW}) = \rho \int_0^b \int_{y\tan\Lambda_L}^{c + y\tan\Lambda_T} y^2 + x^2 \left(t_r - \frac{t_r - t_t}{b} y\right) dx\, dy$$

$$= I_{1x} + I_{1y}$$

$$I_{1xz} = \rho \int_0^{b\sin\theta} \int_{\frac{z\tan\Lambda_L}{\sin\theta}}^{C_r + \frac{z\tan\Lambda_T}{\sin\theta}} xzt \sin\theta \, dx dz \quad +$$

$$\rho \int_0^{b\cos\theta} \int_{\frac{y\tan\Lambda_L}{\cos\theta}}^{C_r + \frac{y\tan\Lambda_T}{\cos\theta}} xzt \cos\theta \, dx dy$$

$$= t_r \rho \int_0^{b\sin\theta} \int_{\frac{z\tan\Lambda_L}{\sin\theta}}^{C_r + \frac{z\tan\Lambda_T}{\sin\theta}} xz \sin\theta \, dx dz \quad + t_r \rho \int_0^{b\cos\theta}$$

$$\int_{\frac{y\tan\Lambda_L}{\cos\theta}}^{C_r + \frac{y\tan\Lambda_T}{\cos\theta}} xz \cos\theta \, dx dy \; - \frac{\rho}{b}(t_r - t_t) \int_{\frac{z\tan\Lambda_L}{\sin\theta}}^{C_r + \frac{z\tan\Lambda_T}{\sin}} \int_0^{b\sin\theta} xz^2 \, dz dx$$

$$- \frac{\rho}{b}(t_r - t_t) \int_0^{b\cos\theta} \int_{\frac{y\tan\Lambda_L}{\cos\theta}}^{C_r + \frac{y\tan\Lambda_L}{\cos\theta}} xy^2 \tan\theta \, dx dy$$

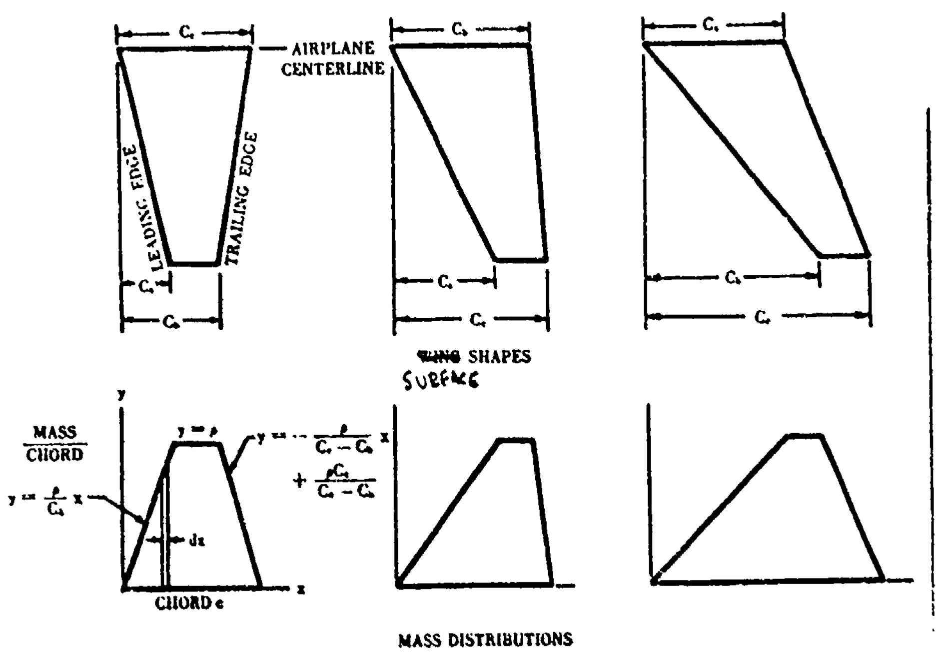
$$= \frac{W}{V} t_r \sin\theta \left[\frac{C_r^2 b^2}{4} + \frac{C_r b^3}{3} \tan\Lambda_T + \frac{b^4}{8} (\tan^2\Lambda_T - \tan^2\Lambda_L) \right]$$

$$- \frac{W}{V} (t_r - t_t) \sin\theta \left[\frac{C_r^2 b^2}{6} + \frac{C_r b^3}{4} \tan\Lambda_T + \frac{b^4}{10} (\tan^2\Lambda_T - \tan^2\Lambda_L) \right]$$

NOTE: Equations are correct for any $\pm\Lambda_T$

2. Longitudinal and spanwise surface center of gravity location

Longitudinal centroid:



$$\Sigma m = \int_0^{C_a} \frac{\rho}{C_a} x\,dx + \int_{C_a}^{C_b} \rho\,dx - \int_{C_b}^{C_c} \frac{\rho}{C_c - C_b} x\,dx + \int_{C_b}^{C_c} \frac{\rho C_c}{C_c - C_b}\,dx = \frac{\rho(-C_a + C_b + C_c)}{2}$$

$$\Sigma mx = \int_0^{C_a} \frac{\rho}{C_a} x^2 dx + \int_{C_a}^{C_b} \rho x\,dx - \int_{C_b}^{C_c} \frac{\rho}{C_c - C_b} x^2 dx + \int_{C_b}^{C_c} \frac{\rho C_c}{C_c - C_b} x\,dx = \frac{\rho(-C_a^2 + C_b^2 + C_bC_c + C_c^2)}{6}$$

$$I = \Sigma mx^2 = \int_0^{C_a} \frac{\rho}{C_a} x^3 dx + \int_{C_a}^{C_b} \rho x^2 dx - \int_{C_b}^{C_c} \frac{\rho}{C_c - C_b} x^3 dx + \int_{C_b}^{C_c} \frac{\rho C_c}{C_c - C_b} x^2 dx$$

$$= \frac{\rho(-C_a^3 + C_b^3 + C_b^2 C_c + C_b C_c^2 + C_c^3)}{12}$$

$$I_o = K_o\left[I - \frac{(\Sigma mx)^2}{\Sigma m}\right]$$

where

$K_o = 0.703$ for any wing

$K_o = 0.771$ for any horizontal or vertical stabilizer

Assuming that $I_{oy} = K_o I - K_o \frac{(\Sigma MX)^2}{\Sigma M}$ and knowing that $\bar{x} = \frac{\Sigma MX}{\Sigma M}$

We have $I_{oy} = K_o I - \Sigma M K_o \bar{\bar{x}}^2$. Since $\bar{\bar{x}}^2$ is multiplied by K_o we assume that $\bar{\bar{x}}$ is multiplied by $\sqrt{K_o}$ $\bar{\bar{x}} = \dfrac{2\rho \; (-C_a^2 + C_b^2 + C_c C_b + C_c^2)}{(-C_a + C_b + C_c)}$

$$\bar{\bar{x}} = (-C_a^2 + C_b^2 + C_c C_b + C_c^2)\sqrt{(K_o)}$$

(Where $\bar{\bar{x}}$ = XS1, XS2, or XS3)

Spanwise centroid:

Using diagram in Section (1) we have:

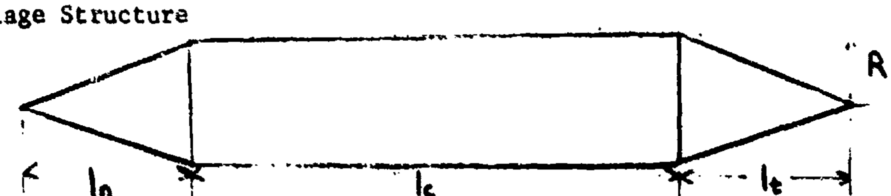
$$\bar{\bar{y}} = \frac{1}{V}\int_0^b \int_{y\tan\Lambda_L}^{c + y\tan\Lambda_T} y\left(t_r - \frac{t_r - t_t}{b}y\right) dx\,dy$$

$$= \frac{1}{V}\int_0^b y\left[c + y\tan\Lambda_T\right]\left[t_r - \frac{t_r - t_t}{b}y\right] - y\left[y\tan\Lambda_L\right]\left[t_r - \frac{t_r - t_t}{b}y\right] dy$$

$$= \frac{b^2}{V}\left[t_r\left(\frac{c}{2} + \frac{b}{3}(\tan\Lambda_T - \tan\Lambda_L)\right) - (t_r - t_t)\left(\frac{c}{3} + \frac{b}{4}(\tan\Lambda_T - \tan\Lambda_L)\right)\right]$$

(Where $\bar{\bar{y}}$ = YS1, YS2, or YS3)

(3) Fuselage Structure



R, l_n, l_c and l_t are chosen to best fit the fuselage geometry of the aircraft.

$$S_n = \pi R\sqrt{R^2 + l_n^2}$$

$$S_c = 2\pi R\, l_c$$

$$S_t = \pi R\sqrt{R^2 + l_t^2}$$

Distributing weight according to surface area:

$$W_n = \frac{S_n (W_s)}{S_n + S_c + S_t}$$

$$W_c = \frac{S_c (W_s)}{S_n + S_c + S_t}$$

$$W_t = \frac{S_t (W_s)}{S_n + S_c + S_t}$$

(4) Fuselage distributed contents

I_o for a right - cylindrical open ended shell is given in Section 9.

Translating this and defining the terms in different notation:

$$I_x = W_{dc} R^2 + W_{dc} (Z_b)^2$$

$$I_y = \frac{W_{dc}}{2} [R^2 + \frac{1}{6} (XS4 - CREW c.g.)^2] + W_{dc} (\frac{XS4 - CREW}{2} c.g. +$$

$$CREW c.g.)^2 + W_{dc} (Z_b)^2$$

$$I_z = I_y - W_{dc} (Z_b)^2$$

(5) Fuselage volumes of mass

I_o for a right circular shell and solid rectangle are given in Section 9.

Translating these to the remote axes and changing the notation gives:

Right circular cylinderical shell

$$I_x = W_{vo} R_v^2 + W_{vo} (Z)^2$$

$$I_y = \frac{W_{vo}}{2} (R_v^2 + \frac{1_v^2}{6}) + W_{vo} (X^2 + Z^2)$$

$$I_z = \frac{W_{vo}}{2} (R_v^2 + \frac{1_v^2}{6}) + W_{vo} (X)^2$$

Rectangular solid:

$$I_x = \frac{W_{vo}}{12} (2R_v^2 + 2 R_v^2) + W_{vo} (Z)^2$$

$$I_y = \frac{W_{vo}}{2} (1_v^2 + 2R_v^2) + W_{vo} (X^2 + Z^2)$$

$$I_z = \frac{W_{vo}}{12} (1_v^2 + 2R_v^2) + W_{vo} (X)^2$$

(6) Fuselage point masses

For point masses, the inertia about the center of the mass is so small that it can be neglected. For pitch and yaw we just translate the mass to each respective axis:

$$I_x = W_p (Y^2 + Z^2)$$

$$I_y = W_p (X^2 + Z^2)$$

$$I_z = W_p (X)^2$$

(PITCH) I_y fuselage structure $= I_y$ nose cone $+ I_y$ cylinder $+ I_y$ tail cone

(See Section 9)

$$I_{y\ nose} = \frac{W_n}{4}\left(R^2 + \frac{2}{9}l_n^2\right) + W_n\left(\frac{2}{3}l_n\right)^2 = \frac{W_n}{4}\left(R^2 + 2l_n^2\right)$$

$$I_y \text{ cylinder} = \frac{W_c}{2}\left(R^2 + \frac{l_c^2}{6}\right) + W_c\left(\frac{1}{2}l_c + l_n\right)^2 = \frac{W_c}{2}\left(R^2 + \frac{l_c^2}{6}\right) + W_c\left(\frac{1}{4}l_c^2 + l_c l_n + l_n\right)^2$$

$$I_{y\ tail} = \frac{W_t}{4}\left(R^2 + \frac{2}{9}l_t 2\right) + W_t\left(\frac{1}{3}l_t + l_n + l_c\right)^2$$

$$= \frac{W_t}{4}\left(R^2 + \frac{2}{9}l_t^2\right) + W_t\left(\frac{1}{9}l_t^2 + l_c^2 + l_n^2 + \frac{2}{3}l_t l_c + \frac{2}{3}l_t l_n\right.$$

Adding these three together:

$$I_y = \frac{R^2}{4}\left(W_n + 2W_c + W_t\right) + l_n^2\left(\frac{Wn}{2} + W_c + W_t\right) + l_c^2\left(\frac{1}{3}W_c + W_t\right) + \frac{1}{6}l_t^2 W_t +$$
$$l_c l_n\left(W_c + 2W_t\right) + \frac{2}{3}l_t l_c W_t + \frac{2}{3}l_t l_n W_t + W_s\left(Z_b\right)^2$$

(Roll)

$$I_{x\ nose} = \frac{W_n R}{2}, \quad I_x \text{ cylinder} = W_c R^2, \quad I_{x\ tail} = \frac{W_t R^2}{2}$$

Adding these three together:

$$I_x = \frac{R^2}{2}\left(W_n + 2W_c + W_t\right) + W_s\left(Z_b\right)^2$$

(YAW)

$$I_z = I_{t_4} - W_s\left(Z_b\right)^2$$

Using the I_{ox} equations for a solid cone and a solid right-cylinder and translating them to the remote axes gives an alternate approach to I_x:

$$I_x = \frac{W_{pc}}{2} R^2 + \frac{3}{10} W_{pnc} R^2 + (W_{pnc} + W_{pc} \, (Z_b)^2$$

7) Internal wing fuel tank centroid

Using the diagram in Section 1

$$XF1 = \frac{1}{V} \int_0^b \int_{y \tan\Lambda_L}^{c + y\tan\Lambda_L} x \left(t_r - \frac{t_r - t_t}{b} y\right) dx\,dy$$

$$= \frac{1}{V} \int_0^b (c + y\tan\Lambda_T)^2 \left(t_r - \frac{t_r - t_t}{b} y\right) - \frac{y^2 \tan^2\Lambda_L}{2} \left(t_r - \frac{t_r - t_t}{b} y\right) dy$$

$$= \frac{b}{V} \left\{ \left[\left(t_r \left(\frac{c^2}{2} + \frac{bc\tan\Lambda_T}{2} + \frac{b^2}{6} (\tan^2\Lambda_T - \tan^2\Lambda_L)\right)\right)\right] - \left[(t_r - t_t) \left(\frac{c^2}{4} + \frac{cb\tan\Lambda_T}{3} + \frac{b^2}{8} (\tan^2\Lambda_T - \tan^2\Lambda_L)\right)\right] \right\}$$

YF1 is the same as the spanwise centroid for surfaces derived in section 2. v(volume) was derived in section 2.

(8) Internal fuselage fuel inertia

$$V = \frac{W_{ff}}{\rho}$$

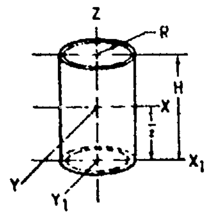
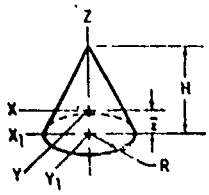
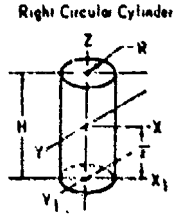
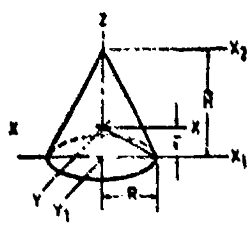
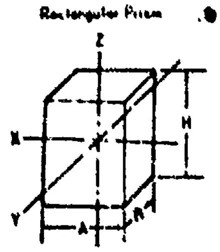
$$R_f = \frac{V}{\Pi l_f}$$

See Section 9 for solid cylinder equation

I_{ox} is assumed equal to 0.

$$I_{oz} = I_{oy} = \frac{W_{ff}}{12} \left(3(R_f)^2 + l_f^2\right) = \frac{W_{ff}}{12} \left(\frac{3W_{ff}}{\pi \rho l_f} + l_f^2\right)$$

(9) Center of gravity, inertia and surface area of various geometric shapes.

Figure	General Properties	Moment of Inertia
Lateral Cylindrical Shell 	Surface Area = $2\pi RH$ Centroid = $\bar{z} = \frac{H}{2}$	$I_x = I_y = \frac{W}{2}(R^2 + \frac{H^2}{6})$ $I_z = WR^2$ $I_{x_1} = I_{y_1} = \frac{W}{6}(3R^2 + 2H^2)$
Lateral Surface of a Circular Cone 	Surface Area = $\pi R\sqrt{R^2 + H^2}$ Centroid = $\bar{z} = \frac{H}{3}$	$I_x = I_y = \frac{W}{4}(R^2 + \frac{2}{9}H^2)$ $I_z = \frac{WR^2}{2}$ $I_{x_1} = I_{y_1} = \frac{W}{12}(3R^2 + 2H^2)$
Right Circular Cylinder 	Volume = $\pi R^2 H$ Centroid = $\bar{z} = \frac{H}{2}$	$I_x = I_y = \frac{W}{12}(3R^2 + H^2)$ $I_{x_1} = I_{y_1} = \frac{W}{12}(3R^2 + 4H^2)$ $I_z = \frac{WR^2}{2}$
Right Circular Cone 	Volume = $\frac{\pi R^2 H}{3}$ Centroid = $\bar{z} = \frac{H}{4}$	$I_x = I_y = \frac{3W}{20}\left(R^2 + \frac{H^2}{4}\right)$ $I_{x_1} = I_{y_1} = \frac{W}{20}(3R^2 + 2H^2)$ $I_z = \frac{3W}{10}R^2$ $I_{x_2} = \frac{3W}{20}(R^2 + 4H^2)$
Rectangular Prism 	Volume = ABH Centroid = $\bar{x} = \frac{A}{2}$, $\bar{y} = \frac{B}{2}$, $\bar{z} = \frac{H}{2}$	$I_x = \frac{W}{12}(B^2 + H^2)$ $I_y = \frac{W}{12}(A^2 + H^2)$ $I_z = \frac{W}{12}(A^2 + B^2)$

*U.S.Government Printing Office: 1979 — 657-002/17