Progress of Theoretical Physics, Vol. 55, No. 1, January 1976

Inevitable Surface Dependence of Some Operator Products and Integrability

Kazuyasu SHIGEMOTO, Yukio TAGUCHI,* Azuma TANAKA and Kunio YAMAMOTO**

Department of Physics, Osaka University, Toyonaka, Osaka 560
**Department of Physics, University of Osaka Prefecture, Sakai, Osaka 591*
***Institute of Physics, College of General Education, Osaka University Toyonaka, Osaka 560*

(Received June 13, 1975)

In general even in local theory the operator products at the same space-time point must be considered as a limit of non-local products. It is natural to confine non-locality on a space-like surface. In this case some operator products with three or more constituents possess an inevitable and purely quantum-mechanical surface dependence. Taking the pion-nucleon system as an example, we explicitly calculate in the order of g^2 this kind of the surface dependence of the interaction Hamiltonian. In order to obtain a consistent theory, this surface is required to be identified with the space-like surface in the Tomonaga-Schwinger equation. Then the interaction Hamiltonian needs an additional, non-canonical and surface-dependent term, which can be derived uniquely from the canonical Hamiltonian. The integrability of the Tomonaga-Schwinger equation is proved by taking account of this surface dependence together with the gradient term in the equal-time commutator.

§ 1. Introduction and preliminaries

In general, a product of three or more operators at the same space-time point, say

$$A(x)B(x)C(x), \tag{1·1}$$

must be considered as the local limit of a non-local product such that

$$\lim_{x_i \to x} A(x_1)B(x_2)C(x_3). \tag{1·2}$$

This fact itself is well known and nothing new. The important thing which we want to point out in this paper is that, in some cases even in the limit, we cannot neglect the dependence of (1·2) on the surface passing through x_1, x_2 and x_3. If this is the case, the product cannot be specified by the constituents alone. The specification of the surface is indispensable.

Now we note the following points.

A) This surface dependence shows up only if (1·2) is singular in the local limit.

B) The surface dependence, if it exists, is of course closely related to the transformation property of the product under the Lorentz group.

C) The transformation property is the one which should be determined in the process of studying the covariance of the theory.

D) The covariance of the theory is guaranteed if and only if the interaction Hamiltonian satisfies the integrability condition[1] of the Tomonaga-Schwinger equation.*)

Therefore, for our purpose, how to decide the interaction Hamiltonian is the first consideration.

When we calculate a Feynman amplitude, which appears in some matrix elements of the S-matrix or those of Heisenberg operators, we often encounter the fact that the correct amplitude is not obtained immediately from the formal and naive use of Feynman rules. To remedy such a situation, there are several method[2)~5)] which we call the regularization methods. The amplitude thus obtained does not depend on the choice of the regularization.

At this stage, we want to note the following additional points:

E) The naive use of Feynman rules may not give the correct answer only if the Feynman amplitude is singular in the sense that it contains the divergent subgraph. Then the naive amplitude involves the indefinite terms depending on the method of the manipulation.

F) The indefinite terms never vanish. The regularized amplitude is the one obtained by subtracting these terms from the naive one.

G) The existence of these terms violates the transformation properties under the Lorentz and/or the gauge group, as expected of the amplitude. Then, these terms are non-covariant and unphysical.

H) The Feynman rules used in the naive manipulation for the amplitude are the calculation rules based on the canonical interaction Hamiltonian.

I) The subtraction of some terms from the S-matrix is possible only through the introduction of new interaction Hamiltonian.

Consider the case where the regularization is required in the evaluation of the S-matrix. Then, from A)$\sim$I), we expect[6] that the interaction Hamiltonian H which satisfies the integrability condition is not the canonical Hamiltonian H_c but

$$H_C + H_{ND} \tag{1·3}$$

where H_{ND} is the new interaction Hamiltonian introduced to subtract the non-covariant terms. We have used the subscript ND since in our formulation it is normal dependent as will be seen later.

It is already known that the above expectation is correct in the two-dimensional quantum electrodynamics[7] and in the Thirring model.[8] In this paper we shall show by a direct calculation that the same is true in the four-dimensional π-N system which is more realistic but involves divergences higher than those of the

*) Throughout this paper, we discuss in the interaction representation.

two-dimensional models.

It is not trivial to extend the two-dimensional case to the four-dimensional one. In the former, the non-canonical gradient term appears in some equal-time commutators and it has been necessary and sufficient to take it into account. In the latter, to take the gradient term into account is, of course, important but that alone is insufficient. We must also consider the surface dependence mentioned in the beginning of this section.

In § 2 we shall see how this kind of the surface dependence appears and how the interaction Hamiltonian satisfies the integrability condition. In § 3 we shall briefly discuss the result of § 2. Appendices A, B and C will be devoted to the proofs of some equations presented in § 2.

When all the non-covariant terms in a matrix element of a Heisenberg operator cannot be subtracted by the new interaction Hamiltonian, the usual definition of the Heisenberg operator is inadequate. This problem will be discussed elsewhere.

§ 2. Interaction Hamiltonian, surface dependence of some operator products and integrability

The covariant system is described by the Tomonaga-Schwinger equation

$$H\{x, \sigma(x)\}\, U(\sigma, \sigma_0) = i\frac{\delta}{\delta\sigma(x)} U(\sigma, \sigma_0), \tag{2·1}$$

where $H\{x, \sigma(x)\}$ is the interaction Hamiltonian density and $\sigma(x)$ is a space-like surface passing through the point x. The Hamiltonian density depends on both the point x and the surface $\sigma(x)$. The consistency condition for (2·1) is

$$[H\{x, \sigma(x)\}, H\{y, \sigma(y)\}] + i\frac{\delta H\{x, \sigma(x)\}}{\delta\sigma(y)} - i\frac{\delta H\{y, \sigma(y)\}}{\delta\sigma(x)} = 0\,, \tag{2·2}$$

which is the so-called integrability condition.

Now we confine ourselves to the discussion of the π-N system. The canonical interaction Hamiltonian is

$$H_c(x) = ig\bar{\psi}(x)\gamma_5\psi(x)\phi(x) \tag{2·3}$$

which corresponds to (1·1). In order to clarify the definition of the operator product (2·3), we replace it by a limit of a non-local product like (1·2). A possible redefinition of (2·3) in the limiting form is

$$H_c\{x, \sigma(x)\} = ig \lim \int_{\sigma(x)} d^3x_1 d^3y_1 d^3z_1 F(x; x_1, y_1, z_1)\, \bar{\psi}(x_1)\gamma_5\psi(y_1)\phi(z_1), \tag{2·4}$$

where $F(x; x_1, y_1, z_1)$ is a form factor describing the non-locality. In principle we can take any surface as $\sigma(x)$ on the right-hand side of (2·4). However, in order that the right-hand side is the part of the interaction Hamiltonian, $\sigma(x)$ should be identified with the space-like surface on which the Hamiltonian may depend. If we apply (2·4) to the Feynman diagram having no divergence, it gives the same results as (2·3) does. In the flat-surface limit, a simple example of (2·4) is

$$H_c(x,\text{flat}) = ig \lim \int_{\text{flat}} d^3\Delta f(\Delta^2)\bar{\psi}(x+\Delta)\gamma_5\psi(x-\Delta)\phi(x) \qquad (2\cdot5)$$

with

$$\lim \int_{\text{flat}} d^3\Delta f(\Delta^2) = 1\,. \qquad (2\cdot6)$$

$H_c(x,\text{flat})$ thus defined is Hermitian.

Now we are in a position to obtain the Hamiltonian, which should be added, in the sense of I) in § 1, to cancel the non-covariant terms in the S-matrix. The concrete calculation in Appendix A shows that it depends on the unit normal $n_\mu(x)$ at the point x on the surface σ and has the form in the order of g^2,

$$H_{ND}\{x, n(x)\} = \frac{g^2}{32\pi^2}\Big[\bar{\psi}(x)\gamma\cdot(\vec{\partial}-\overleftarrow{\partial})\psi(x) + \frac{2}{3}\phi(x)\Box\phi(x)$$
$$+\bar{\psi}(x)(n\cdot\gamma n\cdot\vec{\partial}-\overleftarrow{\partial}\cdot nn\cdot\gamma)\psi(x) + \frac{2}{3}\phi(x)n_\mu n_\nu\partial_\mu\partial_\nu\phi(x)\Big]. \qquad (2\cdot7)^{*)}$$

As has been discussed in § 1, our next task is to prove that

$$H\{x,\sigma(x)\} = H_c\{x,\sigma(x)\} + H_{ND}\{x, n(x)\} \qquad (2\cdot8)$$

is the integrable interaction Hamiltonian. For simplicity, we prove (2·2) in the flat-surface limit. Then we examine

$$[H_c(x,\text{flat}), H_c(y,\text{flat})]\delta(x_0-y_0), \qquad (2\cdot9)$$

$$\left[\frac{\delta H_c\{x,\sigma(x)\}}{\delta\sigma(y)}\right]_{\text{flat}}\delta(x_0-y_0) - (x\leftrightarrow y) \qquad (2\cdot10)$$

and

$$\left[\frac{\delta H_{ND}\{x,n(x)\}}{\delta\sigma(y)}\right]_{\text{flat}}\delta(x_0-y_0) - (x\leftrightarrow y) \qquad (2\cdot11)$$

up to the order of g^2.

As will be proved in Appendices B and C, the results for the expressions (2·9) and (2·10) are given by

$$[H_c(x,\text{flat}), H_c(y,\text{flat})]\delta(x_0-y_0)$$
$$= \frac{g^2}{12\pi^2}[\bar{\psi}(x)\gamma_4(\vec{\partial}_i-\overleftarrow{\partial}_i)\psi(x)]\partial_i\delta^4(x-y) - (x\leftrightarrow y) \qquad (2\cdot12)$$

and

*) We denote the four-dimensional scalar product by $a\cdot b$ or $a_\mu b_\mu$, whereas the three-dimensional scalar product by $a_i b_i$. Thus, $a\cdot b = a_\mu b_\mu = a_i b_i - a_0 b_0$.

$$iU^\dagger(x_0,\infty)\left[\frac{\delta H_c\{x,\sigma(x)\}}{\delta\sigma(y)}-(x\leftrightarrow y)\right]_{\text{flat}}\delta(x_0-y_0)\,U(x_0,-\infty)$$

$$=\frac{g^2}{32\pi^2}\Big[\bar\psi(x)\Big\{\gamma_i(\vec\partial_4-\overleftarrow{\partial}_4)-\frac{5}{3}\gamma_4(\vec\partial_i-\overleftarrow{\partial}_i)\Big\}\psi(x)$$

$$+\frac{4}{3}\phi(x)\partial_4\partial_i\phi(x)\Big]\partial_i\delta^4(x-y)-(x\leftrightarrow y). \tag{2·13}$$

The reason why we consider the quantities sandwiched in between $U^\dagger(x_0,\infty)$ and $U(x_0,-\infty)$ in (2·13) is that (2·2) is the operator equation in the Hilbert space spanned by the physical states. In fact, such quantities appear directly in the consistency condition of (2·1). For (2·12), the consideration of sandwiching gives no effect since the commutator is already of the order of g^2.

The left-hand side of (2·12) vanishes if we calculate it naively. However, the rigorous estimation gives us purely quantum-mechanical*) gradient terms written down on the right-hand side. This is just like the Schwinger term in the commutator between the components of a vector. The left-hand side of (2·13) also vanishes if we calculate it naively. Again the rigorous estimation gives us the purely quantum-mechanical*) gradient terms written down on the right-hand side. These come from the surface dependence due to the definition of the product discussed in § 1.

The right-hand side of (2·13) does not contain the term with the higher derivatives of $\delta^4(x-y)$ so that $H_c\{x,\sigma(x)\}$ (2·4) is independent of the curvature of $\sigma(x)$.

The expression (2·11) can be evaluated by using (2·7) and

$$\left[\frac{\delta n_\mu(x)}{\delta\sigma(y)}\right]_{\text{flat}}\delta(x_0-y_0)=-\delta_{\mu i}\partial_i\delta^4(x-y). \tag{2·14}$$

The result is

$$i\left[\frac{\delta H_{ND}\{x,n(x)\}}{\delta\sigma(y)}-(x\leftrightarrow y)\right]_{\text{flat}}\delta(x_0-y_0)$$

$$=-\frac{g^2}{32\pi^2}[\bar\psi(x)\{\gamma_i(\vec\partial_4-\overleftarrow{\partial}_4)+\gamma_4(\vec\partial_i-\overleftarrow{\partial}_i)\}\psi(x)$$

$$+\frac{4}{3}\phi(x)\partial_4\partial_i\phi(x)\Big]\partial_i\delta^4(x-y)-(x\leftrightarrow y). \tag{2·15}$$

In (2·15), again we have no problem of sandwiching by $U^\dagger$ and U since H_{ND} is of the order of g^2.

Thus the integrability condition (2·2) is established in the flat-surface limit, since

*) If we write $\hbar$ explicitly, the right-hand side is proportional to $\hbar^2$.

$$
\begin{aligned}
&(2.12)+(2.13)+(2.15)\\
&=\frac{g^2}{32\pi^2}\Big[\bar{\psi}\Big\{(0+1-1)\gamma_i(\vec{\partial}_4-\overleftarrow{\partial}_4)+\Big(\frac{8}{3}-\frac{5}{3}-1\Big)\gamma_4(\vec{\partial}_i-\overleftarrow{\partial}_i)\Big\}\psi\\
&\quad+\Big(0+\frac{4}{3}-\frac{4}{3}\Big)\phi\partial_4\partial_i\phi\Big]\partial_i\delta^4(x-y)-(x\leftrightarrow y)\\
&=0\,. \qquad (2\cdot16)
\end{aligned}
$$

§ 3. Discussion

In the preceding section, we have studied how the interaction Hamiltonian in the quantum theory should be constructed from the canonical and classical Hamiltonian. At that time we faced the problem of the definition of the products of the operators at the same space-time point. This problem is peculiar to the quantum theory. To solve this, we have assumed that the local product is the limit of a non-local product spread over a space-like surface.

As a result of this limiting procedure, non-commutativity arises between some operator products and surface dependence appears in some operator products. On the other hand, the Hamiltonian itself is by nature surface dependent. Then, we have two surfaces; the surface appearing in the definition of the operator product and the surface necessary to specify the Hamiltonian. It is our assertion that these two surfaces should be identical. Consequently we have been able to construct the consistent quantum theory uniquely for the given classical theory by taking the non-commutativity and the surface dependence into account.

This is a support for our standpoint that the interaction Hamiltonian is the limit from the space-like direction of the non-local product of the constituent operators. Since the operators are separated space-likely, they commute with each other. Therefore, when we construct quantum field theory, the problem with respect to the order of the operators does not arise. Furthermore in our theory there is no ambiguity to obtain quantum-mechanical results from the given classical theory. These are the characteristic features which should be required for the correct theory, because we know that the quantum-mechanical results can be obtained without ambiguity by Feynman rules with the regularization.

Next we want to remark on inevitability of the surface dependence. Clearly this dependence arises as a consequence of the limiting procedure on the special surface. If the limit is the symmetrized one in the four-dimensional space-time, this dependence disappears. However, in this case the Hamiltonian has the non-locality in the time direction. Furthermore the unique separation of the indefinite term stated in E) in § 1 is not easy. It seems to us that the symmetrized limit formulation is impossible. Then we conclude that the surface dependence is inevitable.

In extracting the surface dependence it is essential to consider the quantity sandwiched by $U^\dagger$ and U. Then, U in our theory is not unitary. This is the outcome of taking all the limits in H_c simultaneously. The meaning of the limit in H_c should be understood in this way. The spirit of the simultaneous limit corresponds exactly to that of the dimensional regularization,[4] where the number of dimensions in all quantities is brought simultaneously to four after all calculations are done. This correspondence is clearer in the previous work.[5] The problem of non-unitary U may not arise in the formulation with the independent limit. However, in our opinion, the independent limit formulation is impossible for the singular case such as the π-N system. If possible, we should have a corresponding regularization method.

The non-covariant terms discussed in this paper are all in one-loop level. New non-covariant terms do not exist. For example, although the vertex correction diverges, it is free from the non-covariant term. The interaction Hamiltonian (2·8) with (2·7) is correct in the one-loop level, namely up to the order of g^2.

Appendix A

——Separation of indefinite terms and determination of H_{ND}——

As stated in D) in §1, the indefinite, non-covariant and unphysical terms appear only if the Feynman diagram contains the divergent subgraph. In order to obtain these terms up to the order of g^2, we examine the second-order corrections to the nucleon and the pion propagators.

The correction to the nucleon propagator due to the interaction Hamiltonian (2·5) in the flat surface is given by

$$\sum{}^{(2)}(p)\,\delta^4(p-p')=\frac{ig^2}{16\pi^4}\lim\int_{\text{flat}}d^3\Delta d^3\Delta' f(\Delta^2)f(\Delta'^2)\int d^4x\,d^4y$$
$$\times\gamma_5 S_F\{(x-\Delta)-(y+\Delta')\}\gamma_5\Delta_F(x-y)\,e^{-ip'\cdot(x+\Delta)+ip\cdot(y-\Delta')}\,. \tag{A·1}$$

Then,

$$\sum{}^{(2)}(p)=-\frac{ig^2}{16\pi^4}\lim\int_{\text{flat}}d^3\Delta\, d^3\Delta' f(\Delta^2)f(\Delta'^2)$$
$$\times\int_0^1 d\alpha\int d^4k\,\frac{i\gamma\cdot k+i\alpha\gamma\cdot p+m}{(k^2+M^2-i\epsilon)^2}e^{-i(k+p+\alpha p)\cdot\bar{\Delta}}\,, \tag{A·2}$$

where

$$\bar{\Delta}=\Delta+\Delta'\,, \tag{A·3}$$

$$M^2=\alpha(1-\alpha)p^2+(1-\alpha)m^2+\alpha\mu^2\,. \tag{A·4}$$

By virtue of the exponential factor, we can carry out k-integrations without any ambiguity by using the integration formulas in the Appendix of our previous work.[9] Omitting the terms which vanish in the local limit and using

$$\lim \int_{\text{flat}} d^3\Delta\, d^3\Delta' f(\Delta^2) f(\Delta'^2) \frac{\bar{\Delta}_i}{\bar{\Delta}^2} = 0\,, \tag{A·5}$$

$$\lim \int_{\text{flat}} d^3\Delta\, d^3\Delta' f(\Delta^2) f(\Delta'^2) \frac{\bar{\Delta}_i \bar{\Delta}_j}{\bar{\Delta}^2} = \frac{1}{3}\delta_{ij}\,, \tag{A·6}$$

we have

$$\begin{aligned}\Sigma^{(2)}(p) &= -\frac{g^2}{8\pi^2} \lim \int_{\text{flat}} d^3\Delta\, d^3\Delta' f(\Delta^2) f(\Delta'^2) \\ &\quad \times \left[\int_0^1 d\alpha (i\alpha\gamma\cdot p + m) \log \frac{\gamma M|\bar{\Delta}|}{2} - \frac{\gamma\cdot\bar{\Delta}}{\bar{\Delta}^2} + \frac{3}{2} i \frac{\gamma\cdot\bar{\Delta} p\cdot\bar{\Delta}}{\bar{\Delta}^2} \right] \\ &= -\frac{g^2}{8\pi^2} \lim \left[\int_{\text{flat}} d^3\Delta\, d^3\Delta' f(\Delta^2) f(\Delta'^2) \int_0^1 d\alpha (i\alpha\gamma\cdot p + m) \log \frac{\gamma M|\bar{\Delta}|}{2} \right. \\ &\quad - \frac{ig^2}{16\pi^2} \gamma_i p_i\,, \end{aligned} \tag{A·7}$$

where $\gamma = 1.781$ is the Euler constant.

In our formulation, the indefinite and non-covariant term never contains the time-component. This is the natural consequence of the limiting procedure from the space direction. We can uniquely separate the term composed of only the space-components and having wrong transformation property. Thus only the last term of (A·7) is indefinite and non-covariant. More detailed discussion has been done in the earlier studies.[5),9)]

Similarly, for the correction to the pion propagator,

$$\begin{aligned}\Pi^{(2)}(p)\delta^4(p-p') &= \frac{-ig^2}{16\pi^4} \lim \int_{\text{flat}} d^3\Delta\, d^3\Delta' f(\Delta^2) f(\Delta'^2) \int d^4x\, d^4y \\ &\quad \times \mathrm{Tr}[\gamma_5 S_F\{(y-\Delta')-(x+\Delta)\}\gamma_5 S_F\{(x-\Delta)-(y+\Delta')\}] e^{ip\cdot x - ip'\cdot y} \\ &= -\frac{g^2}{4\pi^2} \lim \int_{\text{flat}} d^3\Delta\, d^3\Delta' f(\Delta^2) f(\Delta'^2) \left[\frac{1}{\bar{\Delta}^2} - m^2 - \frac{p^2}{6} \right. \\ &\quad \left. + 2\int_0^1 d\alpha \{m^2 + 3p^2\alpha(1-\alpha)\} \log \gamma M'|\bar{\Delta}| \right] \delta^4(p-p') \\ &\quad + \frac{g^2}{24\pi^2} p_i^2 \delta^4(p-p'), \end{aligned} \tag{A·8}$$

where

$$M'^2 = m^2 + p^2\alpha(1-\alpha). \tag{A·9}$$

The last term is non-covariant.

The normal dependent Hamiltonian to cancel out the non-covariant terms in (A·7) and (A·8) is

$$\int d^3x H_{ND}(x, \text{flat}) = \frac{g^2}{16\pi^2} \int d^3x \left[\bar{\psi}(x)\gamma_i\partial_i\psi(x) + \frac{1}{3}\phi(x)\partial_i^2\phi(x) \right]. \tag{A·10}$$

From this, we obtain

$$H_{ND}(x,\text{flat})=\frac{g^2}{32\pi^2}\left[\bar{\psi}(x)\gamma_i(\vec{\partial}_i-\overleftarrow{\partial}_i)\psi(x)+\frac{2}{3}\phi(x)\partial_i^2\phi(x)\right] \tag{A·11}$$

as the simplest Hermitian form. In the curved surface this becomes (2·7).

Appendix B

——Proof of (2·12)——

Using the canonical commutation relation

$$\{\psi(x),\bar{\psi}(y)\}\delta(x_0-y_0)=\gamma_4\delta^4(x-y), \tag{B·1}$$

we have

$$\begin{aligned}&[H_c(x,\text{flat}),H_c(y,\text{flat})]\delta(x_0-y_0)\\&=\lim\int_{\text{flat}}d^3\Delta\, d^3\Delta' f(\Delta^2)f(\Delta'^2)g^2[\bar{\psi}(x+\Delta)\gamma_4\psi(y-\Delta')\delta^4\{(x-\Delta)-(y+\Delta')\}\\&\quad-\bar{\psi}(y+\Delta')\gamma_4\psi(x-\Delta)\delta^4\{(x+\Delta)-(y-\Delta')\}]\phi(x)\phi(y).\end{aligned} \tag{B·2}$$

The right-hand side has nonvanishing contribution only from the singular part of the operator products. Since the singularities of $\bar{\psi}(x)\gamma_4\psi(y)$ and $\phi(x)\phi(y)$ are

$$\bar{\psi}(x)\gamma_4\psi(y)\delta(x_0-y_0)\approx 0\,, \tag{B·3}$$

$$\phi(x)\phi(y)\delta(x_0-y_0)\approx\frac{1}{4\pi^2}\frac{1}{(x-y)^2}\delta(x_0-y_0) \tag{B·4}$$

at $x=y$, (B·2) becomes

$$\begin{aligned}&\frac{g^2}{4\pi^2}\lim\int_{\text{flat}}d^3\Delta\, d^3\Delta' f(\Delta^2)f(\Delta'^2)\frac{1}{\bar{\Delta}^2}\bar{\psi}(x)\gamma_4[-1+\bar{\Delta}_i(\vec{\partial}_i-\overleftarrow{\partial}_i)]\psi(x)\\&\quad\times\bar{\Delta}_i\partial_i\delta^4(x-y)-(x\leftrightarrow y)\end{aligned} \tag{B·5}$$

with $\bar{\Delta}$ (A·3). By applying (A·5) and (A·6) to (B·5) we obtain (2·12).

Appendix C

——Proof of (2·13)——

The canonical interaction Hamiltonian, considered as a local limit of a non-local product, may have a surface dependence as is seen in (2·4). Assuming that this surface is identical to the space-like surface appearing in the Tomonaga-Schwinger equation, we have

$$\begin{aligned}&i\left[\frac{\delta H_c\{x,\sigma(x)\}}{\delta\sigma(y)}-(x\leftrightarrow y)\right]_{\text{flat}}\delta(x_0-y_0)\\&=-ig\lim\int_{\text{flat}}d^3\Delta f(\Delta^2)\,[\bar{\psi}(x+\Delta)\overleftarrow{\partial}_4\gamma_5\psi(x-\Delta)\phi(x)\delta^4(x+\Delta-y)\end{aligned}$$

$$+\bar{\psi}(x+\varDelta)\gamma_5\{\partial_4\psi(x-\varDelta)\}\phi(x)\delta^4(x-\varDelta-y)]-(x\leftrightarrow y)\,. \tag{C·1}$$

Using the free field equations of ψ and $\bar{\psi}$ in (C·1), we obtain, up to the order of g^2

$$\begin{aligned}
&iU^\dagger(x_0,\infty)\left[\frac{\delta H_c\{x,\sigma(x)\}}{\delta\sigma(y)}-(x\leftrightarrow y)\right]_{\text{flat}}\delta(x_0-y_0)\,U(x_0,-\infty)\\
&=ig^2\lim\int_{\text{flat}}d^3\varDelta\, d^3\varDelta' f(\varDelta^2)f(\varDelta'^2)\int d^4z\\
&\quad\times[\{\bar{\psi}(x+\varDelta)\,(\gamma_i\overleftarrow{\partial}_i-m)\gamma_4\gamma_5 S_F(x-z-\bar{\varDelta})\gamma_5\varDelta_F(x-z)\psi(z-\varDelta')\\
&\quad+\bar{\psi}(z+\varDelta')\gamma_5 S_F(z-x-\bar{\varDelta})\,(-\gamma_i\overleftarrow{\partial}_i-m)\gamma_4\gamma_5\varDelta_F(z-x)\psi(x-\varDelta)\}\delta^4(x+\varDelta-y)\\
&\quad+\{\bar{\psi}(x+\varDelta)\gamma_5\gamma_4(\gamma_i\vec{\partial}_i+m)S_F(x-z-\bar{\varDelta})\gamma_5\varDelta_F(x-z)\psi(z-\varDelta')\\
&\quad+\bar{\psi}(z+\varDelta')\gamma_5 S_F(z-x-\bar{\varDelta})\gamma_5\gamma_4(\gamma_i\vec{\partial}_i+m)\psi(x-\varDelta)\varDelta_F(z-x)\}\delta^4(x-\varDelta-y)\\
&\quad-\mathrm{Tr}\{\gamma_4\gamma_5 S_F(x-z-\bar{\varDelta})\gamma_5 S_F(z-x-\bar{\varDelta})\,(-\gamma_i\overleftarrow{\partial}_i-m)\}\phi(x)\phi(z)\delta^4(x+\varDelta-y)\\
&\quad-\mathrm{Tr}\{\gamma_5\gamma_4(\gamma_i\vec{\partial}_i+m)S_F(x-z-\bar{\varDelta})\gamma_5 S_F(z-x-\bar{\varDelta})\}\phi(x)\phi(z)\delta^4(x-\varDelta-y)]\\
&\quad-(x\leftrightarrow y)
\end{aligned} \tag{C·2}$$

with $\bar{\varDelta}$ (A·3), where ∂_i is the differentiation with respect to the argument of the nearest propagator or operator. There is a one-loop diagram in each term of (C·2). We can perform the integration on the loop momentum by the standard manner as in Appendix A.

Noting that only the linear and higher singularities on $\bar{\varDelta}$ contribute to (C·2), neglecting the term which vanishes in the limit and using (A·5) and (A·6), we get (2·13).

References

1) S. Kanesawa and S. Tomonaga, Prog. Theor. Phys. **3** (1948), 1, 101.
2) For the regularization by the artificial fields, W. Pauli and F. Villars, Rev. Mod. Phys. **21** (1949), 434.
3) For the normal product method, W. Zimmermann, *Lectures on Elementary Particles and Quantum Field Theory*, vol. 2 (MIT Press, Cambridge, Mass., 1970), p. 397; Ann. of Phys. **77** (1973), 536, 570.
4) For the regularization by the continuous dimension, G. 't Hooft and M. Veltman, Nucl. Phys. **44B** (1972), 189.
5) For the regularization based on the integrability condition, K. Yamamoto, Prog. Theor. Phys. **52** (1974), 304.
6) K. Yamamoto, Prog. Theor. Phys. **46** (1971), 1002.
7) Y. Taguchi and K. Yamamoto, Prog. Theor. Phys. **46** (1971), 1003.
8) Y. Taguchi, A. Tanaka and K. Yamamoto, Prog. Theor. Phys. **52** (1974), 1042.
9) K. Kikuchi, Y. Taguchi and K. Yamamoto, Prog. Theor. Phys. **50** (1973), 1397.