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INFERRING LABOR INCOME RISK AND PARTIAL INSURANCE FROM ECONOMIC CHOICES

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INFERRING LABOR INCOME RISK AND PARTIAL INSURANCE FROM ECONOMIC CHOICES

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This paper uses the information contained in the joint dynamics of individuals' labor earnings and consumption-choice decisions to quantify both the amount of income risk that individuals face and the extent to which they have access to informal insurance against this risk. We accomplish this task by using indirect inference to estimate a structural consumption–savings model, in which individuals both learn about the nature of their income process and partly insure shocks via informal mechanisms. In this framework, we estimate (i) the degree of partial insurance, (ii) the extent of systematic differences in income growth rates, (iii) the precision with which individuals know their own income growth rates when they begin their working lives, (iv) the persistence of typical labor income shocks, (v) the tightness of borrowing constraints, and (vi) the amount of measurement error in the data. In implementing indirect inference, we find that an auxiliary model that approximates the true structural equations of the model (which are not estimable) works very well, with negligible small sample bias. The main substantive findings are that income shocks are moderately persistent, systematic differences in income growth rates are large, individuals have substantial amounts of information about their income growth rates, and about one-half of income shocks are smoothed via partial insurance. Putting these findings together, the amount of uninsurable lifetime income risk that individuals perceive is substantially smaller than what is typically assumed in calibrated macroeconomic models with incomplete markets.

KEYWORDS: Labor income risk, idiosyncratic shocks, partial insurance, indirect inference estimation, heterogeneous income profiles, persistence.

1. INTRODUCTION

THE GOAL OF THIS PAPER is to use individuals' consumption–savings decisions to learn about the uninsurable labor-income risks that they face. Income fluctuations alone, even in the absence of measurement problems, overstate this risk because they do not reveal whether individuals anticipate these fluctuations or can use informal mechanisms, beyond self-insurance through borrowing and lending, to insure against these fluctuations. Instead, we use the joint dynamics of individuals' labor earnings and consumption choices to infer both what

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they know about the income risks they face and how well they can use informal mechanisms to insure these risks.

Specifically, we build a life-cycle consumption–savings model with constant relative risk aversion (CRRA) utility, potentially binding borrowing constraints, partial insurance, and a realistic retirement pension system. We assume that the slopes of individuals' labor income profiles (i.e., their income growth rates) vary in the population but that individuals have imperfect information about their own growth rates. Each individual enters the labor market with a prior belief about his own growth rate and then updates his beliefs over time in a Bayesian fashion. A key parameter is the precision of the initial prior belief: this parameter determines the extent to which an individual has advance information about the slope of his future income path.[2] In addition, we assume that some part of the surprise (or shock) to an individuals' labor income can be insured via informal mechanisms that we do not model explicitly. A key parameter here is the fraction of this surprise that can be insured. Finally, we also allow for several types of measurement error, a pervasive feature of individual-level data on income and consumption. Along all three of these dimensions—the amount of advance information about income growth rates, the extent of partial insurance, and the size of measurement error—consumption-choice decisions play a critical role because data on income alone cannot identify any of them.

We use a simulation method—indirect inference—to estimate these key parameters as well as the tightness of borrowing constraints, the discount factor, and the parameters governing labor-income dynamics. Rather than select an arbitrary set of unconditional moments upon which to base estimation, indirect inference focuses instead on the parameters of an auxiliary model that plays the role of a reduced form for the structural model. In particular, we use an auxiliary model that approximates the joint dynamics of income and consumption implied by the structural consumption–savings model. The indirect inference estimator chooses the values of the structural parameters so that the parameters of this auxiliary model, estimated using either the observed data or data simulated from the structural model, are as close as possible. In effect, the indirect inference estimator seeks to find the best-fitting set of auxiliary-model parameters subject to the cross-equation restrictions that the structural model places on these parameters.[3]

The key findings of our analysis regarding the nature of income risk are that (i) informal mechanisms insure about one-half of a given income surprise,

[2]The modeling of this learning process builds on Guvenen (2007). As we discuss in the next section, however, the analysis in the present paper differs in several important ways from the one in that paper.

[3]Thus our estimation methodology follows in the spirit of Sargent (1978), who uses aggregate rather than microdata to estimate a "rational expectations version of Friedman's time-series consumption model … by imposing the pertinent restrictions across the stochastic processes for consumption and income."

(ii) systematic differences in income growth rates are large, (iii) individuals have substantial amounts of information about their future income prospects, and (iv) the typical income shock is not very persistent. Taken together, these findings deliver the main substantive conclusion of this paper: the amount of uninsurable income risk perceived by individuals upon entering the labor market is substantially smaller than what is typically assumed in calibrated incomplete markets models that do not account for partial insurance and advance information about income growth rates (see Figure 7 in Section 6.2).

This paper is related to a growing literature that uses panel data to study the transmission of income shocks to consumption when markets are incomplete. Important examples include Hall and Mishkin (1982) and, more recently, Blundell, Pistaferri, and Preston (2008), Kauffmann and Pistaferri (2009), Krueger and Perri (2009), Kaplan and Violante (2010), and Heathcote, Storesletten, and Violante (2014). This paper contributes to this literature in the following ways. First, we estimate the amount of partial insurance in a model where the underlying income risk can have a richer structure than what is considered in these papers. In particular, we allow for growth-rate heterogeneity and Bayesian learning as well as persistent shocks that are not restricted to follow a random walk. This structure allows us to provide a more comprehensive picture of the sources of income uncertainty (arising either from genuine shocks or from learning) as well as the extent of insurance against this background. Second, with the exception of Heathcote, Storesletten, and Violante (2014), these papers derive estimable equations first and then impose partial insurance on top of these equations. Instead, in this paper, partial insurance is modeled as a transfer in the budget constraint and the implications for the consumption–savings choice are worked out explicitly. Kaplan and Violante (2010) share some similarities with our paper: that paper incorporates nonpermanent shocks and retirement into the model of Blundell, Pistaferri, and Preston (2008) but does not allow for growth-rate heterogeneity with Bayesian learning.[4] Finally, Heathcote, Storesletten, and Violante (2014) allow for endogenous labor supply and derive structural equations that allow them to quantify the degree of partial insurance using data on labor hours, in addition to consumption and income used here.

Although the framework in this paper shares some common elements with that in Guvenen (2007), there are three important differences. First, one major goal of the present paper is to distinguish between information about future

[4]A related set of papers includes Deaton and Paxson (1994), Blundell and Preston (1998), and, more recently, Primiceri and van Rens (2009). These papers use structural models similar to those above but employ repeated cross sections of consumption and income, rather than a panel. Furthermore, they assume permanent shocks and rule out profile heterogeneity. Primiceri and van Rens (2009) is a partial exception to this statement in that their framework potentially allows for systematic patterns in income growth, although this is conditional on shocks being permanent. They also do not estimate how much individuals know about their own growth rates.

earnings prospects and partial insurance, whereas Guvenen (2007) restricts insurance opportunities to self-insurance only. Second, Guvenen (2007) takes all of the parameters of the income process as given (estimated in another paper from income data alone) and uses the consumption data to calibrate the value of one parameter: the amount of prior information regarding one's own income growth rate. In contrast, this paper brings consumption data to bear on the estimation of the entire vector of structural parameters, which contains all of the parameters of the income process, prior beliefs, preferences, and borrowing constraints as well as several types of measurement error. Third, that paper focuses exclusively on the life-cycle mean and variance profiles of consumption (using repeated cross sections); here, we use the joint dynamics of consumption and income using panel data to conduct a formal structural estimation. In Sections 2.2 and 6.3, we show how critical these joint dynamics are for identifying parameters, especially those pertaining to partial insurance. This more thoroughgoing analysis leads us to conclude that the amount of prior information about income growth is quite a bit larger than what Guvenen (2007) gleans from life-cycle profiles. This larger amount of prior information, in combination with a significant amount of partial insurance, leads us to conclude that the amount of uncertainty perceived by a 25-year-old individual about his future income at age 55 is about *one-third* of what was found in that paper.

From a methodological perspective, an important precursor to this paper is Gourinchas and Parker (2002), who estimate a life-cycle consumption–savings model using the method of simulated moments. Their main focus is on whether such a model can explain the hump-shaped consumption profile over the life cycle. These authors estimate the income-process parameters from income data first and, in a second step, estimate risk aversion and the time discount factor from consumption data, whereas we estimate income-process parameters and preference parameters using both data sources jointly. Finally, there is a small but growing literature that uses indirect inference to estimate structural economic models in a variety of fields, including labor economics, finance, macroeconomics, and industrial organization; a nonexhaustive list includes Smith (1993), Magnac, Robin, and Visser (1995), Bansal, Gallant, and Tauchen (2007), Nagypal (2007), Li (2010), Low and Pistaferri (2012), Collard-Wexler (2013), and Bagger, Fontaine, Postel-Vinay, and Robin (2014). Among these, our philosophy for selecting the auxiliary model is in the same spirit as Magnac, Robin, and Visser (1995) and Bagger et al. (2014), who use an auxiliary model that mimics the true structural equations of their model. To our knowledge, this paper is the first to use indirect inference to estimate a fully specified consumption–savings model.

The next section describes a linear-quadratic version of the consumption–savings problem that permits an analytical solution, thereby allowing us to characterize theoretically the information content in consumption choices. Section 3 then lays out the full model used in estimation. Sections 4 and 5 describe the data and the indirect inference methodology. Section 6 presents the results and extensions. Section 7 concludes. The Supplemental Material (Guvenen and Smith (2014)) comprises Appendixes A–D and additional data and programs.

2. A FRAMEWORK FOR INFERRING INCOME RISK

Let the log labor income of individual i with t years of labor market experience be given by

$$y_t^i = \underbrace{g(t,\ \text{observables}, \dots)}_{\text{common life-cycle component}} + \underbrace{\left[\alpha^i + \beta^i t\right]}_{\text{profile heterogeneity}} + \underbrace{\left[z_t^i + \varepsilon_t^i\right]}_{\text{stochastic component}}, \tag{1}$$

where $z_t^i = \rho z_{t-1}^i + \eta_t^i$, and η_t^i and ε_t^i are zero-mean innovations that are independent and identically distributed (i.i.d.) over time and across individuals, with standard deviations of σ_η and σ_ε, respectively. The initial value of the persistent process, z_0^i, is drawn from a distribution with zero mean and a standard deviation of σ_{z_0}.

The terms in the first set of brackets capture the life-cycle variation in labor income that is common to all individuals with given observable characteristics. The second component captures potential *individual-specific* differences in income growth rates (as well as in levels). Such differences would be implied, for example, by a human capital model with heterogeneity in the ability to accumulate skills.[5] Finally, the terms in the last bracket represent the stochastic variation in income, which is modeled here as the sum of an (autoregressive) AR(1) component and a purely transitory shock. This specification encompasses (or differs only in minor ways from) the processes estimated in the literature.

2.1. *Bayesian Learning About Income Profiles*

We begin by laying out a framework that allows for various possibilities regarding individuals' perceptions of their future income risk. For example, an individual is likely to have more information than the econometrician about his β^i at the time he enters the labor market and will update those beliefs as information is revealed in his income realizations. We model this process by assuming that an individual enters the labor market with some prior belief about his own β^i, which is then refined over time in a Bayesian fashion (following Guvenen (2007)). The prior variance of this belief distribution measures how uncertain individuals are about their own β^i when they enter the labor market and is therefore a key parameter for determining the amount of perceived income risk.

[5]See, for example, the classic paper by Ben-Porath (1967). For more recent examples of such a human capital model, see Guvenen and Kuruscu (2010) and Huggett, Ventura, and Yaron (2011).

Time 0: Prior Beliefs and Variance (Uncertainty). Imagine that, for each individual, nature draws two random variables at time zero, β_k^i and β_u^i, with $\beta_k^i \perp \beta_u^i$, $\mathbb{E}(\beta_k^i) = \mathbb{E}(\beta_u^i) = 0$, and variances denoted with $\sigma^2_{\beta_k}$ and $\sigma^2_{\beta_u}$, respectively. The income growth rate is given by $\beta^i = \beta_k^i + \beta_u^i$, implying $\sigma^2_\beta = \sigma^2_{\beta_k} + \sigma^2_{\beta_u}$. The key distinction between the two components is that individual i observes the realization of β_k^i, but not of β_u^i (hence, the subscripts indicate *k*nown and *u*nknown, respectively). Then the prior mean is $\hat{\beta}^i_{1|0} = \beta_k^i$, and the prior variance is $\sigma^2_{\beta,0} = \sigma^2_{\beta_u}$. To express the amount of prior uncertainty in relation to the heterogeneity in income growth rates, it is useful to define

$$\lambda \equiv \frac{\sigma_{\beta,0}}{\sigma_\beta},$$

which is simply the fraction of the population dispersion of income growth rates that represents uncertainty on the part of individuals at the time they enter the labor market. Two polar cases deserve special attention. When $\lambda = 1$, individuals do not have any *private prior information* about their income growth rate (i.e., $\sigma^2_{\beta,0} = \sigma^2_\beta$ and $\hat{\beta}^i_{1|0} = \overline{\beta}$ for all i, where $\overline{\beta}$ is the population average). At the other extreme, when $\lambda = 0$, β^i is revealed completely at time zero, and hence the individual faces no prior uncertainty about its value.

Updating Beliefs Over the Life Cycle. We now cast the optimal learning process as a Kalman filtering problem, which yields convenient recursive updating formulas for beliefs. Each individual knows his own α^i, observes his income, y_t^i, and must learn about $\mathbf{S}_t^i \equiv (\beta^i, z_t^i)$.[6] The *state equation* describes the evolution of the vector of state variables that is unobserved by the decision maker:

$$\underbrace{\begin{bmatrix} \beta^i \\ z^i_{t+1} \end{bmatrix}}_{\mathbf{S}^i_{t+1}} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & \rho \end{bmatrix}}_{\mathbf{F}} \underbrace{\begin{bmatrix} \beta^i \\ z^i_t \end{bmatrix}}_{\mathbf{S}^i_t} + \underbrace{\begin{bmatrix} 0 \\ \eta^i_{t+1} \end{bmatrix}}_{\boldsymbol{\nu}^i_{t+1}}.$$

The *observation equation* expresses the observable variable in the model—in this case, log income net of the fixed effect (denoted $\tilde{y}_t^i$)—as a linear function

[6]The assumption that α^i is observable is fairly innocuous here because the uncertainty regarding this parameter is resolved very quickly, even when the individual has substantial prior uncertainty, as shown in Guvenen (2007). Because knowing α^i gives some information about β^i (as long as the two parameters are correlated), it is natural to think of λ as already incorporating this information, which is the interpretation adopted from now on. One way to think about this interpretation is that β_k^i captures all the correlation between α^i and β^i, and $\beta_u^i \perp \alpha^i$. It is easily shown that there is an upper bound to λ (which we denote with $\lambda^{\max}$) that captures this minimum information obtained from α^i alone; it depends on σ^2_α, σ^2_β, and $\mathrm{corr}(\alpha, \beta)$ and equals 1 when $\mathrm{corr}(\alpha, \beta) = 0$ and 0 when $\mathrm{corr}(\alpha, \beta) = \pm 1$.

of the hidden state and the transitory shock:

$$\tilde{y}_t^i \equiv y_t^i - \alpha^i = [\,t \quad 1\,]\begin{bmatrix} \beta^i \\ z_t^i \end{bmatrix} = \mathbf{H}_t' \mathbf{S}_t^i + \varepsilon_t^i.$$

Both innovations $(\eta_t^i, \varepsilon_t^i)$ have i.i.d. normal distributions and are independent of each other. Each individual's prior belief over (β^i, z_1^i) is represented by a multivariate normal distribution with mean $\hat{\mathbf{S}}_{1|0}^i \equiv (\hat{\beta}_{1|0}^i, \hat{z}_{1|0}^i)$ and covariance matrix

$$P_{1|0} = \begin{bmatrix} \sigma_{\beta,0}^2 & 0 \\ 0 & \sigma_{z,0}^2 \end{bmatrix}.$$

After observing $(\tilde{y}_t^i, \tilde{y}_{t-1}^i, \dots, \tilde{y}_1^i)$, the posterior belief about $\mathbf{S}_t^i$ is normally distributed with a mean vector $\hat{\mathbf{S}}_t^i$ and covariance matrix $\mathbf{P}_t$. Associated with this belief distribution is a one-period-ahead *forecast* (distribution), characterized by a mean vector and a covariance matrix, $\hat{\mathbf{S}}_{t|t-1}^i$ and $\mathbf{P}_{t|t-1}$, obtained from beliefs as $\hat{\mathbf{S}}_{t|t-1}^i = \mathbf{F}\hat{\mathbf{S}}_{t-1}^i$ and $\mathbf{P}_{t|t-1} = \mathbf{F}\mathbf{P}_{t-1}\mathbf{F}' + \mathbf{Q}$, where $\mathbf{Q}$ is the covariance matrix of ν_t^i. With this notation, we can define a key variable—the *perceived innovation* to (log) income—as

$$\hat{\xi}_t^i \equiv \tilde{y}_t^i - \mathbb{E}_{t-1}(\tilde{y}_t^i) = \tilde{y}_t^i - (\hat{\beta}_{t|t-1}^i t + \hat{z}_{t|t-1}^i), \tag{2}$$

which does not necessarily have the same sign as the true innovation to income, η_t^i—a point that will play a crucial role below. The recursive Kalman updating formulas are given by

$$\hat{\mathbf{S}}_t^i = \hat{\mathbf{S}}_{t|t-1}^i + \mathbf{K}_t \times \hat{\xi}_t^i, \tag{3}$$

$$\mathbf{P}_t = (I - \mathbf{K}_t \mathbf{H}_t') \times \mathbf{P}_{t|t-1}, \tag{4}$$

where $\mathbf{K}_t \equiv \mathbf{P}_{t|t-1}\mathbf{H}_t[\mathbf{H}_t'\mathbf{P}_{t|t-1}\mathbf{H}_t + \sigma_\varepsilon^2]^{-1}$ is the (optimal) Kalman gain. Conditional on period $t-1$ beliefs, next period's log income (net of α^i) is normally distributed as

$$\tilde{y}_t^i | \hat{\mathbf{S}}_{t-1}^i \sim \mathcal{N}(\mathbf{H}_t'\hat{\mathbf{S}}_{t|t-1}^i, \mathbf{H}_t'\mathbf{P}_{t|t-1}\mathbf{H}_t + \sigma_\varepsilon^2). \tag{5}$$

2.2. *Understanding Identification: A Stylized Linear-Quadratic Framework*

Before delving into the details of the full estimation, it is useful to provide a better understanding of the sources of identification. For example, if individuals indeed differ in their income growth rates (i.e., $\sigma_\beta > 0$), how would this fact be revealed in their consumption-choice behavior? Similarly, can we detect the extent of an individual's prior uncertainty (that is, λ) about his β^i by observing the response of his consumption to income movements? And, finally, what kind of empirical relationship would allow us to measure the degree of partial insurance, θ, in the presence of these other features?

For this purpose, we begin with a stylized life-cycle model of the consumption–savings decision. Specifically, (i) individuals have quadratic utility over consumption, (ii) the time discount factor, δ, is the reciprocal of the gross interest rate, $1+r$, (iii) there are no borrowing constraints, and (iv) there is no retirement. Finally, we assume a simpler form of the income process in (1),

$$(6) \qquad Y_t^i = \alpha^i + \beta^i t + z_t^i,$$

where the income *level* (instead of its logarithm) is linear in the underlying components, and we set $\varepsilon_t^i \equiv 0$.[7] Under these assumptions, the consumption–savings problem can be written as

$$(7) \qquad V_t^i(\omega_t^i, \hat{\beta}_t^i, \hat{z}_t^i) = \max_{C_t^i, a_{t+1}^i} \left\{ -(C_t^i - C^*)^2 + \frac{1}{1+r} \mathbb{E}_t\left[V_{t+1}^i(\omega_{t+1}^i, \hat{\beta}_{t+1}^i, \hat{z}_{t+1}^i)\right] \right\}$$

$$(8) \qquad \text{s.t.} \quad C_t^i + a_{t+1}^i = \omega_t^i,$$

$$(9) \qquad \omega_t^i = (1+r)a_t^i + Y_t^{\text{disp},i},$$

and the Kalman recursions ((3) and (4) adapted to the level specification for income (6)), where C_t^i is the consumption level, C^* is the consumption "bliss" point, a_t^i is holdings of risk-free bonds, and ω_t^i is wealth (or "cash-on-hand").

Partial Insurance. Modeling partial insurance in an economy with learning requires some care. It seems plausible to assume that the informal risk-sharing mechanisms available in the society (which allow partial insurance) are subject to the same informational constraints faced by the individuals themselves. This means that insurance can only be based on perceived shocks (e.g., $\hat{z}_t$, $\hat{\xi}_t^i$) rather than on the true but unobservable sources of uncertainty (e.g., β^i or η). With this in mind, we specify disposable income as

$$(10) \qquad Y_t^{\text{disp},i} = Y_t^i - \theta \hat{\xi}_t^i,$$

where θ is the partial insurance parameter, which measures the fraction of the perceived income innovation that is insured, and $\hat{\xi}_t^i$ is now reinterpreted as the innovation in the *level* of income, that is, $Y_t^i - (\hat{\beta}_{t|t-1}^i t + \hat{z}_{t|t-1}^i)$. So, for example, when the realization of Y_t^i is lower than what was expected in the previous

[7] Closed-form solutions such as those below can still be derived in the presence of transitory shocks and retirement. We abstract from them here only for clarity of exposition.

period, $\hat{\xi}_t^i$ will be negative (by definition), and disposable income will be higher than labor income, thanks to the positive partial insurance term: $-\theta\hat{\xi}_t^i > 0$.

The framework described above is a much simplified version of the full model we estimate in Section 5. It is, however, general enough that it encompasses three special cases of interest that we will refer to in the rest of the paper. First, without any further restrictions imposed, the framework has a heterogeneous-income-profiles (HIP; following Guvenen (2007)) process with Bayesian learning about individual income slopes, that is, the HIP model. A second important benchmark is obtained when $\sigma_\beta \equiv 0$, in which case there is no heterogeneity in profiles and no Bayesian learning. Thus, the framework reduces to the standard RIP (restricted-income-profiles) model that is extensively studied in the literature. Finally, a third case of interest is when $\sigma_\beta > 0$ and $\lambda = 0$. In this case, individuals face a HIP process, but each individual knows his β^i at the time he enters the labor market. The only source of uncertainty in this case is the idiosyncratic shocks, as in the RIP model. This is an intermediate case between the HIP and RIP models.

2.2.1. *Information in Consumption Growth*

For clarity of exposition, in this section we abstract from partial insurance by setting $\theta \equiv 0$. In the next section, we reintroduce it and show that the conclusions of the current section extend to the case with partial insurance.

For the problem described by equations (6)–(9) with $\theta = 0$, optimal consumption choice satisfies

$$\Delta C_t^i = \varphi_t \left[\sum_{s=0}^{T-t} \gamma^s (\mathbb{E}_t - \mathbb{E}_{t-1}) Y_{t+s}^i \right], \tag{11}$$

where $\gamma \equiv 1/(1+r)$ and $\varphi_t \equiv (1-\gamma)/(1-\gamma^{T-t+1})$ is the annuitization factor. After substituting (6) into (11), some tedious but straightforward manipulations yield

$$\Delta C_t^i = \Phi(t; T, r)\big(\hat{\beta}_t^i - \hat{\beta}_{t-1}^i\big) + \Psi(t; T, \rho, r)\big(\hat{z}_t^i - \rho\hat{z}_{t-1}^i\big), \tag{12}$$

where Φ_t and Ψ_t are some age-dependent positive coefficients.[8] (In what follows, for clarity, we suppress all the arguments of Φ and Ψ except t, unless it creates confusion.) Finally, substituting equation (3) into (12) yields a key structural equation in this framework,

$$\Delta C_t^i = \Pi_t \times \hat{\xi}_t^i, \tag{13}$$

where Π_t is the response coefficient (formula given in Appendix A). This equation basically says that consumption changes proportionally to the perceived

[8] Full formulas are provided in Appendix A.

innovation to income, which, as we shall see, may or may not have the same sign as η_t^i.

If we eliminate income growth heterogeneity by setting $\sigma_\beta \equiv 0$ (and thereby also eliminate learning), the resulting (RIP) model implies

$$\Delta C_t^i = \Psi_t \times \eta_t^i. \tag{14}$$

The last two equations can be used to understand some of the key differences between the two frameworks.[9] When $\sigma_\beta \equiv 0$, only the current shock, η_t^i, matters for consumption response, whereas in the HIP model, the entire history of shocks matters—through beliefs. As a result, two individuals hit by the same η_t^i may react differently depending on their histories. We now present three examples that help explain the intuition behind the identification of two key parameters—σ_β and λ. For this purpose, we specialize to the case where $\rho = 1$, which makes the exposition much simpler (although the main conclusions we reach below hold for $\rho < 1$ as well). We also assume $\overline{\beta} = 0$, again, without loss of generality and for clarity of exposition.

EXAMPLE 1—Consumption Growth Depends Negatively on Past Income Growth: Consider Figure 1, which plots the income paths of two individuals up to period 6. Individual 1 experiences a faster average income growth rate in

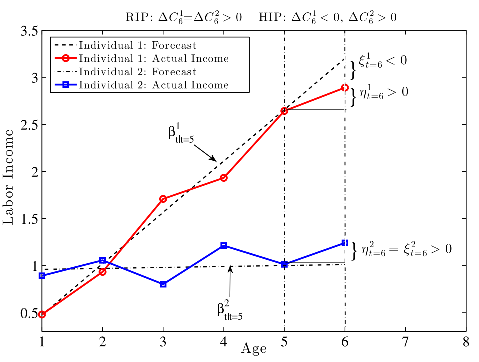

FIGURE 1.—Information about σ_β and λ in consumption changes.

[9]Before moving further, it is important to stress that equation (12) is obtained by fully solving the consumption–savings model and, therefore, requires (i) using the Euler equation, (ii) imposing the budget constraint, and (iii) specifying a stochastic process for income. In this sense, the analysis here is in the spirit of Hall and Mishkin (1982) (and, more recently, Blundell, Pistaferri, and Preston (2008)), who derive the full consumption function (as we do here), rather than Hall (1978), who requires only the Euler equation to hold. Therefore, by imposing stronger restrictions, the current approach allows us to estimate the parameters of the income process in addition to the preference parameters (which is all one could estimate with the Euler equation approach).

the first five periods than individual 2, but observes precisely the same rise in income between periods 5 and 6 ($\Delta Y_6^1 = \Delta Y_6^2$). If these income paths are generated by a process with $\sigma_\beta = 0$ (and thus $\beta^1 = \beta^2$), then the consumption choice of both individuals will satisfy equation (14), implying $\Delta C_6^1 = \Delta C_6^2 = \eta_6 > 0$. Instead, if the true income process has $\sigma_\beta > 0$ (HIP process), individual 1 will have formed a belief that his income growth rate is higher than that of individual 2 and will forecast his income to be on the (dashed) trend line. (Obviously, this remains true when $\lambda = 0$ and so when beliefs are perfectly accurate from the beginning.) Therefore, even though his income increases, it is below the expected trend ($\hat{\xi}_6^1 < 0$), which causes him to revise down his beliefs about β^1 and, consequently, reduce his consumption level, from equation (13). In contrast, based on his past income growth (which is nearly zero), individual 2 is positively surprised to see any increase in his income between periods 5 and 6 ($\hat{\xi}_6^2 > 0$) and will increase his consumption. Thus, we have $\Delta C_6^1 < 0$ and $\Delta C_6^2 > 0$ in response to the same income change for both agents.

The following proposition summarizes these implications in a more rigorous form. First, define $\Delta \overline{C}_t^i \equiv \mathbb{E}(\Delta C_t^i | \beta^i, \Delta Y_t^i)$. In words, $\Delta \overline{C}_t^i$ is the average consumption growth of an individual with slope parameter β^i and income growth ΔY_t^i between $t-1$ and t, where the expectation is taken over all possible histories up to $t-1$ (of initial signals and income realizations: $\{Y_1^i, Y_2^i, \ldots, Y_{t-1}^i, \hat{\beta}_{1|0}^i\}$). In other words, $\Delta \overline{C}_t^i$ can be thought of as the consumption change of a *typical* individual with β^i and who observed ΔY_t^i.

PROPOSITION 1—Information in Consumption Growth: *In the life-cycle certainty-equivalent model with permanent shocks described above, consumption growth satisfies the following properties*:

(i) *Controlling for current income growth, consumption growth will, on average, be a decreasing function of an individual's* β^i, *that is*, $\frac{\partial \Delta \overline{C}_t^i}{\partial \beta^i} < 0$ *for all* t.

(ii) *Although the prediction in* (i) *is true for all values of* λ (*including when* $\lambda = 0$ *and, hence, when there is no learning*), *the relationship becomes stronger as* λ *rises, that is*, $\frac{\partial^2 \Delta \overline{C}_t^i}{\partial \beta^i \partial \lambda} < 0$ *for all* t.

(iii) *Similarly* (*holding everything else constant*), *the response of consumption growth to income growth becomes stronger as* λ *increases*: $\frac{\partial^2 \Delta \overline{C}_t^i}{\partial \Delta Y_t^i \partial \lambda} > 0$.

See Appendix A for all of the omitted proofs and derivations.

To understand the empirical content of the proposition, note that even though β^i is not observed by the econometrician, in any given period, past and future income growth rates will be positively correlated with β^i. Therefore, the empirical relationship predicted by part (i) is that controlling for ΔY_t^i, consumption growth will be a decreasing function of the past income growth rate, which is observable by the econometrician. The second part of the proposition then implies that this negative dependence on the past income growth becomes stronger as individuals receive signals that are less informative at the beginning of life (i.e., a higher λ). Similarly, part (iii) implies that the response coefficient of consumption growth to income growth contains valuable information about the initial prior uncertainty faced by individuals. Loosely speaking, this is because when the initial signal is not very informative, optimal learning will result in a larger updating of beliefs about β^i in response to a given income realization, ΔY_t^i, which will in turn cause a larger change in consumption. Finally, it is easy to see that when $\sigma_\beta \equiv 0$, consumption growth will not depend on an individual's past or future income growth. As we shall see in the next section, we will use these observations to write an auxiliary model that captures the way in which consumption growth depends on past and future income growth rates, as well as how it responds to contemporaneous income growth, to infer the values of σ_β and λ as well as other parameters.

2.2.2. *Information in Consumption Levels*

We next turn to the information revealed in the levels of consumption and begin with the following useful lemma. (See Appendix A for the proofs.)

LEMMA 1: *The consumption decision rule can be solved in closed form as a linear function of the state vector* $(\omega_t^i, \hat{\beta}_t^i, \hat{z}_t^i)$:

$$C_t^i = \varphi_t \omega_t^i + \gamma \Phi_{t+1} \hat{\beta}_t^i + \gamma \rho \Psi_{t+1} \hat{z}_t^i. \tag{15}$$

This expression clearly shows that, at every point in time, consumption choice reveals valuable information about individuals' perceived future income prospects as reflected in $(\hat{\beta}_t^i, \hat{z}_t^i)$. We now state the key result of this section and then present two examples to illustrate how this information can be used.

PROPOSITION 2—Information in Consumption Levels: *Controlling for an individual's current income and assets, the consumption level is an increasing function of the individual's beliefs about his income growth rate,* $\hat{\beta}_t^i$. *This prediction holds true regardless of how much individuals know about their true income growth rate, that is, for all* $\lambda \in [0, 1]$. *However, if* $\sigma_\beta = 0$, *consumption depends only on current income and assets.*

PROOF: We present the proof in a way that is helpful for understanding the two examples that follow. Consider two individuals with $Y_t^1 = Y_t^2$, $\omega_t^1 = \omega_t^2$, and $\hat{\beta}_t^1 > \hat{\beta}_t^2$. Then we have

$$\begin{aligned} 0 = Y_t^1 - Y_t^2 &= (\hat{\beta}_t^1 - \hat{\beta}_t^2)t + (\hat{z}_t{}^1 - \hat{z}_t{}^2) \\ \Rightarrow \quad (\hat{\beta}_t^1 - \hat{\beta}_t^2)t &= -(\hat{z}_t{}^1 - \hat{z}_t{}^2). \end{aligned} \tag{16}$$

Taking the difference of the consumption level of each individual as given in equation (15) and using (16), we get

$$C_t^1 - C_t^2 = \gamma\Phi_{t+1}(\hat{\beta}_t^1 - \hat{\beta}_t^2) + \gamma\rho\Psi_{t+1}(\hat{z}_t^{\,1} - \hat{z}_t^{\,2}) = \gamma(\hat{\beta}_t^1 - \hat{\beta}_t^2)[\Phi_{t+1} - \rho t\Psi_{t+1}]. \tag{17}$$

Since $\hat{\beta}_t^1 - \hat{\beta}_t^2 > 0$ by hypothesis, $C_t^1 - C_t^2 > 0$ if and only if $\Phi_{t+1} - \rho t\Psi_{t+1} > 0$. The proof of Lemma A.1 in Appendix A establishes that $\Phi_{t+1} - t\Psi_{t+1} > 0$, which straightforwardly implies that $\Phi_{t+1} - \rho t\Psi_{t+1} > 0$ as well, since $\rho \leq 1$. Thus, controlling for current income and assets, an individual's consumption is higher if his (perceived) income growth prospect, $\hat{\beta}_t^i$, is higher. In contrast, when $\sigma_\beta = 0$, we have $\beta^1 = \hat{\beta}_t^1 = \beta^2 = \hat{\beta}_t^2 = \overline{\beta}$, which in turn implies from (17) that the two individuals will have the same consumption levels. *Q.E.D.*

To better understand the empirical content of the proposition, it is useful to study the following two examples. But first note that at any point in time, $\hat{\beta}_t^i$ depends on two things: (i) an individual's initial belief ($\hat{\beta}_{1|0}^i$) and (ii) the path of income realizations up to time t. Both examples consider two individuals with $\beta^1 > \beta^2$. In the first example, individuals enter the labor market with the same prior belief but experience different income paths consistent with their own β^i. The second example considers the opposite situation, with different priors but the same income history.

EXAMPLE 2—Past Income Growth Affects Current Consumption Level: Figure 2 plots a particular realization of income paths for two individuals. In this example, both individuals experience different growth rates up to period 3, but have $Y_3^1 = Y_3^2$. Now, if the true data-generating process has $\sigma_\beta = 0$ (i.e., $\beta^1 = \beta^2$) and individuals perceive it as such, then both individuals' forecasts

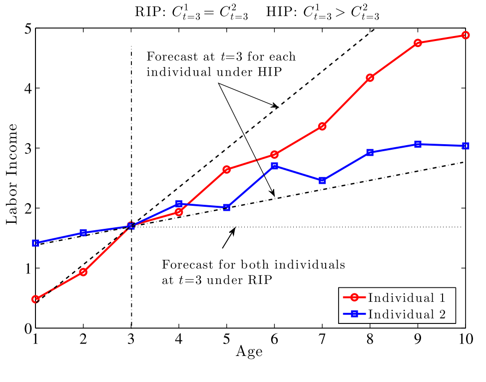

FIGURE 2.—Information about σ_β in consumption levels.

of their future income would be the same: $\mathbb{E}_3(Y^1_{3+s}) = Y^1_3 = Y^2_3 = \mathbb{E}_3(Y^2_{3+s})$ at all horizons $s \geq 0$ (shown in Figure 2 with the horizontal dotted line). Furthermore, if both individuals started life with no wealth, it is easy to see that $C^1_3 = Y^1_3 = Y^2_3 = C^2_3$. In contrast, when $\sigma_\beta > 0$, individuals know that they can have different β^i's and will use the past income growth to form beliefs about their own β^i. Based on the high past income growth (and the same prior beliefs), individual 1 will expect a higher β^i and, therefore, a much higher lifetime income than individual 2. (And if $\lambda = 0$, then each will know his β^i with certainty from the beginning.) Therefore, the first individual will have a higher consumption *level* than individual 2 at the same age, despite having the same income level.

EXAMPLE 3—Dependence of Consumption Level on Future Income Growth Reveals Prior Information: One can turn the same argument around to see how the level of consumption can also be informative about the degree of private information, λ. To show this, we turn to Figure 3, which is a slight variation of Figure 2. Consider two individuals with $\beta^1 > \beta^2$, who nevertheless experience the same income realizations up to period 3. Now if $\lambda = 1$, then both individuals will have the same beliefs in period 3 and, therefore, will choose the same consumption level. If, on the other hand, individuals have some prior information (i.e., $\lambda < 1$), the individual who starts out with a higher prior belief ($\hat{\beta}^1_{1|0} > \hat{\beta}^2_{1|0}$) will also have a higher belief at time t ($\hat{\beta}^1_3 > \hat{\beta}^2_3$) and, therefore, have a higher consumption level. This implication stands in contrast to the RIP model, which predicts no such dependence on past income levels (beyond what is captured by current income and assets). Moreover, since $\hat{\beta}^1_{1|0}$ is positively correlated with the true β^1, and both individuals in this example observed the same *past* income paths, the individual's prior belief will be correlated with his *future* income growth. Thus, controlling for current income and assets, and the past income path, the correlation between current consumption and future in-

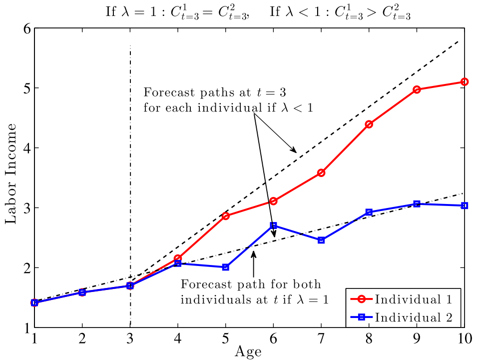

FIGURE 3.—Information about prior uncertainty.

come growth also reveals how much prior information the individual has. This is a useful prediction, as it shows how the observable variation in consumption and income can be used to infer the amount of prior information, which is unobservable.

These three examples illustrate how one can use the structural equations—such as (13), (14), and (15) that hold true exactly in a somewhat simplified version of the economic model to be estimated—to choose an auxiliary model. Indirect inference allows one to think in terms of these rich dynamic relationships instead of a set of moments (means, covariances, etc.). Below we shall write a parsimonious auxiliary model that captures these dynamic relationships to identify the key parameters of the income process.

The results of this section illustrate some important advantages of using the information revealed by intertemporal choices, such as consumption–savings, over using panel data on income alone. One difficulty of the latter approach is that identification between different income processes partly depends on the behavior of the *higher* order autocovariances of income (Guvenen (2009) contains a detailed discussion of this point). In contrast, because of its forward-looking nature, even short-run movements in consumption and the immediate response of consumption to income innovations contain information about the perceived *long-run* behavior of the income process, as can be seen from (13) and (15).

2.2.3. *Reintroducing Partial Insurance*

We now remove the restriction on θ and summarize the implications of partial insurance in the following proposition.

PROPOSITION 3—Partial Insurance: *With partial insurance, optimal consumption growth is given by*

$$\Delta C_t^i = (\Pi_t - \theta\varphi_t) \times \hat{\xi}_t^i. \tag{18}$$

The parameter θ is identified from the age profile of the response of consumption to income surprises.

To understand the empirical content of the proposition, first notice that the effect of partial insurance on consumption is separable from the effects of learning (captured by Π_t). Thanks to this separability, all of the results established in the previous section (that is, Propositions 1 and 2 and Lemma 1) continue to hold in the presence of partial insurance, with "disposable income" now playing the role of "income." Second, for a sufficiently low interest rate and/or long enough horizon, the annuitization factor φ_t is nearly constant (especially up to age 55, which is the age range we will be focusing on), implying that the effect of partial insurance is flat over the life cycle. In contrast, the effect of learning (Π_t) is either monotonically increasing or inverse U-shaped depending on the parameterization. Therefore, the age patterns of the two effects are distinctly different in response to a given shock $\hat{\xi}_t^i$. Third, it is clear that (as long as $\theta > 0$), the total response coefficient is now smaller than without partial insurance (but always positive), so, as expected, the response of consumption to income is muted. In Section 6.3, we examine sample paths for different individuals to illustrate the effects of partial insurance and how they depend on other variables, such as income level, wealth, age, and so on.

To summarize, the results of this section show that the dynamics of consumption and income contain rich information that can allow us to identify partial insurance as well as various aspects of the income process and individuals' prior information about their future income growth prospects.

3. THE FULL CONSUMPTION–SAVINGS MODEL

We now describe the full model estimated in the empirical analysis. Compared to the stylized framework in the previous section, the most significant changes are that here we relax the quadratic utility assumption, add a retirement period, and introduce borrowing constraints.

Specifically, in each period an individual faces an age-dependent probability of death, denoted by $p^{\mathrm{d}}_{t,t+1}$, and can live up to at most age T. An individual works for the first R ($< T$) years of his life, after which time he is retired. Preferences over consumption are given by the CRRA specification. As before, individuals can borrow and lend at the constant interest rate r, subject to an age-dependent lower limit as specified below. The relevant state variables for this dynamic problem are cash-on-hand (assets plus labor income), ω_t^i, and the vector of mean beliefs, $\hat{\mathbf{S}}_t = (\hat{\beta}_t^i, \hat{z}_t^i)$. Therefore, the dynamic program is

$$
\begin{aligned}
&V_t^i\big(\omega_t^i, \hat{\beta}_t^i, \hat{z}_t^i; \alpha^i\big) \\
&\quad = \max_{C_t^i, a_{t+1}^i} \left\{ \frac{(C_t^i)^{1-\phi}}{1-\phi} + \delta_{t+1}\mathbb{E}_t\big[V_{t+1}^i\big(\omega_{t+1}^i, \hat{\beta}_{t+1}^i, \hat{z}_{t+1}^i; \alpha^i\big)\big] \right\} \\
&\quad \text{s.t.} \quad C_t^i + a_{t+1}^i = \omega_t^i, \\
&\qquad\qquad \omega_t^i = (1+r)a_t^i + Y_t^{\mathrm{disp},i}, \\
&\qquad\qquad a_{t+1}^i \geq \underline{a}_t, \quad \text{and Kalman recursions (3) and (4)}
\end{aligned}
$$

for $t = 1, \dots, R-1$, where V_t^i is the value function of a t-year-old individual; $\underline{a}_t$ is an age-dependent borrowing limit, which will be specified in a moment. The discount factor embeds the survival probability: $\delta_{t+1} \equiv \overline{\delta}(1 - p^{\mathrm{d}}_{t,t+1})$, where $\overline{\delta}$ is the pure time discount factor.[10] Disposable income, $Y_t^{\text{disp},i}$, is income inclusive of the partial insurance via informal mechanisms, as described by equations (19) and (20) below. The evolution of the vector of beliefs and its covariance matrix are governed by the Kalman recursions ((3) and (4)). Finally, the expectation is taken with respect to the conditional distribution of $\tilde{y}_{t+1}^i$ (5).

Partial Insurance. In the full model, we use the log specification for income given in (1) and thus modify the way partial insurance works by expressing it in logs:

$$y_t^{\text{disp},i} \equiv y_t^i - \theta \hat{\xi}_t^i, \tag{19}$$

where $\hat{\xi}_t^i$ is as given by equation (2). An alternative expression can be obtained by substituting the expression for $\hat{\xi}_t^i$ from equation (2) into (19), yielding

$$\begin{aligned} y_t^{\text{disp},i} &= y_t^i - \theta\big(\tilde{y}_t^i - \mathbb{E}_{t-1}(\tilde{y}_t^i)\big) = y_t^i - \theta\big((y_t^i - \alpha^i) - (\mathbb{E}_{t-1}(y_t^i) - \alpha^i)\big) \\ &= (1-\theta) y_t^i + \theta \mathbb{E}_{t-1}(y_t^i). \end{aligned}$$

Now it can be seen that disposable income is a convex combination of actual income and the expected income in period t. When $\theta = 0$, there is no partial insurance. When $\theta = 1$, disposable income equals expected income, so any shock in period t is completely insured via informal mechanisms. Thus, θ provides a useful measure of partial insurance. The level of disposable income is obtained by exponentiating (19) and adding an income floor:[11,12]

$$Y_t^{\text{disp},i} = \underline{Y} + \exp(y_t^{\text{disp},i}). \tag{20}$$

Borrowing Constraints. As discussed above, the tightness of the borrowing constraints can have a potentially large impact on the estimates of the income

[10]Clearly, some individuals will die with debt as long as $\underline{a}_t < 0$. Since we are not conducting a general equilibrium analysis, it is not necessary to model explicitly how this debt is disposed of. The probability of death is very low up to age 55, so the behavior up to that age is little affected. After retirement, we assume the presence of perfect annuity markets.

[11]The process in (1) is richer than most of the specifications used to calibrate incomplete markets models, yet it still allows meaningful empirical identification. Although one could postulate even more general processes (for example, allowing for separate permanent and persistent shocks components, or considering household-specific quadratic terms), empirical identification would be problematic given the limited size of Panel Study of Income Dynamics (PSID) samples. Guvenen, Karahan, Ozkan, and Song (2014) consider such richer specifications but have access to a substantially larger and cleaner data set from administrative records. However, they do not have any consumption data, which is the focus of the present paper.

[12]The purpose of the income floor, $\underline{Y} > 0$, is to make sure that income realizations are never too close to zero, which can happen since all the terms in y_t^i are normally distributed and, thus, have no lower bound. The possibility of a zero income state would make borrowing effectively impossible (since the household would have no funds to pay back its debt), which does not seem realistic given the sizable uncollateralized borrowing by households observed in the data.

process parameters. Therefore, rather than picking a (an arbitrary) value for $\underline{a}_t$ beforehand, we estimate the borrowing limit along with the rest of the structural parameters. Our starting point is the *natural* borrowing limit, which is essentially the *loosest* limit that still guarantees full repayment by the last period (T) even if the household gets the lowest income realization in every period. Here, this limit would be $\underline{a}_t = \sum_{\tau=1}^{T-t} \gamma^\tau \min(Y_\tau) = \underline{Y}\frac{1-\gamma^{T-t}}{1-\gamma}$. Although this is a conceptually clean and useful benchmark, it has the somewhat questionable implication that households face a looser constraint when young rather than when old, which is the opposite of what we seem to observe in real life. To capture this possibility in a simple fashion (without introducing the complications of default and credit rating), we assume that banks use a potentially higher interest rate to discount households' future labor income during working years in calculating their borrowing limit, but simply apply the risk-free rate for discounting retirement income. That is, we define

$$\underline{a}_t \equiv \underline{Y}\left[\sum_{\tau=1}^{R-t}(\psi\gamma)^\tau + \psi^{R-t+1}\sum_{\tau=R-t+1}^{T-t}\gamma^\tau\right],$$

where $\psi \in [0, 1]$ measures the tightness of the borrowing limit. When $\psi = 0$, no borrowing is allowed against future labor income; when $\psi = 1$, households can borrow up to the natural limit. This specification generates borrowing limits that become looser (tighter) with age when ψ is sufficiently low (close to 1). The tightness parameter ψ will be estimated in the empirical analysis.

Retirement Period. During retirement, households receive annual pension payments from a retirement system that mimics the salient features of the U.S. Social Security Administration's Old-Age Insurance Benefits System. Since there is no uncertainty (or learning) after retirement, the problem simplifies significantly:

$$V_t^i(\omega_t^i; Y) = \max_{c_t^i, a_{t+1}^i}\left[\frac{(C_t^i)^{1-\phi}}{1-\phi} + \delta_{t+1}V_{t+1}^i(\omega_{t+1}^i; Y)\right], \tag{21}$$

$$\text{s.t.} \quad Y^i = Y(Y_R^i; \overline{Y}), \quad \text{and (8) and (9) hold}$$

for $t = R, \dots, T$, with $V_{T+1} \equiv 0$.

Social Security System. The pension system in the model—captured by the function Y—mimics the U.S. Social Security system, with one notable difference. In the actual U.S. pension system, retirement income is tied to households' average labor income during the working years (denoted by $\overline{Y}^i$).[13] Adopting this exact structure here, however, would add another state variable—$\overline{Y}^i$—to the dynamic problem above, increasing the already high

[13]More precisely, the average is taken over the 35 working years with the highest earnings.

computational burden of the estimation. So, instead, we adopt the same functional form used in the U.S. system for the $Y(\cdot)$ function, but rather than use $\overline{Y}^i$, we instead use the *predicted* average income given the worker's income at retirement age (Y_R^i). This is accomplished by first running the cross-sectional regression $\overline{Y}^i = k_0 + k_1 Y_R^i$ and then using the predicted average income implied by this regression, which we denote by $\hat{Y}(Y_R^i)$.[14] This structure does not add a state variable but recognizes the empirical relationship between average income and the income at retirement age implied by each stochastic process. Letting $\overline{Y}$ denote the economy-wide average lifetime labor income and defining $\tilde{Y}_R^i \equiv \hat{Y}(Y_R^i)/\overline{Y}$, the pension function is given by

$$Y(Y_R^i; \overline{Y}) = \overline{Y} \times \begin{cases} 0.9\tilde{Y}_R^i, & \text{if } \tilde{Y}_R^i \leq 0.3, \\ 0.27 + 0.32(\tilde{Y}_R^i - 0.3), & \text{if } 0.3 < \tilde{Y}_R^i \leq 2, \\ 0.81 + 0.15(\tilde{Y}_R^i - 2), & \text{if } 2 < \tilde{Y}_R^i \leq 4.1, \\ 1.13, & \text{if } 4.1 \leq \tilde{Y}_R^i. \end{cases}$$

4. THE DATA

This section discusses the data used in the empirical analysis and provides definitions for key variables. The unit of analysis in this paper is a married household—so both income and consumption are measured at the household level.

Constructing a Panel of Imputed Consumption. The Panel Study of Income Dynamics (PSID) has a long panel dimension but covers limited categories of consumption expenditures, whereas the Consumer Expenditure Survey (CE) has detailed expenditures over a short period of time (four quarters). As a result, most previous work has either used food expenditures (e.g., Hall and Mishkin (1982), Altug and Miller (1990), and Hayashi, Altonji, and Kotlikoff (1996)) as a measure of nondurable consumption (available in PSID) or resorted to using repeated cross sections from the CE under additional assumptions.

Blundell, Pistaferri, and Preston (2006) developed a structural method that imputes consumption expenditures for PSID households using information from the CE survey. These authors showed that several statistics calculated using the imputed measure compare quite well with their counterparts from the CE data. However, because the CE data set is available on a continuous basis only after 1980, their method was tailored to generate imputed consumption for the PSID from 1980 to 1992. In this paper, we modify and extend the method proposed by these authors by also using the information in the (large)

[14]We run this regression using data simulated from the structural model given the values of its parameters.

1972–1973 CE waves to obtain an imputed panel that covers the period 1968–1992. We also conduct a detailed validation study, examining a broader set of statistics to show that the method works well for this longer sample. Appendix C.3 contains the details.

Measure of Household Labor Income. The household labor-income data come from the PSID. We restrict attention to households that are in the nationally representative core sample, whose head is between the ages of 25 and 55 (inclusive), and has nonmissing data on food expenditures and head and wife's labor income. In the PSID, households report their total taxable income, which includes labor income, transfers, and financial income of all the members in the household. The measure of labor income that we use subtracts financial income from this measure, and, therefore, includes the labor income of the head and wife as well as several categories of transfer income (unemployment benefits, Social Security income, pension income, worker's compensation, welfare payments, child support, financial help from relatives, and so on). We then subtract the labor portion of income taxes paid by each household. A more complete description of the sample selection criteria we use and other details on the PSID (such as the method for estimating taxes) are contained in Appendix C.

Converting the Data to Per-Adult Equivalent Units. We adjust both the imputed consumption and income measures for demographic differences across households, since such differences have no counterpart in our model. This is accomplished by regressing each variable on family size, a race dummy, region dummies, a dummy indicating whether the head is employed, a dummy indicating residence in a large city, and a set of cohort dummies.[15] We then use the residuals of these regressions—which are interpreted as consumption and income per-adult equivalent—in the analysis below.

5. ECONOMETRIC APPROACH

We now describe the method—indirect inference—used to estimate the parameters of the structural model laid out in the previous section. Indirect inference is a simulation-based estimation method whose hallmark is the use of an "auxiliary model" to capture aspects of the data upon which to base the estimation. Indirect inference permits significant flexibility in choosing an auxiliary model: it can be any statistical model relating the model variables to each other provided that each structural parameter has an independent effect on at least one (reduced-form) parameter of the auxiliary model.[16] Like other simulation-based methods, indirect inference can, therefore, accommodate many realistic

[15]Each cohort is defined by 5-year bands based on the birth year of each head of household, for example, those born between 1951 and 1955, 1956 and 1960, and so forth.

[16]More formally, indirect inference is consistent if the mapping from structural parameters to the parameters of the auxiliary model has full rank near the true structural parameter vector. For more details, see Smith (1993) and Gourieroux, Monfort, and Renault (1993).

features (such as borrowing constraints, a rich specification for utility functions) that methods based on linear-quadratic models, which form the basis of most research on consumption–savings decisions, cannot. We elaborate on this point in the next section and then in the following section describe the auxiliary model that we use in estimation.

5.1. *A Brief Digression: Identifying Risk Aversion and Borrowing Constraints*

The standard method for estimating consumption–savings models since Hall and Mishkin (1982) has been to derive explicit structural expressions that link observable variables (such as consumption and income) to unobservable variables (such as persistent and transitory income shocks). It is useful to contrast this approach with the indirect inference method we employ in this paper. To this end, consider the certainty-equivalent consumption–savings model described above, but in keeping with earlier work (Hall and Mishkin (1982), Blundell, Pistaferri, and Preston (2008)), suppose that the income process is the sum of a permanent and a transitory shock, which implies $\Delta Y_t = \eta_t + \Delta\varepsilon_t$, where $\eta_t, \varepsilon_t \sim$ i.i.d. Here, it can be shown that $\Delta C_t = \eta_t + \varphi_t \varepsilon_t$. These two equations can be jointly used to estimate the ratio of shock variances $(\sigma_\eta^2/\sigma_\varepsilon^2)$, which is a measure of the persistence, or durability, of shocks. To see how, consider the regression

$$(22)\qquad \Delta C_t = \pi \times \Delta Y_t + \text{ error}, \quad \text{where} \quad \pi \equiv \frac{1+\varphi_t(\sigma_\varepsilon^2/\sigma_\eta^2)}{1+2(\sigma_\varepsilon^2/\sigma_\eta^2)}.$$

Thus, $\sigma_\eta^2/\sigma_\varepsilon^2$ can be identified by estimating π from this regression. When income shocks are permanent (i.e., $\sigma_\varepsilon^2 = 0$), we get $\pi = 1$, and consumption moves in lockstep with income from (22). At the other extreme, when $\sigma_\eta^2 = 0$, $\pi = \varphi_t/2 \approx 0$, implying that consumption fluctuations will be much smoother than those in income. This identification strategy is commonly used in the literature, where a set of coefficients (such as π above) is typically estimated using a minimum-distance method. Notice, however, that even in this simple example, inference is feasible only because we have two exact expressions that link ΔC_t and ΔY_t to η_t and ε_t, which were used to derive the expression for π. It follows that the consistency of the estimates relies—potentially critically—on the validity of the assumptions we made above (quadratic utility, no borrowing constraints, etc.) to make the derivation of these two equations feasible.

To illustrate some of the potential difficulties with this approach, now consider the example shown in the left panel of Figure 4. The line marked with squares is income, and the dashed line marked with circles is consumption. (Ignore the dashed-dotted line for now.) The main observation that is obvious in this picture is that the individual's consumption and income move almost one-for-one during most of his life (from 25 to 57). Thus, using (22)

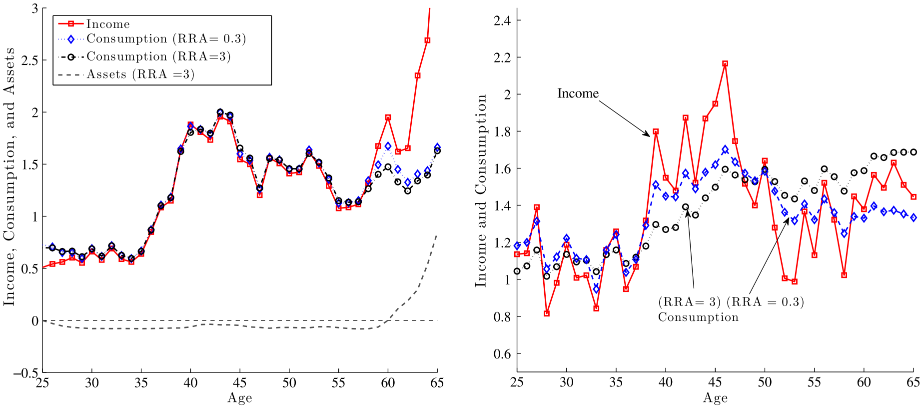

FIGURE 4.—Inferring persistence of shocks using generalized method of moments (GMM) moment conditions.

for inference would lead one to conclude that income shocks are nearly permanent. Indeed, a method of moments estimation using the coefficient above (i.e., $\sigma(\Delta C_t, \Delta Y_t)/\sigma^2(\Delta Y_t)$) as the only moment yields $\sigma_\eta^2/\sigma_\varepsilon^2 = 27.2$—that is, income shocks are almost completely permanent, when in fact the true ratio used in the simulation was $\sigma_\eta^2/\sigma_\varepsilon^2 = 2$!

To understand the source of this substantial bias, now consider the dashed-dotted line, which plots the asset position (scaled to fit in the figure): the household is right up against the constraint up to the mid-50s, after which time savings for retirement starts to kick in. But borrowing constraints had to be ignored to derive (22), which turns out to be critical for this household. In other words, for this individual, consumption moves one-for-one with his income because he is not able to borrow any further—not because income shocks are permanent and he chooses to fully accommodate these shocks. Although this example is clearly an extreme case, it sounds a cautionary note that this common assumption has the potential to bias inference if a nonnegligible fraction of households are borrowing constrained.

A second example is shown in the right panel of Figure 4. A particular income path is plotted here along with the consumption path that would be chosen by an individual when his risk aversion is, respectively, 3 (circles) and 0.3 (diamonds).[17] As could be expected, when the individual has higher risk aversion, the consumption path is much smoother than when he is more risk tolerant. Now assume that the econometrician observes the consumption path of the risk tolerant individual, but since risk aversion is not observable, he assumes a value of 3 as a reasonable figure. (When equation (22) above is derived

[17]Although assets have not been plotted to save space, the individual is never constrained in either case.

using quadratic utility, an assumption about the value of risk aversion is made automatically.) In this case, observing the strong response of consumption to income, the econometrician would be led to overestimate the persistence of income shocks. Of course, the opposite case would arise if the econometrician assumes a risk aversion lower than the true value. The indirect inference approach we use in this paper, like other simulation-based methods, allows us to relax several of these assumptions, model and estimate the tightness of the borrowing constraints and the time discount factor, and explore the effects of different risk aversion parameters on the estimates of income dynamics parameters.

5.2. *A Parsimonious and Feasible Auxiliary Model*

Although indirect inference shares a basic similarity to the method of simulated moments (MSM), it differs from it in its use of an auxiliary model to generate moment conditions. In particular, indirect inference allows one to think in terms of the dynamic structural relationships that characterize most economic models (such as (13) and (15)) but are difficult to express as simple unconditional moments. In this section, we describe a set of linear equations suggested by the linear-quadratic approximation to our structural model to serve as an auxiliary model. Because the auxiliary model is linear, it clearly cannot be an exact representation of the nonlinear structural model, but it can approximate it more closely than MSM typically can using simple unconditional moments (see, for example, Magnac, Robin, and Visser (1995)). Moreover, the approach bears some similarities to the early literature on estimation of linear rational expectations models subject to the "cross-equation" restrictions imposed by the structural model on the coefficients of the (linear) reduced-form equations.

In light of Propositions 1 and 2, equations (13) and/or (15) are ideal candidates to form the basis of an auxiliary model. For example, the response coefficient in equation (13) is $\Pi_t(\lambda, \sigma_\beta, \sigma_{\alpha\beta}, \sigma_\eta, r, \rho; R, T)$, which depends on several key variables that we wish to estimate.[18] The presence of the unobserved beliefs $\hat{\beta}^i_{t|t-1}$ and $\hat{z}^i_{t|t-1}$, however, makes it impossible to use these equations directly as an auxiliary model. But, as discussed in Section 2.2, current beliefs depend both on an individual's prior beliefs before beginning to work and on income realizations over the working life. Furthermore, prior beliefs are likely to be correlated with future income realizations. We can, therefore, use leads and lags of income as proxies for beliefs, leading to the following equation for consumption that depends only on observables:

$$(23)\qquad \begin{aligned} c_t &= \mathbf{a}'\mathbf{X}_{c,t} + \epsilon^c_t \\ &= a_0 + a_1 y_{t-1} + a_2 y_{t-2} + a_3 y_{t+1} + a_4 y_{t+2} + a_5 \overline{y}_{1,t-3} \end{aligned}$$

[18]The dependence of Π on λ, σ_β, $\sigma_{\alpha\beta}$, and σ_η can be seen from the formulas for A_t and B_t.

$$+ a_6\overline{y}_{t+3,R} + a_7\Delta y_{1,t-3} + a_8\Delta y_{t+3,R} + a_9 c_{t-1}$$
$$+ a_{10}c_{t-2} + a_{11}c_{t+1} + a_{12}c_{t+2} + \epsilon_t^c,$$

where c_t and y_t are the log of consumption and log of income, respectively; $\Delta y_{\tau_1,\tau_2}$ and $\overline{y}_{\tau_1,\tau_2}$ are, respectively, the average of the growth rate and the average of the level of log income from time τ_1 to τ_2; and $\mathbf{a}$ and $\mathbf{X}_{c,t}$ denote the vectors of coefficients and regressors. The use of logged variables in this regression seems natural given that the utility function is CRRA and income is log normal. By adding past and future income growth rates as well as past and future income levels, this regression captures the predictions made by the HIP and RIP models. Leads and lags of consumption capture the dynamics of consumption around the current date.

To complete the auxiliary model, we add a second equation with y_t as the dependent variable and use all of the income regressors above as right-hand-side variables:

$$(24) \qquad y_t = \mathbf{b}'\mathbf{X}_{y,t} + \epsilon_t^y$$
$$= b_0 + b_1 y_{t-1} + b_2 y_{t-2} + b_3 y_{t+1} + b_4 y_{t+2} + b_5\overline{y}_{1,t-3} + b_6\overline{y}_{t+3,R}$$
$$+ b_7\Delta y_{1,t-3} + b_8\Delta y_{t+3,R} + \epsilon_t^y.$$

We divide the population into two age groups—those between 25 and 38 years of age, and those between 39 and 55 years of age—and allow the coefficients of the auxiliary model to vary across the two groups.[19] For each age group, the auxiliary model has 22 regression coefficients (13 in the first equation and 9 in the second), two residual variances, and the correlation between the two residuals for a total of 25 parameters. With two age groups, this yields a total of 50 reduced-form parameters that determine the likelihood of the auxiliary model.

The goal of our estimation procedure, then, is to choose the parameters of the structural model so that the auxiliary model parameters estimated using the observed data are "close" to those estimated using data simulated from the model. Appendix B describes the metric we use to measure the distance between two sets of auxiliary-model parameters; we minimize this metric by maximizing a "Gaussian objective" constructed out of this auxiliary model subject to the restrictions that the structural model imposes on its parameters (full formulas are provided in Appendix B). Using a simple example, Appendix A.4 shows that our approach is asymptotically identical to minimizing a quadratic

[19]Although the auxiliary model would correspond to the structural equations in (13) and (15) more closely if the coefficients were varying freely with age, this would increase the number of parameters in the auxiliary model substantially. We have experimented with having one or three age groups, but found the small sample performance to be better with the specification adopted here (see Appendix B.4).

form in the difference between the two sets of parameters with a specific choice for the weighting matrix.

Appendix B.4 presents the results of a Monte Carlo study to gauge the ability of the auxiliary model described above to identify the structural parameters. This study shows that it works very well, with minimal bias and tight confidence intervals. Alternative auxiliary models with more or less parsimony also perform reasonably well, though not as well as the one we use to obtain our empirical results.

5.3. *Empirical Preliminaries*

Preset Values. Working life is $R = 41$ years, and the retirement duration is 15 years ($T = 80$). The price of the one-period discount bond is set to 0.95, implying an interest rate of $r = 1/0.95 - 1 \approx 5.26\%$. The common life-cycle profile of log income ($g(\cdot)$ in (1)) is captured by feeding into the model the empirical profile computed from our PSID sample. The income floor, $\underline{Y}$, is set to 5% of average income in this economy. In consumption–savings models without a stochastic interest rate or portfolio decision, such as ours, empirical identification between risk aversion, ϕ, and the time discount factor, $\overline{\delta}$, is tenuous at best.[20] Therefore, in our benchmark case, we fix ϕ at 2 and estimate $\overline{\delta}$. Later we will conduct detailed sensitivity analyses with respect to the values of the parameters preset in this section (including ϕ, r, and $\underline{Y}$). Age-dependent death probabilities, $p^d_{t,t+1}$, are taken from Bell and Miller (2002) for males.

Notice that the initial draw of the persistent process, z_0, determines the initial dispersion of the persistent shock. In the benchmark estimation, we set $\sigma_{z_0} \equiv 0$, so that all individuals start with $z^i_0 \equiv 0$, so $z^i_1 \equiv \eta^i_1$, and $\text{var}_i(y^i_1) = \sigma^2_\alpha + \sigma^2_\beta + \sigma^2_\eta + \sigma^2_\varepsilon$. In the robustness analysis, we relax this assumption and estimate σ_{z_0}, and find that it is quite small.

Measurement Error. We add measurement error to simulated consumption and income data,

$$y^{i,*}_t = y^i_t + u^{i,y}_t,$$
$$c^{i,*}_t = c^i_t + \overline{u}^{i,c} + u^{i,c}_t,$$

where $y^{i,*}_t$ and $c^{i,*}_t$ are measured variables of household i, and $u^{i,y}_t$ and $u^{i,c}_t$ are zero-mean random variables that are independent over time, with standard deviations of σ_y and σ_c.[21] Notice that we have also added a second term to

[20]While this point has been noted in the literature, we further discuss and illustrate it in Appendix D.6.

[21]The variable $u^{i,c}_t$ contains the imputation error, which is heteroskedastic owing to the mechanics of the imputation procedure (a point also observed by Blundell, Pistaferri, and Preston (2006)). In our estimation, we will impose stationarity in variances to deal with the computational burden, but further work is needed on the effects of this assumption.

consumption, $\overline{u}^{i,c}$, which is an individual fixed measurement error with potentially nonzero mean in the cross section, μ_{c_0}, and standard deviation σ_{c_0}. This fixed effect is needed for two reasons. First, and most important, recall that we regress both income and consumption on a set of demographics to convert these variables into per-adult equivalent terms. One effect of this adjustment is to introduce level differences between consumption and income, the magnitudes of which vary across households. This fixed effect captures such differences. Second, the model described above abstracts from initial wealth differences across households. These differences in wealth would also drive a household-specific wedge between the levels of income and consumption. The fixed effect is also a simple way to capture these differences in initial wealth levels.

Missing Observations. In the observed data set, we include only households with at least five observations between the ages of 25 and 55 (of the head), for a total of 2,235 households with an average of 12 observations on each (for a total of 26,441 household-year observations). With more than half of the observations missing compared to a fully balanced panel, one question is "How does one run the regressions in (23) and (24)?" For missing values of regressors, we simply use values that are constructed or "filled in" using a reasonable procedure.[22] However, on the left-hand side of regressions, we only use actual (i.e., not filled in) observations. As will become clear in the Monte Carlo analysis, a strength of the indirect inference method is that the particular filling-in method is not critical for the estimation as long as the same procedure is applied consistently to real and simulated data (as we do). As an extreme example, if we simply fill in all missing values with zeros, the estimates would still be consistent, as we show in the Monte Carlo analysis.

Matching the Wealth-to-Income Ratio. The auxiliary model specified above does not explicitly target the amount of savings and wealth generated by the estimated model. One goal of this paper, however, is to provide estimates of income processes (together with combinations of time discount factor, risk aversion, and borrowing constraints) that can be used for calibrating life-cycle models to be used in quantitative macroeconomic analysis. For this purpose, it is important to make sure that the estimated model yields a reasonable amount of savings.

To ensure this, we add one static moment condition—the scaled squared difference between the median wealth-to-income ratio (denoted WY) in the PSID and that implied by the simulated data ($10 \times (WY_{\text{PSID}} - W_{\text{SIM}})^2$)—to the objective function that the indirect inference procedure is minimizing. To calculate the empirical target, WY_{PSID}, we use the wealth supplement of PSID, available in 1984 and 1989. We use households in our estimation sample that were also present in the PSID in 1984 and/or 1989, and use net worth as the measure of wealth (see Appendix C.2 for the precise definition). The median value of WY

[22]Appendix B.3 contains the details.

is 0.99 in 1984 and 1.17 in 1989, averaging 1.08, which we take as our empirical target.[23]

6. ESTIMATION RESULTS

In this section, we present the estimation results. In Section 6.1, we discuss the structural parameter estimates. In Section 6.2, we compare the implications of the estimated model to the data for life-cycle patterns of income and consumption that has received a lot of attention in the literature. Section 6.3 studies simulated paths for consumption and income under different estimated models to shed light on how the joint dynamics of consumption and income can be informative about various model parameters.

6.1. *Structural Parameters*

The vector of structural parameters that is estimated is

$$(\sigma_\alpha, \sigma_\beta, \mathrm{corr}_{\alpha\beta}, \rho, \sigma_\eta, \sigma_\varepsilon; \lambda, \theta, \overline{\delta}, \psi; \sigma_y, \mu_c, \sigma_c, \sigma_{c_0}).$$

The parameter estimates are reported in Table I.

Parameters of the Income Process: ρ, σ_η, σ_ε, σ_α, σ_β, $\mathrm{corr}_{\alpha,\beta}$. The first column reports the results from our benchmark model described in Section 3. First, the AR(1) process has an estimated annual persistence of 0.756 and an innovation standard deviation of 22.7% (log percent), both estimated precisely. Below, we present sensitivity analyses that show that the estimates of persistence are quite robust. Therefore, we conclude that the joint dynamics of consumption and income data do not lend support to permanent shocks as a reasonable representation of income shocks. That said, these results should not be interpreted as evidence against the existence of permanent shocks—as even the most casual observation tells us that such shocks exist. Rather, these results indicate that when income shocks are modeled as a univariate persistent process, the *typical* shock received by a *typical* household is better represented as having a moderate persistence.

Turning to the fixed heterogeneity across households, first, the dispersion of the fixed (level) effects in income, σ_α, is 0.288. This figure is consistent with the estimates in the existing literature. The standard deviation of income *growth rates*—a key parameter of interest—is estimated to be 1.76%, which is substantial. For example, by age 55, an individual with a β^i that is 1 (2) standard deviation(s) above the mean will earn 1.68 times (2.80 times) the median income. Moreover, both σ_α and σ_β are estimated precisely, with t-statistics exceeding 10. Finally, the correlation between these two parameters ($\mathrm{corr}_{\alpha\beta}$) is

[23]When we include all households up to age 65, the corresponding ratios are 1.19 and 1.43.

TABLE I

ESTIMATING THE FULL CONSUMPTION–SAVINGS MODEL[a]

Data:	Income and Consumption			Income
Partial Insurance?	*Benchmark* Insure $\hat{\xi}$ (1)	Yes Insure $\hat{z}$ (2)	Self-Insurance ($\theta \equiv 0$) (3)	(4)
Income Processes Parameters (can be identified with income data alone)				
σ_α	0.288	0.286	0.265	0.298
	(0.017)	(0.017)	(0.022)	(0.038)
σ_β	1.764	1.881	1.660	1.343
	(0.137)	(0.131)	(0.118)	(0.271)
$\text{corr}_{\alpha\beta}$	−0.127	−0.140	−0.112	0.558
	(0.102)	(0.090)	(0.121)	(0.289)
ρ	0.756	0.755	0.768	0.783
	(0.023)	(0.021)	(0.025)	(0.022)
σ_η	0.227	0.427	0.196	0.200
	(0.007)	(0.012)	(0.005)	(0.005)
σ_ε	0.100	0.004	0.008	0.147
	(0.016)	(0.018)	(0.021)	(0.005)
Economic Model Parameters (need consumption data)				
λ (prior uncertainty)	0.438	0.429	0.345	–
	(0.045)	(0.042)	(0.074)	–
θ (partial insurance)	0.451	0.552	0.00*	–
	(0.028)	(0.031)	–	
ψ (borrowing constraint)	0.582	0.859	0.855	–
	(0.040)	(0.048)	(0.083)	
$\overline{\delta}$ (subjective time discount factor)	0.953	0.955	0.956	–
	(0.001)	(0.001)	(0.001)	–
Measurement Error and Transitory Shocks (need consumption data)				
σ_y	0.165	0.146	0.148	–
	(0.006)	(0.005)	(0.007)	–
σ_c	0.355	0.356	0.356	–
	(0.007)	(0.006)	(0.002)	–
σ_{c_0}	0.430	0.429	0.427	–
	(0.011)	(0.011)	(0.009)	
Max % constrained...	17.4%	21.8%	19.5%	
...at age	29	27	27	
$\underline{a}_{25}/\mathbb{E}(Y^i)$	0.08	0.13	0.11	–
$\underline{a}_{55}/\mathbb{E}(Y^i)$	0.06	0.11	0.09	–

[a]Standard errors (in parentheses) are obtained via parametric bootstrap with 140 repetitions. * θ is restricted to be zero.

small but negative, –0.13, though it cannot be statistically distinguished from zero. We can, however, easily reject a strong correlation of either sign.

The parameters discussed so far can all be identified with income data alone. To better understand what consumption data bring, it is instructive to compare these estimates to those obtained by using only the income regression (24), reported in column 4. The main noticeable differences are in the estimates of σ_β and $\text{corr}_{\alpha\beta}$. The remaining parameters are very similar across the two cases, suggesting that they are pinned down very well with income data and that there is no conflicting information in consumption data. The estimate of σ_β is lower, at 1.34%, compared to the benchmark case, but now $\text{corr}_{\alpha\beta}$ is positive and fairly large: 0.56. As a result, the rise in income inequality generated by heterogeneous profiles in this case is, if anything, slightly larger than in the benchmark case (by 4 log points). Finally, notice that the standard errors of these two parameters are significantly higher when estimated with income data alone, suggesting that the two parameters are not identified very precisely with income data alone. Consumption data are especially informative about these two key parameters.

Parameters Identified With Consumption Data: λ, θ, $\overline{\delta}$, ψ. Now we return to column 1 in Table I and discuss the structural estimates pertaining to the economic model. First, notice that all four parameters—and especially λ and θ, which are of key interest—are precisely estimated with *t*-statistics exceeding 10. Thus, the joint dynamics of consumption and income contain sufficiently rich information to tightly pin down these parameters.

The estimated value of $\lambda = 0.438$ reveals a modest amount of prior uncertainty regarding individuals' growth rate. To see this, notice that the component of income growth that is predictable by households at time zero represents a substantial fraction of the total dispersion of β^i in the population: $\sigma^2_{\beta_k}/\sigma^2_\beta = 1 - \lambda^2 = 0.81$. The remaining uncertainty is small but nonnegligible, as we show in the next section.

The partial insurance parameter, θ, is estimated to be 0.451, implying that almost one-half of income surprises are smoothed away through informal mechanisms in the data. As we shall see later, this availability of partial insurance further reduces the already moderate amount of income risk implied by the estimates of the income process parameters and λ. As noted earlier, Blundell, Pistaferri, and Preston (2008) also estimated the extent of partial insurance, although their framework is rather different from the current one. While their estimates vary quite a bit across samples and cohorts, their baseline estimate is that about 35% of permanent shocks are insured along with almost 95% of transitory shocks. In our specification, the income surprise, $\hat{\xi}$, includes both transitory and persistent components. So the fact that our estimate of $\theta = 0.45$ is somewhat higher is broadly consistent with their results.

The parameter ψ is estimated to be 0.582, implying that individuals are able to borrow against only about 60 cents of each dollar of future minimum income at every future date and state. Another useful measure is the maximum debt that a household is allowed to carry as a fraction of average income: $\underline{a}_t/\mathbb{E}(Y^i)$. For a household whose head is 25 years old, this limit is 8% of (the economy-wide) average annual income and remains quite flat (still 6% at age 55). Finally, in the estimated model, the fraction of households who are constrained peaks at age 29 (17.4% of households) and stays between 6.5% and 13.5% between ages 30 and 40. The fraction constrained falls to about 5% or less beyond age 40.

The estimated time discount factor is 0.953, which implies that $\overline{\delta}(1+r)$ is slightly above unity. However, because average income is growing over the life cycle and individuals further discount the future due to the possibility of death, the average individual is impatient—in the sense of Deaton (1991)—according to these estimates. Although the standard error on $\overline{\delta}$ is extremely small, this is conditional on the fixed value of ϕ: unfortunately, the estimate of $\overline{\delta}$ is quite sensitive to the preset value of risk aversion. In Appendix D.6, we conduct a detailed sensitivity investigation with respect to the value of ϕ and show that our results are robust.

Measurement Errors and Transitory Shocks: σ_ε, σ_y, σ_c, σ_{c_0}, μ_c. With consumption data, in principle, we can tell transitory shocks apart from i.i.d. measurement error in income, since consumption should respond to the former but not to the latter. In practice, however, because the response of consumption to transitory shocks is proportional to its annuitized value—which is small—this response is rather weak, and identification is a problem empirically. In this framework, however, borrowing constraints are binding for a nonnegligible fraction of households. As a result, these households' consumption would move one-for-one with transitory shocks, allowing us to distinguish these shocks from pure measurement error. We estimate the standard deviation of the transitory measurement error in income to be about 16.5% annually, whereas true i.i.d. shocks to income have a standard deviation of about 10% annually. Finally, the transitory measurement error in consumption has a standard deviation of 35.5% and includes the noise introduced by the imputation method. Furthermore, the fixed effect in measured consumption has a standard deviation of 43%, and both components are estimated with extremely high precision.

Partial Insurance: An Alternative Specification. The baseline model featured partial insurance against income *surprises*, for example, $\hat{\xi}$. An important implication is that once a household's expectations adjust to incorporate an income shock, insurance also ceases to exist. To see what this means, consider the case without learning and assume shocks are permanent: $\lambda \equiv 0$ and $\rho \equiv 1$. Now consider a 10% permanent shock in period t: $\eta_t = -0.10$. Income will drop by 10% at all future dates, but the only "unexpected shock" happens at t. After that, expectations fully adjust and there is no surprise. Consequently, partial insurance entails a compensation of $0.10 \times \theta$ in period t and 0 in subsequent periods. It is easy to see that the same effect holds true more generally when $\lambda > 0$ and/or $\rho < 1$. This specification may not be unreasonable especially, for example, in a society where individuals display habit formation preferences, in which case the disutility associated with a consumption drop is largest on impact. This formulation would provide front-loaded insurance precisely to alleviate those short-run pains.

That said, an alternative plausible way to think about partial insurance is that it might insure a given shock at all future dates (e.g., disability insurance). To capture this idea, we modify equation (19) so that now a fraction θ of the entire (perceived) persistent component, $\hat{z}_t^i$, is insured:

$$y_t^{\text{disp},i} \equiv y_t^i - \theta \hat{z}_t^i. \tag{25}$$

Column 2 in Table I reports the results from this specification. Starting at the top panel, the first four parameters of the stochastic process, σ_α, σ_β, $\text{corr}_{\alpha\beta}$, and ρ, are very little changed from the benchmark specification (column 1). However, the two innovation variances are quite different now: σ_η is now 0.43, almost twice its baseline value. At the same time, notice that the partial insurance parameter is estimated to be $\theta = 0.55$. So about half of the total *level* of z_t is not transmitted from income to consumption. Thus, the after-insurance component of z_t is about 0.20—little changed from the baseline.[24] It is interesting to note that the consumption equation in the auxiliary model contains so much information that it is able to raise the estimate of σ_η well above what is implied by income data alone.

Turning to λ, its estimated value is virtually unchanged from the baseline, indicating that it is robust to this modification to the way partial insurance is modeled. Overall, these results are quite similar to the benchmark case. But perhaps this should not be surprising: z_t is not a very persistent process, so it reverts to its mean rather quickly. Therefore, its largest effect is upon impact, which is also how partial insurance worked in the benchmark case.

Self-Insurance Model: Shutting Down Partial Insurance. Next, we examine the case when households have no ability to smooth consumption over and above self-insurance. To this end, we reestimate the model by restricting $\theta \equiv 0$. Column 3 of Table I reports the results. Compared with the baseline specification, three differences are worth noting: (i) σ_β is slightly lower (1.66% vs. 1.76%); (ii) λ is also lower (0.345 vs. 0.438); (iii) combining these two pieces implies that the prior uncertainty (as measured by $\sigma_{\beta,0}$) is about one-third higher in the baseline model compared with the self-insurance model: $0.438 \times 1.76 \approx 0.77$ versus $0.345 \times 1.66 \approx 0.57$. Perhaps this result should not be too surprising: imposing $\theta \equiv 0$ implies that none of the idiosyncratic risk is insurable (beyond self-insurance), so the estimation procedure lowers the overall amount of risk faced by households to compensate for this lack of partial insurance by estimating a smaller amount of growth rate uncertainty.

Overall, however, the differences between the two sets of estimates are not very large. These results are intuitively consistent with equation (18), which

[24]Heathcote, Storesletten, and Violante (2014) build a rich life-cycle model with endogenous labor supply, progressive taxation, and partial insurance, and estimate that about 60% of permanent shocks are vanquished through these three channels and only about 40% are transmitted to consumption. These figures are broadly consistent with the estimated partial insurance here, although the persistence of shocks here is estimated to be quite a bit less than permanent.

showed that the response of consumption growth to income surprises was separable from the terms involving beliefs and learning (Π_t). So while θ is precisely estimated, it does not interact with the other parameters of the model (with the exception of λ), at least for the specifications that we consider in this paper.

Estimating Initial Dispersion in z. Finally, we relax $\sigma_{z_0} = 0$ and reestimate the benchmark specification with this additional parameter, σ_{z_0}. The estimated standard deviation is 0.046, which is fairly small (for example, compared with the annual innovation standard deviation of $\sigma_\eta = 0.227$). Consequently, most of the other parameter estimates remain largely unchanged. The only significant changes are in λ, from 0.438 in benchmark to 0.473; θ, from 0.451 to 0.484; and in σ_β, from 1.76 to 1.70.[25] However, even these changes are quite small and do not alter any of the substantive conclusions drawn above.

6.2. *Model–Data Comparison: Life-Cycle Profiles of Income and Consumption*

We now evaluate the fit of the estimated model to the U.S. data along three dimensions that are important benchmarks in the incomplete-markets literature. The first two are the evolution of the within-cohort variance of log income and log consumption over the life cycle. The third one is the average life-cycle profile of log consumption. (The average life-cycle profile of income is matched by construction.) We first discuss the benchmark estimates and then turn to the alternative specifications estimated above.

Figure 5 plots the variance of log income and consumption using our PSID estimation sample, after cohort effects are taken out following the usual procedure in the literature (e.g., Deaton and Paxson (1994)). First, in the left panel,

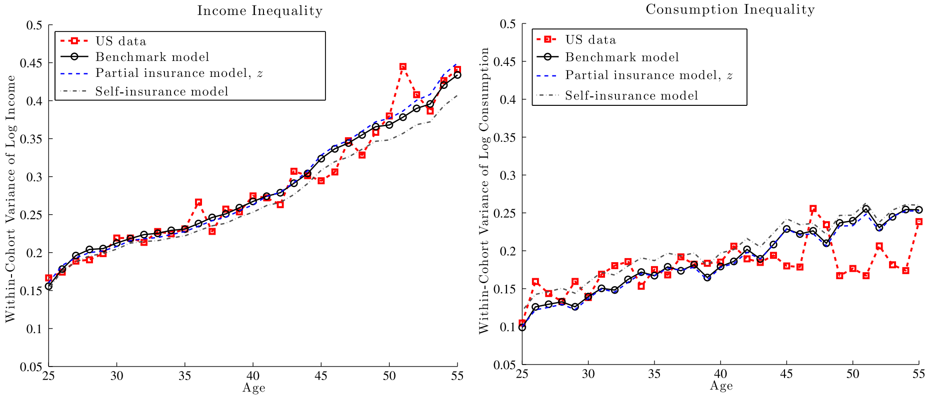

FIGURE 5.—Within-cohort income and consumption inequality: data versus estimated models.

[25]Other key parameter values are $\sigma_\alpha = 0.284$, $\rho = 0.758$, $\sigma_\eta = 0.229$, $\text{corr}_{\alpha\beta} = -0.118$, $\overline{\delta} = 0.953$, and $\psi = 0.554$.

the income variance (red dashed-dotted line marked with squares) increases by about 30 log points from age 25 to 55, consistent with figures reported in other studies (see, e.g., Storesletten, Telmer, and Yaron (2004a) and Heathcote, Perri, and Violante (2010)). The estimated benchmark model (black solid line with circles) agrees with the trend in the data quite well, even though these variances were never directly targeted in the estimation. Second, turning to consumption inequality, we see that in the right panel a similar pattern is revealed: the simulated model matches up with the data very well from ages 25 to 45. There is some evidence that consumption inequality grows further after that age in the model, but the data counterpart is quite noisy, making a clear judgement difficult.[26] Third, Figure 6 plots the average life-cycle profile of consumption. The estimated model tracks the empirical counterpart from the PSID quite well until about age 49, after which point average consumption in the data starts declining, whereas the model counterpart continues to grow, although at a slower rate.[27]

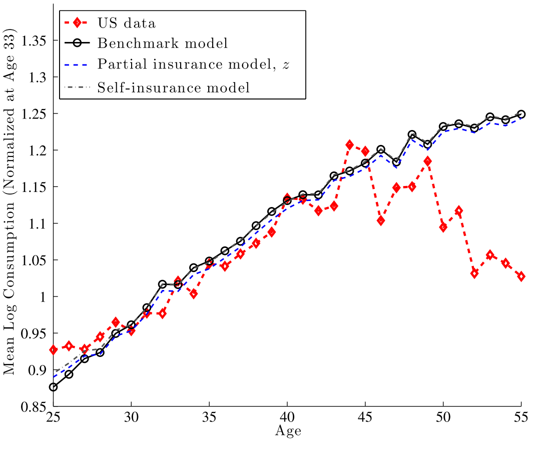

FIGURE 6.—Mean log consumption profile over the life cycle: model versus U.S. data.

[26]In Figure 5, the rise in the cross-sectional variance of consumption in the U.S. data is about 10 log points, which is small compared with that reported in earlier papers, such as Deaton and Paxson (1994), but is consistent with more recent papers that use CE data with a longer time span, such as Heathcote, Perri, and Violante (2010), Kaplan (2012), and Aguiar and Hurst (2013). In turn, this small rise in consumption inequality found here also explains in part why our structural estimation found a small value of λ, implying a smaller overall amount of uncertainty. Again, despite not having any terms that capture the rise in the variance of consumption explicitly, the auxiliary model attempts to be consistent with the small rise, which in turn requires a small amount of income risk perceived by households.

[27]Aguiar and Hurst (2013) showed that the steep decline in average consumption after age 50 is largely accounted for by the decline in work-related expenditures later in the life cycle. Because our model assumes inelastic labor supply, this margin of adjustment is not present in our model, which is probably why it misses the decline in consumption expenditures later in life.

Overall, the model does a fairly good job of matching these three salient aspects of life-cycle income and consumption patterns, despite the fact that these do not appear as explicit moments in the estimation procedure. Furthermore, all three versions of the estimated model appear to fit these profiles, without significant differences among them. In the next section, we will show that looking at household-level dynamics of consumption and income reveal a very different picture and allows us to clearly distinguish between partial- and self-insurance models.

Quantifying Uninsurable Income Uncertainty. As stated in the Introduction, a central goal of this paper is to quantify the magnitude of lifetime income risk *perceived* by households at different points in life, which we are now ready to do. The model we estimated contains three components that affect the *rise* in perceived income risk over time: (i) the AR(1) process, which is standard; (ii) the uncertainty about β^i: (iii) the existence of partial insurance. Figure 7 plots the forecast variance of predicted log income at different horizons, from the perspective of a 25-year-old: $\mathbb{E}_{25}((y_{25+t}^{\text{disp}} - \hat{y}_{25+t|25}^{\text{disp}})^2)$ for $t = 1, 2, \ldots, 30$. The graph contains four lines. We start from the very top line (dashed line marked with circles), which shows the maximum amount of uncertainty that is possible in this model—essentially if all the rise in income inequality over the life cycle represented income risk. It is computed by using the benchmark estimates from column 1 of Table I *but* imposing $\theta = 0$ and $\lambda = \lambda^{\max}$.[28] This is a useful benchmark, against which we compare each estimated model.

The second line from the top (dashed line) plots the amount of perceived income risk in the self-insurance model. As seen here, this profile overlaps very

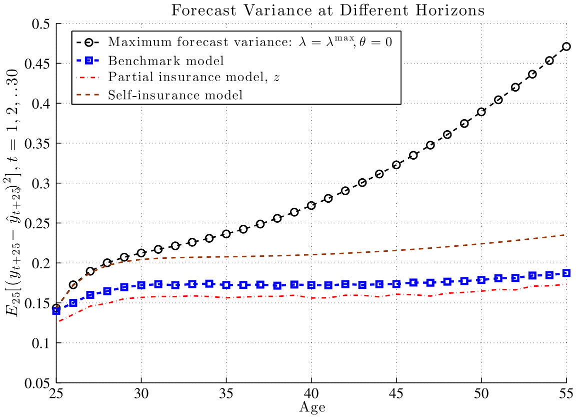

FIGURE 7.—Uninsurable income uncertainty.

[28]Recall from footnote 6 that $\lambda^{\max}$ is defined as the maximum amount of prior uncertainty possible. Given the low value of $\text{corr}_{\alpha\beta}$ in the benchmark case (−0.127), $\lambda^{\max}$ is 0.992, very close to 1.

well with the first (top) line in the first five years of the life cycle, implying that all the rise in inequality up to age 30 is considered as risk by households. This is largely because heterogeneity in β^i contributes little to income differences early on, which are instead driven by persistent income shocks. However, notice from the graph that the absolute amount of this risk is quite small. After age 30, however, inequality keeps rising—now almost entirely due to heterogeneous growth rates—whereas perceived risk tapers off because of the small initial uncertainty regarding β^i. Overall, between ages 30 and 55, total income dispersion rises by 25 log points, whereas perceived risk rises by only 3 log points.

Now we move to the benchmark model (line marked with squares), which adds partial insurance (and has slightly different parameter estimates too). As could be expected, now the overall uncertainty is even lower because part of the risk will be insured through informal channels. This mainly affects the uncertainty in the younger ages though, so the rise after age 35 is similar to the self-insurance model (only slightly lower).

Overall, according to our benchmark estimates, a 25-year-old household perceives that less than one-fifth (5 of the 32 log points) of the increase in income dispersion until age 55 is due to uncertainty, with the rest being predictable and/or insurable by households. Notice that this is not a statement about the *level* of uncertainty, but about its *rise* over the life cycle. In contrast, the standard RIP process estimated in the literature together with a self-insurance model would attribute *all* of the 32 log points rise in inequality to risk. Furthermore, if we include the level of initial uncertainty (the intercept in the figure) in the computation, then the forecast of a 25-year-old household for age 55 income has a standard deviation of about 20 log points, whereas the total dispersion at that age is more than 45 log points. Thus, our estimates imply that uncertainty about future income is less than half of the amount of inequality at that age.

The richer model studied here allows us to separate known heterogeneity from risk as well as the part of risk that is insurable, which reveals a much smaller amount of risk than is currently used to calibrate models in the incomplete markets literature.

At this point, it is also useful to compare the estimates of uncertainty regarding growth rates to the value that Guvenen (2007) gleaned from within-cohort variances and without allowing for partial insurance. The calibrated standard deviation of prior beliefs in that paper is $\sigma_{\beta,0} = 0.012$.[29] The corresponding number in this paper is $\sigma_{\beta,0} = 0.0077$. To map these numbers into figures that are easy to interpret, we compute the part of forecast variance that is due to growth rate uncertainty only. In the current paper, this forecast variance for predicting income at age 65 is only 8.4 log points, compared with 23.1 log points

[29]The notation for λ differs between the two papers, so they are not directly comparable.

using Guvenen's (2007) numbers. This difference is quite substantial, corroborating our conclusion that accounting for the full joint dynamics of consumption and income significantly reduces our estimate of income uncertainty.

6.3. *Inspecting the Response of Consumption to Income*

One conclusion that we draw from the life-cycle profiles in the previous section is that the three versions of the estimated model have broadly similar implications for life-cycle patterns. In other words, if we were to focus simply on the moments summarized in these three graphs, it would have been difficult to identify and precisely estimate partial insurance along with the other parameters. The reason we are able to do so here is precisely because we use the joint dynamics of consumption and income.

We now take a closer look at the sources of identification. Figure 8 plots six figures in two columns. Each column corresponds to a different household and plots (from top to bottom) the simulated paths of income, annual consumption growth, and wealth over the life cycle. Looking at the left column, we see that household 1 has a fairly high income. The income profile is also hump-shaped and reaches about three times the median income at ages 45–50. As seen in the right column, household 2 has a low and flat income profile, earning about half of the median income. The middle panel shows three lines. The dashed line is consumption growth in the self-insurance model. The solid black line displays the *gap* between consumption growth under the benchmark (partial insurance) model and under the self-insurance model, $\Delta C_t^{\theta\xi} - \Delta C_t^{\theta=0}$, where the superscript $\theta\xi$ indicates the benchmark model. The dashed–dotted line displays the same gap, this time for the alternative partial insurance specification (indicated by label superscript θz). Notice that, as expected, both gaps are perfectly negatively correlated with $\Delta C_t^{\theta=0}$, implying that consumption growth under partial insurance is always smoother than under self-insurance. However, the difference is sometimes large and sometimes small. For example, in the left column, household 1 would experience a consumption decline of 35% from age 31 to 32 under self-insurance, but instead experiences a decline of only $35\% - 12\% = 23\%$ under partial insurance. The flip side, of course, is that at age 34, consumption growth is 36% with self-insurance and only 22% under partial insurance.

A key point to observe is that after about age 40, the difference between partial insurance and self-insurance declines significantly—the solid line fluctuates near zero. Looking at the bottom panel makes the reason clear: because this household experiences positive income shocks, its asset holdings rise significantly after about age 40, which makes self-insurance very feasible and effective. In this case, what can be attained in terms of consumption smoothing by partial insurance can also be effectively achieved by self-insurance. Turning to the right panel, though, we see a different picture: the persistently low income of this household does not allow it to accumulate much wealth until later in

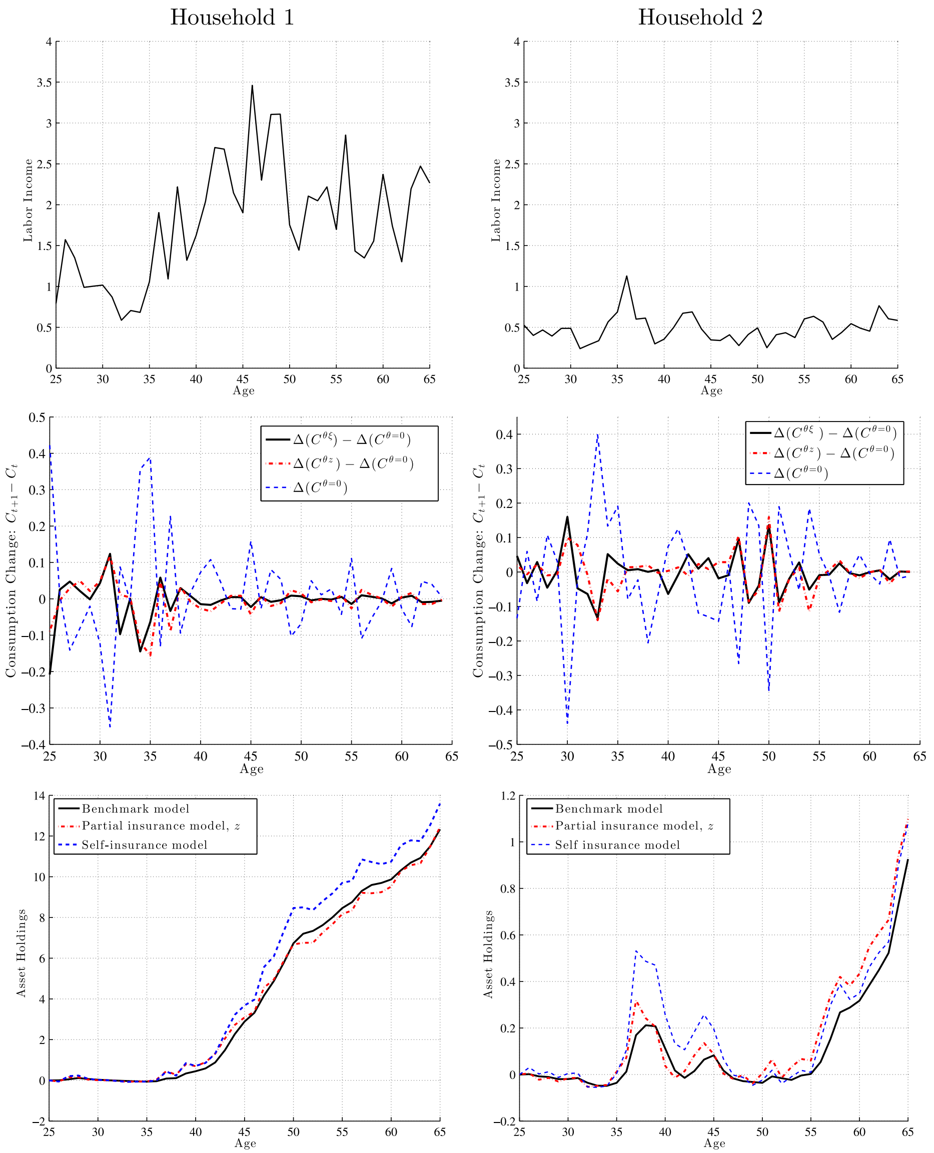

FIGURE 8.—Consumption response to income shocks: two sample paths.

the working life (about age 55). Consequently, partial insurance allows one to achieve a smoother consumption path than self-insurance at later ages. For example, from ages 50 to 51, consumption declines by 34% under self-insurance but by only 21% with partial insurance.

Inspecting these paths reveals the strength of the panel data analysis of the joint dynamics of consumption and income. Focusing on precisely how a given household's consumption responds to a particular income shock and how this relationship varies by the household's age, past and future average income, income growth rate, and so on, allows us to precisely pin down the parameters of our model, including the degree of partial insurance. This is an important advantage over some earlier papers that focused only on life-cycle profiles, such as Guvenen (2007) and Storesletten, Telmer, and Yaron (2004b), among others.

6.4. *Inspecting the Auxiliary Model*

Before we conclude the analysis of the estimated model, it is also instructive to look at the auxiliary-model coefficients. It is useful to know whether the data coefficients are significant as well as how the estimated model does in matching those data coefficients. Table II displays the 50 coefficients of interest for the benchmark model (44 regression coefficients and 6 elements of the covariance matrix). Throughout the table, boldface indicates that the coefficient is highly significant (a t-statistic of 5.0 or more).[30] The first row reports the data coefficients for the income equation of the young group: all regressors, except for the last two, are very significant. The next row displays their counterparts from the simulated model. For each coefficient, a test is conducted under the null that the model and data coefficients are equal to each other. Rejection with a t-statistic greater than 5 is indicated with the triple superscript daggers († † †). Similarly, rejections at the 1% and 5% levels are indicated with double superscript daggers (††) and a single superscript dagger (†), respectively.

In the income equation, the leads and lags of y, as well as the past and future average income, are almost always very significant (with t-statistics exceeding 30). The coefficients from the simulated data match their data counterparts well: for the 11 coefficients in this regression that are very significant (in bold), one cannot reject equality between the model- and data-implied coefficients at a 5% level or higher. Further, for only 3 out of the 18 coefficients in this regression can one reject that the model and data coefficients are statistically different. Overall, the estimated structural model does a fairly good job of matching the auxiliary-model coefficients in the income equation.

The consumption regression (23) is more challenging in many ways: the left-hand-side variable is now the output of a complex structural model that filters through the income shocks into the consumption choice. Moreover, consumption data contain not only measurement error (as did income data), but also imputation error. Inspecting the coefficients, we see that the lagged income term is highly significant and the model matches them quite well. Another set

[30]We compute the standard errors using a nonparametric bootstrap (sampling individuals with replacement).

TABLE II

COEFFICIENTS OF THE AUXILIARY MODEL: BENCHMARK ESTIMATED MODEL VERSUS U.S. DATA

	Constant	y_{t-1}	y_{t-2}	y_{t+1}	y_{t+2}	$\bar{y}_{1,t-3}$	$\bar{y}_{t+3,T}$	$\Delta y_{1,t-3}$	$\Delta y_{t+3,T}$	c_{t-1}	c_{t-2}	c_{t+1}	c_{t+2}
PANEL A: INCOME EQUATION													
Young Group													
(1) Data	**−0.036**[a]	**0.346**	**0.360**	**0.077**	**0.097**	**−0.098**	**0.150**	0.037	−0.022				
(2) Model	−0.024††[b]	0.359	0.381	0.086	0.092	−0.088	0.107††	−0.022††	−0.011				
Middle-Age Group													
(3) Data	0.006**	**0.418**	**0.358**	**0.111**	**0.093**	−0.027	0.043**	0.031	0.028				
(4) Model	−0.002††	0.429	0.399	0.109	0.095	−0.055	0.007††	−0.005	0.052				
PANEL B: CONSUMPTION EQUATION													
Young Group													
(5) Data	−0.007	**0.108**	0.042*	−0.023	−0.005	−0.045*	−0.017	0.030	−0.002	**0.248**	**0.262**	**0.178**	**0.175**
(6) Model	−0.021††	0.092	0.124††	−0.025	−0.034	−0.088	0.004	0.006	0.015	0.211††	0.205†††	0.247†††	0.228†††
Middle-Age Group													
(7) Data	−0.004	**0.136**	0.046**	−0.014	−0.040*	−0.082**	0.012	0.030	0.028	**0.270**	**0.260**	**0.177**	**0.187**
(8) Model	0.007††	0.097	0.083	0.008	−0.054	0.041†††	−0.059††	0.025	0.037	0.201†††	0.210††	0.224††	0.256†††

PANEL C: RESIDUAL VARIANCES AND CORRELATIONS

	Young Group			*Middle-Age Group*		
	$\sigma^2(\epsilon_t^y)$	$\sigma^2(\epsilon_t^c)$	$\rho(\epsilon_t^y, \epsilon_t^c)$	$\sigma^2(\epsilon_t^y)$	$\sigma^2(\epsilon_t^c)$	$\rho(\epsilon_t^y, \epsilon_t^c)$
(9) Data	**0.222**	**0.396**	**0.117**	**0.235**	**0.379**	**0.114**
(10) Model	0.216	0.390	0.121	0.223††	0.388	0.108

[a] Boldface indicates that the estimated parameter has a t-statistic greater than 5.0; superscript asterisks (** and *) indicate that the coefficient is significant at a 1% and 5% level, respectively. The largest t-statistic for parameter estimates is 34.5, and for the residual variances it is 67.81.

[b] Parameters estimated using simulated data are marked with one, two, or three superscript dagger (†) signs if the t-statistic for the hypothesis test of equality between simulated and data coefficients is greater than 1.96, 2.57, and 5.0, respectively. The largest t-statistic for these hypothesis tests is 6.56.

of coefficients that are also very significant is the leads and lags of consumption. Here, we see a fairly clear rejection. As much as the estimation procedure tries to bring these coefficients close together (as evidenced by the fact that the model coefficients are of the right order of magnitude and always have the correct sign), a statistically significant gap still remains.[31] Finally, panel C displays the variance–covariance matrix of the regression residuals, whose elements are very precisely estimated. Consequently, the Gaussian objective function puts significant weight on these terms. Out of the six elements in panel C, the estimated model fails to match only one of them (the correlation of residuals across equations for the young group).

To sum up, the estimated structural model matches several very significant coefficients of the auxiliary model quite well, but also falls short in matching the coefficients on lagged and future consumption, resulting in a statistical rejection.

6.5. *Robustness*

Despite many appealing features of simulation-based structural estimation methods, a drawback is that the estimates depend *potentially* on all the assumptions made on the structural model. Consequently, it is essential to examine whether the parameter estimates we obtain are sensitive to the key features of the model that have been fixed in advance. In an earlier version of the paper, we conducted extensive robustness analyses by varying a number of key assumptions made so far. These included (i) an alternative method for filling in missing observations, (ii) considering a higher income floor $\underline{Y}$, (iii) a lower interest rate, (iv) fixing (rather than estimating) the borrowing constraints, and (v) using all data available up to age 65. We found that the results were overall quite robust to these changes and none of the substantive conclusions reached in the paper was altered by these changes. For completeness, we included these results in Appendix D in the Supplemental Material.

7. DISCUSSIONS AND CONCLUSIONS

Partial Insurance versus Advance Information: Can We Tell Them Apart?

An ongoing debate in the literature attempts to understand whether partial insurance can be disentangled from advance information (typically about one-period-ahead income realizations) and identified using consumption and income data (Blundell, Pistaferri, and Preston (2008), Kauffmann and Pistaferri (2009), and Kaplan and Violante (2010)). The general conclusion is concisely

[31] One conjecture is that some sort of temporal dependence in the utility function, such as habit formation, could help resolve this problem by increasing the correlation of consumption at lower lags compared with the benchmark model. Exploring alternative utility function specifications is beyond the scope of this paper and is left for future work.

summarized by Kauffmann and Pistaferri (2009, p. 392): "[D]ata on income and consumption are not sufficient to separately identify advance information that consumers may have about their income from the extent of consumption insurance against income innovations." It should be noted that this debate typically revolves exclusively around moments based on consumption *changes.* In this section, we revisit this question and show that while we confirm the lack of identification between insurance and advance information using consumption changes alone, the *levels* of consumption—as used in our auxiliary model—are clearly able to separate the two. The basic argument can be explained most easily in a stylized example, although it is easy to generalize.

For this purpose, consider a two-period model with quadratic utility, no time discounting, no borrowing constraints, and a zero net interest rate,

$$\max_{C_1,C_2}\left[-\left(C_1 - C^*\right)^2 - \mathbb{E}\left(C_2 - C^*\right)^2\right]$$
$$\text{s.t.}\quad C_1 + C_2 = Y_1 + Y_2^{\text{disp}},$$

where only income in period 2 is assumed to be "partially insurable." Further, we assume that income (before partial insurance) is given as $Y_2 = Y_1 + \eta$. We model advance information as follows. Suppose that at time 1, the individual receives a signal about his future income, so that his expectation is given by $\mathbb{E}^{\text{AI}}(Y_2) = (1 - \alpha)Y_2 + \alpha Y_1$. (The superscript AI—advance information—indicates that the expectation is conditional on information beyond Y_1.) The parameter α measures the amount of advance information. When $\alpha = 1$, there is no advance information, $\mathbb{E}^{\text{AI}}(Y_2) = \mathbb{E}(Y_2) = Y_1$, and when $\alpha = 0$, the signal is fully revealing, $\mathbb{E}^{\text{AI}}(Y_2) = Y_2$. This latter case is the same as a model with no uncertainty. For partial insurance, we use the same structure as in Section 3. That is, disposable income is given by

$$\begin{aligned}Y_2^{\text{disp}} &= Y_2 - \theta\left(Y_2 - \mathbb{E}^{\text{AI}}(Y_2)\right) = (1 - \theta)Y_2 + \theta\mathbb{E}^{\text{AI}}(Y_2)\\ &= Y_2 - \alpha\theta(Y_1 - Y_2).\end{aligned}$$

Optimal consumption choices can be shown to be

$$C_1 = \frac{(1 + \alpha)Y_1 + (1 - \alpha)Y_2}{2} \quad \text{and} \tag{26}$$
$$C_2 = \left[\frac{1}{2} - \alpha\left(\frac{1}{2} - \theta\right)\right]Y_1 + \left[\frac{1}{2} + \alpha\left(\frac{1}{2} - \theta\right)\right]Y_2.$$

The formula for C_1 does not involve θ, so observing C_1, Y_1, and Y_2 identifies α; then C_2 identifies θ straightforwardly. The auxiliary model we used in this paper (23) contains current consumption on the left-hand side and the levels of both past and future income on the right-hand side, which captures precisely the type of information that is needed according to these formulas.

Now, let us compute the consumption change:

$$C_2 - C_1 = \alpha(1-\theta)(Y_2 - Y_1). \tag{27}$$

Notice that the two key parameters, α and θ, appear in (27) multiplicatively and, therefore, cannot be separately identified. This is precisely analogous to the results found in the literature that the response of consumption growth to income growth identifies a mixture of partial insurance and advance information.[32]

These results suggest that we can introduce a signal (advance information) into our model that yields information about one-period-ahead income and estimate it along with the partial insurance already included. Conceptually, our framework is perfectly capable of dealing with such an analysis, particularly because our auxiliary model uses information contained not only in the growth rates of consumption (which, as we have shown, cannot distinguish partial insurance from advance information), but also in the levels of consumption. In this paper, we have not undertaken this extension because it introduces another state variable, increasing computational complexity by an order of magnitude. However, this is high on our future research agenda.

Conclusions

The joint dynamics of consumption and labor income contain rich information about the economic environment that individuals inhabit. In this paper, we have studied how such information can be extracted from choice data to shed light on different aspects of lifetime income risk. The framework that we analyzed encompasses a number of earlier papers that also attempted to interpret the joint dynamics of consumption and income data through the lens of structural models, such as Gourinchas and Parker (2002), Guvenen (2007), Blundell, Pistaferri, and Preston (2008), and Kaplan and Violante (2010). While our estimate of growth-rate heterogeneity in the population is quite similar to what was used in Guvenen (2007), the amount of growth-rate *uncertainty* we estimate here is much smaller than what was found in that paper. Our results on partial insurance are broadly consistent with Blundell, Pistaferri, and Preston's (2008) findings that up to one-half of persistent shocks are insured through informal channels. Our analysis also addresses Kaplan and Violante's (2010) critique that the estimated effects of partial insurance could be partly attributable to the nonpermanence of income shocks: we left the persistence of income shocks unrestricted, but, instead, estimated it along with the rest of the model parameters.

[32]Generalizing this example to a multiperiod setting would introduce assets into the level formulas, but indirect inference is easily able to deal with that by using appropriate proxies (such as, e.g., long-run averages of income) for wealth.

While in this paper we have focused entirely on consumption–savings choices, the estimation method we use is general enough to accommodate a variety of other static or intertemporal decisions. Economic decisions that involve large fixed costs (and, hence, are made infrequently, such as fertility choice, house purchases, etc.) are likely to be especially forward-looking and, therefore, are useful for inferring the nature and amount of risk. We believe that the indirect inference methodology used in this paper can be fruitfully used in these alternative implementations.

Substantively, we find that (i) income shocks have moderate persistence—much less than a unit root; (ii) income growth rates display significant cross-sectional heterogeneity; (iii) individuals have much better information about their own income growth rates than what can be predicted by the observable variables typically available to the econometrician; and, finally, (iv) there is a substantial amount of partial insurance available to households, over and above what they can achieve on their own through self-insurance. Combining these pieces, the main conclusion of our analysis is that the amount of uninsurable lifetime income risk that households perceive is substantially less than what is typically assumed in calibrated macroeconomic models with incomplete markets.

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