The Geneva Papers on Risk and Insurance, 20 (July 1981), 48-52

Inferring Risk Tolerance from Deductibles in Insurance Contracts *

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Of the numerous results provided over the past 30 years by the theory of decision under uncertainty, none has proved more helpful to me in solving practical problems than the following elegant proposition due to Arrow : "*Proposition :* If an insurance company is willing to offer any insurance policy against loss desired by the buyer at a premium which depends only on the policy's actuarial value, then the policy chosen by a risk-averting buyer will take the form of 100 percent coverage above a deductible minimum.

Note : The premium will, in general, exceed the actuarial value ; it is only required that two policies with the same actuarial value will be offered by the company for the same premium [1]."

The purpose of this note is to show that Arrow's proposition is helpful to economists not only in solving daily life problems [2] but also in drawing inference about the degree of risk aversion of insurance purchasers. This new approach is applicable to a broader class of consumers than the prevailing alternative based upon asset portfolio composition [3]. Also, it seems to suggest an order of magnitude for the relative risk aversion measure which is substantially higher than estimates based on portfolio composition.

* The result presented in this note was obtained during the IMSSS workshop at Stanford University under support from NSF grant SOC 75-21820-A01. Helpful comments from Louis Phlips are gratefully acknowledged.

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[1] Arrow ([1971], p. 212). For a more recent contribution to the same topic and additional references, see Raviv [1979].

[2] Arrow's proposition came to my attention at a very opportune time, when I had just become a house owner and was trying to use the results of Mossin [1968] to decide on a value assessment for which to insure the house against fire. The insurance contract made no provision for a deductible. After reading Arrow's paper, I insured the house for its replacement cost, and took a deductible indirectly by assessing the furniture below replacement cost. After writing the present note, I did revise downward the insurance coverage on the furniture, thereby increasing the implied deductible.

[3] See, e.g., Blume and Friend [1975].

Let W	=	initial wealth of the individual
X		his loss, a random variable with distribution $\Phi(X)$
$I(X)$		amount of insurance paid if loss X occurs
P		insurance premium
$Y(X)$		wealth of the individual after paying the premium, incurring the loss, and receiving the insurance benefit
$u[Y(X)]$		cardinal utility function for terminal wealth, assumed twice differentiable concave,
and let E		denote the expected value operator.

Arrows's result states that if $P = P[E\,I(X)]$, then any optimal policy must stipulate

(1) $$I(X) = \text{Max}\,(X-D, 0)$$

where D is a constant — the " deductible ".

Under (1),

(2) $$E\,I(X) = E(X-D \mid X \geqslant D) = \int_{X \geqslant D} (X-D)d\Phi(X),$$

so that $E\,I(X)$ is a function of D — say $\bar{I}(D)$. Accordingly, $P = P(D)$ and

(3) $$Y(X) = W - P(D) - \min\,(X, D),$$

(4) $$E\,u[Y(X)] = \int_{X \leqslant D} u[W-P(D)-X]d\Phi(X) + \int_{X \geqslant D} u[W-P(D)-D]d\Phi(X) =_{\text{def}} \bar{u}(D).$$

The optimal insurance contract (deductible) is obtained by maximizing (4) with respect to D. For ease of notation, define

(5) $$u[W - P(D) - X] = u_X,\ u[W - P(D) - D] = u_D.$$

The first order condition for an interior maximum is [4]

(6) $$0 = \frac{d\bar{u}(D)}{dD} = \int_{X \leqslant D} u'_X \left(-\frac{dP}{dD}\right)d\Phi(X) + \int_{X \geqslant D} u'_D \left(-\frac{dP}{dD} - 1\right)d\Phi(X).$$

[4] The second order condition is satisfied; it follows from risk aversion and $P[E\,I(X)] > E\,I(X)$ that $W \geqslant D \geqslant 0$ at the optimum.

For $X \leqslant D$, I will use the approximation

$$(7) \qquad u'_X \approx u'_D - (X-D)u''_D,$$

and rewrite (6) as

$$0 = -\frac{dP}{dD}[u'_D \int_{X \leqslant D} d\Phi(X) - u''_D \int_{X \leqslant D} (X-D)d\Phi(X)] - (\frac{dP}{dD} + 1)u'_D \int_{X \geqslant D} d\Phi(X)$$

$$(8) \qquad 0 = -(\frac{dP}{dD} + \int_{X \geqslant D} d\Phi(X))u'_D + \frac{dP}{dD} u''_D \int_{X \leqslant D} (X-D)d\Phi(X),$$

$$(9) \qquad -\frac{u''_D}{u'_D} = -\frac{\frac{dP}{dD} + \int_{X \geqslant D} d\Phi(X)}{\frac{dP}{dD} \int_{X \leqslant D} (X-D)d\Phi(X)}.$$

It follows from (2) that

$$(10) \qquad \frac{dP}{dD} = -\frac{dP}{d\bar{I}} \int_{X \geqslant D} d\Phi(X) =_{\text{def}} -k \int_{X \geqslant D} d\Phi(X),$$

thereby defining k as one plus the (marginal) loading factor. Substituting into (9), we obtain

$$(11) \qquad -\frac{u''_D}{u'_D} = \frac{1-k}{k \int_{X \leqslant D} (X-D)d\Phi(X)} = \frac{k-1}{kD + k[\int_{X \leqslant D} (D-X)d\Phi(X) - D]} > \frac{k-1}{kD}$$

$$(12) \qquad R_D =_{\text{def}} -\frac{Y(D)u''_D}{u'_D} > \frac{k-1}{k} \frac{Y(D)}{D}.$$

Formula (12) places a lower bound on the relative risk aversion measure, R, evaluated at the minimal wealth level $Y(D) = W - P(D) - D$. To aid intuition, let $k = 2$, a reasonable figure for many contracts of insurance against loss. Then a lower bound for R is given by half the ratio of $Y(D)$ to D, i.e. by the reciprocal of twice the uninsured fraction of wealth. The figure $R = 2$, sometimes found in the literature, would imply deductibles as high as one fourth of the consumer's wealth $Y(D)$, or equivalently one fifth of the consumers initial wealth W (if $P(D)$ is small in relation to W).

Table 1 gives the lower bounds for R corresponding to alternative assumptions about the values of k and $D/Y(D)$. The striking feature of this table is that reasonable assumptions suggest rather large values of R.

Table 1. — Lower bounds for R as a function of k and D/Y(D)

k \ $D/Y(D)$	1/3	1/4	1/5	1/10	1/15	1/20
2	1.5	2	2.5	5	7.5	10
5/3	1.2	1.6	2	4	6	8
3/2	1	1.3	1.7	3.3	5	6.7
5/4	.6	.8	1	2	3	4

In order to explain the cumbersome term $k[\int_{X \leq D} (D-X)d\Phi(X) - D]$ in (11) and to gain insight in the degree of tightness of the lower bound, assume that the loading factor is constant so that

$$P(D) = k\,E\,I(X) = k \int_{X \geq D} (X-D)d\Phi(X). \tag{13}$$

Under that assumption,

$$k \int_{X \leq D} (X-D)d\Phi(X) = k[\int_{X \geq 0} Xd\Phi(X) - D] - P(D) = P(0) - kD - P(D) \tag{14}$$

$$-\frac{Y(D)u''_D}{u'_D} = \frac{(k-1)Y(D)}{kD-[P(0)-P(D)]}. \tag{15}$$

When the savings on the insurance premium due to the deductible, $P(0) - P(D)$, is small in relation to the deductible itself, then the lower bound in formula (12) will be close to the more exact formula (15).

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