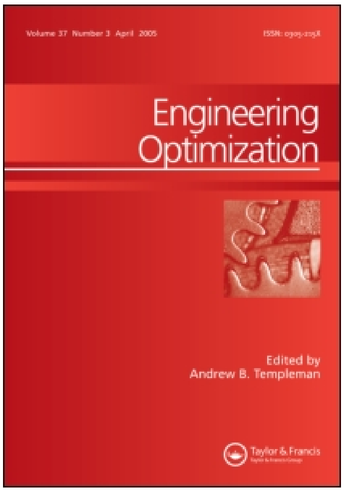

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Infield logistics planning for crop-harvesting operations

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Crop-harvesting operations are typically carried out with combine harvesters. The harvested product is transferred to one or more tractors every time the combine harvester's storage capacity is reached. The efficiency of the process can be significantly improved by computing optimal routes and interactions for the harvest vehicles in the field. Furthermore, an automated method for generating itineraries for the harvest vehicles facilitates the planning for autonomous agricultural vehicles. The infield logistics problem is formulated as an integer linear programming vehicle routing problem with additional turn penalty constraints, but, because of the high number of decision variables, it is not possible to solve cases of realistic field size. The solution time of the infield logistics problem is considerably reduced by reformulating it as a modified minimum-cost network flow problem. This specific structure allows the exact solution of intermediate-size planning problems in a much shorter time period. The result of solving the infield logistics problem with the proposed modelling approaches is a set of itineraries ('tours'), covering the entire field. Each 'tour' is characterized by the combine harvester's start and end points and the positions where the combine harvester needs to be unloaded. The planning models minimize non-productivity (*i.e.* the time when a combine harvester travels in a field without harvesting). The results indicate that coordination between combine harvesters and tractors is also improved.

Keywords: crop harvesting; harvest vehicles; planning of itinerary; vehicle routing; minimum-cost network flow

1. Introduction

Over the last few decades, substantial advancement in the technological development of agricultural equipment has been observed. The early focus of original equipment manufacturers was on increasing the engine thrust of self-propelled agricultural vehicles (see for example, Hilliard 1972). Gradually their focus shifted towards improving the harvest operations for cutting, threshing, and cleaning performed by combines. The recent interest is in improving the use of the harvest vehicles by efficiently planning the harvest process (Sørensen 2003, Foulds *et al.* 2005).

Crops are gathered by combine harvesters (Figure 1), which follow specific patterns to harvest the field. The supporting logistical activities of transferring the grain from the combine harvesters and transporting the product from the field to a depot are performed with the help of tractor trailers. Crop-harvesting operations, in most real-life cases, are planned according to the experience of the

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Figure 1. Crop harvesting with multiple harvest vehicles.

farmers operating the vehicles. Delays are often experienced, due to poor cooperation between the combine harvesters and the tractors, increasing the overall duration of the harvest.

Crop-harvesting operations require precise routing guidelines for the harvest vehicles. This article proposes a practical planning approach for harvesting a field with several capacitated combine harvesters and tractors. The goal of the planning is to generate 'minimum cost' itineraries that can be followed by one or several combine harvesters available for harvesting. Each itinerary, or 'tour', starts and ends at the unloading positions in the field and takes into account the capacity restrictions of the combine harvesters. The article has five sections. After this introductory Section 1, Section 2 elaborates on the planning considerations for the crop-harvesting process. The research is situated within its environment and relevant literature is considered. In Section 3, the modelling efforts are discussed and mathematical programming formulations are presented. The models are tested with a number of crop harvesting cases and the results are discussed. A comparison of the proposed modelling approaches is presented in Section 4, and Section 5 concludes the article.

2. The crop-harvesting process

2.1. *Operational scheduling*

Crop harvesting is often carried out with several combine harvesters and tractors. The allocation of combine harvesters and tractors to the fields can be performed before the harvesting season by means of a higher-level planning tool. Various planning methodologies have been developed to facilitate harvest-scheduling and the resource-allocation decisions. Fokkens and Puylaert (1981) were among the first to formulate a linear programming model for the management of harvest operations on a larger-scale grain farm. Their model minimizes the total cost of harvesting, including the operational cost of a single field and the transportation cost between different fields. The outcome of the model provides decision support for the allocation of the harvest vehicles to the fields and the transfer of combine harvesters from one field to another. A survey of different mathematical modelling approaches for various farm operations is provided by Glen (1987). His review elaborated on the mathematical models for machinery selection, cropping policies and farm operations scheduling. Sørensen (2003) presented a method to determine the machinery and the workforce requirements for harvesting a field. This method is based on a study of machine performance versus crop condition. The operational model helps to determine the optimal workforce and machinery size, with the objective of reducing the overall operational costs for a field.

Recently, Foulds and Wilson (2005) proposed an operational scheduling method for harvesting rapeseed and hay fields. They found that the duration of operation depends upon the interaction and combination of constrained resources allocated to a job. The scheduling of harvest operations, therefore, is truly complex. They presented an integer programming model and heuristics to construct operational harvest schedules. These models are based on resource-levelling methods and produce significant improvements over the previously used schedules. Basnet *et al.* (2006) extended this approach to scheduling operations at more than one field. Harvest contractors can use the developed approach to plan harvest operations and determine the sequence in which fields should be visited during the harvesting season.

Although scientific research has led to the development of good algorithms, it can be observed that, in reality, most infield harvest operations of combine harvesters and tractors are still performed without any detailed planning, and the efficiency of the process relies heavily on the experience of the workers performing the operations.

2.2. *Logistics planning issues*

Combine harvesters harvesting a crop in a field need to follow a specific route. The path followed by a combine harvester should be optimal with regard to the distance travelled, subject to the operational constraints of the field.

Considerable research has been conducted on various path-planning problems, particularly for robot navigation and autonomous agricultural vehicles. Stentz (1994) developed the 'D-Algorithm' for the task of path planning for a mobile robot equipped with sensors. The algorithm is designed to find, in real time, the optimal path in a directed graph and allows dynamic planning of the path whenever the robot senses changes in its environment. A fuzzy-logic and genetic-algorithms-based technique is presented by Pratihar *et al.* (1999) to generate obstacle-free paths for mobile robots. Path planning with the objective to completely cover an area resembles the geometric Travelling Salesman Problem. Several variants of this geometric algorithm have been proposed (Arkin *et al.* 2000).

Optimal covering tour problems with consideration for turn costs have been investigated by Arkin *et al.* (2005). This problem is experienced in many actual routing scenarios of automatic inspection, spray painting, milling, lawn mowing, and the like. The problem with turn minimization is proved to be NP-complete (on the basis of the well-known difficulty of deciding whether a grid graph has a Hamiltonian cycle) and efficient approximation algorithms have been proposed for finding a minimum number of turn tours, particularly for milling and lawn-mowing applications (Arkin *et al.* 2000, 2005).

Path planning, in order to determine feasible paths for agricultural vehicles in a field, has been investigated by Sørensen *et al.* (2004). In their study, the problem was related to covering tour problems like the Chinese Postman Problem and the Rural Postman Problem. Since these problems are NP-hard in nature, a heuristic was proposed for finding a solution. Bochtis *et al.* (2007) proposed a multi-travelling salesman problem for planning a fleet of combine harvesters operating in a field. Ryerson and Zhang (2007) conducted a feasibility study to determine the applicability of a genetic algorithm for path planning of agricultural vehicles. Although, this methodology did not result in completely optimized paths, the approach achieved 90% coverage of the field. Recently, Oksanen and Visala (2007) proposed an area-coverage planning algorithm for agricultural operations. Their algorithm included procedures for the division of the coverage region into sub-regions, the selection of sequence of those sub-regions, and the generation of a path that covers each sub-region taking into account the desired working direction.

As reported in this section, different path-planning and covering-tour approaches are developed mainly for robotics and autonomous agricultural vehicles. These approaches do not focus on a specific operation or process performed with these vehicles. Therefore, the existing path-planning methods cannot be used without modifications to generate harvest patterns for a field. Firstly, combine harvesters operating in a field have a limited bin capacity and need to unload grain into a tractor trailer at regular intervals in order to continue harvesting. Secondly, some additional constraints must be taken into account to deal with the accessibility requirements for the unloading of combine harvesters. And thirdly, sometimes the fieldwork pattern imposes additional constraints that need to be considered in order to make the planning realistic. Thus, a path-planning approach should consider the very practical implications of the process under study. A good path plan for crop harvesting ideally results in the identification of a set of itineraries, considering the capacity limits of combine harvesters and the requirement that each tour starts and ends at feasible positions.

2.3. *Crop-harvesting scenarios*

There are two main scenarios under which combine harvesters and tractors can operate and coordinate for crop-harvesting operations: continuous and intermittent harvesting.

2.3.1. *Continuous harvesting*

In continuous harvesting, a tractor trailer approaches the combine harvester in the field to unload grain once a combine harvester reaches its specified bin capacity. For the grain transfer, either the combine harvester stops in the field and unloads the grain to the tractor trailer, or the transfer is achieved by moving the tractor in parallel with the combine harvester while the combine harvester continues reaping.

2.3.2. *Intermittent harvesting*

In this scenario, tractors cannot approach the combine harvesters operating in the field. For the grain-transfer operation, a combine harvester has to stop the harvesting operation and is obliged to travel to the tractor located at a fixed position in the field. After unloading the grain, the combine harvester returns and continues the harvesting operation.

2.4. *Logistics planning requirement for crop harvesting*

In the crop-harvesting process, two issues are important. The first issue is the determination of optimal covering tours for combine harvesters operating in the field. The second issue is to identify feasible positions for the grain transfer between the combine harvesters and tractors. Both planning issues require routing decisions. The overall aim of the planning is to minimize the total distance travelled by the combine harvesters to harvest a crop field, thus minimizing the duration of harvesting process.

3. Modelling the crop-harvesting problem

This section explains the development of two integer linear programming (IP) formulations for infield logistics planning of crop harvesting. The infield logistics problem is first reformulated as a modified vehicle routing problem (VRP). Next, the problem is modelled based on a minimum cost network flow problem (MCNFP) to improve the computation times.

3.1. *Test cases and problem representation*

Both the VRP and the MCNFP based infield logistics planning models are evaluated by means of a number of test cases. The test cases comprise crop fields of different dimensions. Table 1 lists the different test cases—six without and two with obstacles in their area. In order to verify the planning results of the models, scaled-size crop fields are used. After this, fields between 1 and 5 hectares in area are considered. Each problem instance is characterized by the field area, the obstacles in the field, and the available combine harvesters. The field is represented by a polygonal area defining the boundaries in which the combine harvesters can travel. An obstacle in the field is represented by a polygon within the field that encloses the obstacle. After representing the field and the obstacles, the remaining area is converted into a grid of equally spaced vertices $v \in V$, where each vertex v_i represents the centre point of a cell. The result is a grid graph of a field $G(V, E)$ with vertex set V and arc set E. Each cell approximates the area covered by the combine harvester when standing still. The crop yield from a cell is determined on the basis of the estimated density of the crop.

This particular method of problem representation is used as an input for the path-planning algorithms. A path is defined as a sequence of vertex transitions and is considered optimal if the sum of transition costs is minimal across all the possible sequences through the graph.

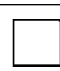
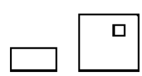
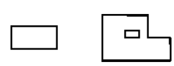
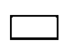
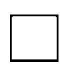
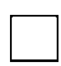
3.2. *Modelling as a VRP*

The goals of infield logistics planning for crop harvesting are (1) minimizing the non-productive distance travelled by the combine harvesters in the field and (2) identifying feasible grain-transfer positions in the field, taking into account the limited capacity of the combine-harvester bin. Harvest logistics planning can thus be divided into a bin-packing problem (harvesting with a minimal number of capacitated combine harvesters) and a travelling-salesperson problem (minimizing the travel distance of the combine harvesters). Since the VRP lies at the intersection of both problems, reformulation of harvest logistics planning problem into a VRP is possible (see for example, Dantzig *et al.* 1959).

3.2.1. *The VRP*

The VRP is a well-known NP-hard problem. In the VRP, a set of delivery–collection routes are designed for a fleet of vehicles to serve a number of customers from a central depot. The

Table 1. Test cases for harvest logistics planning models.

Test case	Field size	Field vertices	Combine-harvester paths to completely cover the field area	Field shape
1	Scaled field 1	30	2	
2	Scaled field 2	42	2	
3	1 hectare	167	3	
4	2 hectare	332	6	
5	4 hectare	662	12	
6	5 hectare	827	14	

customers are distributed with a travel distance c_{ij} between them. The objective of the VRP is to find a minimum-distance tour for each vehicle, such that each customer is served exactly once by a vehicle and the route of each vehicle starts and ends at the depot. Several variants of the VRP exist and different solutions have been proposed. Overviews are given by Laporte *et al.* (2000) and Cordeau *et al.* (2002). Mazzeo and Loiseau (2004) solved the problem with ant colonization and compared the results with other metaheuristics. An overview of exact approaches is given by Toth and Vigo (2000).

Adaptations of the standard VRP allow the inclusion of time windows, site-dependencies, multiple-depots, open routes, and such like. The interested reader is referred to Pisinger and Ropke (2007). The capacitated version has the highest relevance for harvest-logistics planning. In the capacitated vehicle routing problem (CVRP), each vehicle has a specific capacity and, once a vehicle reaches its limit, it returns to the depot. In the infield logistics problem, the combine harvesters operating in the field are the vehicles with capacity constraints. The combine harvesters must unload into the tractor every time the bin is filled up with the harvested crops. The yield from the crop is spread over the set of vertices V and each field vertex $v_i \in V$ must be covered by a combine harvester.

3.2.2. *Problem formulation*

The infield logistics planning problem for crop harvesting is modelled as a CVRP with additional turn penalty constraints. The following variables are used:

- i, j, h: vertex indices
- k: combine harvester path index
- c_{ij}: travel distance between vertex i and j
- p_i: turn penalty at vertex i
- A_i: yield from vertex i
- C_k: capacity of combine harvester k
- I: set of all vertices
- K: set of all combine harvester paths

The following variables are determined by the model:

- x_{ijk}: binary variable indicating if vertex i is followed by vertex j in a combine harvester path k
- y_{ijk}: binary variable to introduce the turn penalty

The mathematical model becomes:

$$\min \quad \sum_{i=0}^{i=I} \sum_{j=0}^{j=I} \sum_{k=1}^{k=K} c_{ij} x_{ijk} + p \sum_{i=1}^{i=I} \sum_{k=1}^{k=K} y_{ijk} \tag{1}$$

$$s.t. \quad \sum_{j=1}^{j=I} \sum_{k=1}^{k=K} x_{ijk} = 1 \qquad \forall i \in I \tag{2}$$

$$\sum_{j=1}^{i=I} x0jk = 1 \qquad \forall k \in K \tag{3}$$

$$\sum_{i=1}^{i=I} xi0k = 1 \qquad \forall k \in K \tag{4}$$

$$\sum_{i=0}^{i=I} x_{ihk} - \sum_{j=0}^{j=I} x_{hjk} = 0 \qquad \forall h \in I_0; \forall k \in K; i \neq h; \quad h \neq j \tag{5}$$

$$\sum_{i=0}^{i=I} A_i \sum_{j=1}^{j=I} x_{ijk} \leq C_k \qquad \forall k \in K \tag{6}$$

$$\begin{aligned}
&x_{(i-n,i,k)} + x_{(i,i+1,k)} \leq 1 + y_{(i,i+1,k)}\\
&x_{(i-n,i,k)} + x_{(i,i-1,k)} \leq 1 + y_{(i,i-1,k)}\\
&x_{(i+n,i,k)} + x_{(i,i+1,k)} \leq 1 + y_{(i,i+1,k)}\\
&x_{(i+n,i,k)} + x_{(i,i-1,k)} \leq 1 + y_{(i,i-1,k)}\\
&x_{(i-1,i,k)} + x_{(i,i+n,k)} \leq 1 + y_{(i,i+n,k)}\\
&x_{(i-1,i,k)} + x_{(i,i-n,k)} \leq 1 + y_{(i,i-n,k)}\\
&x_{(i+1,i,k)} + x_{(i,i+n,k)} \leq 1 + y_{(i,i+n,k)}\\
&x_{(i+1,i,k)} + x_{(i,i-n,k)} \leq 1 + y_{(i,i-n,k)} \qquad \forall i \in I; \forall k \in K
\end{aligned} \tag{7}$$

$$u_i - u_j + Cx_{ijk} + (C - A_i - A_j)x_{ijk} \leq C - A_i \qquad \forall i, j \in I_0; \forall k \in K; i \neq j \tag{8}$$

$$A_i \leq u_i - C \qquad \forall i \in I$$

$$x_{ijk} \in \{0, 1\} \qquad \forall i, j \in I; \forall k \in K \tag{9}$$

The objective function (1) minimizes the total distance travelled and the weighted number of turns in the field. Constraints (2) ensure that every vertex in the field is covered by a combine-harvester tour. Constraints (3 and 4) also make sure that all the available combine harvesters are used. The number of combine harvesters K (combine harvester paths) can be preset, based on the approximate crop yield and the combine-harvester bin capacity. Constraints (5) are the flow balance constraints. Capacity constraints (6) guarantee that the combine-harvester bin is never filled beyond its capacity. As the distance c_{ij} between the adjacent field vertices is constant, turn penalty constraints (7) are used to eliminate unnecessary turns in a field tour. These constraints are evaluated based on the 'if–then' constraints (Winston 2004). They make sure that, if a right angle turn is made (*i.e.* three consecutive vertices, for example $i - n, i, i + 1$ in a tour form a turn as shown in Figure 2), then the penalty cost p is added to the objective value of the model.

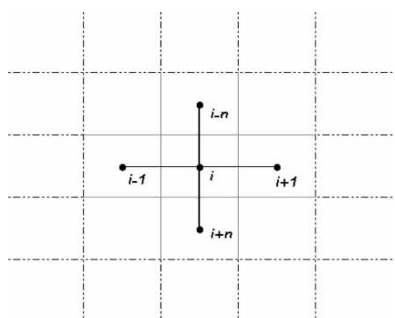


Figure 2. Turn penalty at vertex i.

Thus, the number of turns in a tour is reduced to the bare minimum. The Miller–Tucker–Zemlin sub-tour elimination constraints (8) prevent the generation of sub-tours and ensure continuity in a path. Equation (9) limits variables to binary values.

3.2.3. *Computational results*

The proposed modelling approach is verified with the test cases of the scaled crop fields. Each problem instance is represented on a grid graph and its IP formulation is generated in the C language. The IP models are solved using the ILOG Cplex v10.01 integer linear programming optimizer on a 3.4 GHz Intel work station with 1 GB RAM. Figure 3 shows a result of solving the problem (test case 2) with the CVRP model. The planning model assigns field vertices to combine harvester paths, taking into account crop yield and combine-harvester bin capacity. As can be seen, each combine-harvester path indicates the starting position for a combine harvester, the area to be covered in the field, and the position where the combine harvester is expected to reach capacity. Depending on the crop-harvesting process, the problem structure can be adapted to specify, in advance, the preferred end positions for the combine harvester tours. For example, in intermittent harvesting, where the combine harvesters operating in the field cannot be approached by tractor for the grain-transfer operation, it is desirable that a combine harvester always ends its tour (with a full bin) at the border of the field, near the tractor location. Thus, by identifying the end positions near the border of the field in the problem, the combine-harvester tours always end at one of the indicated positions suitable for grain transfer.

The planning results for the scaled crop fields provide an indication of the paths to be followed in the real field and the expected grain-transfer positions. However, as can be observed from Table 2, computation times increase tremendously with increases in field size. With the proposed

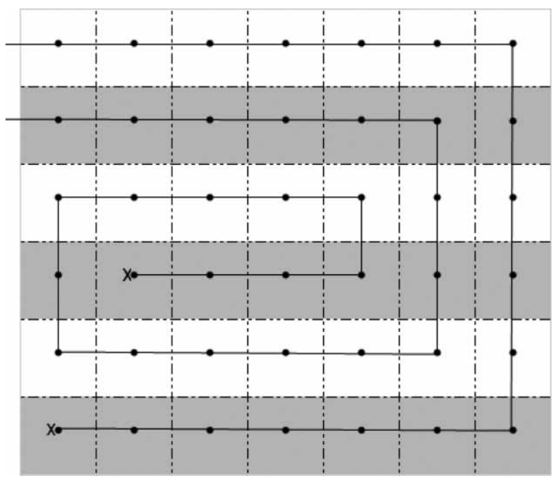


Figure 3. Harvest patterns generated by the vehicle routing problem (VRP) model. Note: X: grain-transfer point.

Table 2. Computation results of the vehicle routing problem (VRP) model.

Test case	Field size	Combine-harvester paths to completely cover the field area	Solution computation time (s)
1	Scaled field 1	2	8925.77
2	Scaled field 2	2	43167.41

CVRP approach, problems of realistic sizes cannot be solved exactly. To overcome this limitation, an alternative IP formulation is presented.

3.3. *Modelling as an MCNFP*

The infield logistics problem is reformulated based on an MCNFP. The specific structure of the MCNFP allows us to solve large problem instances in much less time and, therefore, it can be used to tackle real-world problems (Frangioni and Manca 2006, Hamacher *et al.* 2007).

3.3.1. *The MCNFP*

The MCNFP involves shipping of a commodity through a single connected network at minimum cost, such that the total flow does not exceed the arc capacities. Given a directed graph G (V, E) with a vertex set V and an arc set E, an upper u_{ij} and a lower bound l_{ij} on the flow through an arc, a non-negative arc cost c_{ij} and a specification of the net flow b_i generated at vertex v_i, the problem is defined as

$$\min \sum_{all\ arcs} c_{ij} x_{ij} \tag{10}$$

$$\text{s.t.} \quad \sum_j x_{ij} - \sum_k x_{ki} = b_i \quad (\textit{for each vertex i in the network})$$

$$l_{ij} \leq x_{ij} \leq u_{ij} \quad (\textit{for each arc in the network})$$

The problem has many applications, from scheduling in the public-transport sector to finding an optimal way to route information through a capacitated communication network (Kamath *et al.* 1995). The objective in each case is minimization of the total cost of flow from the source to the destination, while obeying the capacity constraints defined on the arcs in the network. Numerous, very efficient, algorithms have been proposed for solving network flow problems. Frangioni and Manca (2006) presented a study comparing the performance of a number of those algorithms.

3.3.2. *Problem formulation*

The infield logistics problem is modelled as a variant of a MCNFP formulation. Some modifications in the representation of the problem instance (as in Section 3.1) are required. Firstly, each vertex $v_i \in V$, representing a cell in the field G (V, E), is split up into two vertices, indexed i' and i''. This allows us to set a lower boundary on the flow through a cell, guaranteeing that all cells in a field are included in the solution. Secondly, the yield from the original vertex $v_i \in V$ is placed on the newly defined arc from vertex i' to i''. Thus, the field is represented by a grid network composed of predefined arcs E in a directed graph G (V, E). The flow capacity $x_e (lb \leq x_e \leq ub)$, specifying lower and upper bounds, respectively, and the flow costs c_e, are associated with each arc $e_{ij} \in E$ in the network. The set of vertices V is categorized into source vertices, intermediate vertices, and sink vertices. The flow starts from a source vertex and travels through intermediate vertices towards a sink vertex, where it is finally absorbed. In harvest logistics planning, the combine harvesters are the commodities that need to travel through the capacitated network (representing the field). The total supply is equal to the number of combine harvesters K required to completely harvest the field.

The flow capacities of all arcs of type i'–i'' in the grid network are set $1 \leq x_e \leq 1$, to ensure that all cells are covered by at least one of the combine harvesters. The flows through 'external arcs' between different cells in the network are bounded by $0 \leq x_e \leq 1$. Whether a particular external arc is to be used for the flow is determined by the model, minimizing the total flow cost through the network.

Several combine harvesters operating in the field should be able to start their trip from any of the source vertices $S \subset V$ in the field. Similarly, when the bin of the combine harvester is filled up, the combine harvester should be able to finish its trip at one of the sink vertices $F \subset V$ in the field. These requirements are incorporated by introducing a new super-source vertex s and a super-sink vertex f in the network. An arc (s, v) is then added for each possible source vertex, where $v \in S$. Also, for every possible sink vertex $u \in F$, an arc (u, f) is added to the network. This results in a grid network with the set of vertices $V \cup \{s, f\}$. All vertices in V are intermediate vertices with the net flow b_i equal to zero.

In general, routing problems need to be constrained in order to prevent the generation of sub-tours (*i.e.* a closed loop that does not start and end at the source and the sink vertices respectively). In this formulation, sub-tours are avoided by the specific way of defining the problem. Firstly, in the field network representation, arcs are defined in alternate directions and no arc points back to its tail, as displayed in Figure 4. Secondly, higher flow costs c_{ij} are used for arcs between cells in different rows. This way of formulating prevents the generation of sub-tours in rectangular and L-shaped fields.

The variables used are:

- i, j, h: arc index
- k : combine harvester path index
- c_{ij}: flow cost through an arc
- b_i: net supply at vertex i
- *ub*: upper bound on flow through an arc
- *lb*: lower bound of flow through an arc
- $\{s, f\}$: source and sink vertices
- V: set of all intermediate vertices
- A_i: yield from arc (i'–i'')
- E: set of all arcs

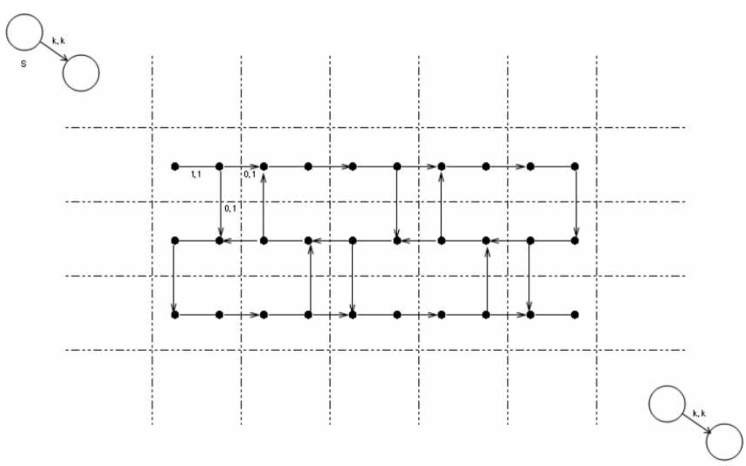


Figure 4. Grid network representation of a field.

- C_k: bin capacity of combine harvester k
- K: set of all combine harvester paths

The variable to be determined by the model is:

- x_{ijk}: binary variable indicating if arc (i, j) is included in the combine harvester path k

The mathematical model is formulated as follows:

$$\min \quad \sum_{(i,j)\in E} \sum_{k=1}^{k=K} c_{ij} x_{ijk} \tag{11}$$

$$\text{s.t.} \quad \sum_{k=1}^{k=K} x_{ijk} \le ub \quad \begin{cases} 1, \forall\, arcs\ between\ (i, j) \\ K, \forall\, arcs\ connecting\ \{s, f\} \end{cases} \quad \forall (i, j) \in E \tag{12}$$

$$\sum_{k=1}^{k=K} x_{ijk} \ge lb \quad \begin{cases} 1, \forall\, arcs\ between\ (i' - i'') \\ 0, \forall\, arcs\ between\ (i, j) \\ K \forall\, arcs\ connecting\ \{s, f\} \end{cases} \quad \forall (i, j) \in E \tag{13}$$

$$\sum_{(h,j)\in E} x_{hjk} - \sum_{(i,h)\in E} x_{ihk} = bi \quad \begin{cases} +K, \forall s \\ -K, \forall f \\ 0, \forall V \end{cases} \quad \forall k \in K \tag{14}$$

$$A_i \sum_{(i'-i'')\in E} x_{ijk} \le C_k \qquad \forall k \in K \tag{15}$$

$$x_{ijk} \in \{0, 1\} \qquad \forall (i, j) \in E; \forall k \in K \tag{16}$$

The objective (11) is to minimize the flow costs of all combine harvesters. Constraints (12) set upper bounds on the flow through the arcs in the network. Note that the upper boundary is equal to 1 for all the intermediate arcs. Lower boundaries on the flow through the arcs are assigned by constraints (13). The lower boundary is set to 1 for all arcs of type i'–i'' and zero for all external arcs i–j. Constraints (14) are the flow conservation constraints. The left-hand side of these constraints indicates the net flow at a vertex. The net flow is equal to zero at all intermediate vertices. The flow entering the network via the super-source vertex s is equal to the number of combine harvesters K (combine harvester paths) for a field. The flow collected at the super-sink f must be equal to the flow leaving the source vertex. The number of combine harvesters K required to completely harvest a field can be determined on the basis of the approximate yield of the crop and the capacity of the combine-harvester bin. For harvest logistics planning, the MCNFP formulation is modified by two additional constraints. Constraints (15) are the capacity constraints to make sure that the combine harvester bin is never filled beyond its available capacity and constraints (16) limit the variables to binary values. Notice that the formulation (11–16) is not a pure MCNFP. To solve the IP to optimality, the branch-and-bound technique is used. The branch-and-bound technique usually requires lengthy computation times, but, due to the specific MCNFP structure, the search trees were limited and the solution found in a comparatively short length of time.

3.3.3. *Planning algorithm*

The algorithm for the infield logistics planning problem is composed of three phases:

(1) field network generation
(2) mixed integer programming (MIP) formulation
(3) MIP solution

The grid network for a field is created following the modelling approach described in Section 3.3.2. In order to solve the planning problems of larger field sizes, the network generation process is automated. The network-grid generator is coded in the C language. After reading the input parameters (*i.e.* field dimension and arc costs), the program quickly generates a connected network with the required number of arcs, along with their respective flow capacities and costs. The generated network representing the harvest logistics planning problem of a field is then used as an input for the planning model. The linear programming formulation of the problem is generated in C. Additional information about the combine-harvester bin capacity and the approximate crop yield are user-defined. For solving the problem, the ILOG Cplex v10.01 optimizer is used. All computations were performed on a 3.4 GHz Intel work station with 1 GB RAM.

3.3.4. *Computational results*

The MCNFP approach was evaluated with a number of test cases. Both intermittent and continuous harvesting were considered. For intermittent harvesting, the sink vertices are located next to the fixed location of the tractor (along the edge of the network). For continuous harvesting, any intermediate vertex can be the sink vertex. For those situations, the grain transfer is performed by approaching the combine harvester in the field. The planning algorithm uses the indicated source and sink vertices as a reference for the start and end positions of the combine-harvester tours. In many practical situations, it might be necessary to take into account the prior knowledge of field characteristics (*e.g.* field entrances and exits and the fieldwork pattern). These specifications can be included in planning by specifying them in the problem representation of a field. Afterwards, the planning algorithm can be used to find the best solution. Solving the infield logistics problem with the modified MCNFP model gives results very similar to those obtained with the CVRP planning model. The set of itineraries generated covers the entire area of the field, with the minimum total distance and number of turns. The computation times required to solve the test cases are shown in Table 3. Compared to the CVRP model, these computation times are much shorter. This makes it possible to find exact solutions for intermediate-size planning problems.

Table 3. Computation results of the MCNFP model.

Test Case	Field Size	Combines (combine paths) to completely cover the field area	Solution time CPU(sec.)
1	Scaled_field_1	2	0.03
2	Scaled_field_2	2	0.05
3	1 hectare	3	1.80
4	2 hectare	6	185.61
5	4 hectare	12	1277.03
6	5 hectare	14	36309.9

Figure 5 shows the harvest patterns for various infield logistics problems. Each tour indicates the starting vertex for an unloaded combine harvester and a vertex where the harvester bin is expected to be filled with grain. The tour ends at one of the predetermined end vertices suitable for grain transfer. This prior knowledge about the expected grain transfer positions also improves the cooperation of the tractors with the combine harvesters.

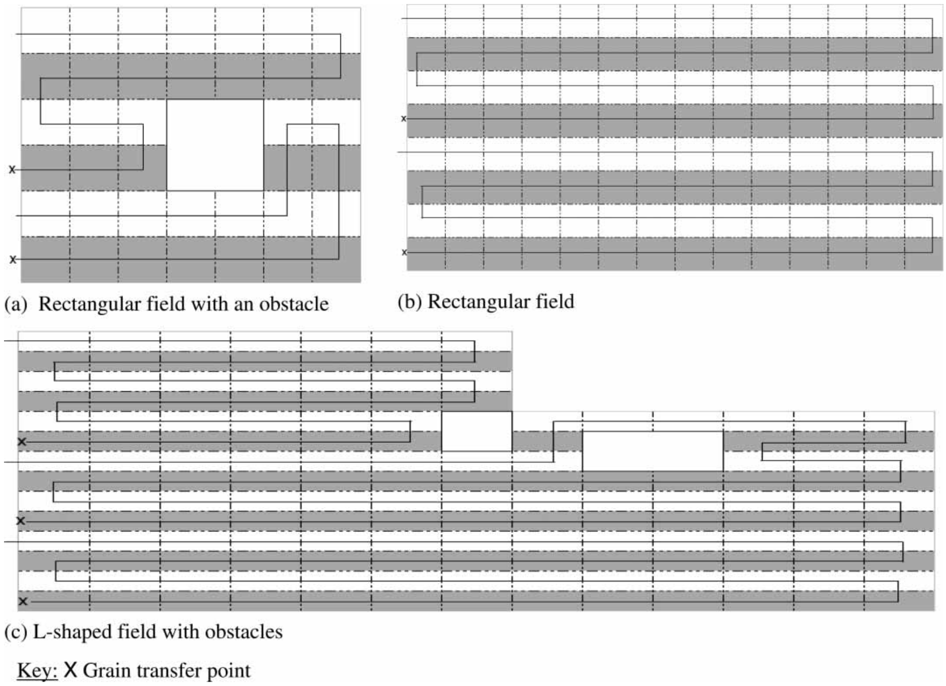


Figure 5. Crop harvesting patterns generated by the MCNFP model.

4. Comparison of the modelling approaches

The CVRP and the MCNFP planning approaches were evaluated with the selected test cases (see Section 3.1). ILOG Cplex v10.01 integer linear programming optimizer was used to obtain the planning results. Careful analysis of the results reveals that the quality of the solution (*i.e.* the number of turns in a tour and the total distance travelled) is the same with both planning approaches. However, the solution times required by the modified MCNFP planning model are much shorter than the solution times required by the CVRP model, thus allowing the cases of realistic field size to be solved exactly. It seems that the branch-and-bound application is limited for the modified MCNFP approach. When the CVRP-based optimization approach is applied to solve problems of large field size, a very long computation time is required. Interrupting the branch-and-bound process provides a solution, but unfortunately, such intermediate results are not often optimal with respect to the number of turns in the field. Therefore, the planning problems for the crop fields between 1 and 5 hectares in area are solved only with the MCNFP-based planning method. This approach generates good results for intermediate-size fields. However, the computation times needed to obtain the exact solutions with the commercial solver still tend to increase for the fields of 5 hectares or larger. Ongoing research is, therefore, focusing on field area planning and segmentation. A large crop field can be split into smaller field segments. The proposed MCNFP planning method can then be used to obtain feasible harvest patterns for each segment.

5. Conclusion

Path planning for robotics and agricultural vehicles has been investigated in order to find the shortest paths and covering tours in an area. However, for crop-harvesting processes, the existing planning approaches are insufficient to generate the harvest patterns. This is mainly because the combine harvesters operating in a field have a limited bin capacity and need to unload grain to the tractor trailer at regular intervals. The infield logistics planning requirements are taken into account and the problem is modelled as a vehicle-routing problem with additional turn penalty constraints. Good results indicating a set of itineraries for the crop field and feasible grain-transfer positions for the combine harvesters are obtained, but the solution times increase tremendously for larger problem instances. To overcome this, the infield logistics planning is reformulated using a minimum-cost network flow problem. This model allows the planning problems of intermediate field size to be solved exactly. The quality of the solution, that is to say the total distance travelled and the number of turns in the field, is the same with both planning models. The determination of grain-transfer positions also improves the coordination between the combine harvesters and the tractors. However, for fields of 5 hectares or larger, the solution times required by the minimum-cost network flow approach also tend to increase. The results obtained from both modelling approaches provide useful insight into infield logistics problems. The minimum-cost network flow approach in particular is very useful for generating exact solutions for problems of intermediate size. Such solutions can serve as benchmark results for evaluating heuristic approaches to the planning problem.

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