

“Infinite Bandwidth” Long Slot Array Antenna

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Abstract—This work deals with the properties of a long slot array fed by an array of periodically located feeds spaced at a Nyquist interval. The study begins with the rigorous Green’s function (GF) of a single long slot which was extended to an infinite array structure. After this, the input impedance properties of the array when fed by an array of δ sources are described by simple formulas, which explicitly show a theoretically unlimited bandwidth when there is no backplane.

Index Terms—Green’s Function, long slots, magnetic dipoles, phased array, ultra wideband.

I. INTRODUCTION

IN [1], an innovative wide-band antenna structure composed of an array of long slots periodically fed at Nyquist intervals [Fig. 1 and the cross section in Fig. 2(a)] has been reported. A prototype test array was manufactured, measured and patented, showing its potential for multifunction radar applications. The potential lies in the fact that the observed bandwidth is extremely large and the polarization purity appears to be outstanding with respect to other standard wide-band radiators.

When the ground plane and the embedded long slots are removed, the densely packed feed elements, Fig. 2(b), look like an array of connected dipoles, as discussed in [2], [3]. The array concept here is indeed similar. The objective of this study is to determine the wide-band properties of the long slot aperture rigorously, and derive parametric design curves for hardware development. In this paper, we focus on the Green’s function (GF) of the array shown in Fig. 1 without a backing reflector or dielectric layers. These latter could be added to make the array unidirectional, but they tend to limit the bandwidth, which will be discussed in a separate paper. First, we derive the analytical expression for the magnetic currents on the ground plane over the long slots, and then compute the active input impedance. We will analytically demonstrate that the bandwidth of such an array is theoretically infinite. To accomplish this, we extended to an infinite periodic array the formalism reported in [4], in which we derived the magnetic current of a single slot etched in a ground plane between two different dielectrics and excited by a single electric delta source. Note that the procedure provides directly the GF of the structure, without requiring a matrix inversion as in standard unit cell method of moments (MoM) based approaches. Following the same methodology, or just applying the duality principle [5] to the results derived here, one can demonstrate that the connected dipole array in Fig. 2(b), has a wide bandwidth also when there is no ground plane.

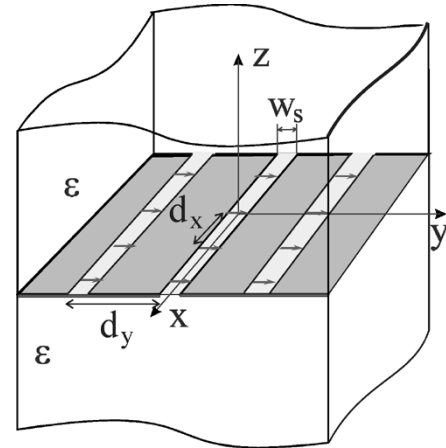


Fig. 1. Geometry of the infinite array of long slots excited at Nyquist intervals.

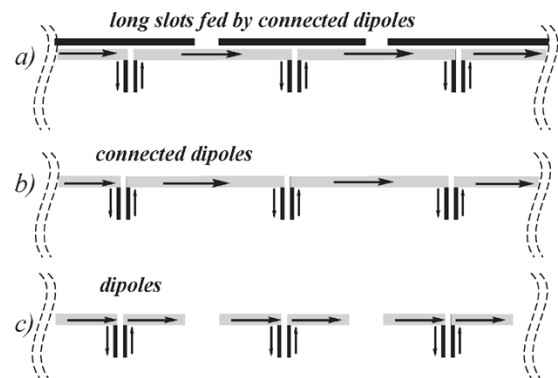


Fig. 2. Geometries of (a) an array of long slots fed by connected dipoles, (b) an array of connected dipoles, and (c) an array of isolated dipoles.

II. GREEN’S FUNCTION OF MAGNETIC CURRENTS

The geometry under investigation, consists of an infinite set of infinitely extended (x -oriented) slots which are etched out in a ground plane between two homogeneous dielectric half-spaces of the same permittivity ϵ_r . The slots are centered in $y_n = n_y d_y$. The cross section width w_s of each slot is uniform in x and small in terms of a wavelength at the maximum frequency of interest. Each of the slots is excited by an array of y -oriented electric current elements of the same length of the slot width w_s , placed at periodic locations $x_n = n d_x$. The δ sources in this section are assumed to be infinitesimally thin but their actual width is considered in Section III.

By invoking the equivalence principle, the slot regions are replaced by metallic surfaces infinitely thin and perfectly conducting, with two unknown magnetic current distributions $(-n \times E) \pm \mathbf{m}(x, y) = \mathbf{m}_{n_y}(x, y)$ above and below the ground plane, respectively. These currents have equal amplitudes and opposite signs to ensure continuity of the tangent electric field through the slots. Due to the small gap of the slots the magnetic

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currents can be assumed to be perfectly polarized along x , ($\mathbf{m} = m_x \hat{\mathbf{i}}_x$). In order to derive the magnetic currents on the slots, an integral equation that enforces the continuity of the tangential magnetic field on the axis of each slot is introduced. In this case the integral equation is completely scalar and can be expressed for each and every slot aperture of index n_{y0}

$$\int_S \int g(x - x', n_{y0}d_y - y') m(x', y') dx' dy' = \sum_{n_x=-\infty}^{\infty} f_{n_x}(x) \quad (1)$$

with

$$f_{n_x}(x) = I_e \delta(x - n_x d_x) \cdot e^{-jk_{x0}n_x d_x} e^{-jk_{y0}n_{y0}d_y}$$

where S represents the ensemble of slot apertures on the ground plane, k_{x0} , k_{y0} are the excitation's phase shifts along x and y , respectively. The left-hand side (LHS) of (1) represents the difference between the magnetic fields at $(x, y = 0, z = 0)$ radiated in the two half-spaces by the magnetic currents $\mathbf{m}_2 = \mathbf{m}$ and $\mathbf{m}_1 = -\mathbf{m}$, respectively. The pertinent GF, g , is the superposition of two contributions associated to the upper and the lower media. Since the two media are both grounded half-spaces, $g(x, y) = 2g_{xx}^{fs} + 2g_{yy}^{fs} = 4g_{xx}^{fs}$ where g_{xx}^{fs} represents the pertinent free space magnetic GF. Note that the factor 2 results from the application of image theorem to each of the two half space problems. The right-hand side (RHS), instead, represents the difference between the magnetic field impressed above and below the ground plane. This difference is associated with a phased array of delta current sources of amplitude I_e and phase shift along x and y , k_{x0} , k_{y0} , respectively. The magnetic current distribution is then assumed to be characterized by a separable space-dependence with respect to x and y ; i.e.,

$$m(x', y') = \sum_{n_y=-\infty}^{\infty} m_{n_y}(x', y') \quad (2)$$

and

$$\begin{aligned} m_{n_y}(x', y') &= v_{n_y}(x') \cdot m_t(y' - n_y d_y) \\ &= v_0(x') e^{-jk_{y0}n_y d_y} \cdot m_t(y' - n_y d_y) \end{aligned} \quad (3)$$

since in the y direction the problem is periodic and the excitation is uniform in amplitude and progressive in phase. The transverse y -dependence of the magnetic current in each slot is required to satisfy the quasi-static edge-singularities $m_t(y') = -(2/w_s \pi)(1 - (2y/w_s)^2)^{-1/2}$ where the normalization constant, $-(2/w_s \pi)$, has been chosen in such a way that $v(x')$ in (3) represents a voltage drop across each slot at any point x' . The function $m_t(y')$ possesses a closed-form Fourier transform

$$M_t(k_y) = FT\{m_t(y')\} = -J_0\left(k_y \frac{w_s}{2}\right) \quad (4)$$

where J_0 is the Bessel function of zeroth order.

Focusing on a specific slot ($n_{y0} = 0$) one can express (1) explicitly as a function of $v_0(x')$

$$\sum_{n_y=-\infty}^{\infty} e^{-jk_{y0}n_y d_y} H(x, y = 0, n_y) \quad (5)$$

$$= \sum_{n_x=-\infty}^{\infty} \delta(x - n_x d_x) I_e e^{-jk_{x0}n_x d_x} \quad (6)$$

where

$$H(x, y = 0, n_y) = \int_{-\infty}^{\infty} \int_{n_y d_y - \frac{w_s}{2}}^{n_y d_y + \frac{w_s}{2}} 4g_{xx}^{fs}(x - x', -y') \quad (7)$$

$$\cdot v_0(x') m_t(y' - n_y d_y) dx' dy' \quad (8)$$

Both the LHS and RHS of this equation can be concisely expressed in the spectral domain proceeding as in [4], using Poisson summation formula and recalling the Fourier component of the free space GF of the magnetic field

$$G_{xx}^{fs} = -\frac{1}{k_0 \zeta_0} \frac{1}{2} \frac{k_0^2 - k_x^2}{\sqrt{k_0^2 - k_x^2 - k_y^2}} \quad (9)$$

where k_0 and ζ_0 are the free-space wave number, and impedance, respectively. Note that, by duality [5], the GF of the magnetic field for the slot case is that of the electric field \mathbf{E}_x given in [6], Eq. 8(b), where the ground plane is removed and $\sin(k_z z')$ is replaced by the decaying exponential term with z' set to zero. Let $V_0(k_x)$ represent the Fourier transform of the unknown magnetic current distribution in the zeroth cell, $V_0(k_x) = FT\{v_0(x)\}$ and define $k_{xm} = (k_{x0} - (2\pi m_x/d_x))$ and $k_{ym} = (k_{y0} - (2\pi m_y/d_y))$. Then, the LHS becomes

$$\text{LHS} = \frac{1}{2\pi} \int_{-\infty}^{\infty} V_0(k_x) e^{-jk_{x0}x} D_{\infty}(k_x) dk_x \quad (10)$$

where

$$D_{\infty}(k_x) = \frac{1}{d_y} \sum_{m_y=-\infty}^{\infty} 4G_{xx}^{fs}(k_x, k_{ym}) M_t\left(k_{ym} \frac{w_s}{2}\right) \quad (11)$$

and the RHS becomes

$$\text{RHS} = I_e \int_{-\infty}^{\infty} \frac{1}{d_x} \sum_{m_x=-\infty}^{\infty} \delta(k_x - k_{xm}) e^{-jk_{x0}x} dk_x. \quad (12)$$

Since LHS and RHS are identically equal for each x one can equate the respective spectra, which leads to the relationship for the spectral components

$$V_0(k_x) = \frac{d_y}{d_x} \frac{2\pi I_e \sum_{m_x=-\infty}^{\infty} \delta(k_x - k_{xm})}{D_{\infty}(k_{xm})}. \quad (13)$$

Finally, this spectrum can be inversely Fourier transformed in closed form, leading to an explicit spatial expression

$$v_0(x) = \frac{d_y k_0 \zeta_0}{d_x 2} \sum_{m_x=-\infty}^{\infty} \frac{I_e e^{-jk_{xm}x}}{(k_0^2 - k_{xm}^2)} \sum_{m_y=-\infty}^{\infty} \frac{J_0\left(k_{ym} \frac{w_s}{2}\right)}{\sqrt{k_0^2 - k_{xm}^2 - k_{ym}^2}}. \quad (14)$$

Eq. (14), provides the closed form expression for the magnetic currents induced on the zeroth slot by the periodically located elementary sources. Using (2) and (3) the magnetic current

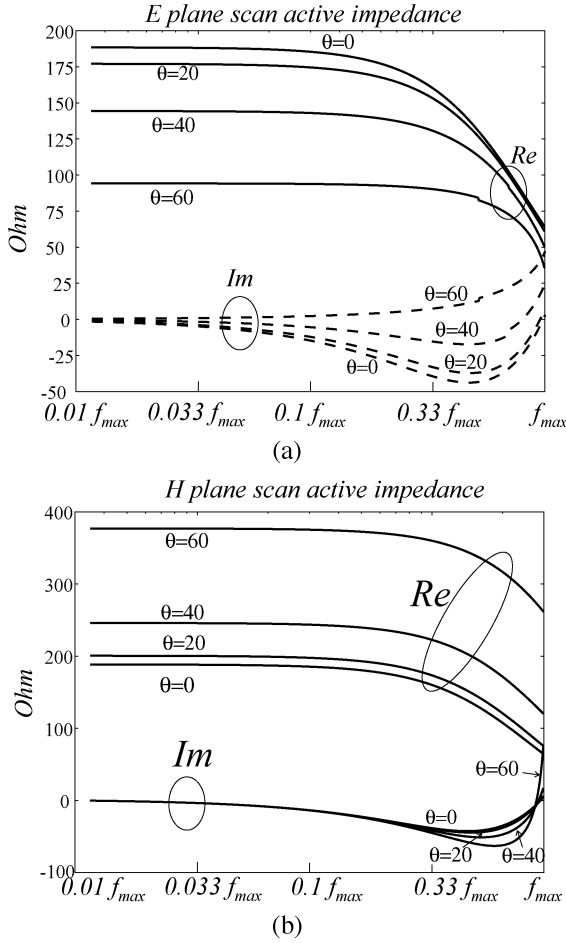


Fig. 3. Parametric active impedance. The frequency f_{\max} is the frequency at which the periods $d_x = d_y = \lambda_{\max}/2$, with $\lambda_{\max} = c/f_{\max}$. $w_s = \lambda_{\max}/10$, $t = \lambda_{\max}/10$. (a) Scan in the E-plane. (b) Scan in the H-plane.

can be calculated in all slots. It represents the GF of the phased array under analysis.

III. ACTIVE IMPEDANCE

The impedance is the key parameter for the qualitative and quantitative description of a phased array. In order to define an active impedance an array of δ -gap type of feeds is considered. Proceeding as in [7], the active impedance at any feed port can be derived as the impedance at center $n_x = 0$, $n_y = 0$ and in the spectral domain is expressed as

$$z_{in} = z_{in}(t, k_{xo}, k_{yo}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{sinc}^2\left(\frac{k_x t}{2}\right) V_0(k_x) dk_x \quad (15)$$

where t is the δ -gap of the magnetic delta source, i.e., the width of the electric current element across the slot. Substituting in this expression the actual spectrum of the voltage distribution from (13) (with $I_e = 1$)

$$z_{in} = \frac{d_y}{d_x} \frac{k_0 \zeta_0}{2} \sum_{m_x=-\infty}^{\infty} \frac{\text{sinc}^2\left(\frac{k_{xm} t}{2}\right)}{(k_0^2 - k_{xm}^2)} \sum_{m_y=-\infty}^{\infty} \frac{J_0(k_{ym} \frac{w_s}{2})}{\sqrt{k_0^2 - k_{xm}^2 - k_{ym}^2}}. \quad (16)$$

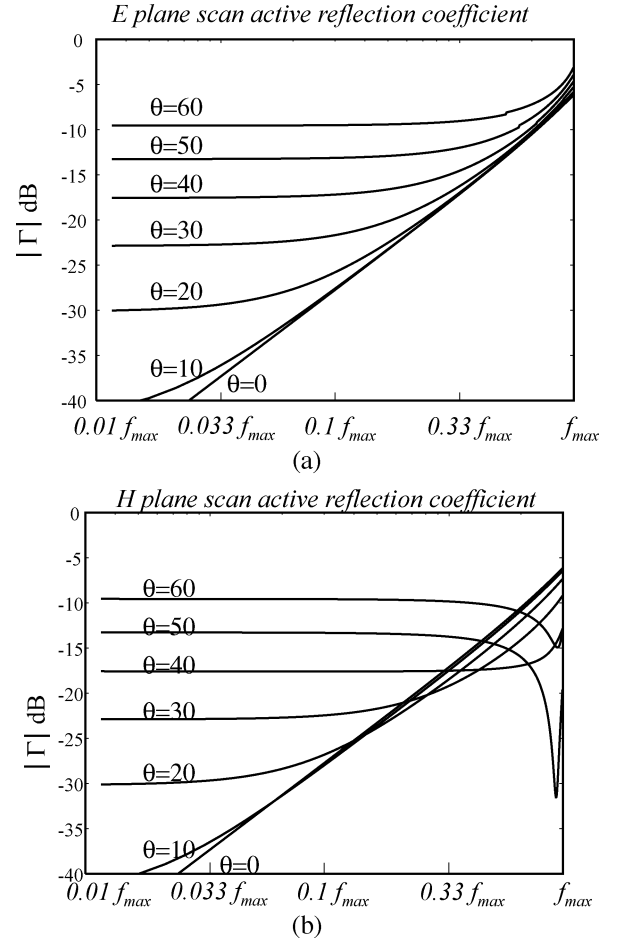


Fig. 4. Parametric reflection coefficient active reflection coefficient with respect to a feed impedance of 188 ohm. The configuration is the same as that in Fig. 3. (a) Scan in the E-plane. (b) Scan in the H-plane.

From this expression one can derive some very significant results, that could not be observed investigating this geometry with a purely numerical method. When investigating one can consider a first-order approximation of the impedance expression, i.e., only the $m_x = 0$ and the $m_y = 0$ in the expansion (16). The validity of this approximation for low frequencies is evident since if one considers the limit for $k_0 \rightarrow 0$ the $m_x = 0$ and the $m_y = 0$ become the dominant terms. Then, for small arguments, the Bessel function and the sinc function can both be approximated by 1 and introducing explicitly the scanning angles ($k_{x0} = k_0 \sin\theta \cos\phi$; $k_{y0} = k_0 \sin\theta \sin\phi$) one obtains

$$z_{in} \approx \frac{d_y}{d_x} \frac{\zeta_0}{2} \frac{\cos\theta}{1 - \sin^2\theta \cos^2\phi}. \quad (17)$$

Note that the low frequency dominant term of the impedance presents no frequency dependence, which demonstrates that this array antenna exhibits an infinite relative bandwidth. The dependence on the scan angles is in agreement with that formulated in [8], valid at fixed frequency points. The valid range of the approximation (17) can be determined by the complete numerical evaluation of the impedance expression (16). The results are summarized in Fig. 3 where (a) is for E-plane parametric scans, and (b) for H-plane parametric scans. The frequency f_{\max} is the frequency at which the feed element spac-

ings $d_x = d_y = \lambda_{\max}/2$, with $\lambda_{\max} = c/f_{\max}$ and c is the speed of light in free space. In this example, the width of the slot is $w_s = \lambda_{\max}/10$ and the width of the current source $t = \lambda_{\max}/10$. In Fig. 4(a) and (b) the active reflection coefficients are computed, referred to $188 \text{ ohm} = \zeta_0/2$ input feeds, on the E- and H-planes, respectively. Provided the small width approximation is valid $w_s < \lambda_{\max}/5$, the effect of the width is secondary (only observable for frequencies close to f_{\max}). Also, the length of the gap t has a secondary impact on the imaginary part of the impedance. Before concluding it is worth mentioning that the infinite relative bandwidth behavior is a consequence of the fact that the feeds are arrayed in two dimensions. It could not be achieved with just one single slot periodically excited as already observed for the dipole case in [3]. This point and more detailed derivations will be discussed in a future publication.

IV. CONCLUSION

The GF characterizing the magnetic currents of an infinite array of long slot excited at a Nyquist interval has been derived rigorously, and expressed in analytical form. From the magnetic currents the active impedance of the array when excited by δ -gap sources is also derived. It is shown that this array theoretically possesses infinite bandwidth when the impedance of the sources is $z_0 = \frac{d_y \zeta_0}{2d_x}$. Similar performances can be demonstrated for an array of connected dipoles radiating in free space by applying the duality principle to the formulation presented here.

As a drawback, this array radiates in two directions $z > 0$ and $z < 0$, when there is no backplane. The inclusion of a backing reflector or other surfaces to make it unidirectional has already been developed and will be the object of a future publication.

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