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Inflation, and Primordial Black Holes as Dark Matter

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Abstract

We discuss the hypothesis that a large (or even a major) fraction of dark matter in the Universe consists of primordial black holes (PBHs). PBHs may arise from adiabatic quantum fluctuations appearing during inflation. We demonstrate that the inflation potential $V(\varphi)$ leading to formation of a great number of PBHs should have a feature of the "plateau" type in some range $\varphi_1 < \varphi < \varphi_2$ of the inflation field φ. The mass-spectrum of PBHs for such a potential is calculated.

1 Introduction

The nature of dark matter (DM) in the Universe is one of the greatest puzzles of modern cosmology. The DM may consist of baryons, weakly interacting massive exotic particles predicted by GUT, primordial black holes or some combination of these species.

In this paper we shall consider the hypothesis that the DM consists mainly of primordial black holes (PBHs) (The first works on PBH are [1], [2] see also [3] and [4]).

Recently the possible discovery of microlensing of stars in the Large Magellanic Cloud by massive compact halo objects (MACHOs) with probable masses ~ 0.1 solar mass was reported [5], [6]. It was supposed (among other possibilities) that

such objects might be black holes. We would like to emphasize that black holes with masses of the order of $0.1M_\odot$ can only be of primordial origin. Thus this discovery gives additional arguments to consider the possibility of the PBH nature of DM.

Let us consider the conditions for PBH formation in the Early Universe. The simple estimates (see for example [4], [7]) show that for the formation of PBHs with total mass density close to the critical one ($\Omega_{PBH} \approx 1$) and with mass M_{PBH} around $0.1M_\odot$ one needs an rms amplitude $\delta_{rms}(0.1M_\odot)$ of the Gaussian distribution of the scalar metric fluctuations of the order of $\delta^{crit}_{rms}(0.1M_\odot) \approx 0.06$. This estimate depends on Ω_{PBH} and M_{PBH} only logarithmically. For example, $\delta^{crit}_{rms} = 0.04$ at 10^{15}g and $\delta^{crit}_{rms} = 0.08$ at 10^6M$_\odot$. On the other hand the COBE measurements of the anisotropy of the Cosmic Microwave Background Radiation and other satellite, balloon and ground-based radio telescope measurements, and also deep surveys of galaxy distributions strongly indicate that on scales of galaxies and greater scales (up to the horizon scale) the amplitude of δ_{rms} was significantly less, probably around $10^{-5} \div 5 \times 10^{-6}$.

It is worth noting that COBE-data are compatible with a power spectrum of the adiabatic perturbations $P(k) \propto k^n$ with $n = 1.15^{+0.45}_{-0.65}$ (see [8]). This means that a direct extrapolation of the COBE-data to smaller scales even with the maximal possible value $n \approx 1.6$, can give δ_{rms} great enough for the formation of essential number of black holes only for M_{PBH} less than 10^{15}g [4]. However such small PBHs would have evaporated a long time ago and could not contribute to DM.

Notice that if we believe that the main part of PBH has some specific mass M_* then the spectrum of the primordial fluctuations must have a decrease or a cut off from the side of smaller mass at $M_{PBH} \approx M_*$.

Thus for the hypothesis of PBH DM one needs the following behaviors of the spectrum of the primordial scalar metric perturbations. The rms amplitude must be of the order of 10^{-5} at large scales, must increase by a factor 10^4 at the scales corresponding to the masses of PBH and must decrease at less scales.

In the inflationary scenario of the Early Universe the spectrum of the primordial perturbations is determined by the potential $V(\varphi)$ of the scalar field φ ("inflaton"). The purpose of our paper is the following. We shall demonstrate that an inflation potential $V(\varphi)$ leading to formation of a great number of PBHs must have a feature of the "plateau" type in some range $\varphi_1 < \varphi < \varphi_2$, and we shall calculate the mass spectrum of PBHs for such a $V(\varphi)$.

Qualitatively the conclusion about the plateau in $V(\varphi)$ follows from a well known estimate for the spectrum of primordial metric fluctuations in the model of the chaotic inflation assuming the friction-dominated and slow-roll conditions, $|\ddot{\varphi}| \ll H|\dot{\varphi}|$ and $\dot{\varphi}^2 \ll V(\varphi)$ respectively. Here the dot denotes differentiation with respect to time, and H is the expansion rate. The power spectrum $P(k)$ in this case can be written as follows [9]:

$$P(k) \sim k \frac{V^3}{\left(\frac{\partial V}{\partial \varphi}\right)^2}|_{k=aH(\varphi)} , \quad (1)$$

where $H(\varphi)$ is the value of the Hubble parameter at the moment when the Universe has the value φ of the inflaton field, a - is the scale factor. If the potential $V(\varphi)$ has a plateau in the range $\varphi_1 < \varphi < \varphi_2$, $V(\varphi)$ ≈const and $\frac{\partial V}{\partial \varphi} \to 0$, then the spectral amplitude $P(k)$ is strongly increased (see the formula (1)). Outside the range $\varphi_1 < \varphi < \varphi_2$, $V(\varphi)$ has a standard (for example a power low) shape. In the range $k \ll k_2$ and $k \gg k_1$ where $k_i = a(\varphi_i)H(\varphi_i)$, corresponding $P(k)$ has also a standard shape (for example it can be the Harrison-Zeldovich spectrum $P(k) = A^2 k$, with $A \approx 5 \times 10^{-6}$).

The structure of the paper is the following. In section 2 the modification of the inflation scenario with the plateau type peculiarity in $V(\varphi)$ is discussed, and we calculate the distortion of the spectrum of the primordial metric fluctuations due to this peculiarity. Section 3 is devoted to the analysis of the mass spectrum of the PBHs. In section 4 we discuss the possible role of the "gas" of PBHs in the origin of the large scale structure of the Universe, and summarize the main conclusions.

2 Spectrum of scalar metric perturbations in the inflationary scenario with a "plateau" in the potential $V(\varphi)$

The simple approach in inflaton based on one scalar field φ is to specify the physics by choosing an appropriate form for $V(\varphi)$ and assuming the friction-dominated and slow roll conditions [9]:

$$|\ddot{\varphi}| \ll 3H|\dot{\varphi}| , \qquad (\dot{\varphi})^2 \ll 2V(\varphi) , \quad (2)$$

where $H = \dot{a}/a$; $a(t)$ is the scale factor. In this regime Fourier components of the scalar metric perturbations are δ - correlated random values with a Gaussian distribution.

Our task is to increase the spectral amplitude in some range $k_2 < k < k_1$, where k is a wave number, without changing the standard spectrum of perturbations outside this range. We propose to introduce the potential $V(\varphi)$ of the inflaton φ which is depicted in Fig.1. This potential has a plateau in the range $\varphi_1 < \varphi < \varphi_2$ and is a power-law type outside of this range.

There are two breaks of the potential at $\varphi = \varphi_1$ and $\varphi = \varphi_2$. We suppose that these breaks are smoothed out in small ranges $\Delta\varphi_1 \ll \varphi_1$ and $\Delta\varphi_2 \ll \varphi_2$ around φ_1 and φ_2 correspondingly (see Fig.1).

The conditions (2) are violated in these ranges. Starobinsky has pointed out [10] that this violation results is a non-monotonic spectrum of perturbations. In the vicinities of breaks of the potential $V(\varphi)$, but outside of $\Delta\varphi_1$ and $\Delta\varphi_2$, the potential can be described by

$$V(\varphi \sim \varphi_i) = V(\varphi_i) + v(x_i) , \quad v(x_i) = \begin{cases} A_i^+ x_i & \text{if} \quad x_i \gg \varphi_i , \quad x_i > 0 , \\ A_i^- x_i & \text{if} \quad |x_i| \gg \varphi_i , \quad x_i < 0 , \end{cases} \quad (3)$$

where $x_i = \varphi - \varphi_i$; $i = 1, 2$; $\varphi_2 > \varphi_1$, or it can be rewritten as

$$V(\varphi \sim \varphi_i) = V(\varphi_i) + A_i^- x_i + (A_i^+ - A_i^-)\Theta(x_i)x_i , \quad (4)$$

where $\Theta(x)$ is the Heaviside-function. Notice, that in the general case the shape of the potential at $\varphi_1 < \varphi < \varphi_2$ can be complicated enough. However, for our purpose it is enough to choose $V(\varphi)$ =const at $\varphi_1 < \varphi < \varphi_2$. In this model $A_1^-, A_2^+ \neq 0$; $A_2^- = A_1^+ = 0$. Evolution of the scalar field φ is governed by the equation [9]:

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} , \quad (5)$$

$$3H^2 = 8\pi(\dot{\varphi}^2 + V(\varphi)) ,$$

where $\dot{\varphi} \equiv d\varphi/dt$. In the case of evolution of φ when $\varphi > \varphi_2$, from (5) we have

$$\dot{\varphi} \approx -\frac{A_2^+}{3H} . \quad (6)$$

We suppose that $|A_2^+ - A_2^-| > H^2\Delta\varphi_2$ and $A_2^+ \cdot \Delta\varphi_2 \ll V(\varphi_2)$. In this case for the dynamics of φ one can neglect $v(x_2)$ in (3) compared with $V(\varphi_2)$ and the regime $a(t) \propto \exp(H \cdot t)$ goes on after the field φ passes the break of the potential at φ_2.

After this passage the field evolves along the plateau, and solution of the equations (5) can be written as

$$\dot{\varphi} = -\frac{A_2^+}{3H} e^{-3H(t-t_2)} , \quad (7)$$

where t_2 is the moment of time, when $\varphi(t_2) = \varphi_2$. At the moment when φ comes to the point $\varphi_1 = \varphi(t_1)$ its "velocity" is

$$\dot{\varphi}|_{t \simeq t_1} \approx -\frac{A_2^+}{3H}\left(\frac{k_2}{k_1}\right)^3 , \quad (8)$$

where $k_i = a(t_i)H(t_i)$; $i = 1, 2$. Using (6) and (8) we have the following expression for $\partial^2 V/\partial\varphi^2$ which determines the dynamics of the generation of adiabatic perturbations in the vicinity of the break of the potential [10]:

$$\frac{\partial^2 V}{\partial \varphi^2} = \frac{3H}{a}\left[\delta(\zeta - \zeta_2) - \frac{A_1^-}{A_2^+}\left(\frac{k_1}{k_2}\right)^3 \delta(\zeta - \zeta_1)\right] , \quad (9)$$

where $\zeta = \int \frac{dt}{a}$, and ζ_1 and ζ_2 correspond to t_1 and t_2 (see [11], [12], [13], [14]).

Let us consider the origin of adiabatic metric perturbations at the epoch of inflation. We write down the spectrum of perturbations in the following form

$$P(k) = A^2 k D(k) \ , \tag{10}$$

where A^2k is the Harrison-Zeldovich spectrum and $D(k)$ describes the deviation from it. Using the method described by Demiansky *et al.* [14] one can obtain the following expression for $D(k)$ after the field φ passes both breaks in the potential

$$\begin{aligned}
&D(k) = D_1 D_2 + D_{int} \ ,\\
&D_j = 1 + \frac{3C_j}{y_j}\left[\left(1-\frac{1}{y_j^2}\right)\sin 2y_j + \frac{2}{y_j}\cos 2y_j\right] +\\
&+\frac{9}{2}C_j^2\frac{1}{y_j^2}\left(1+\frac{1}{y_j^2}\right)\left[1+\frac{1}{y_j^2}+\left(1-\frac{1}{y_j^2}\right)\cos 2y_j - \frac{2}{y_j}\sin 2y_j\right],\\
&D_{int} = 4(a_1 - d_1([(a_1-d_1)(b_2+a_2d_2) - (1-b_1)(d_2 - a_2b_2)] \ , \\
&a_j = I_m(\alpha_j) \ , \quad b_j = Re(\beta_j) \ , \quad d_j = I_m(\beta_j) \ , \quad j = 1,2 \ ,\\
&\alpha_i = 1 - \frac{3i}{2}C_j y_j^{-1}(1+y_j^{-2}) \ ,\\
&\beta_i = \frac{3i}{2}C_j \exp(2iy_j) y_j^{-1}(1+iy_j^{-1}) \ ,
\end{aligned} \tag{11}$$

$y_j = R_j k$, R_j is the wavelength of the perturbation entering the horizon at the moment when $\varphi = \varphi_j$; $C_2 = 1$; $C_1 = -\frac{A_1^-}{A_2^+}\left(\frac{k_1}{k_2}\right)^3$; $j = 1,2$. Asymptotic behaviors of $D(k)$ are

$$D(0) = \left(\frac{A_1^-}{A_2^+}\right)^2, \qquad D(\infty) = 1. \tag{12}$$

The function $D(k)$ is depicted in Fig.2. Notice the oscillations in the spectrum related with each break in the potential $V(\varphi)$. These oscillations were discussed in [14], [15].

For the hypothesis of PBH DM the case $\left(\frac{A_1^-}{A_2^+}\right) \le 1$ is especially interesting. Under this condition, at $\frac{k}{k_2} > 4$, and $D(k)$ can be described with accuracy better than 5% by the fitting formula

$$D(k) \approx \left[1 + \frac{A_1^-}{A_2^+}\gamma^3\right]^2 \left[1 + 3\frac{\sin 2R_2}{kR_2}\right], \qquad \gamma = \frac{k_1}{k_2}, \qquad \gamma \gg 1 \ . \tag{13}$$

One can see in Fig.2 that in the case $\gamma \gg 1$ at $k_2 < k < k_1$ there is a great increase of the spectral amplitude by a factor $D^{1/2}(k) \sim \frac{A_1}{A_2}\gamma^3$. The two lowest curves in Fig.2 show the character of approach of $D(k)$ to its asymptotic value at $k \to 0$. One can see that between the long wavelength asymptotic of $D(k)$ and the range of the strong increase of the spectral amplitude there is a range where the amplitude is suppressed.

3 Mass-spectrum of PBH.

In our approach to the calculation of the mass-spectrum of PBHs we focus on the peaks of the Gaussian random field of the primordial adiabatic metric perturbations [1][1].

PBH can arise when the space scale of the peak scalar metric perturbations of the order of one becomes smaller than a particle horizon $\lambda_H \approx t$ but still is greater than the Jeans' radius $\lambda_J = \lambda_H/\sqrt{3}$. Masses of PBHs, M_{BH}, are proportional to the moment t_{BH} of their formation $M_{BH} \propto t_{BH}$. Shapes of the peaks of the fluctuations play an important role in the formation of PBHs [16], [17]. Some shapes can result in the dissipation of the peaks due to pressure gradients.

Zabotin and Naselsky [17] have pointed out that in the case of the Harrison-Zeldovich spectrum of the primordial fluctuations (and spectra which are close to them) the most probable distribution of the matter inside the peak is favorable for the formation of PBHs. Moreover one can calculate the mass-spectrum of PBHs in the framework of the model of the homogeneous collapse proposed in the work [18].

Note that the process of PBH formation is certainly nonlocal because it includes a volume of the radius R. In order to take into account this nonlocality one needs to use the characteristics averaged over the sphere with a Gaussian filtering function [19].

Besides that, it is necessary to take into account that for the perturbations with the wavelength more than the particle horizon evolution of the density contrast $\delta(\vec{r},t)$ can be written in the form $\delta(\vec{r},t) = \delta(\vec{r})\Phi(t)$ where $\delta(\vec{r})$ is determined by the spectrum of the initial metric perturbations and $\Phi(t)$ corresponds to the growing mode of gravitational instability.

Because of this factorization it is enough to analyze the statistical behavior of peaks of the function $\delta(\vec{r})$.

Following the work [19], let us introduce a new field $F(\vec{r},R)$ which is the result of Gaussian smoothing of the random field $\delta(\vec{r})$ on the scale R

$$f(\vec{r},R) = \frac{1}{(2\pi R^2)^{3/2}} \int d^3\vec{r}\,'\delta(\vec{r}\,')\exp\left(-\frac{|\vec{r}-\vec{r}\,'|^2}{2R^2}\right), \tag{14}$$

and consider the correlation function

$$C(R,x) = \frac{1}{2\pi^2}\int_0^\infty dkk^2 P(k)\exp(-k^2R^2)\frac{\sin kx}{kx} \ ,$$

where $x^2 = \sum_{\alpha=1}^3 \left(x_1^{(\alpha)} - x_2^{(\alpha)}\right)^2$. For the notation of $x_l^{(\alpha)}$, the upper indices $\alpha = 1,2,3$ label coordinates, and the lower ones $l = 1,2$ label points in space.

[1] For an alternative approach to the mechanism for PBH formation see the work by Dolgov and Silk [20].

In order to make further calculations the dispersion $C_0(R) \equiv C(R,0)$ is required. For analytic estimates we shall use the approximation (13) for the spectrum $P(k)$, assuming $P(k) = 0$ in the ranges $k < k_2$ and $k > k_1$. In this approximation we obtain

$$C_0(R) \approx \frac{A^2}{4\pi^2 R^4}\gamma^6 \left[1 + 6\,{}_1F_1\left(3, \frac{3}{2}, -\frac{R_2^2}{R^2}\right)\right], \qquad (15)$$

where ${}_1F_1(a,b,x)$ is the degenerated hypergeometric function and A is the amplitude of the Harrison-Zeldovich spectrum on large scales (it can be normalized to COBE-data).

Formula (15) is correct for the range $R_1 \leq R \leq R_2$. For the range $R \ll R_1$ and $R \gg R_2$, the dispersion $C_0(R)$ is negligible compared with the dispersion in the range $R_1 \leq R \leq R_2$, and we can put $C_0(R) \simeq 0$ at $R \ll R_1$ and $R \gg R_2$.

Thus the spectrum of Gaussian random matter density perturbations smoothed by the Gaussian filter on the scale R can be given by the following simple formula:

$$C_0(R) = \varepsilon^2 \left(\frac{R_1}{R}\right)^4 F(R_2/R) , \qquad (16)$$

where $\varepsilon^2 = (A^2/4\pi^2)(\gamma^6/R_1^4)$ is a measure of the spectral amplitude at $R = R_1$ and

$$F(R_2/R) = 1 + 6\,{}_1F_1\left(3, \frac{3}{2}, -\frac{R_2^2}{R^2}\right).$$

During its formation each PBH absorbs mass from the region with the co-moving scale $R \propto M_{PBH}^{1/2}$. Since $F(R_2/R)$ varies in the limits $O(1)$, one can use the following approximate estimate of the fraction of the total matter $\beta(R)$ collapsing into PBHs with mass M_{PBH}

$$\beta(R) \simeq \varepsilon \left[F\left(\frac{R_2}{R}\right)\right]^{1/2} \exp\left[-\frac{1}{18\varepsilon^2 F\left(\frac{R_2}{R}\right)}\right]. \qquad (17)$$

We performed numerical computations of $\beta(R)$ using the equation (10) and (11) and estimate for calculation $\beta(R)$ given by [4]. The results of these computations are presented on Fig.3 for $\gamma = 16.5; 18$ and 20. As seen from Fig.3 the asymptotic rough analytic formula (25) is valid at $R_1/R \leq R_2$ only. Numerical computations show two maxima which correspond two scales R_1 and R_2 of the initial spectrum $P(k)$ of perturbations. The mass-spectrum of PBHs is determined by the function $\beta(M_{BH})$

$$F(M_{BH}) \propto \frac{1}{M_{BH}}\frac{d}{dM_{BH}}\left[\beta(M_{BH})M_{BH}^{-1/2}\right], \quad M_{BH} \propto R^2 .$$

Thus varying the main parameters of the model one can vary the possible values for PBH masses in very broad limits.

4 Astronomical consequences of the hypothesis about PBH DM and concluding remarks

We have demonstrated that under some conditions on the inflation potential $V(\varphi)$ (see section 3) the matter density of PBHs could be great enough to make up a considerable or even major part of DM in the modern Universe. This imposes a lower limit on the possible parameter R_1 of the model. Indeed, PBHs could not have masses $M_{BH} \leq 10^{15}$g. Such PBHs must evaporate due to Hawking's process, and this gives a strong observational constrain on their density $\Omega_{BH} < 10^{-8}$, see [3]. For $M_{BH} \geq 10^{20}$g and up to scales of the clusters of galaxies constraints come only from the inequality $\Omega_{BH} \leq 1$ in the modern Universe [3].

One can consider models with $\Omega_{tot} = 1$ and with DM consisting mainly of PBHs, which implies $\Omega_{PBH} \approx 1$, or more complicated models with $\Omega_{PBH} < 1$ and with a Λ-term or some HDM, see [21].

In Fig.3 we show straight lines corresponding to the conditions $\Omega_{PBH} = 1$ for the modern Universe and which are tangential to the spectrum $\beta(R)$ of different models. The tangent points determine the corresponding effective masses M_{PBH} of the models. As may be seen from Fig.3, for all interesting ranges of M_{PBH}, the parameter γ is $\gamma \simeq 15 - 20$. The parameter γ determines the distribution of masses of PBHs, and thus it could be a possible test on the nature of DM.

In this paper we do not analyze special behaviors of the formation of the large scale structure of the Universe (LSS) in the framework of the PBH DM model. We note only the following. The main properties of the LSS in this model probably are the same as in different versions of the standard CDM model due to the fact that the masses of PBHs are much smaller than the masses of the LSS.

We want to point out that the condition $\Omega_{PBH} \approx 1$ for PBHs with small masses can be satisfiesd only by very "delicate" adjustment of the parameters of the theory. Indeed, in order that the total mass contained in PBHs be close to the critical value it is necessary that the fraction of the total mass contained in them be sufficiently small (but have some well defined value!) at the period of PBH formation [1], [22]. Perhaps the explanation of this "fine tuning" of parameters could be related to the anthropic principle.

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Figure captions.

FIg.1. Schematic representation of the potential $V(\varphi)$ of the scalar field φ ("inflaton"). The potential has a "plateau" in the range $\varphi_1 < \varphi < \varphi_2$ and is power-law type outside of this range. The breaks of the potential are smoothed out in small ranges $\Delta\varphi_1 \ll \varphi_1$ and $\Delta\varphi_2 \ll \varphi_2$ around φ_1 and φ_2 correspondingly.
Fig.2. Result of computations of $D(x)$, $x = kR_1$. The dashed, dashed dotted, dashed-triple dotted and solid lines correspond to $\gamma = 20$, $\gamma = 10$, $\gamma = 5$ and $\gamma = 2$ respectively.
Fig.3. Function $\beta(y)$,$y = R/R_1$. The solid, dashed and dashed-dotted lines correspond to $\gamma = 20$, $\gamma = 18$ and $\gamma = 16.5$ respectively. Straight lines which are tangential to $\beta(y)$ of different models correspond to the conditions $\Omega_{PBH} = 1$ for the modern Universe. The tangent points determine the corresponding M_{PBH}.

$M_{PBH} \simeq 10^8 M_\odot$ at $\gamma = 20.0$
$M_{PBH} \simeq 10^2 M_\odot$ at $\gamma = 18.0$
$M_{PBH} \simeq 10^{-6} M_\odot$ at $\gamma = 16.5$

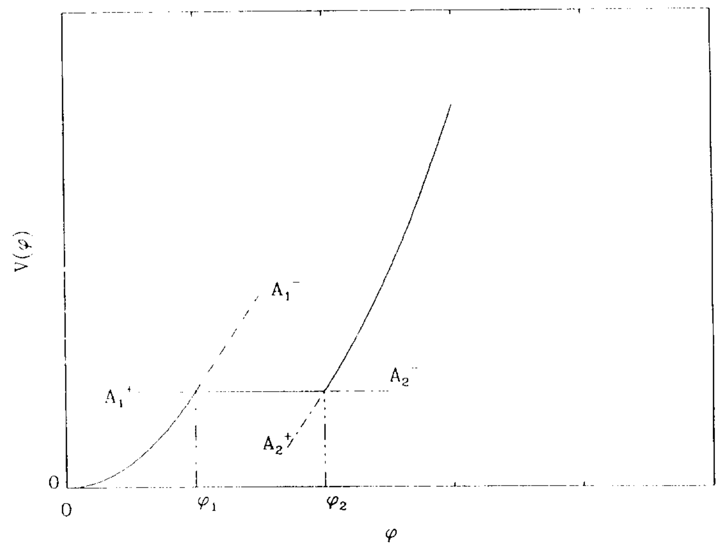
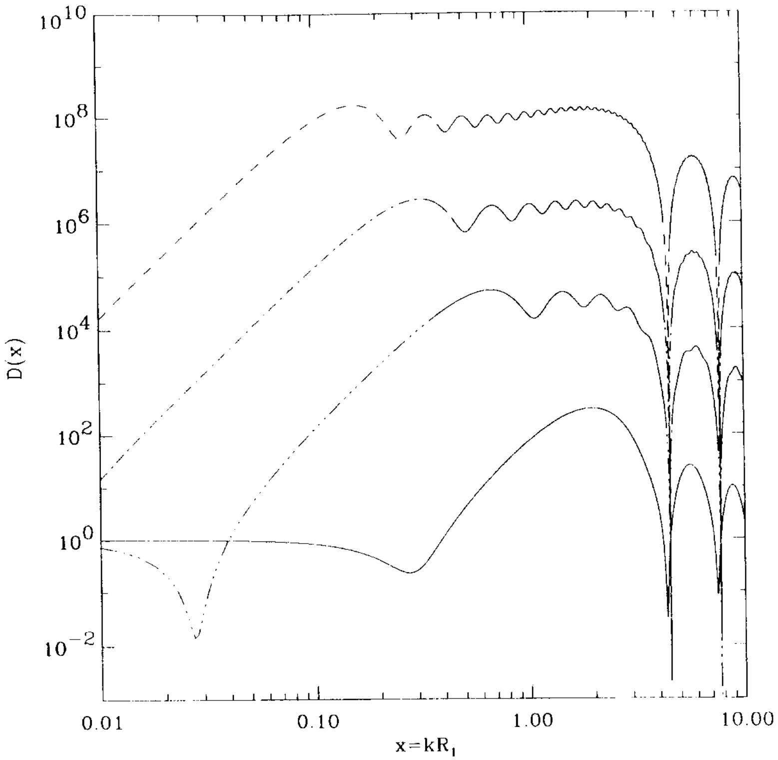
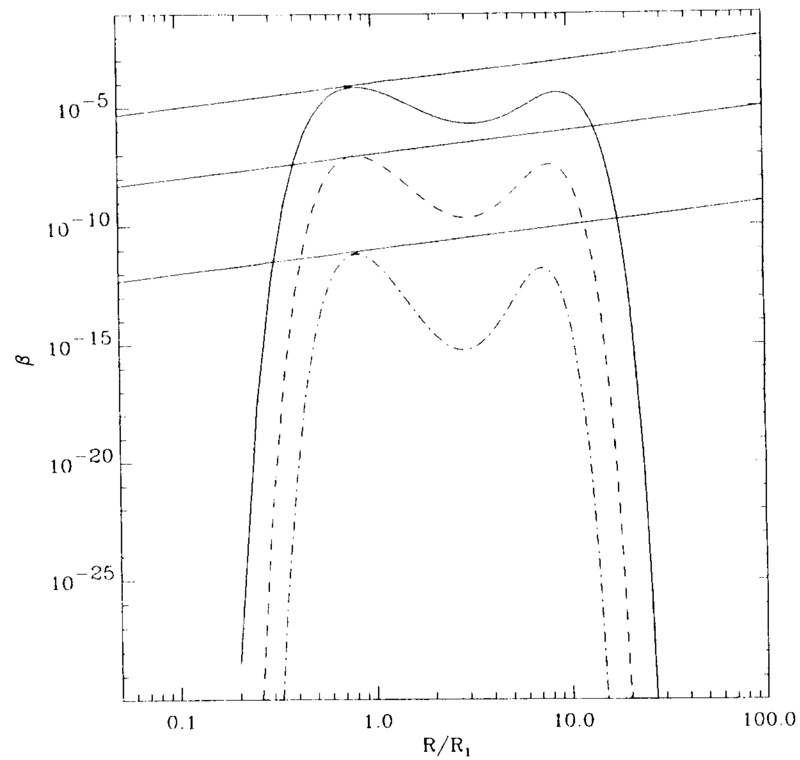


Fig. 1



10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
D(x)
0.01
0.10
1.00
10.00
$x=kR_1$

Fig. 2



10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
β
0.1
1.0
10.0
100.0
R/R_1

Fig. 3