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INFLATIONARY FINANCE
UNDER DISCRETION AND RULES

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Abstract

Inflationary finance involves first, the tax on cash balances from expected inflation, and second, a capital levy from unexpected inflation. From the standpoint of minimizing distortions, these capital levies are attractive, ex post, to the policymaker. In a full equilibrium two conditions hold: 1) the monetary authority optimizes subject to people's expectations mechanisms, and 2) people form expectations rationally, given their knowledge of the policymaker's objectives. The outcomes under discretionary policy are contrasted with those generated under rules. In a purely discretionary regime the monetary authority can make no meaningful commitments about the future behavior of money and prices. Under an enforced rule, it becomes possible to make some guarantees. Hence, the links between monetary actions and inflationary expectations can be internalized. There is a distinction between fully-contingent rules and rules of simple form. A simple rule allows the internalization of some connections between policy actions and inflationary expectations, but discretion permits some desirable flexibility of monetary growth.

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Introduction

Inflationary finance has often been studied in the context of a once-and-for-all choice of the monetary growth rate. See, for example, Bailey (1956) and Phelps (1973). The nominal interest rate is the tax rate levied on real cash balances. The effects of this tax on governmental revenues and excess burden can be analyzed in a manner similar to that for taxes on labor income, produced commodities, etc. The special characteristics of money concern the tendency for monopoly on the supply side, and the possibility that the marginal cost of production (for real balances) is near zero.

Cagan (1956, pp. 77, ff.) observed that surprise inflation is also a source of government revenue. The discussions in Auernheimer (1974) and Sjaastad (1976) link surprise inflation to unanticipated capital losses on real cash (or, more generally, on any governmental obligations whose payment streams are fixed in nominal terms). The revenue obtained through surprise inflation amounts to an ex post capital levy. As with other capital levies, this form of tax--when not foreseen--can raise revenue at little deadweight loss.

Since people understand the attractions of ex post capital levies, they will attempt to forecast, ex ante, the government's tendency to exploit such situations. So, in deciding to hold money or other nominal liabilities of the government, people will take into account the government's power to engineer ex post "surprises," which depreciate the real value of these claims. (Default on public debts and high rates of taxation on "old" capital are similar phenomena.) The expected cost of holding money takes into consideration the likelihood of these subsequent capital losses.

Further, in making optimal forecasts of these losses, people essentially have to model the monetary authority's behavior. Specifically, people would project the reaction of monetary growth to a set of state variables, which include the position of the money-demand function. In a full rational expectations equilibrium, no systematic inflation surprises can occur. Prices may be higher or lower than predicted because of the realizations of some random events, but the government cannot be systematically generating proceeds from surprise inflation.

Although a regular pattern of inflation surprises cannot arise in equilibrium, the government's capacity to create these shocks, ex post, influences the position of the equilibrium for the growth rates of money and prices. If the policymaker could commit himself in advance to resist the ex post benefits from surprise inflation, then the equilibrium rates of monetary growth and inflation are likely to be lower. Monetary and price rules provide such commitments to varying degrees. In contrast, a purely discretionary regime has no scope for these types of restrictions on subsequent monetary behavior. This type of distinction between rules and discretion has been stressed by Kydland and Prescott (1977).

Calvo (1978) studied a model in which the government was interested only in maximizing its revenue from inflationary finance. In a purely discretionary setup, where people have rational expectations, there is no finite equilibrium solution for monetary growth and inflation. In the present paper the government's objective is modified to trade off costs of inflation against the benefits from revenues. A finite equilibrium is then determined under purely discretionary monetary policy. The implied growth rates of money and prices depend on some stochastic state variables, which include the government's valuation of revenues (which may vary with the overall level of government spending or with aggregate business conditions), and the position of the money-demand function.

The results under discretion are contrasted to those obtainable from monetary rules. I consider rules that allow for a full set of state-contingent responses, as well as those of simple form that do not allow for full contingencies. The constant-growth-rate rule is of the latter type. Although state-contingent rules can unambiguously outperform discretion (abstracting from costs of enforcing the rules), the ranking of discretion and simple rules is uncertain. The rules have the benefit of internalizing all linkages between monetary behavior and inflationary expectations. However, discretion (and fully-contingent rules) allow for a flexible response to changes in circumstances, which has some value in the model under study.

Setup of the Model

It is convenient (for me) to develop the model in a framework of discrete time. The length of the period is denoted by τ, which has units of time, say of years. Since the length of the period plays no economic role in the present model, I ultimately focus on the results when τ is allowed to approach zero.

The government determines the nominal money stock at the start of period t to be the amount M_t. No private issues of money are considered. The growth rate of money from period t-1 to t is $\mu_t \equiv (1/\tau)\log(M_t/M_{t-1})$, which has units of per year. The general price level for period t is P_t, so that real balances are M_t/P_t. I focus here on holdings of money, although the approach can be applied also to government bonds that have prescribed nominal payouts. The inflation rate from period t-1 to t is $\pi_t \equiv (1/\tau)\log(P_t/P_{t-1})$, which is measured per year.

Let I_t denote the information set that each person has during period t. (The government will have equivalent information when setting the money stock, M_t.) This information set includes knowledge of P_t and M_t. Based on this information, everyone calculates the expected rate of inflation from period t to t+1,

$$(1) \qquad \pi^e_{t+1} \equiv E(\pi_{t+1}|I_t) = \frac{1}{\tau}[E(\log P_{t+1}|I_t) - \log P_t].$$

Demand for real balances during period t depends inversely on π^e_{t+1}, and positively on a stochastic scale variable, A_t. I use the semi-logarithmic form, as in Cagan (1956),

$$(2) \qquad M_t/P_t = A_t \cdot \exp(-\alpha\pi^e_{t+1}).$$

The logarithm of the scale variable, A_t, is generated from a random-walk process,

$$(3) \qquad \log A_t = \log A_{t-1} + a_t,$$

where a_t is white noise. The variance of a_t, σ^2_a, is proportional to the period length, τ. People know the value of A_t at the start of period t--that is, A_t and a_t are contained in the information set, I_t.

Taking logarithmic first differences of equation (2), and using equation (3) and the definitions of μ_t and π_t, leads to the condition,

$$(4) \qquad \pi_t \cdot \tau = \mu_t \cdot \tau + \alpha(\pi^e_{t+1} - \pi^e_t) - a_t.$$

Note that $\pi^e_t = (1/\tau)[E(\log P_t | I_{t-1}) - \log P_{t-1}]$. Equation (4) indicates that the inflation rate, π_t, is above (below) the monetary growth rate, μ_t, to the extent of the proportionate fall (rise) in real money demanded from period t-1 to t. The change in real money demanded reflects either a change in expected inflation, $\pi^e_{t+1} - \pi^e_t$ (multiplied by the proportionate sensitivity of money demand, α), or a proportionate change in the scale variable, $a_t = \log(A_t/A_{t-1})$.

The government's real revenue from money creation for period t is

$$(5) \qquad R_t = (M_t - M_{t-1})/P_t = (M_t/P_t) - (M_{t-1}/P_{t-1})(P_{t-1}/P_t).$$

Substituting for the real-balance terms from equation (2), and using the condition, $P_{t-1}/P_t = \exp(-\pi_t \cdot \tau)$, leads to the expression,

$$(6) \qquad R_t = A_t \cdot \exp(-\alpha\pi^e_{t+1}) - A_{t-1} \cdot \exp(-\alpha\pi^e_t - \pi_t \cdot \tau).$$

For given inflationary expectations between periods t and t+1, π^e_{t+1}, there is a given amount of real cash that people will hold during period t, $(M_t/P_t) = A_t \cdot \exp(-\alpha\pi^e_{t+1})$. Given this term, governmental revenue will be higher if the real value of money carried over from the previous period, $M_{t-1}/P_t = A_{t-1} \cdot \exp(-\alpha\pi^e_t - \pi_t \cdot \tau)$, can be reduced. Therefore, if π^e_{t+1} could be held fixed (and for a given value of the prior expectation, π^e_t), revenue would be enhanced by choosing monetary growth, μ_t, in order to engineer a higher value of π_t. Higher inflation depreciates more real cash, which allows the policymaker to provide more new real cash as a replacement. Of course, this procedure works best when π^e_{t+1} is insulated from the determination of μ_t and π_t--that is, when people do not anticipate a repetition of the high rate of depreciation in the real value of their money. The model has to clarify the interconnections among anticipated inflation rates, π^e_{t+1} and π^e_t, monetary growth, μ_t, and inflation, π_t. This analysis requires a theory of expectations, as well as a model of the policymaker's incentives to choose different rates of monetary expansion. The discussion deals first with the objectives of the policymaker. The solution to this problem--which private agents are assumed to understand--ultimately provides the basis for the theory of expectations.

The Policymaker's Objective

The government likes more revenues from money creation--perhaps because the alternatives are either extra revenue obtained from distorting taxation or less (valuable) public expenditures. For any period the policymaker values each unit of real revenue from money issue by the positive amount, θ_t. This valuation would be higher in times when alternative revenue sources require greater welfare losses at the margin. Possibly, θ_t would be especially high during wartime, in other periods where government expenditures have risen sharply from previous levels, or in recessions. In this model changes in the logarithm of the θ_t parameter are regarded as unpredictable--that is, I use the random-walk specification,

(7) $$\log\theta_t = \log\theta_{t-1} + \varepsilon_t,$$

where ε_t is white noise, and independent of a_t in equation (3). The variance of ε_t, σ_ε^2, is proportional to the period length, τ. People know the value of θ_t at the start of period t--that is, θ_t and ε_t are included in the information set, I_t.

The policymaker is also concerned with costs from inflation. The usual liquidity costs, which are associated with the quantity of real money demanded during period t, involve the expected inflation rate between periods t and t+1, π_{t+1}^e. The actual inflation rate between periods t-1 and t, π_t, is assumed to entail separate costs from the standpoint of the policymaker. There might be costs of changing prices, per se, as well as losses from unexpected inflation. The costs of inflation, as perceived by the policymaker, are written as the function, $\phi(\pi_t, \pi_{t+1}^e)$. These costs have units of commodities per year. The total costs incurred over period t, which has length τ, are equal to $\tau\cdot\phi(\pi_t, \pi_{t+1}^e)$.

Overall, the government's objective concerns the net costs of inflation for each period, Z_t, as calculated from

$$(8) \qquad Z_t = \tau\cdot\phi(\pi_t, \pi_{t+1}^e) - \theta_t R_t$$

$$= \tau\cdot\phi(\pi_t, \pi_{t+1}^e) - \theta_t[A_t\cdot\exp(-\alpha\pi_{t+1}^e) - A_{t-1}\cdot\exp(-\alpha\pi_t^e - \pi_t\cdot\tau)],$$

where the formula for R_t has been substituted from equation (6). The cost function satisfies the properties, $\partial\phi/\partial\pi_t > 0$ and $\partial\phi/\partial\pi_{t+1}^e > 0$. The valuation placed on revenue, $\theta_t R_t$, is subtracted from the costs ascribed to actual and expected inflation. In general the policymaker would seek to minimize the expected sum of net costs--that is, of the Z_t's--where each period's cost is expressed in present-value terms. The expectations of the Z_t's would be evaluated as of some appropriate starting date.

The Choice of Monetary Growth under Discretion

The analysis of discretionary policy follows the general line of argument presented in Barro and Gordon (1981). The policymaker selects the monetary growth rate, μ_t, for each period. Under discretion, there are no possibilities for prior constraints or commitments that would restrict the subsequent choices of monetary growth rates.

At date t the expectation of inflation that was formed last period, π_t^e, cannot be altered. Suppose for the moment that current and future inflationary expectations, π_{t+1}^e, π_{t+2}^e, ..., are also treated as givens by the policymaker during period t. The choice of μ_t then determines π_t from equation (4), given the exogenous disturbance, a_t. The value for μ_t also determines the real revenue for period t, R_t, from equation (6) (given the values of A_t and A_{t-1}). Therefore, we can compute the full effect of μ_t on the contemporaneous net cost, Z_t, in equation (8). With the various π_{t+i}^e's held fixed (and future μ_{t+i}'s restricted by current actions), there are no effects in this model of the choice of μ_t on future Z_{t+i}'s. Therefore, the optimizing policymaker would choose μ_t in order to minimize the contemporaneous net cost, Z_t.

More generally, the π_{t+i}^e's will be related to expectations of future μ_{t+i}'s. Hence, the π_{t+i}^e's can be held fixed above only if the choice of μ_t has no implications for expectations of the μ_{t+i}'s. In the present context each μ_{t+i} will emerge from a minimization problem that starts from date t+i. The choice of μ_t and the resulting value for π_t do nothing to alter the objective characteristics of this problem for any future period. Therefore, it is reasonable to look for an equilibrium solution to the model where the μ_{t+i}'s are invariant with the selection of μ_t. (Note that there is nothing to learn about in this model--in particular, people already know the government's objective and "technology.") With the μ_{t+i}'s held fixed, it is reasonable also to hold fixed the π^e_{t+i}'s when μ_t varies. (In other words there will be a sensible equilibrium that satisfies this condition.)

For a discretionary policymaker, the choice of μ_t follows from the first-order condition, $(\partial Z_t/\partial \mu_t)|_{\pi^e_t,\ \pi^e_{t+1},\ \ldots} = 0$, as

$$(9) \qquad \partial\phi/\partial\pi_t - \theta_t A_{t-1}\cdot \exp(-\alpha\pi^e_t - \pi_t\cdot\tau) = 0.$$

This calculation uses the conditions, $(\partial\pi_t/\partial\mu_t)|_{\pi^e_t,\ \pi^e_{t+1}} = 1$ (from equation (4)), and $(\partial\pi^e_{t+1}/\partial\mu_t)|_{\pi^e_{t+1}} = 0$. (The period length , τ, appears multiplicatively in both terms in equation (9) and has been cancelled out.) Rewriting equation (9) yields the condition,

$$(10) \qquad \partial\phi/\partial\pi_t = \theta_t A_{t-1}\cdot \exp(-\alpha\pi^e_t - \pi_t\cdot\tau) = \theta_t(M_{t-1}/P_{t-1})\cdot \exp(-\pi_t\cdot\tau).$$

The monetary growth rate, μ_t, is chosen so as to equate the marginal cost of (actual) inflation to the marginal benefit from additional revenue. The latter quantity equals the valuation per unit of revenue, θ_t, times the marginal effect of μ_t on revenue, R_t. With π^e_{t+1} and π^e_t held fixed, it follows from equations (5) and (6) that $\partial R_t/\partial\mu_t = \tau(M_{t-1}/P_{t-1})\cdot \exp(-\tau_t\cdot\tau)$. Notice that--because π^e_{t+1} is held fixed when μ_t changes--the marginal cost of expected inflation does not enter into the first-order condition. The discretionary policymaker also does not consider any effects of monetary behavior--via influences on expected inflation--on the real money demanded for periods t-1 or t.

In order to obtain some closed-form results, I specialize the cost function to

(11) $$\phi(\pi_t, \pi^e_{t+1}) = (k_1/b)\cdot\exp(b\pi_t) + (k_2/b)\cdot\exp(b\pi^e_{t+1}); \quad k_1, k_2, b > 0.$$

The exponents, b, could differ in the two cost terms without affecting most of the results. Equation (11) implies positive and increasing marginal costs for actual and expected inflation. (π_t and π^e_{t+1} are also entered in separable form in this example.) With this specification of costs, equation (10) implies

(12) $$k_1\cdot\exp(b\pi_t) = \theta_t A_{t-1}\cdot\exp(-\alpha\pi^e_t - \pi_t\cdot\tau).$$

Rearranging terms and taking logarithms of both sides yields

(13) $$(b+\tau)\pi_t + \alpha\pi^e_t = \log(\theta_{t-1}A_{t-1}/k_1) + \varepsilon_t,$$

where recall that $\varepsilon_t = \log(\theta_t/\theta_{t-1})$. Given the formula for π_t in equation (13), the implied choice for μ_t (given π^e_{t+1}, π^e_t and a_t) can be determined from equation (4).

Rational Expectations

Equations (4) and (13) determine actual rates of inflation and monetary growth, π_t and μ_t, as functions of exogenous variables and the expected inflation rates, π^e_t and π^e_{t+1}. In order to close the model, these expectations must be determined. I use a rational expectations condition,

$\pi^e_{t+1} = E(\pi_{t+1}|I_t)$ and $\pi^e_t = E(\pi_t|I_{t-1})$. The information set, I_t, includes observations of all variables for date t, including M_t, P_t, θ_t and A_t. People also use knowledge of the model's structure in order to calculate these expectations. Most importantly, everyone understands that the policymaker chooses μ_t in each period in order to minimize costs--Z_t from equation (8)--while treating π^e_t and π^e_{t+1} as givens. Hence, they know that equation (13) holds. In a full equilibrium, people's inflationary expectations, π^e_{t+1} and π^e_t, will be consistent with well-informed projections of the policymaker's future choices of monetary growth. Although the policymaker is not forced to accord with these expectations (which will appear as givens to the policymaker during each period), the first-order condition in equation (10) will guide the choices to this result in equilibrium. That is, the policymaker will not be motivated in equilibrium to deviate systematically from people's prior expectations.

People form the expectation, π^e_t, based on the information available in period t-1. The variables, θ_{t-1} and A_{t-1}, are known at this date. Therefore, taking expectations of equation (13), conditional on the information set I_{t-1}, the expected inflation rate must satisfy

$$(14) \qquad \pi^e_t = (1/b+\alpha+\tau)\cdot\log(\theta_{t-1}A_{t-1}/k_1),$$

where I have used the condition, $E(\varepsilon_t|I_{t-1}) = 0$. The formula for π^e_t can be substituted back into equation (13) in order to calculate the actual inflation rate, π_t. The form of equation (14) can be updated by one period to determine π^e_{t+1}, the anticipated rate of inflation between periods t and t+1. Then, the results for π^e_t, π^e_{t+1} and π_t can be substituted into equation (4) in order to determine the monetary growth rate, μ_t.

The period length, τ, is now assumed to be negligible relative to the parameters, b and α. As $\tau \rightarrow 0$, the full solution to the model is as follows:

$$(15) \qquad \pi_t = (1/b+\alpha)\cdot\log(\theta_{t-1}A_{t-1}/k_1) + (1/b)\varepsilon_t,$$

$$(16) \qquad \pi_t^e = (1/b+\alpha)\cdot\log(\theta_{t-1}A_{t-1}/k_1),$$

$$(17) \qquad \pi_{t+1}^e = (1/b+\alpha)\cdot\log(\theta_t A_t/k_1),$$

$$(18) \qquad \mu_t = (1/b+\alpha)\cdot[\log(\theta_{t-1}A_{t-1}/k_1) - \alpha\varepsilon_t/\tau + ba_t/\tau],$$ [1]

where recall that $\varepsilon_t = \log(\theta_t/\theta_{t-1})$ and $a_t = \log(A_t/A_{t-1})$.

The solution for monetary growth and inflation is an equilibrium in the following sense. First, at a point in time, the policymaker treats π_t^e as given and regards π_{t+1}^e as generated from the expectations mechanism in equation (17)--in particular, π_{t+1}^e is invariant with the choice of μ_t. Given π_t^e and π_{t+1}^e, and the relation of π_t to μ_t from equation (4), the setting for μ_t in equation (18) satisfies the policymaker's first-order condition for minimizing Z_t, as specified in equation (13). Second, the expectations mechanisms in equations (16) and (17) are rational--that is, $\pi_t^e = E(\pi_t|I_{t-1})$ and $\pi_{t+1}^e = E(\pi_{t+1}|I_t)$, given that monetary growth for each period follows the form of equation (18).

Notice from equation (10) that the marginal cost of inflation, $\partial\phi/\partial\pi_t$, must be positive in equilibrium. Therefore, if $k_1 = 0$--so that no costs are attached by the policymaker to actual inflation, π_t--then no finite equilibrium values for π_t and μ_t are determined in equations (15) and (18). This case corresponds to the one explored by Calvo (1978), where the discretionary policymaker considers only the maximization of revenue, R_t.

Within a discretionary setup where the policymaker attempts to minimize net costs in each period, Z_t, the results provide a positive theory of monetary growth and inflation. As long as $k_1 > 0$, inflation and monetary growth depend on the two variables that move around in the model, θ and A. A higher level for θ_{t-1} (given the value of $\varepsilon_t = \log(\theta_t/\theta_{t-1})$) means a greater value placed on revenues. Hence, in an equilibrium where the first-order condition from equation (13) is satisfied, the values of π_t and μ_t are higher.[2] Similarly, a higher level for real money demand, A_{t-1} (given $a_t = \log(A_t/A_{t-1})$), encourages the revenue objective relative to the cost of inflation (if the cost parameter, k_1, is held constant[3]). Therefore, π_t and μ_t rise with A_{t-1}. A higher value for the cost parameter, k_1, reduces π_t and μ_t.

Although expectations are rational, surprises do occur because of the stochastic terms in the model. The amount of unexpected inflation follows from equations (15) and (16) as

$$(19) \qquad \pi_t - \pi_t^e = (1/b)\varepsilon_t.$$

When the valuation of revenue rises unexpectedly--that is, $\varepsilon_t = \log(\theta_t/\theta_{t-1}) > 0$--inflation is unexpectedly high. This result obtains because the policymaker is motivated by the rise in θ to incur a higher marginal cost of inflation, which means a higher value for π_t. The positive value for ε_t means also a rise in the expected inflation rate--that is, $\pi_{t+1}^e - \pi_t^e > 0$ from equations (16) and (17). Because of the reduction in real money demanded, we need a downward shift in μ_t in order to satisfy equation (4) (hence, the term, $-\alpha\varepsilon_t/\tau$, in the expression for μ_t in equation (18)[4]). Expected future rates of monetary expansion rise with ε_t.

A surprise shift in the level of real money demanded, $a_t = \log(A_t/A_{t-1})$, does not generate unexpected inflation (although it does produce unanticipated monetary growth). Given A_{t-1}, a higher value for a_t means a higher level of A_t. Revenue rises from equation (6) in accordance with the term, $(M_t/P_t) = A_t \cdot \exp(-\alpha\pi^e_{t+1})$. However, with π^e_{t+1} treated as invariant with the choice of μ_t, a shift in this term leaves unaltered the value of π_t that satisfies the policymaker's first-order condition. The term, a_t, does influence the relation between monetary growth and inflation in equation (4). This element raises μ_t for a given value of π_t. A positive value for a_t also raises anticipated future rates of monetary growth, which means an increase in π^e_{t+1}. This element lowers μ_t for a given value of π_t in equation (4). On net, a_t leads to an (unexpected) expansion of μ_t.[5]

The model determines the variance of the inflation rate over various horizons. Using equation (15) and the random-walk specifications for $\log(\theta_t)$ and $\log(A_t)$ (from equations (3) and (7)), the variance of the inflation rate is

$$\text{(20)} \qquad \mathrm{VAR}(\pi_t | I_0) = (1/b+\alpha)^2(t-1)(\sigma^2_\varepsilon+\sigma^2_a) + (1/b)^2\sigma^2_\varepsilon,$$

where I_0 is the information available in period zero. The variance of the inflation rate depends on the variances of the two innovation terms, σ^2_ε and σ^2_a. The forecast variance for π_t increases linearly with the distance in time, t. The variance for the log of any future price level, $\log(P_t)$, can also be determined as a function of σ^2_ε, σ^2_a and t.[6]

Discretion versus Rules

Constant State Variables

In the discretionary regime the policymaker holds fixed current and future inflationary expectations when deciding on period t's rate of monetary growth. Since these expectations are, in fact, related to monetary behavior, the discretionary policymaker is unable to internalize some effects of his actions. Recall the form of the objective function from equation (8),

$$Z_t = \tau\cdot\phi(\pi_t, \pi^e_{t+1}) - \theta_t[A_t\cdot\exp(-\alpha\pi^e_{t+1}) - A_{t-1}\cdot\exp(-\alpha\pi^e_t - \pi_t\cdot\tau)].$$

The discretionary policymaker has no way to consider the effects of monetary behavior on inflationary expectations, which influence costs, $\phi(\pi_t, \pi^e_{t+1})$, and revenues, through the terms, $\exp(-\alpha\pi^e_{t+1})$ and $\exp(-\alpha\pi^e_t)$.

The difference between discretion and policy rules--or commitments--can best be clarified by initially suppressing the variations over time in the state variables, θ_t and A_t. Assume now that $\theta_t = \theta$, and $A_t = A$, so that $\epsilon_t = a_t = 0$ holds for all t. The discretionary solutions now dictate constant rates of monetary growth and inflation. The results follow from equations (15)--(18) as

$$(21) \qquad \pi = \pi^e = \mu = (1/b+\alpha)\cdot\log(\theta A/k_1).$$

Suppose that the policymaker could restrict himself in advance to a particular value for future monetary growth. That is, the policymaker rules out the possibility of altering his subsequent behavior, even if such alterations would appear advantageous, ex post. In this case there would be a linkage between the policymaker's choice of (constant) monetary growth, μ, and the values of inflationary expectations. Namely, $\pi_t^e = \mu$ would hold for all periods t. Rather than treating expectations as givens at any point in time, the policymaker would now choose monetary growth subject to the condition that $\pi_t^e = \mu$ holds for all t.[7] Because the state variables are constant, it also follows here (from equation (4)) that $\pi = \mu$, and (from equation (6)) that $R/\tau = \mu(M/P)$.

The policymaker's objective still entails minimization of net costs--which are now the same for all periods (see, however, n. 7 above)--

$$Z = \tau \cdot \phi(\pi, \pi^e) - \theta R$$
$$(22) \qquad = \tau(k_1/b) \cdot \exp(b\pi) + \tau(k_2/b) \cdot \exp(b\pi^e) - \tau\theta\mu A \cdot \exp(-\alpha\pi^e).$$

Given that $\pi^e = \pi = \mu$ holds, the optimizing value for the monetary growth rate, denoted by μ^*, satisfies the first-order condition,

$$(k_1 + k_2) \cdot \exp(b\mu^*) = \theta A(1-\alpha\mu^*) \cdot \exp(-\alpha\mu^*).$$

This condition can be rewritten as

$$(b+\alpha)\mu^* = \log(\frac{\theta A}{k_1+k_2}) + \log(1-\alpha\mu^*).$$

If $\alpha\mu^*$ is much less than one (which means that μ^* is well below the purely revenue-maximizing rate), then $\log(1-\alpha\mu^*) \approx -\alpha\mu^*$. The approximate solution for μ^* can then be written as

$$(23) \qquad \mu^* \approx \frac{1}{(b+2\alpha)} \cdot \log\left(\frac{\theta A}{k_1+k_2}\right), \text{ if } \alpha\mu^* \text{ is much less than one.}$$

Since people anticipate this monetary behavior under a rule, their expectations, $(\pi^e)^*$, accord with μ^* and with the actual inflation rate--that is, $\pi^* = (\pi^e)^* = \mu^*$.

Compare the choice of monetary growth under a rule, μ^*, with the value of μ from equation (21), which arises under discretion. The growth rates of money and prices tend to be higher under discretion for two reasons.[8] First, by neglecting the interplay between monetary behavior and (future) inflationary expectations, the discretionary policymaker ignores the costs of expected inflation. Therefore, the cost parameter k_2 is absent from equation (21), but is present in equation (23). Second, the discretionary policymaker neglects the negative effect of higher anticipated inflation--associated with higher expected rates of monetary expansion--on real money demanded. This factor enters into the rules solution of equation (23) through the parameter, 2α. For the discretionary solution in equation (21), α appears instead of 2α.

Viewed alternatively, suppose that μ^* were conjectured to be the equilibrium rate of monetary growth under discretion. This rate of monetary expansion minimizes net costs in equation (22), given that π^e and π move one-to-one with μ. But, if π^e (at all dates) were set by the formula in equation (23), the discretionary policymaker--who is not bound to set $\mu = \mu^*$--would not find this rate of monetary growth to be advantageous. Rather, a (surprisingly) higher value for monetary growth, which depreciates more of the initial real balances (in each period), would be preferred.[9] The increase in costs due to added inflation would be more than outweighed by the gains from additional revenue. Of course, agents would not then maintain their inflationary expectations at the value indicated in equation (23). The full equilibrium under the assumption of an unrestricted policymaker is the discretionary outcome that is specified in equation (21).[10]

Net costs, Z, are lower under the monetary rule--μ^* in equation (23)--than under discretion--μ in equation (21). It is unclear whether revenues are higher under rules or discretion. Since monetary growth is higher under discretion (n. 8 above), revenues will also surely be higher under discretion if the rate μ from equation (21) is below the purely revenue-maximizing rate, which is $1/\alpha$ in this model. Using equation (21), this condition holds if $(\theta A/k_1) < \exp[(b+\alpha)/\alpha]$. (Because the discretionary policymaker does not internalize the full consequences of his monetary behavior, it is conceivable that he would choose a monetary growth rate that exceeds the revenue-maximizing rate.)

Changes in the State Variables

Given constant state variables, θ and A, discretion and rules generate constant values for monetary growth, which are μ and μ^* respectively. When θ and A move randomly over time, the discretionary choice of monetary growth also moves randomly, as shown in equation (18). What does a monetary rule dictate when the state variables change? Suppose that we interpret a rule as a commitment to a prescribed functional relation between μ_t and the pertinent state variables--θ_t, A_t, ε_t, a_t, etc. Then, we could in principle solve for the rule that minimizes the expected sum of net costs--that is, the Z_t's--in present-value terms. The expectations would be evaluated as of some starting date (see below). Although I have not figured out how to determine the best contingent rule of this type in the present model, it is clear that such a rule would outperform discretion--in particular, it would be possible to deliver a lower level of Z_t, contingent on any configuration for the state variables.[11] Basically, the contingent rule differs from discretion only because it allows the policymaker to internalize the relation between monetary behavior and inflationary expectations. The consideration of this linkage can only allow for better preformance, regardless of whether θ and A are constants or variables. (Note also that discretion is one form of "rule" that is admissable to a policymaker who can choose among all possible contingent forms of action.)

Alternatively, it might be argued that only simple commitments can be adequately monitored and enforced. (The fact that I have not figured out the optimal contingent rule in the present model may or may not be evidence in this context.) Rules with a full set of contingencies may be infeasible. Then, the operational choice would be between discretion and simple rules. Discretion has the advantage of responding "flexibly" to movements in the state variables. (This flexibility may be valuable in a model, such as the present one, where it would be desirable to alter monetary behavior in response to new information.) However, the nature of the response and the choice of average monetary growth under discretion are "wrong," because they fail to internalize the connection between monetary behavior and inflationary expectations. That is, the discretionary policymaker lacks the valuable option of making binding promises (long-term contracts with the public) about his future actions. A simple rule exercises this option, but misses the flexibility of response to changes in the state variables.

To illustrate, suppose that $\log\theta_t$ again moves as the random walk,

$$(24) \qquad \log\theta_t = \log\theta_{t-1} + \varepsilon_t.$$

For simplicity, continue to assume that $A_t = A$ applies for all t. Suppose that the only admissable form of rule is a constant-growth-rate rule. At some starting date, 0, the policymaker makes a once-and-for-all, binding selection of the monetary growth rate, $\hat{\mu}$. Since A_t is constant, we also have--for all periods after period 0--$\hat{\pi} = \hat{\pi}^e = \hat{\mu}$. Further, revenue per unit of time is constant in all periods after period 0 at the amount, $\widehat{R/\tau} = \hat{\mu}\widehat{(M/P)} = \hat{\mu}A\cdot\exp(-\alpha\hat{\mu})$. Net costs for any period $t \geq 1$ are therefore given by

$$(25) \qquad Z_t = \frac{\tau(k_1+k_2)}{b}\cdot\exp(b\hat{\mu}) - \tau\theta_t\hat{\mu}A\cdot\exp(-\alpha\hat{\mu}).$$

The expected net costs for any period $t \geq 1$, as calculated at date 0, are

$$(26) \qquad E(Z_t|I_0) = \frac{\tau(k_1+k_2)}{b}\cdot\exp(b\hat{\mu}) - \tau\hat{\mu}A\cdot\exp(-\alpha\hat{\mu})\cdot E(\theta_t|I_0),$$

where I_0 is the information available at date zero.

Suppose that ε_t in equation (24) is normally distributed with zero mean and variance, $\sigma_\varepsilon^2 = \sigma^2\cdot\tau$. The parameter, σ^2, represents the variance over a period of unit length. Then, conditioned on I_0, $\log(\theta_t)$ is normally distributed with mean $\log(\theta_0)$ and variance $\sigma^2\cdot\tau t$. (Note that t is measured in periods and τ in years per period, so that τt is measured in years.) The expectation of θ_t is then

$$(27) \qquad E(\theta_t | I_0) = \theta_0 \cdot \exp(\frac{1}{2}\sigma^2 \cdot \tau t),$$

where I have used the standard formula for the mean of a log-normal variate (Aitchison and Brown, 1957, Ch. 2). Note that a random walk for $\log(\theta_t)$ means a drift in θ_t at rate $\frac{1}{2}\sigma^2$ per year.

Suppose that we neglect matters for period 0 (as we did above in n. 6), which would otherwise bring in the initial state of inflationary expectations, π_0^e.[12] (The policymaker may be able to "cheat once" by picking $\hat{\mu} \neq \pi_0^e$.) Assume that the policymaker cares about the sum of expected present values of net costs from period 1 onward. That is, $\hat{\mu}$ is chosen to minimize the quantity,

$$(28) \qquad \begin{aligned} V &\equiv \sum_{t=1}^{\infty} E(Z_t | I_0) \cdot \exp(-r \cdot \tau t) \\ &= \sum_{t=1}^{\infty} \left[\left(\frac{\tau(k_1+k_2)}{b} \right) \cdot \exp(b\hat{\mu} - r\tau t) - \tau\hat{\mu}A\theta_0 \cdot \exp[-\alpha\hat{\mu} - (r - \frac{1}{2}\sigma^2)\tau t] \right], \end{aligned}$$

where r is an exogenous, constant real discount rate, expressed per unit of time. I assume that $r > \frac{1}{2}\sigma^2$, which generates a finite sum on the right side of equation (28). (That is, the discounting outweighs the drift in θ_t.) As $\tau \to 0$, the sum can be evaluated as

$$(29) \qquad V = \frac{(k_1+k_2)}{rb} \cdot \exp(b\hat{\mu}) - \hat{\mu}A\theta_0 \cdot \exp(-\alpha\hat{\mu})/(r - \frac{1}{2}\sigma^2).$$

Assuming that $\alpha\hat{\mu}$ is much less than one (that is, that $\hat{\mu}$ is well below the purely revenue-maximizing rate), the value of $\hat{\mu}$ that minimizes V is given by

$$(30) \qquad \hat{\mu} \approx \frac{1}{(b+2\alpha)} \cdot \log\left[\frac{\theta_0 A}{(k_1+k_2)} \cdot \left(\frac{r}{r-\frac{1}{2}\sigma^2}\right)\right].$$

A comparison of equation (30) with equation (23) indicates that the new feature is the variance term, σ^2. If σ^2 is higher, relative to r, the value of the monetary growth rate, $\hat{\mu}$, is increased. Essentially, the policymaker recognizes that the mean of θ_t advances over time in accordance with the value of σ^2. Given the assumed inability to match μ_t to θ_t at each date, the policymaker compensates partially by picking a higher value for $\hat{\mu}$ at the start. Of course, as time passes, the rate $\hat{\mu}$ is likely to depart farther and farther from the rate that would have been chosen on a fully contingent basis. There might eventually be sufficient pressure to generate a breakdown in the constant-growth-rate rule. But, in order to analyze this situation, we would have to specify precisely the conditions that motivate a departure of monetary growth from the value $\hat{\mu}$. We would then have a rule that allows for limited contingencies, although not for a full set of these contingencies.

It is possible to compare the outcomes from a constant-growth-rate rule, as specified in equation (30), with those generated under discretion, as shown in equation (21). It is now uncertain which method of operation delivers the lower sum of expected present values of net costs (say, from period 1 onward). The expected costs can be calculated explicitly for the two cases, although the results are hard to interpret. Generally, the relative sizes of the two expected costs depend on the parameter values, α, b, A, k_1, k_2, σ^2 and r. For example, a high value for k_2 tends to favor the simple rule, because discretion does not internalize the responses associated with this parameter. However, a higher value for σ^2 favors discretion, because the flexibility of adjustment becomes more significant as σ^2 rises.

Concluding Remarks

The interaction between monetary behavior and inflationary expectations determines the time path of inflation. In order to understand this interaction and the mechanisms by which expectations are generated, we have to model the money-supply process. In the present analysis monetary behavior reflects the government's desire for inflationary finance, subject to some concern about the costs of inflation. The outcomes depend heavily on the characteristics of monetary institutions. Specifically, there are important distinctions between discretion and rules. In a purely discretionary regime the monetary authority can make no meaningful commitments about the future behavior of money and prices. Under an enforced rule--such as a serious gold standard or statutes that restrain monetary growth--it becomes possible to make these commitments, which amount to long-term contracts with holders of the government's liabilities. Thereby, the policymaker can internalize some linkages between monetary actions and inflationary expectations--some of these connections are missing under discretion.

The analysis distinguishes fully-contingent rules from commitments of a simple form, such as a constant-growth-rate-rule for the money supply. Abstracting from costs of erecting and enforcing the rules, discretionary monetary policy is always inferior to a fully-contingent rule. The comparison with limited rules is ambiguous. The rules allow the internalization of some links between policy actions and inflationary expectations, but discretion permits some desirable "flexibility" of monetary growth.

Generally, monetary institutions are important in two respects. First, the characteristics of these institutions--such as discretion versus rules--matter for predicting the actual course of monetary growth and inflation. That is, institutional features matter at the level of positive analysis. Secondly, it may be that economists' normative analysis of governmental policy is usefully directed toward changes in institutions, rather than to day-to-day operating procedures within a given regime. Whether this is so depends on a positive theory of institutional choice, which is not well developed.

One specific issue needs more attention. The contrast between discretion and rules amounts to the distinction between no promises and fully-binding promises. There seems to be a middle ground where policymakers can build up a reputation--that is, credibility--which is based on past actions. We want to know when these reputational forces will substitute satisfactorily for formal rules of law--that is, for fully articulated monetary or price rules. Reputation seems to work well in many types of private exchanges, as well as in some areas of governmental policy. We want to know when reputation will generate satisfactory outcomes in the context of monetary growth and inflation.

Footnotes

[1]As $\tau \to 0$ the resulting revenue per unit of time, R_t/τ, can be written as

$$(R_t/\tau) = \frac{A_{t-1}}{(b+\alpha)}\left(\frac{\theta_{t-1}A_{t-1}}{k_1}\right)^{-\alpha/(b+\alpha)}\left[\log\left(\frac{\theta_{t-1}A_{t-1}}{k_1}\right) + b\left(\frac{a_t}{\tau}\right) - \alpha\left(\frac{\varepsilon_t}{\tau}\right)\right] = \mu_t(M_t/P_t).$$

[2]If $a_t = \varepsilon_t = 0$, then the equilibrium amount of revenue per unit of time (n. 1, above) rises with θ_{t-1} if and only if $\mu_t < 1/\alpha$--that is, if and only if the monetary growth rate is below the revenue-maximizing value for the context of a once-and-for-all choice of μ.

[3]Secular growth, which leads to a steady increase in real money demand, would tend to be accompanied by a parallel rise in the cost parameter, k_1. These changes leave π_t and μ_t unaltered over time.

[4]As $\tau \to 0$ the variance of ε_t/τ approaches infinity. In other words jumps in the money stock are required in order to maintain equality between money supply and demand at all points in time. However, the policymaker acts in this model to avoid jumps in the price level--see equation (15).

[5]The term, a_t/τ, appears in equation (18). The variance of this term approaches infinity as $\tau \to 0$. Hence, jumps in the money stock can occur, as discussed in n. 4 above.

[6]The variance of the monetary growth rate, μ_t, is infinite. However, the variance of the log of the future money stock, $\log(M_t)$, is finite.

[7]In some initial period where the rule is implemented, say date 0, expectations need not be realized. That is, $\pi_0^e \neq \mu$ may apply. Under reasonable assumptions, this element has a negligible impact on the policymaker's once-and-for-all choice of the monetary growth rate.

[8] Equations (21) and (23) imply

$$\mu - \mu^* = \left(\frac{1}{b+\alpha}\right)\cdot \log\left(\frac{k_1+k_2}{k_1}\right) + \frac{\alpha}{(b+\alpha)(b+2\alpha)}\cdot \log\left(\frac{\theta A}{k_1+k_2}\right).$$

$\mu > \mu^*$ follows unambiguously if $\log(\theta A/k_1+k_2) > 0$--that is, if $\mu^* > 0$ in equation (23).

[9] This preference does not require any difference of opinion between the policymaker and the typical member of the private sector. The valuation of revenues, θ_t, reflects some externality (for example, from distorting taxation). The inflation surprise lowers the welfare loss from this existing distortion.

[10] There may be other equilibria where people condition subsequent inflationary expectations, π^e_{t+1}, ..., on the history of monetary growth, including μ_t. (This conditioning arises in these cases although μ_t affects no objective state variables that appear from date t+1 onward.) These types of reputational equilibria are fragile because they always allow the policymaker to "cheat" in the short run. (Recall from n. 9 above that this cheating may be socially optimal, ex post.) An important issue is the potential for the policymaker to reinstate a reputation for low inflation--this process cannot be too easy, or else cheating would always arise (and the equilibrium would then be the discretionary one from equation (21)). Some discussion of these matters appears in Barro and Gordon (1981, Section 6).

[11] Rules would also deliver a variance for the inflation rate that differs from the one generated under discretion in equation (20). In a larger model the forecast variance for the price level might enter into the cost function, Z in equation (8), or in the money-demand function.

[12] Under reasonable assumptions, this element would not have a significant impact on the policymaker's once-and-for-all choice of monetary growth rate. We would also have to consider whether the implementation of the monetary rule were anticipated at earlier dates--that is, it may be inappropriate to treat π_0^e as an arbitrary parameter.

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