

Inflationary Models and Connections to Particle Physics[†]

Alan H. Guth[‡]

Center for Theoretical Physics

Laboratory for Nuclear Science and Department of Physics

Massachusetts Institute of Technology, Cambridge, Massachusetts 02139 U.S.A.[§]

and

Isaac Newton Institute for Mathematical Sciences,

Clarkson Road, Cambridge CB3 0EH, UK

`guth@ctp.mit.edu`

Abstract

The basic workings of inflationary models are summarized, along with the arguments that strongly suggest that our universe is the product of inflation. The mechanisms that lead to eternal inflation in both new and chaotic models are described. Although the infinity of pocket universes produced by eternal inflation are unobservable, it is argued that eternal inflation has real consequences in terms of the way that predictions are extracted from theoretical models. The ambiguities in defining probabilities in eternally inflating spacetimes are reviewed, with emphasis on the youngness paradox that results from a synchronous gauge regularization technique. To clarify (but not resolve) this ambiguity, a toy model of an eternally inflating universe is introduced. Vilenkin's proposal for avoiding these problems is also discussed, as is the question of whether it is meaningful to discuss probabilities for unrepeatable measurements.

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[§]Present address.

I. Introduction

There are many fascinating issues associated with eternal inflation, which will be the main subject of this talk. You have certainly heard other people talk about eternal inflation, but I feel that the topic is important enough so that you should hear about it in some accent other than Russian. I will begin by summarizing the basics of inflation, including a discussion of how inflation works, and why many of us believe that our universe almost certainly evolved through some form of inflation. This material is certainly not new, but I think it is an appropriate introduction to any volume that focuses on inflationary cosmology. Then I will move on to discuss eternal inflation, first explaining how it works. I will then argue the eternal inflation has important implications, and raises important questions, which should not be dismissed as being merely metaphysical.

II. How Does Inflation Work?

In this section I will review the basics of how inflation works, focusing on the earliest working forms of inflation—*new inflation* [1, 2] and *chaotic inflation* [3]. While more complicated possibilities (e.g. hybrid inflation [4, 5, 6, 7, 8] and supernatural inflation [9]) appear very plausible, the basic scenarios of new and chaotic inflation will be sufficient to illustrate the physical effects that I want to discuss in this article.

The key property of the laws of physics that makes inflation possible is the existence of states of matter that have a high energy density which cannot be rapidly lowered. In the original version of the inflationary theory [10], the proposed state was a scalar field in a local minimum of its potential energy function.* Such a state is called a *false vacuum*, since the state temporarily acts as if it were the state of lowest possible energy density. Classically this state would be completely stable, because there would be no energy available to allow the scalar field to cross the potential energy barrier that separates it from states of lower energy. Quantum mechanically, however, the state would decay by tunneling [12]. Initially it was hoped that this tunneling process could successfully end inflation, but it was soon found that the randomness of false vacuum decay would produce catastrophically large inhomogeneities [10, 13, 14].

This “graceful exit” problem was solved by the invention of the new inflationary universe model [1, 2], which achieved all the successes that had been hoped for in the context of the original version. In this theory inflation is driven by a scalar field perched on a plateau of the potential energy diagram, as shown in Fig. 1. Such a scalar field is generically called the *inflaton*. If the plateau is flat enough, such a state can be stable enough for successful inflation. Soon afterwards Linde showed that the inflaton potential need not have either a local minimum or a gentle plateau; in chaotic inflation [3], the inflaton potential can be as simple as

$$V(\phi) = \frac{1}{2}m^2\phi^2, \tag{2.1}$$

*A similar proposal was advanced by Starobinsky [11], in which the high energy density state was achieved by curved space corrections to the energy-momentum tensor of a scalar field.

provided that ϕ begins at such a large value that it takes a long time for it to relax. For simplicity of language, I will stretch the meaning of the phrase “false vacuum” to include all of these cases; that is, I will use the phrase to denote any state with a high energy density that cannot be rapidly decreased. While inflation was originally developed in the context of grand unified theories, the only real requirement on the particle physics is the existence of a false vacuum state.

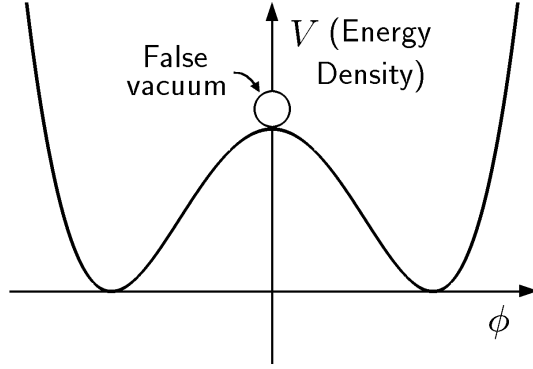


Figure 1: Generic form of the potential for the new inflationary scenario.

The New Inflationary Scenario:

Suppose that the energy density of a state is approximately equal to a constant value ρ_f . Then, if a region filled with this state of matter expanded by an amount dV , its energy would have to increase by

$$dU = \rho_f dV . \quad (2.2)$$

Something would have to supply that energy. Work would have to be done to cause the region to expand, which implies that the region has a negative pressure, which pulls back against whatever is causing the region to expand. The work done by this negative pressure p_f is given by the elementary formula

$$dW = -p_f dV . \quad (2.3)$$

Equating the work with the change in energy, one finds

$$p_f = -\rho_f . \quad (2.4)$$

It is this negative pressure which is the driving force behind inflation. When one puts this negative pressure into Einstein’s equations to find out its gravitational effect, one finds that it leads to a repulsion, causing such a region to undergo exponential expansion. If the region can be approximated as isotropic and homogeneous, this result can be seen from the standard Friedmann-Robertson-Walker (FRW) equations:

$$\frac{d^2 a}{dt^2} = -\frac{4\pi}{3}G(\rho + 3p)a = \frac{8\pi}{3}G\rho_f a . \quad (2.5)$$

where $a(t)$ is the scale factor, G is Newton's constant, and we adopt units for which $\hbar = c = 1$. For late times the growing solution to this equation has the form

$$a(t) \propto e^{\chi t}, \text{ where } \chi = \sqrt{\frac{8\pi}{3}G\rho_f}. \quad (2.6)$$

Of course inflationary theorists prefer not to assume that the universe began homogeneously and isotropically, but there is considerable evidence for the “cosmological no-hair conjecture” [15], which implies that a wide class of initial states will approach this exponentially expanding solution.

So the basic scenario of new inflation begins by assuming that at least some patch of the early universe was in this peculiar false vacuum state. In the original papers [1, 2] this initial condition was motivated by the fact that, in many quantum field theories, the false vacuum resulted naturally from the supercooling of an initially hot state in thermal equilibrium. It was soon found, however, that quantum fluctuations in the rolling inflaton field give rise to density perturbations in the universe [16, 17, 18, 19, 20], and that these density perturbations would be much larger than observed unless the inflaton field is very weakly coupled. For such weak coupling there would be no time for an initially nonthermal state to reach thermal equilibrium. Nonetheless, since thermal equilibrium describes a probability distribution in which all states of a given energy are weighted equally, the fact that thermal equilibrium leads to a false vacuum implies that false vacuum-like states are not uncommon. Thus, even in the absence of thermal equilibrium, even if the universe started in a highly chaotic initial state, it seems reasonable to simply assume that some small patches of the early universe settled into the false vacuum state, as was suggested for example in [21]. The idea that one should consider small patches of the early universe with arbitrary initial configurations of scalar fields was later emphasized by Linde [3] in the context of chaotic inflation. Linde pointed out that even highly improbable initial patches could be important if they inflated, since the exponential expansion could still cause such patches to dominate the volume of the universe. One might hope that eventually a full theory of quantum origins would allow us to calculate the probability of regions settling into the false vacuum, but I will argue in Sec. V that, in the context of eternal inflation, this probability is quite irrelevant.

Once a region of false vacuum materializes, the physics of the subsequent evolution seems rather clear-cut. The gravitational repulsion caused by the negative pressure will drive the region into a period of exponential expansion. If the energy density of the false vacuum is at the grand unified theory scale ($\rho_f \approx (2 \times 10^{16} \text{ GeV})^4$), Eq. (2.6) shows that the time constant χ^{-1} of the exponential expansion would be about 10^{-38} sec. For inflation to achieve its goals, this patch has to expand exponentially for at least 60 e-foldings. Then, because this state is only metastable—the inflaton field is perched on top of the hill of the potential energy diagram of Fig. 1—eventually this state will decay. The inflaton field will roll off the hill, ending inflation. And when it does, the energy density that has been locked in the inflaton field is released. Because of the coupling of the inflaton to other fields, that energy becomes thermalized to produce a hot soup of particles, which is exactly what had always been taken as the starting point of the standard big bang theory before inflation was introduced. From here on the scenario joins onto the standard big bang description. The role of inflation is to replace the postulates of the standard big bang theory with dynamically generated initial

conditions.

The inflationary mechanism produces an entire universe starting from essentially nothing, so one needs to answer the question of where the energy of the universe came from. The answer is that it came from the gravitational field. I am not saying that the colossal energy of the universe was stored from the beginning in the gravitational field. Rather, the crucial point is that the energy density of the gravitational field is literally negative—a statement which is true both in Newtonian gravity and in general relativity. So, as more and more positive energy materialized in the form of an ever-growing region filled with a high-energy scalar field, more and more negative energy materialized in the form of an expanding region filled with a gravitational field. So the total energy remained very small, and could in fact be exactly zero. There is nothing known that places any limit on the amount of inflation that can occur while the total energy remains exactly zero.[†]

Chaotic Inflation:

Chaotic inflation [3] can occur in the context of a much more general class of potential energy functions. In particular, even a potential energy function as simple as Eq. (2.1), describing a scalar field with a mass and no interaction, is sufficient to describe chaotic inflation. Chaotic inflation is illustrated in Fig. 2. In this case there is no state that bears any obvious resemblance to the false vacuum of new inflation. Instead the scenario works by supposing that chaotic conditions in the early universe produced one or more patches in which the inflaton field ϕ was at some high value $\phi = \phi_0$ on the potential energy curve. Inflation occurs as the inflaton field rolls down the hill. As long as the initial value ϕ_0 is sufficiently high on the curve, there will be sufficient inflation to solve all the problems that inflation is intended to solve.

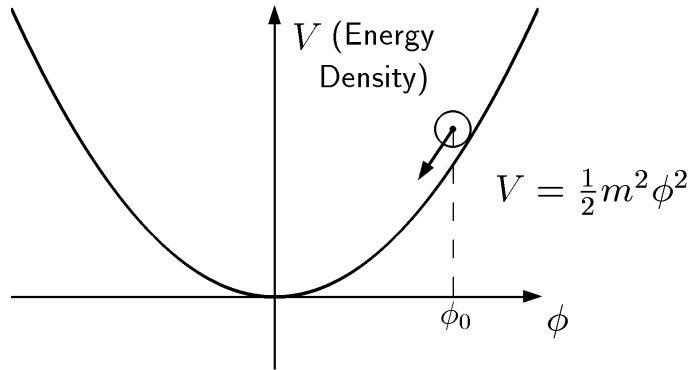


Figure 2: Generic form of the potential for the chaotic inflationary scenario.

The equations describing chaotic inflation can be written simply, provided that we assume that the universe is already flat enough so that we do not need to include a curvature term.

[†]In Newtonian mechanics the energy density of a gravitational field is unambiguously negative; it can be derived by the same methods used for the Coulomb field, but the force law has the opposite sign. In general relativity there is no coordinate-invariant way of expressing the energy in a space that is not asymptotically flat, so many experts prefer to say that the total energy is undefined. Either way, there is agreement that inflation is consistent with the general relativistic description of energy conservation.

The field equation for the inflaton field in the expanding universe is

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{\partial V}{\partial \phi} , \quad (2.7)$$

where the overdot denotes a derivative with respect to time t , and H is the time-dependent Hubble parameter given by

$$H^2 = \frac{8\pi}{3}GV . \quad (2.8)$$

For the toy-model potential energy of Eq. (2.1), these equations have a very simple solution:

$$\phi = \phi_0 - \frac{m}{\sqrt{12\pi G}} t . \quad (2.9)$$

One can then calculate the number N of inflationary e-foldings, which is given by

$$N = \int_{\phi=\phi_0}^{\phi=0} H(t) dt = 2\pi G\phi_0^2 . \quad (2.10)$$

In this free-field model N depends only on ϕ_0 and not on the inflaton mass m . Thus the number of e-foldings will exceed 60 provided that

$$\phi_0 > \sqrt{\frac{60}{2\pi}} M_{\text{P}} \approx 3.1M_{\text{P}} , \quad (2.11)$$

where $M_{\text{P}} \equiv 1/\sqrt{G} = 1.22 \times 10^{19}$ GeV is the Planck mass. Although the value of the scalar field is larger than M_{P} , the energy density can be low compared to the Planck scale:

$$\rho_0 = \frac{1}{2}m^2\phi_0^2 > \frac{60}{4\pi}M_{\text{P}}^2m^2 . \quad (2.12)$$

For example, if $m = 10^{16}$ GeV, then the potential energy density is only $3 \times 10^{-6} M_{\text{P}}^4$. Since it is presumably the energy density that is relevant to gravity, one does not expect this situation to lead to strong quantum gravity effects.

III. Evidence for Inflation

The arguments in favor of inflation are pretty much the same no matter which form of inflation we are discussing. In my opinion, the evidence that our universe is the result of some form of inflation is very solid. Since the term *inflation* encompasses a wide range of detailed theories, it is hard to imagine any alternative. Let me review the basic arguments.

1) The universe is big

First of all, we know that the universe is incredibly large. The visible part of it contains about 10^{90} particles. It is easy, however, to take this fact for granted: of course the universe is big, it's the whole universe! In "standard" Friedmann-Robertson-Walker cosmology, without inflation, one simply postulates that about 10^{90} or more particles were here from the start.

If, however, we try to imagine a theory describing the origin of the universe, it would have to somehow output this number of 10^{90} or more. That is a very big number, and it is hard to imagine it ever coming out of a calculation in which the input consists only geometrical quantities, quantities associated with simple dynamics, and factors of 2 and π . In the inflationary model, the huge number of particles is explained naturally by the exponential expansion, which reduces the problem to explaining 60 or 70 e-foldings of inflation. In fact, it is easy to construct underlying particle theories that will give far more than 70 e-foldings, suggesting that the observed universe is only a tiny speck within the universe as a whole.

2) The Hubble expansion

The Hubble expansion is also easy to take for granted, since it is so familiar. In standard FRW cosmology, the Hubble expansion is part of the list of postulates that define the initial conditions. But inflation offers an explanation of how the Hubble expansion began. The repulsive gravity associated with the false vacuum is exactly the kind of force needed to propel the universe into a pattern of motion in which any two particles are moving apart with a velocity proportional to their separation.

3) Homogeneity and isotropy

The degree of uniformity in the universe is startling. Through careful measurements of the cosmic background radiation, we know that the intensity of this radiation is the same in all directions to an accuracy of 1 part in 100,000. To get some feeling for how high this precision is, we can imagine a marble that is spherical to this accuracy. The surface of the marble would have to be shaped with a tolerance of about 1,000 angstroms, a quarter of the wavelength of light.

Although precision lenses can be ground to quarter-wavelength accuracy, we would nonetheless be shocked if we ever dug up a stone from the ground that was round to this extraordinary accuracy. If such a stone were somehow found, I am confident that we would not accept an explanation of its origin which simply proposed that the stone started out perfectly round. Similarly, in the current era, I do not think it makes sense to consider any theory of cosmogenesis that cannot offer some explanation of how the universe became so incredibly isotropic.

The uniformity of the cosmic background radiation implies that the observed universe had become uniform in temperature by about 300,000 years after the big bang, when the universe cooled enough so that the opaque plasma neutralized into a transparent gas. In standard FRW cosmology, the uniformity could be established by this time only if signals could propagate 100 times faster than light, which is not possible. In inflationary cosmology, however, the uniformity can be created initially on microscopic scales, by normal thermal-equilibrium processes. Then inflation takes over and stretches the regions of uniformity to become large enough to encompass the observed universe.

4) The flatness problem

I find the flatness problem particularly impressive, because the numbers that it leads to

are so extraordinary. The problem concerns the value of the ratio

$$\Omega_{\text{tot}} \equiv \frac{\rho_{\text{tot}}}{\rho_c} , \quad (3.1)$$

where ρ_{tot} is the average total mass density of the universe and $\rho_c = 3H^2/8\pi G$ is the critical density, the density that would make the universe spatially flat. (ρ_{tot} includes any vacuum energy $\rho_{\text{vac}} = \Lambda/8\pi G$ associated with the cosmological constant Λ , if it is nonzero.)

The present value of Ω_{tot} satisfies

$$0.1 \lesssim \Omega_{\text{tot},0} \lesssim 2 , \quad (3.2)$$

but the precise value is not known. Despite the breadth of this range, the value of Ω_{tot} at early times is highly constrained, since $\Omega_{\text{tot}} = 1$ is an unstable equilibrium point of the standard model evolution. If Ω_{tot} was ever *exactly* equal to one, it would remain so forever. However, if Ω_{tot} differed slightly from 1 in the early universe, that difference—whether positive or negative—would be amplified with time. In particular, the FRW equations imply that $\Omega_{\text{tot}} - 1$ grows as

$$\Omega_{\text{tot}} - 1 \propto \begin{cases} t & \text{(during the radiation-dominated era)} \\ t^{2/3} & \text{(during the matter-dominated era)} . \end{cases} \quad (3.3)$$

At $t = 1$ sec, for example, Dicke and Peebles [22] pointed out that Ω_{tot} must have equaled one to an accuracy of one part in 10^{15} . Classical cosmology provides no explanation for this fact—it is simply assumed as part of the initial conditions. In the context of modern particle theory, where we try to push things all the way back to the Planck time, 10^{-43} sec, the problem becomes even more extreme. At this time Ω_{tot} must have equaled one to 58 decimal places!

While this extraordinary flatness of the early universe has no explanation in classical FRW cosmology, it is a natural prediction for inflationary cosmology. During the inflationary period, instead of Ω_{tot} being driven away from 1 as described by Eq. (3.3), Ω_{tot} is driven towards 1, with exponential swiftness:

$$\Omega_{\text{tot}} - 1 \propto e^{-2H_{\text{inf}}t} , \quad (3.4)$$

where H_{inf} is the Hubble parameter during inflation. Thus, as long as there is enough inflation, Ω_{tot} can start at almost any value, and it will be driven to unity by the exponential expansion.

5) Absence of magnetic monopoles

All grand unified theories predict that there should be, in the spectrum of possible particles, extremely massive magnetic monopoles. By combining grand unified theories with classical cosmology without inflation, Preskill [23] found that magnetic monopoles would be produced so copiously that they would outweigh everything else in the universe by a factor of about 10^{12} . A mass density this large would cause the inferred age of the universe to drop to about 30,000 years! In inflationary models, the monopoles can be eliminated simply by arranging the parameters so that inflation takes place after (or during) monopole production, so the monopole density is diluted to a completely negligible level.

6) Anisotropy of the cosmic background radiation

The process of inflation smooths the universe essentially completely, but quantum fluctuations of the inflaton field can generate density fluctuations as inflation ends. Generically these are adiabatic Gaussian fluctuations with a nearly scale-invariant spectrum [16, 17, 18, 19, 20]. New data is arriving quickly, but so far the observations are in excellent agreement with the predictions of the simplest inflationary models. For a review, see for example Bond and Jaffe [24], who find that the combined data give a slope of the primordial power spectrum within 5% of the preferred scale-invariant value.

IV. Eternal Inflation: Mechanisms

Having discussed the mechanisms and the motivation for inflation itself, I now wish to move on the main issue that I want to stress in this article—eternal inflation, the questions that it can answer, and the questions that it raises. In this section I will discuss the mechanisms that make eternal inflation possible, leaving the other issues for the following sections. I will discuss eternal inflation first in the context of new inflation, and then in the context of chaotic inflation, where it is more subtle.

Eternal New Inflation:

The eternal nature of new inflation was first discovered by Steinhardt [25] and Vilenkin [26] in 1983. Although the false vacuum is a metastable state, the decay of the false vacuum is an exponential process, very much like the decay of any radioactive or unstable substance. The probability of finding the inflaton field at the top of the plateau in its potential energy diagram does not fall sharply to zero, but instead trails off exponentially with time [27]. However, unlike a normal radioactive substance such as radium, the false vacuum exponentially expands at the same time that it decays. In fact, in any successful inflationary model the rate of exponential expansion is always much faster than the rate of exponential decay. Therefore, even though the false vacuum is decaying, it never disappears, and in fact the total volume of the false vacuum, once inflation starts, continues to grow exponentially with time, *ad infinitum*.

Fig. 3 shows a schematic diagram of an eternally inflating universe. The top bar indicates a region of false vacuum. The evolution of this region is shown by the successive bars moving downward, except that I could not show the expansion and still fit all the bars on the page. So the region is shown as having a fixed size in comoving coordinates, while the scale factor, which is not shown, increases from each bar to the next. As a concrete example, suppose that the scale factor for each bar is three times larger than for the previous bar. If we follow the region of false vacuum indicated by the top bar as it evolves into the second bar, in about one third of the region the scalar field rolls down the hill of the potential energy diagram, precipitating a local big bang that will evolve into something that will eventually appear to its inhabitants as a universe. This local big bang region is shown in gray and labeled “Universe.” Meanwhile, however, the space has expanded so much that each of the two remaining regions of false vacuum is the same size as the starting region. Thus, if we

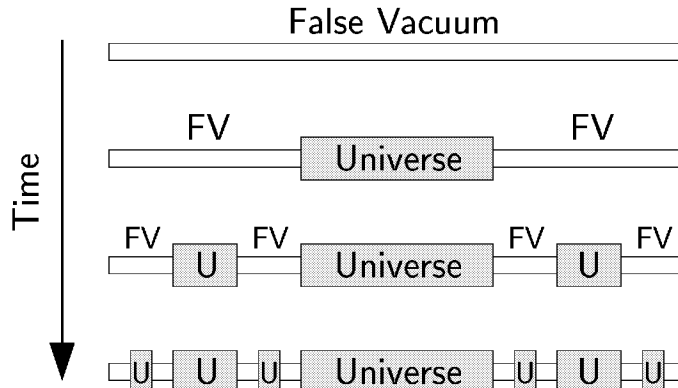


Figure 3: A schematic illustration of eternal inflation.

follow the region for another time interval of the same duration, each of these regions of false vacuum will break up, with about one third of each evolving into a local universe, as shown on the third bar from the top. Now there are four remaining regions of false vacuum, and again each is as large as the starting region. This process will repeat itself literally forever, producing a kind of a fractal structure to the universe, resulting in an infinite number of the local universes shown in gray. These local universes are often called *bubble universes*, but that terminology conveys the unfortunate connotation that the local universes are spherical. While bubbles formed in first-order phase transitions are round [28], the local universes formed in eternal new inflation are generally very irregular, as can be seen for example in the two-dimensional simulation by Vanchurin, Vilenkin, and Winitzki in Fig. 2 of Ref. [29]. I therefore prefer to call them *pocket universes*, to try to avoid the suggestion that they are round.

The diagram in Fig. 3 is of course an idealization. The real universe is three dimensional, while the diagram illustrates a schematic one-dimensional universe. It is also important that the decay of the false vacuum is really a random process, while I constructed the diagram to show a very systematic decay, because it is easier to draw and to think about. When these inaccuracies are corrected, we are still left with a scenario in which inflation leads asymptotically to a fractal structure [30] in which the universe as a whole is populated by pocket universes on arbitrarily small comoving scales. Of course this fractal structure is entirely on distance scales much too large to be observed, so we cannot expect astronomers to actually find it. Nonetheless, one does have to think about the fractal structure if one wants to understand the very large scale structure of the spacetime produced by inflation.

Most important of all is the simple statement that once inflation begins, it produces not just one universe, but an infinite number of universes.

Eternal Chaotic Inflation:

The eternal nature of new inflation depends crucially on the scalar field lingering at the top of the plateau of Fig. 1. Since the potential function for chaotic inflation, Fig. 2, has no plateau, it does not seem likely that eternal inflation can happen in this context.

Nonetheless, Andrei Linde [31] showed in 1986 that chaotic inflation can also be eternal.

The key to eternal chaotic inflation is the role of quantum fluctuations, which is very significant in all inflationary models. Quantum fluctuations are invariably important on very small scales, and with inflation these very small scales are rapidly stretched to become macroscopic and even astronomical. Thus the quantum fluctuations of the inflaton field can have very noticeable effects.

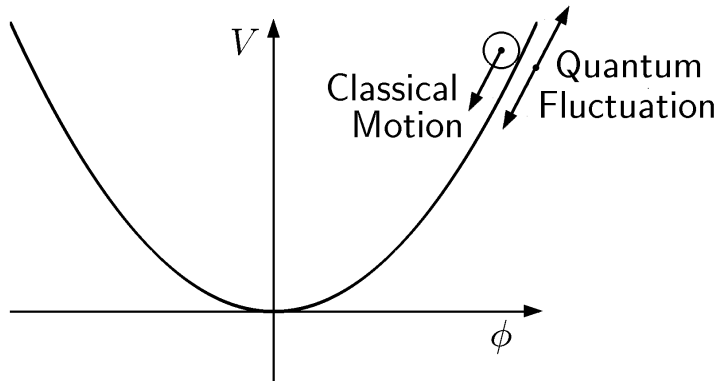


Figure 4: Evolution of the inflaton field during eternal chaotic inflation.

When the mass of the scalar field is small compared to the Hubble parameter H , these quantum evolution of the scalar field is accurately described as a random walk. It is useful to divide space into regions of physical size H^{-1} , and to discuss the average value of the scalar field ϕ within a given region. In a time H^{-1} , the quantum fluctuations cause the scalar field to undergo a random Gaussian jump of zero mean and a root-mean-squared magnitude [32, 33, 16, 34] given by

$$\Delta\phi_{\text{qu}} = \frac{H}{2\pi} . \quad (4.1)$$

This random quantum jump is superimposed on the classical motion, as indicated in Fig. (4).

To illustrate how eternal inflation happens in the simplest context, let us consider again the free scalar field described by the potential function of Eq. (2.1). We consider a region of physical radius H^{-1} , in which the field has an average value ϕ . Using Eq. (2.9) along with Eqs. (2.8) and (2.1), one finds that the magnitude of the classical change that the field will undergo in a time H^{-1} is given in by

$$\Delta\phi_{\text{cl}} = \frac{M_{\text{P}}m}{\sqrt{12\pi}} H^{-1} = \frac{1}{4\pi} \frac{M_{\text{P}}^2}{\phi} . \quad (4.2)$$

Let ϕ^* denote the value of ϕ which is large enough so that

$$\Delta\phi_{\text{qu}}(\phi^*) = \Delta\phi_{\text{cl}}(\phi^*) , \quad (4.3)$$

which implies that

$$\phi^* = \left(\frac{3}{16\pi} \right)^{1/4} \frac{M_{\text{P}}^{3/2}}{m^{1/2}} . \quad (4.4)$$

Now consider what happens to a region for which the initial average value of ϕ is equal to ϕ^* . In a time interval H^{-1} , the volume of the region will increase by $e^3 \approx 20$. At the end of the time interval we can divide the original region into 20 regions of the same volume as the original, and in each region the average scalar field can be written as

$$\phi = \phi^* + \Delta\phi_{\text{cl}} + \delta\phi, \quad (4.5)$$

where $\delta\phi$ denotes the random quantum jump, which is drawn from a Gaussian probability distribution with standard deviation $\Delta\phi_{\text{qu}} = \Delta\phi_{\text{cl}}$. Gaussian statistics imply that there is a 15.9% chance that a Gaussian random variable will exceed its mean by more than one standard deviation, and therefore there is a 15.9% chance that the net change in ϕ will be positive. Since there are now 20 regions of the original volume, on average the value of ϕ will exceed the original value in 3.2 of these regions. Thus the volume for which $\phi \geq \phi^*$ does not (on average) decrease, but instead increases by more than a factor of 3. Since this argument can be repeated, the expectation value of the volume for which $\phi \geq \phi^*$ increases exponentially with time. Typically, therefore, inflation never ends, but instead the volume of the inflating region grows exponentially without bound. The minimum field value for eternal inflation is a little below ϕ^* , since a volume increase by a factor of 3.2 is more than necessary—any factor greater than one would be sufficient. A short calculation shows that the minimal value for eternal inflation is $0.78\phi^*$.

While the value of ϕ^* is larger than Planck scale, again we find that this is not true of the energy density:

$$V(\phi^*) = \frac{1}{2}m^2\phi^{*2} = \sqrt{\frac{3}{64\pi}} m M_{\text{P}}^3, \quad (4.6)$$

which for $m = 10^{16}$ GeV gives an energy density of $1 \times 10^{-4} M_{\text{P}}^4$.

If one carries out the same analysis with a potential function

$$V(\phi) = \frac{1}{4}\lambda\phi^4, \quad (4.7)$$

one finds [35] that

$$\phi^* = \left(\frac{3}{2\pi\lambda}\right)^{1/6} M_{\text{P}}, \quad (4.8)$$

and

$$V(\phi^*) = \left(\frac{3}{16\pi}\right)^{2/3} \lambda^{1/3} M_{\text{P}}^4. \quad (4.9)$$

Since λ must be very small in any case so that density perturbations are not too large, one finds again that eternal inflation is predicted to happen at an energy density well below the Planck scale.

V. Eternal Inflation: Implications

When I told Rocky Kolb that I was going to be talking about eternal inflation, he said, “That’s OK, we can talk about physics later.” So that’s the point I’d like to address here.

In spite of the fact that the other universes created by eternal inflation are too remote to imagine observing directly, I still believe that eternal inflation has real consequences in terms of the way we extract predictions from theoretical models. Specifically, there are four consequences of eternal inflation that I will highlight.

1) Unobservability of initial conditions

First, eternal inflation implies that all hypotheses about the ultimate initial conditions for the universe—such as the Hartle-Hawking [36] no boundary proposal, the tunneling proposals by Vilenkin [37] or Linde [38], or the more recent Hawking-Turok instanton [39]—become totally divorced from observation. That is, one would expect that if inflation is to continue arbitrarily far into the future with the production of an infinite number of pocket universes, then the statistical properties of the inflating region should approach a steady state which is independent of the initial conditions. Unfortunately, attempts to quantitatively study this steady state are severely limited by several factors. First, there are ambiguities in defining probabilities, which will be discussed later. In addition, the steady state properties seem to depend strongly on super-Planckian physics which we do not understand. That is, the same quantum fluctuations that make eternal chaotic inflation possible tend to drive the scalar field further and further up the potential energy curve, so attempts to quantify the steady state probability distribution [40, 41] require the imposition of some kind of a boundary condition at large ϕ . Although these problems remain unsolved, I still believe that it is reasonable to assume that in the course of its perpetual evolution, an eternally inflating universe would lose all memory of the state in which it started.

Although the ultimate origin of the universe would become unobservable, I would not expect that the question of how the universe began would lose its interest. While eternally inflating universes continue forever once they start, they are presumably not eternal into the past. (The word *eternal* is therefore not technically correct—it would be more precise to call this scenario *semi-eternal* or *future-eternal*.) While the issue is not completely settled, it appears likely that eternally inflating universes must necessarily have a beginning. Borde and Vilenkin [42] have shown, provided that certain conditions are met, that spacetimes which are future-eternal must have an initial singularity, in the sense that they cannot be past null geodesically complete. The proof, however, requires the weak energy condition, which is classically valid but quantum-mechanically violated [43]. In any case, I am not aware of any viable model without a beginning, and certainly nothing that we know can rule out the possibility of a beginning. The possibility of a quantum origin of the universe is very attractive, and will no doubt be a subject of interest for some time. Eternal inflation, however, seems to imply that the entire study will have to be conducted with literally no input from observation.

2) Irrelevance of initial probability

A second consequence of eternal inflation is that the probability of the onset of inflation becomes totally irrelevant, provided that the probability is not identically zero. Various authors in the past have argued that one type of inflation is more plausible than another,

because the initial conditions that it requires appear more likely to have occurred. In the context of eternal inflation, however, such arguments have no significance.

To illustrate the insignificance of the probability of the onset of inflation, I will use a numerical example. We will imagine comparing two different versions of inflation, which I will call Type A and Type B. They are both eternally inflating—but Type A will have a higher probability of starting, while Type B will be a little faster in its exponential expansion rate. Since I am trying to show that the higher starting probability of Type A is irrelevant, I will choose my numbers to be extremely generous to Type A. First, we must choose a number for how much more probable it is for Type A inflation to begin, relative to type B. A googol, 10^{100} , is usually considered a large number—it is some 20 orders of magnitude larger than the total number of baryons in the visible universe. But I will be more generous: I will assume that Type A inflation is more likely to start than type B inflation by a factor of $10^{1,000,000}$. Type B inflation, however, expands just a little bit faster, say by 0.001%. We need to choose a time constant for the exponential expansion, which I will take to be a typical grand unified theory scale, $\tau = 10^{-37}$ sec. (τ represents the time constant for the overall expansion factor, which takes into account both the inflationary expansion and the exponential decay of the false vacuum.) Finally, we need to choose a length of time to let the system evolve. In principle this time interval is infinite (the inflation is eternal into the future), but to be conservative we will watch the system for only one second.

We imagine setting up a statistical ensemble of universes at $t = 0$, with an expectation value for the volume of Type A inflation exceeding that of Type B inflation by $10^{1,000,000}$. For brevity, let the term “weight” to refer to the ensemble expectation value of the volume. Thus, the weights of Type A inflation and Type B inflation will begin with the ratio

$$\left. \frac{W_B}{W_A} \right|_{t=0} = 10^{-1,000,000} . \quad (5.1)$$

After one second of evolution, the expansion factors for Type A and Type B inflation will be

$$Z_A = e^{t/\tau} = e^{10^{37}} \quad (5.2)$$

$$\begin{aligned} Z_B &= e^{1.00001 t/\tau} = e^{0.00001 t/\tau} Z_A \\ &= e^{10^{32}} Z_A \approx 10^{4.3 \times 10^{31}} Z_A . \end{aligned} \quad (5.3)$$

The weights at the end of one second are proportional to these expansion factors, so

$$\left. \frac{W_B}{W_A} \right|_{t=1 \text{ sec}} = 10^{(4.3 \times 10^{31} - 1,000,000)} . \quad (5.4)$$

Thus, the initial ratio of $10^{1,000,000}$ is vastly superseded by the difference in exponential expansion factors. In fact, we would have to calculate the exponent of Eq. (5.4) to an accuracy of 25 significant figures to be able to barely detect the effect of the initial factor of $10^{1,000,000}$.

One might criticize the above argument for being naive, as the concept of time was invoked without any discussion of how the equal-time hypersurfaces are to be chosen. I do not know a decisive answer to this objection; as I will discuss later, there are unresolved questions

concerning the calculation of probabilities in eternally inflating spacetimes. Nonetheless, given that there is actually an infinity of time available, it seems reasonable to believe that the form of inflation that expands the fastest will always dominate over the slower forms by an infinite factor.

A corollary to this argument is that new inflation is not dead. While the initial conditions necessary for new inflation cannot be justified on the basis of thermal equilibrium, as proposed in the original papers [1, 2], in the context of eternal inflation it is sufficient to conclude that the probability for the required initial conditions is nonzero. Since the resulting scenario does not depend on the words that are used to justify the initial state, the standard treatment of new inflation remains valid.

3) Inevitability of eternal inflation

Third, I'd like to claim that, since it appears that a universe is in principle capable of eternally reproducing, it is hard to believe that any other description can make sense at all. To clarify this point, let me raise the analogy of rabbits. We all know that rabbits can reproduce—in fact, they reproduce like rabbits. Suppose that you went out into the woods and found a rabbit that had characteristics indicating that it did not belong to any known rabbit species. Then you would have to theorize about how the rabbit originated. You might entertain the notion that the rabbit was created by some unique, mysterious, cosmic event that you hope to someday understand better. Or you could assume that the rabbit was created by the process of rabbit reproduction that we all know so well. I think that we would all consider the latter possibility to be far more plausible. So, I claim that once we become convinced that universes can reproduce like rabbits, then the situations are similar. When we notice that there is a universe and ask how it originated, the same inferences that we made for the rabbit question should apply to this one.

4) Possibility of restoring the uniqueness of theoretical predictions

A fourth consequence of eternal inflation is the possibility that it offers to rescue the predictive power of theoretical physics. All the indications suggest that string theory or M theory describes an elegantly unique theoretical structure, but nonetheless it seems unlikely that the theory possesses a unique vacuum. Since predictions will ultimately depend on the properties of the vacuum, the predictive power of string/M theory may be limited. Eternal inflation, however, provides a hope that this problem can be remedied. Even if many types of vacua are equally stable, it may turn out that a unique state produces the maximum possible rate of inflation. If so, then this state will dominate the universe, even if its expansion rate is only infinitesimally larger than the other possibilities. Thus, eternal inflation might allow physicists to extract unique predictions, in spite of the multiplicity of stable vacua.

VI. Difficulties in Calculating Probabilities

In an eternally inflating universe, anything that can happen will happen; in fact, it will happen an infinite number of times. Thus, the question of what is possible becomes trivial—

anything is possible, unless it violates some absolute conservation law. To extract predictions from the theory, we must therefore learn to distinguish the probable from the improbable.

However, as soon as one attempts to define probabilities in an eternally inflating spacetime, one discovers ambiguities. Since an eternally inflating universe produces an infinite number of pocket universes, the sample space is infinite. The fraction of universes with any particular property is given by the meaningless ratio of infinity divided by infinity. To obtain a well-defined answer, one needs to invoke some method of regularization. The most straightforward form of regularization consists of truncating the space to a finite subspace, and then taking a limit in which the subspace becomes larger and larger.

To understand the nature of the problem, it is useful to think about the integers as a model system with an infinite number of entities. We can ask, for example, what fraction of the integers are odd. Most people would presumably say that the answer is $1/2$, since the integers alternate between odd and even. That is, if the string of integers is truncated after the N th, then the fraction of odd integers in the string is exactly $1/2$ if N is even, and is $(N + 1)/2N$ if N is odd. In any case, the fraction approaches $1/2$ as N approaches infinity.

However, the ambiguity of the answer can be seen if one imagines other orderings for the integers. One could, if one wished, order the integers as

$$1, 3, 2, 5, 7, 4, 9, 11, 6, \dots, \tag{6.1}$$

always writing two odd integers followed by one even integer. This series includes each integer exactly once, just like the usual sequence $(1, 2, 3, 4, \dots)$. The integers are just arranged in an unusual order. However, if we truncate the sequence shown in Eq. (6.1) after the N th entry, and then take the limit $N \rightarrow \infty$, we would conclude that $2/3$ of the integers are odd. Thus, we see that probabilities can depend nontrivially on the method of regularization that is used.

In the case of eternally inflating spacetimes, one might consider a regularization defined by ordering the pocket universes in the sequence in which they form, and then truncating after the N th. However, each pocket universe fills its own future light cone, so no pocket universe forms in the future light cone of another. Any two pocket universes are spacelike separated from each other, so different observers can disagree about which formed first. One can arbitrarily choose equal-time surfaces that foliate the spacetime, and then truncate at some value of t , but this recipe is far from unique. In practice, different ways of choosing equal-time surfaces give different results.

VII. The Youngness Paradox

If one chooses a regularization in the most naive way, one is led to a set of very peculiar results which I call the *youngness paradox*.

Specifically, suppose that one constructs a Robertson-Walker coordinate system while the model universe is still in the false vacuum (de Sitter) phase, before any pocket universes have formed. One can then propagate this coordinate system forward with a synchronous gauge condition,[‡] and one can define probabilities by truncating at a fixed value t_f of the

[‡]By a synchronous gauge condition, I mean that each equal-time hypersurface is obtained by propagating

synchronous time coordinate t . That is, the probability of any particular property can be taken to be proportional to the volume on the $t = t_f$ hypersurface which has that property. This method of defining probabilities was studied in detail by Linde, Linde, and Mezhlumian, in a paper with the memorable title “Do we live in the center of the world?” [44]. I will refer to probabilities defined in this way as synchronous gauge probabilities.

The youngness paradox is caused by the fact that the volume of false vacuum is growing exponentially with time with an extraordinarily short time constant, in the vicinity of 10^{-37} sec. Since the rate at which pocket universes form is proportional to the volume of false vacuum, this rate is increasing exponentially with the same time constant. That means that in each second the number of pocket universes that exist is multiplied by a factor of $\exp\{10^{37}\}$. At any given time, therefore, almost all of the pocket universes that exist are universes that formed very very recently, within the last several time constants. The population of pocket universes is therefore an incredibly youth-dominated society, in which the mature universes are vastly outnumbered by universes that have just barely begun to evolve. Although a mature universe has a larger volume than a young one, this multiplicative factor is of little importance, since in synchronous coordinates the volume no longer grows exponentially once the pocket universe forms.

Probability calculations in this youth-dominated ensemble lead to peculiar results, as discussed in Ref. [44]. These authors considered the expected behavior of the mass density in our vicinity, concluding that we should find ourselves very near the center of a spherical low-density region. Here I would like to discuss a less physical but simpler question, just to illustrate the paradoxes associated with synchronous gauge probabilities. Specifically, I will consider the question: “Are there any other civilizations in the visible universe that are more advanced than ours?”. Intuitively I would not expect inflation to make any predictions about this question, but I will argue that the synchronous gauge probability distribution strongly implies that there is no civilization in the visible universe more advanced than we are.

Suppose that we have reached some level of advancement, and suppose that t_{\min} represents the minimum amount of time needed for a civilization as advanced as we are to evolve, starting from the moment of the decay of the false vacuum—the start of the big bang. The reader might object on the grounds that there are many possible measures of advancement, but I would respond by inviting the reader to pick any measure she chooses; the argument that I am about to give should apply to all of them. The reader might alternatively claim that there is no sharp minimum t_{\min} , but instead we should describe the problem in terms of a function which gives the probability that, for any given region within a pocket universe of the size of our visible universe, a civilization as advanced as we are would develop by time t . I believe, however, that the introduction of such a probability distribution would merely complicate the argument, without changing the result. So, for simplicity of discussion, I will assume that there is some sharply defined minimum time t_{\min} required for a civilization as advanced as ours to develop.

every point on the previous hypersurface by a fixed infinitesimal time interval Δt in the direction normal to the hypersurface.

Since we exist, our pocket universe must have an age t_0 satisfying

$$t_0 \geq t_{\min} . \quad (7.1)$$

Suppose, however, that there is some civilization in our visible universe that is more advanced than we are, let us say by 1 second. In that case Eq. (7.1) is not sufficient, but instead the age of our pocket universe would have to satisfy

$$t_0 \geq t_{\min} + 1 \text{ second} . \quad (7.2)$$

However, in the synchronous gauge probability distribution, universes that satisfy Eq. (7.2) are outnumbered by universes that satisfy Eq. (7.1) by a factor of approximately $\exp\{10^{37}\}$. Thus, if we know only that we are living in a pocket universe that satisfies Eq. (7.1), the probability that it also satisfies Eq. (7.2) is approximately $\exp\{-10^{37}\}$. We would conclude, therefore, that it is extraordinarily improbable that there is a civilization in our visible universe that is at least 1 second more advanced than we are.

Perhaps this argument explains why SETI has not found any signals from alien civilizations, but I find it more plausible that it is merely a symptom that the synchronous gauge probability distribution is not the right one.

VIII. Toy Model of Eternal Inflation

The conceptual issue involved in the youngness paradox can perhaps be clarified by considering a toy model of a highly simplified eternally inflating universe. Suppose that the universe as a whole can be labeled with a global time variable t , and that it consists of a countably infinite set of pocket universes, each of which is labeled by an index i . For simplicity, we let each pocket universe have zero spatial dimensions, so a spacetime point is fully specified by the time t and the index i which indicates the pocket universe in which it is located. We assume that each pocket universe i forms at some time $t_i = n_i\tau$, where n_i is an integer and τ is a fixed time constant characterizing the entire universe. Let the number of pocket universes that form at time $t = n\tau$ be equal to 2^n , for each nonnegative integer n . Assume that each pocket universe exists for a time $T \gg \tau$, and then disappears, and that within each pocket universe the interval from the time of formation to disappearance is uniformly populated with “sentient beings.” Within each pocket universe we can define a relative time, $t_{\text{rel}} \equiv t - t_i$, which measures the amount of time since the formation of the pocket universe.

The difficult question, then, is the following: At what relative time t_{rel} does a *typical* sentient being live? If one answers this question by truncating the spacetime by the criterion

$$t \leq t_c , \quad (8.1)$$

for some cut-off time $t_c = n_c\tau$, then one finds that most of the pocket universes in the truncated spacetime formed within the past few time constants. As $t_c \rightarrow \infty$, the mean value of t_{rel} approaches τ . This method is analogous to the synchronous gauge cut-off discussed above. If, however, one truncates the spacetime by including all pocket universes for which the time of formation

$$t_i \leq t_c , \quad (8.2)$$

then the mean value of t_{rel} is equal to $T/2$ for any t_c . The truncation method of Eq. (8.1) leads to the youngness paradox, in which the probability sample is strongly dominated by universes that are extremely young, while the truncation method of Eq. (8.2) does not.

At this point, I have to admit that I do not understand how to resolve the ambiguities associated with this toy model. It is conceivable that there is no meaningful method of regularization, and that t_{rel} is somehow not susceptible to probabilistic predictions. It is also conceivable that there is something wrong with either the truncation (8.1) or (8.2) or both, and that a correct analysis would lead to a unique probability calculation. It is also conceivable that the regularization has to be specified as part of the theory, so that the truncations (8.1) and (8.2) represent two distinct theories, each of which is logically consistent.

IX. An Alternative Probability Prescription

Since the probability measure depends on the method used to regulate the infinite spacetime of eternal inflation, we are not forced to accept the consequences of the synchronous gauge probabilities. A very attractive alternative has been proposed by Vilenkin [45], and developed further by Vanchurin, Vilenkin, and Winitzki [29]. This procedure is, roughly speaking, analogous to the truncation of Eq. (8.2).

The key idea of the Vilenkin proposal is to define probabilities within a single pocket universe (which he describes more precisely as a connected, thermalized domain). Thus, unlike the synchronous gauge method, there is no comparison between old pocket universes and young ones. To justify this approach it is crucial to recognize that each pocket universe is infinite, even if one starts the model with a finite region of de Sitter space. The infinite volume arises in the same way as it does for the special case of Coleman-de Luccia bubbles [28], the interior of which are open Robertson-Walker universes. From the outside one often describes such bubbles in a coordinate system in which they are finite at any fixed time, but in which they grow without bound. On the inside, however, the natural coordinate system is the one that reflects the intrinsic homogeneity, in which the space is infinite at any given time. The infinity of time, as seen from the outside, becomes an infinity of spatial extent as seen on the inside. Thus, at least for continuously variable parameters, a single pocket universe provides an infinite sample space which can be used to define probabilities. The second key idea of Vilenkin's method is to use the inflaton field itself as the time variable, rather than the synchronous time variable discussed in the previous section.

This approach can be used, for example, to discuss the probability distribution for Ω in open inflationary models, or to discuss the probability distribution for some arbitrary field that has a flat potential energy function. If, however, the vacuum has a discrete parameter which is homogeneous within each pocket universe, but which takes on different values in different pocket universes, then this method does not apply.

The proposal can be described in terms of Fig. 5. We suppose that the theory includes an inflaton field ϕ of the new inflation type, and some set of fields χ_i which have flat potentials. The goal is to find the probability distribution for the fields χ_i . We assume that the evolution of the inflaton ϕ can be divided into three regimes, as shown on the figure. $\phi < \phi_1$ describes the eternally inflating regime, in which the evolution is governed by quantum diffusion. For

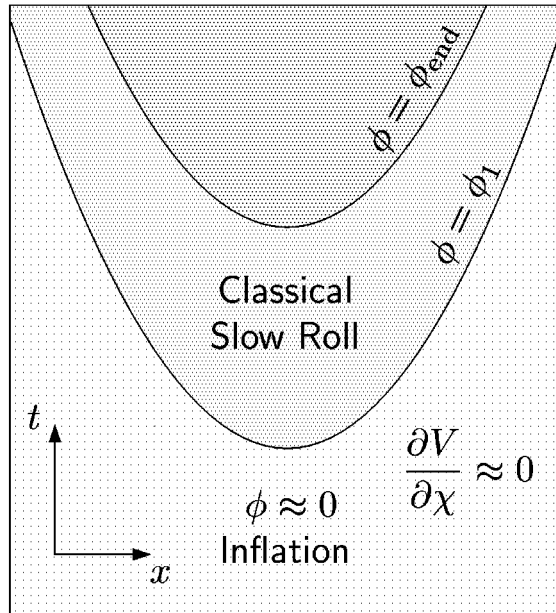


Figure 5: A schematic picture of a pocket universe, illustrating Vilenkin’s proposal for the calculation of probabilities.

$\phi_1 < \phi < \phi_{\text{end}}$, the evolution is described classically in a slow-roll approximation, so that $\dot{\phi} \equiv d\phi/dt$ can be expressed as a function of ϕ . For $\phi > \phi_{\text{end}}$ inflation is over, and the ϕ field no longer plays an important role in the evolution. The χ_i fields are assumed to have a finite range of values, such as angular variables, so that a flat probability distribution is normalizable. They are assumed to have a flat potential energy function for $\phi > \phi_{\text{end}}$, so that they could settle at any value. They are also assumed to have a flat potential energy function for $\phi < \phi_1$, although they might interact with ϕ during the slow-roll regime, however, so that they can affect the rate of inflation.

Since the potential for the χ_i is flat for $\phi < \phi_1$, we can assume that they begin with a flat probability distribution $P_0(\chi_i) \equiv P(\chi_i, \phi_1)$ on the $\phi = \phi_1$ hypersurface. If the kinetic energy function for the χ_i is of the standard form, we take $P_0(\chi_i) = \text{const}$. If, however, the kinetic energy is nonstandard,

$$\mathcal{L}_{\text{kinetic}} = g^{ij}(\chi) \partial_\mu \chi_i \partial^\mu \chi_j , \tag{9.1}$$

as is plausible for a field described in angular variables, then the initial probability distribution is assumed to take the reparameterization-invariant form

$$P_0(\chi_i) \propto \sqrt{\det g} . \tag{9.2}$$

During the slow-roll era, it is assumed that the χ_i fields evolve classically, so one can calculate the number of e-folds of inflation $N(\chi_i)$ as a function of the final value of the χ_i (i.e., the value of χ_i on the $\phi = \phi_{\text{end}}$ hypersurface). One can also calculate the final values χ_i in terms of the initial values χ_i^0 (i.e., the value of χ_i on the $\phi = \phi_1$ hypersurface). One then assumes that the probability density is enhanced by the volume inflation factor $e^{3N(\chi_i)}$. The evolution

from χ_i^0 to χ_i results in a Jacobian factor. The (unnormalized) final probability distribution is thus given by

$$P(\chi_i, \phi_{\text{end}}) = P_0(\chi_i^0) e^{3N(\chi_i)} \det \frac{\partial \chi_j^0}{\partial \chi_k}. \quad (9.3)$$

Alternatively, if the evolution of the χ_i during the slow-roll era is subject to quantum fluctuations, Ref. [29] shows how to write a Fokker-Planck equation which is equivalent to averaging the result of Eq. (9.3) over a collection of paths that result from interactions with a noise term.

The Vilenkin proposal sidesteps the youngness paradox by defining probabilities by the comparison of volumes within one pocket universe. The youngness paradox, in contrast, arose when one considered a probability ensemble of all pocket universes at a fixed value of the synchronous gauge time coordinate—an ensemble that is overwhelmingly dominated by very young pocket universes.

The proposal has the drawback, however, that it cannot be used to compare the probabilities of discretely different alternatives. Furthermore, although the results of this method seem reasonable, I do not at this point find them compelling. That is, it is not clear what principles of physics or probability theory ensure that this particular method of regularizing the spacetime is the one that leads to correct predictions. Perhaps there is no way to answer this question, so we may be forced to accept this proposal, or something similar to it, as a postulate.

X. Probabilities with only one universe?

In discussing a probabilistic approach to cosmology, we need to know whether it makes sense to talk about a probability distribution for a cosmic parameter such as Ω , for which we have only one example to measure. I have certainly heard more than one physicist say that he or she doesn't think that one can meaningfully talk about probabilities for an experiment that can be done only once. The notion that probability requires repetition is very widespread, and I am sure that it is incorporated into many books about probability theory. Nonetheless, I would like to argue that repetition is not at all necessary to make use of probability theory. Instead, I will argue that probability is meaningful whenever one has a *strong* probabilistic prediction, by which I mean a prediction that the probability for some discernible event is either very close to zero or very close to one.

Thus, if a cosmological theory predicts a probability distribution for Ω which is reasonably flat, then there is no strong prediction, and the implications of the theory for Ω do not provide a way of testing the theory. However, if the theory predicts that the probability of Ω lying outside the range of 0.99 to 1.01 is 10^{-6} , then I would claim that the prediction is meaningful and can be used to test the theory.

My point of view can be explained most easily by considering coin flips. If a flip of an unbiased coin is repeated 20 times, the probability of getting 20 successive heads is a very small number, about 10^{-6} . This is an example of what I call a strong prediction. Many common examples of strong predictions involve repetition. However, if 20 unbiased coins are flipped simultaneously in a single experiment, the probability that all will come up heads is

identical, about 10^{-6} . Since the probability of these two results—20 successive heads or 20 simultaneous heads—are both equally small, I would draw the obvious conclusion that we should be equally surprised if either result occurred. It does not matter that the first result involved repetition, while the second did not. Some might argue that the 20 simultaneous coin flips involved the replication of identical experiments, even if they were not performed in succession, so I will take the analogy one step further. Suppose we constructed a roulette wheel that was so finely ruled that the probability of the ball landing on 0 was only 10^{-6} . Again, we should be just as surprised if this result occurred as we would be if 20 successive coins landed heads.

Similarly, if our cosmological theory predicted that the probability of Ω lying outside the range of 0.99 to 1.01 is 10^{-6} , we should be just as surprised if this outcome occurred as we would be if 20 consecutive coins came up heads. In both cases, we would have good cause to question the assumptions that went into calculating the prediction.

XI. Conclusion

In this paper I have summarized the workings of inflation, and the arguments that strongly suggest that our universe is the product of inflation. I argued that inflation can explain the size, the Hubble expansion, the homogeneity, the isotropy, and the flatness of our universe, as well as the absence of magnetic monopoles, and even the characteristics of the nonuniformities. The detailed observations of the cosmic background radiation anisotropies continue to fall in line with inflationary expectations, and the evidence for an accelerating universe fits well with the inflationary preference for a flat universe.

Next I turned to the question of eternal inflation, claiming that essentially all inflationary models are eternal. In my opinion this makes inflation very robust: if it starts anywhere, at any time in all of eternity, it produces an infinite number of pocket universes. Eternal inflation has the very attractive feature, from my point of view, that it offers the possibility of allowing unique predictions even if the underlying string theory does not have a unique vacuum. I have also emphasized, however, that there are important problems in understanding the implications of eternal inflation. First, there is the problem that we do not know how to treat the situation in which the scalar field climbs upward to the Planck energy scale. Second, the definition of probabilities in an eternally inflating spacetime is not yet a closed issue, although important progress has been made. And third, I might add that the entire present approach is at best semiclassical. A better treatment may not be possible until we have a much better handle on quantum gravity, but eventually this issue will have to be faced.

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