

Inflationary scenario from higher curvature warped spacetime

Narayan Banerjee^{1,a}, Tanmoy Paul^{2,b}

¹ Department of Physical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur Campus, Nadia, West Bengal 741246, India

² Department of Theoretical Physics, Indian Association for the Cultivation of Science, 2A & 2B Raja S.C. Mullick Road, Kolkata 700 032, India

Received: 23 August 2017 / Accepted: 24 September 2017 / Published online: 11 October 2017

© The Author(s) 2017. This article is an open access publication

Abstract We consider a five dimensional warped spacetime, in presence of the higher curvature term like $F(R) = R + \alpha R^2$ in the bulk, in the context of the two-brane model. Our universe is identified with the TeV scale brane and emerges as a four dimensional effective theory. From the perspective of this effective theory, we examine the possibility of “inflationary scenario” by considering the on-brane metric ansatz as an FRW one. Our results reveal that the higher curvature term in the five dimensional bulk spacetime generates a potential term for the radion field. Due to the presence of radion potential, the very early universe undergoes a stage of accelerated expansion and, moreover, the accelerating period of the universe terminates in a finite time. We also find the spectral index of curvature perturbation (n_s) and the tensor to scalar ratio (r) in the present context, which match with the observational results based on the observations of Planck (Astron. Astrophys. **594**, A20, 2016).

1 Introduction

Over the last two decades, extra spatial dimensions [2–14] have been increasingly playing a central role in physics beyond the standard model of particle [15] and cosmology [16]. Apart from phenomenological approach, higher dimensional scenarios come naturally in string theory. Depending on geometry, the extra dimensions are compactified under various compactification schemes. Our usual four dimensional universe is considered to be a 3-brane (3 + 1 dimensional brane) embedded within the higher dimensional spacetime and emerges as a four dimensional effective theory.

Among the various extra dimensional models proposed over the last several years, the Randall–Sundrum (RS) warped extra dimensional model [7] earned special attention since it resolves the gauge hierarchy problem without

introducing any intermediate scale (between Planck and TeV scale) in the theory. RS model is a five dimensional AdS spacetime with S^1/Z_2 orbifolding along the extra dimension while the orbifold fixed points are identified with two 3-branes. The separation between the branes is assumed to be of the order of Planck length so that the hierarchy problem can be solved. However, due to the intervening gravity, the aforementioned brane configuration cannot be a stable one. So, like other higher dimensional braneworld scenarios, one of the crucial aspects of the RS model is in stabilizing the interbrane separation (known as the modulus or radion). For this purpose, one needs to generate a suitable radion potential with a stable minimum. Goldberger and Wise (GW) proposed a useful mechanism [17] to construct such a radion potential by imposing a massive scalar field in the bulk with appropriate boundary conditions. Subsequently the phenomenology of the radion field has also been studied extensively in [18–21].

Some variants of the RS model and its modulus stabilization have been discussed in [19–25].

The Standard Big Bang model gives a predictive description of our universe from nucleosynthesis to the present. But back in very early stages of the evolution, the big bang model is plagued with some problems such as the horizon and flatness problems. For a comprehensive review, we refer to [26,27]. In order to resolve these problems, the idea of inflation was introduced by Guth [28], in which the universe had to go through a stage of accelerated expansion after the big bang. It had also been demonstrated that a massive scalar field with a suitable potential plays a crucial role in producing an accelerated expansion of the universe. This resulted in a huge amount of work on inflation based on scalar fields [26,27,29–37].

It is interesting to note that in the extra dimensional models, the modulus field can fulfill the requirement of the scalar field as regards inflation. Thus the cosmology of higher dimensional models [38–45] can be very differ-

^a e-mail: narayan@iiserkol.ac.in

^b e-mail: pul.tnmy9@gmail.com

ent from the usual cosmology of four dimensions where the inflaton field is normally invoked by hand. In our current work, we take advantage of the modulus field of extra dimensions and address the early time cosmology of our universe in the backdrop of the RS two-brane model.

It is well known that the Einstein–Hilbert action can be generalized by adding higher order curvature terms which naturally arise from the diffeomorphism property of the action. Such terms also have their origin in string theory due to quantum corrections. $F(R)$ [46–51], Gauss–Bonnet (GB) [52–54] or more generally Lanczos–Lovelock gravity [55–57] are some of the candidates in higher curvature gravitational theory.

Higher curvature terms become extremely relevant at the regime of large curvature. Thus for RS bulk geometry, where the curvature is of the order of Planck scale, the higher curvature terms should play a crucial role. Motivated by this idea, we consider a generalized version of RS model by replacing the Einstein–Hilbert bulk gravity Lagrangian, given by the Ricci scalar R by $F(R)$ where $F(R)$ is an analytic function of R [58–60]. Recently it has been shown [23] that, for an RS braneworld modified by $F(R)$ gravity, a potential term for the radion field is generated (in the four dimensional effective theory) even without introducing an external scalar field in the bulk and, moreover, the radion potential has a stable minimum for a certain range of parametric space. However, from the cosmological aspect, the important questions that remain in the said higher curvature RS model [23], are:

1. Can the usual four dimensional universe undergo an accelerating expansion at an early epoch, due to the presence of the radion potential generated by a higher curvature term?
2. If such an inflationary scenario is allowed, then what are the dependence of duration of inflation and the number of e-foldings on higher curvature parameter? Moreover, what are the values of n_s and r in the present context?

We aim to address these questions in this work and, motivated by the Starobinsky model [61], the form of $F(R)$ in the five dimensional bulk is taken as $F(R) = R + \alpha R^2$ where α is a constant.

The paper is organized as follows: the following two sections are devoted to brief reviews of the RS scenario and its extension to $F(R)$ model. Section 4 is reserved for determining the solutions of effective Friedmann equations on the brane. In Sects. 5, 6, 7 and 8, we address the consequences of the solutions that are obtained in Sect. 4. Finally the paper ends with some concluding remarks in Sect. 9.

2 Brief description of the RS scenario

The RS scenario is defined on a five dimensional AdS spacetime involving one warped and compact extra space-like dimension. Two 3-branes known as TeV/visible and Planck/hidden brane are embedded in a five dimensional spacetime. If ϕ is the extra dimensional angular coordinate, then the branes are located at two fixed points $\phi = (0, \pi)$ while the latter one is identified with our known four dimensional universe. The opposite brane tensions along with the finely tuned five dimensional cosmological constant serve as the energy-momentum tensor of the RS scenario. The resulting spacetime metric [7] is non-factorizable and expressed as

$$ds^2 = e^{-2kr_c|\phi|} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2. \quad (1)$$

Here, r_c is the compactification radius of the extra dimension. Due to S^1/Z_2 compactification along the extra dimension, ϕ ranges from $-\pi$ to $+\pi$. The quantity $k = \sqrt{\frac{-\Lambda}{12M^3}}$ is of the order of the five dimensional Planck scale M . Thus k relates the 5D Planck scale M to the 5D cosmological constant Λ .

In order to solve the hierarchy problem, it is assumed in the RS scenario that the branes are separated by such a distance that $k\pi r_c \approx 36$. Then the exponential factor present in the metric, which is often called the warp factor, produces a large suppression so that a mass scale of the order of Planck scale is reduced to TeV scale on the visible brane. A scalar mass, e.g. the mass of Higgs boson, is given by

$$m_H = m_0 e^{-k\pi r_c} \quad (2)$$

where m_H and m_0 are physical and bare Higgs masses, respectively.

3 RS like spacetime in F(R) model: four dimensional effective action

In the present paper, we consider a five dimensional warped spacetime with two 3-brane scenario in F(R) model. The form of $F(R)$ is taken as $F(R) = R + \alpha R^2$ where α is a constant with the square of the inverse mass dimension. Considering ϕ as the extra dimensional angular coordinate, two branes are located at $\phi = 0$ (hidden brane) and at $\phi = \pi$ (visible brane), respectively, while the latter one is identified with the visible universe. Moreover, the spacetime is S^1/Z_2 orbifolded along the coordinate ϕ . The action for this model is

$$S = \int d^4x d\phi \sqrt{G} \left[\frac{1}{2\kappa^2} (R + \alpha R^2) + \Lambda + V_h \delta(\phi) + V_v \delta(\phi - \pi) \right] \quad (3)$$

where G is the determinant of the five dimensional metric (G_{MN}), $\Lambda (< 0)$ is the bulk cosmological constant, $\frac{1}{2\kappa^2} = M^3$ and V_h, V_v are the brane tensions on the hidden and visible brane, respectively.

It is well known that a $F(R)$ gravity model can be recast into Einstein gravity with a scalar field by means of a conformal transformation on the metric [23, 49]. Thus the solutions of the five dimensional Einstein equations for the action presented in Eq. (3) can be extracted from the solutions of the corresponding conformally related scalar–tensor (ST) theory and this is discussed in the following two subsections.

3.1 Solutions of field equations for corresponding ST theory

This higher curvature like $F(R)$ model (in Eq. (3)) can be transformed into scalar–tensor theory. The demonstration goes as follows.

Introducing an auxiliary field $A(x, \phi)$, the above action (3) can be equivalently written as

$$S = (1/2\kappa^2) \int d^4x d\phi \sqrt{G} \left[F'(A)(R - A) + F(A) + \Lambda + V_h \delta(\phi) + V_v \delta(\phi - \pi) \right]. \tag{4}$$

By the variation of the auxiliary field $A(x, \phi)$, one easily obtains $A = R$. Plugging back this solution $A = R$ into the action (4), the initial action (3) can be reproduced. At this stage, we perform a conformal transformation of the metric:

$$G_{MN}(x, \phi) \rightarrow \tilde{G}_{MN} = \exp\left(\frac{1}{\sqrt{3}}\kappa\Phi(x, \phi)\right) G_{MN}(x, \phi), \tag{5}$$

M, N run from 0 to 4 and the conformal factor Φ is related to the auxiliary field (A) as $\frac{\sqrt{3}}{2}\kappa\Phi = \ln[F'(A)]$. If R and \tilde{R} are the Ricci scalar formed by G_{MN} and \tilde{G}_{MN} , respectively, then they are related by

$$R = e^{\frac{1}{\sqrt{3}}\kappa\Phi} \left[\tilde{R} + \frac{4\kappa}{\sqrt{3}} \tilde{G}^{MN} \partial_M \partial_N \Phi + \kappa^2 \tilde{G}^{MN} \partial_M \Phi \partial_N \Phi \right].$$

With this above expression and the aforementioned relation between Φ and A , the action (in Eq. (4)) turns out to be [23]

$$S = \int d^4x d\phi \sqrt{\tilde{G}} \left[\frac{\tilde{R}}{2\kappa^2} + \frac{1}{2} \tilde{G}^{MN} \partial_M \Phi \partial_N \Phi - V(\Phi) + \Lambda + \exp\left(-\frac{5}{2\sqrt{3}}\kappa\Phi\right) V_h \delta(\phi) + \exp\left(-\frac{5}{2\sqrt{3}}\kappa\Phi\right) V_v \delta(\phi - \pi) \right], \tag{6}$$

where the quantities in tilde are reserved for the ST theory. $\Phi(x, \phi)$ is the scalar field which corresponds to higher curvature degrees of freedom and

$$V(\Phi) = \frac{1}{2\kappa^2} \left[\frac{A}{F'(A)^{2/3}} - \frac{F(A)}{F'(A)^{5/3}} \right] - \Lambda \left[\exp\left(-\frac{5}{2\sqrt{3}}\kappa\Phi\right) - 1 \right] \tag{7}$$

is the scalar potential, which for the specific choice of $F(R) = R + \alpha R^2$ has the following form:

$$V(\Phi) = \frac{1}{8\kappa^2\alpha} \exp\left(-\frac{5}{2\sqrt{3}}\kappa\Phi\right) \left[\exp\left(\frac{3}{2\sqrt{3}}\kappa\Phi\right) - 1 \right]^2 - \Lambda \left[\exp\left(-\frac{5}{2\sqrt{3}}\kappa\Phi\right) - 1 \right]. \tag{8}$$

This form of $V(\Phi)$ immediately leads to $V'(\Phi)$ as follows:

$$V'(\Phi) = \frac{1}{16\sqrt{3}\kappa\alpha} e^{-\frac{5}{2\sqrt{3}}\kappa\Phi} \left[e^{\frac{6}{2\sqrt{3}}\kappa\Phi} + 4e^{\frac{3}{2\sqrt{3}}\kappa\Phi} - (5 - 40\kappa^2\alpha\Lambda) \right]. \tag{9}$$

Using Eq. (9), one can check that the potential [in Eq. (8)] is stable for the parametric regime $\alpha > 0$. The stable value ($< \Phi >$) and the mass squared (m_Φ^2) of the scalar field (Φ) are given by the following two equations:

$$\exp\left(\frac{3}{2\sqrt{3}}\kappa < \Phi >\right) = \left[\sqrt{9 - 40\kappa^2\alpha\Lambda} - 2 \right] \tag{10}$$

and

$$m_\Phi^2 = \frac{1}{8\alpha} \left[\sqrt{9 - 40\kappa^2\alpha\Lambda} \right] \left[\sqrt{9 - 40\kappa^2\alpha\Lambda} - 2 \right]^{-\frac{2}{3}}. \tag{11}$$

Furthermore, the minimum value of the potential, i.e. $V(< \Phi >)$, is nonzero and serves as a cosmological constant. Thus the effective cosmological constant in scalar–tensor theory is $\Lambda_{\text{eff}} = \Lambda - V(< \Phi >)$ where $V(< \Phi >)$ is

$$V(< \Phi >) = \Lambda + \left[\sqrt{9 - 40\kappa^2\alpha\Lambda} - 2 \right]^{-\frac{5}{3}} \times \left[-\Lambda + (1/8\kappa^2\alpha) \left[\sqrt{9 - 40\kappa^2\alpha\Lambda} - 3 \right]^2 \right].$$

This form of $V(< \Phi >)$ with $\Lambda < 0$ clearly indicates that Λ_{eff} is also negative. Expanding $V(\Phi)$ (in a Taylor series) around $\Phi = < \Phi >$ and retaining up to quadratic order in $\Phi - < \Phi > = \xi$, one finally is left with the following form of the action for the scalar–tensor theory:

$$S = \int d^4x d\phi \sqrt{\tilde{G}} \left[\frac{\tilde{R}}{2\kappa^2} + \frac{1}{2} \tilde{G}^{MN} \partial_M \xi \partial_N \xi - (1/2)m_\Phi^2 \xi^2 + \Lambda_{\text{eff}} + e^{-\frac{5}{2\sqrt{3}}\kappa(< \Phi > + \xi)} V_h \delta(\phi) + e^{-\frac{5}{2\sqrt{3}}\kappa(< \Phi > + \xi)} V_v \delta(\phi - \pi) \right]. \tag{12}$$

For the case of ST theory presented in Eq. (12), ξ can act as a bulk scalar field with the mass given by Eq. (11). Considering a negligible backreaction of the scalar field (ξ) on the background spacetime, the solution of the metric \tilde{G}_{MN} is exactly the same as in the well-known RS model, i.e.,

$$d\tilde{s}^2 = e^{-2kr_c|\phi|} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2, \tag{13}$$

where $k = \sqrt{\frac{-\Lambda_{\text{eff}}}{24M^3}}$ and r_c is the compactification radius of the extra dimension in ST theory. The brane tensions V_h, V_v are given by the following expressions:

$$V_h = 24M^3 k * \exp\left[\frac{5}{2\sqrt{3}}\kappa(< \Phi > + v_h)\right],$$

$$V_v = -24M^3 k * \exp\left[\frac{5}{2\sqrt{3}}\kappa(< \Phi > + v_v)\right],$$

where v_h and v_v are the boundary values of ξ on hidden and visible brane, respectively.

With the metric presented in Eq. (13), the scalar field equation of motion in the bulk is the following:

$$-\frac{1}{r_c^2} \partial_\phi [\exp(-4kr_c|\phi|) \partial_\phi \xi] + m_\Phi^2 \exp(-4kr_c|\phi|) \xi(\phi) = 0, \tag{14}$$

where the scalar field ξ is taken as a function of the extra dimensional coordinate only [17]. Considering the nonzero value of ξ on branes, Eq. (14) has the general solution

$$\xi(\phi) = e^{2kr_c|\phi|} [A e^{vkr_c|\phi|} + B e^{-vkr_c|\phi|}] \tag{15}$$

with $v = \sqrt{4 + m_\Phi^2/k^2}$. Moreover, A and B are obtained from the boundary conditions, $\xi(0) = v_h$ and $\xi(\pi) = v_v$ as follows:

$$A = v_v e^{-(2+v)kr_c\pi} - v_h e^{-2vkr_c\pi}$$

and

$$B = v_h (1 + e^{-2vkr_c\pi}) - v_v e^{-(2+v)kr_c\pi}.$$

Upon substitution of the forms of A and B into Eq. (15), one finds that

$$\xi(0) = v_h,$$

$$\xi(\pi) = v_v \left[1 - e^{-2vkr_c\pi} + \frac{v_h}{v_v} e^{-(3v-2)kr_c\pi} \right].$$

The above values of $\xi(0)$ and $\xi(\pi)$ match with the boundary condition (i.e. $\xi(0) = v_h$ and $\xi(\pi) = v_v$) by neglecting the subleading powers of $e^{-kr_c\pi}$, as has been done earlier by the authors in [17].

3.2 Solutions of field equations for original F(R) theory

Recall that the original higher curvature $F(R)$ model is represented by the action given in Eq. (3). Solutions of the metric (G_{MN}) for this $F(R)$ model can be extracted from the solutions of corresponding scalar–tensor theory (Eqs. (13) and (15)) with the help of Eq. (5). Thus, the line element in $F(R)$ model turns out to be

$$ds^2 = e^{-\frac{\kappa}{\sqrt{3}}\Phi(\phi)} \left[e^{-2kr_c|\phi|} \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2 \right] \tag{16}$$

where $\Phi(\phi) = < \Phi > + \xi(\phi)$ and $\xi(\phi)$ is given by Eq. (15).

In order to introduce radion (or modulus) field, r_c is replaced by a field $T(x)$ [18] and this new field is assumed to be a function of the brane coordinates only. Then the metric takes the following form:

$$ds^2 = e^{-\frac{\kappa}{\sqrt{3}}\Phi(x,\phi)} \left[e^{-2kT(x)|\phi|} g_{\mu\nu}(x) dx^\mu dx^\nu - T(x)^2 d\phi^2 \right] \tag{17}$$

where $g_{\mu\nu}(x)$ is the induced on-brane metric and $T(x)$ is known as the radion field. Moreover, $\Phi(x, \phi)$ is obtained from Eq. (15) by replacing r_c by $T(x)$.

Plugging back the solutions presented in Eq. (17) into the original five dimensional F(R) action (in Eq. (3)) and integrating over ϕ yield the four dimensional effective action as follows [23]:

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left[M_{(4)}^2 R_{(4)} + \frac{1}{2} g^{\mu\nu} \partial_\mu \Psi \partial_\nu \Psi - U_{\text{rad}}(\Psi) \right] \tag{18}$$

where $M_{(4)}^2 = \frac{M^3}{k} \left[\sqrt{9 - 40\kappa^2 \alpha \Lambda} - 2 \right]^{1/2}$ is the four dimensional Planck scale, $R_{(4)}$ is the Ricci scalar formed by $g_{\mu\nu}(x)$. Moreover, $\Psi(x) = \sqrt{\frac{24M^3}{k}} \left[1 + \frac{20}{\sqrt{3}} \alpha k^2 \kappa v_h \right] e^{-k\pi T(x)} = f e^{-k\pi T(x)}$ (with $f = \sqrt{\frac{24M^3}{k}} \left[1 + \frac{20}{\sqrt{3}} \alpha k^2 \kappa v_h \right]$) is the canonical radion field and $U_{\text{rad}}(\Psi)$ is the radion potential; it has the following form [23]:

$$U_{\text{rad}}(\Psi) = \frac{20}{\sqrt{3}} \frac{\alpha k^5}{M^6} \Psi^4 \left[\left(v_h - \frac{\kappa v_h^2}{2\sqrt{3}} + \frac{\kappa v_h v_v}{2\sqrt{3}} \right) \times (\Psi/f)^\sigma - v_v \right]^2 \tag{19}$$

where the terms proportional to $\sigma \left(= \frac{m_\Phi^2}{k^2} \right)$ are neglected [17,18]. It may be observed that $U_{\text{rad}}(\Psi)$ goes to zero as α tends to zero. This is expected because, for $\alpha \rightarrow 0$, the action contains only the Einstein part which does not produce any potential term for the radion field [18]. Thus, for five

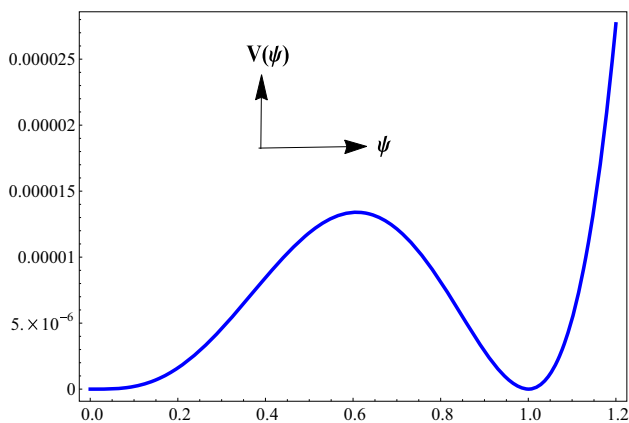


Fig. 1 $U_{\text{rad}}(\Psi)$ vs. Ψ for $M = k = 1$ (in Planck units), $\sigma = 0.04$, $\kappa v_v = 10^{-7}$, $\frac{v_h}{v_v} = 1.2$ and $\alpha = \frac{1}{M^2}$

dimensional warped geometric model, the radion potential is generated from the higher order curvature term αR^2 .

The potential in Eq. (19) has a minimum at

$$\Psi_{\text{min}} = \langle \Psi \rangle = \left[\frac{v_v f^\sigma}{\left(v_h - \frac{\kappa v_h^2}{2\sqrt{3}} + \frac{\kappa v_v v_h}{2\sqrt{3}} \right)} \right]^{1/\sigma} \tag{20}$$

and a maximum at

$$\Psi_{\text{max}} = \left[\left(\frac{2}{2 + \sigma} \right) \left(\frac{v_v f^\sigma}{v_h - \frac{\kappa v_h^2}{2\sqrt{3}} + \frac{\kappa v_v v_h}{2\sqrt{3}}} \right) \right]^{1/\sigma},$$

respectively, as long as $\alpha > 0$. Moreover, Eq. (19) clearly indicates that $U_{\text{rad}}(\Psi)$ goes to zero at $\Psi = 0$. In Fig. 1, we give a plot of $U_{\text{rad}}(\Psi)$ against Ψ .

Due to the presence of $U_{\text{rad}}(\Psi)$, the radion field has a certain dynamics governed by effective field equations. In this scenario, our motivation is to investigate whether the dynamics of the radion field (Ψ) can trigger an inflationary scenario on the brane scale factor. Motivated by this idea, we try to solve the cosmological equations (Friedmann equations) obtained from the four dimensional effective action S_{eff} , which is discussed in the next section.

4 Solutions of effective Friedmann equations

We consider for the on-brane metric ansatz the flat FRW one, i.e.

$$ds_{(4)}^2 = g_{\mu\nu}(x) dx^\mu dx^\nu = dt^2 - a^2(t) [dx^2 + dy^2 + dz^2]$$

where $a(t)$ is the scale factor of the visible universe. The effective field equations [obtained from the effective action presented in Eq. (18)] take the following form:

$$H^2 = \frac{1}{3} \left[U_{\text{rad}}(\Psi) + \frac{1}{2}(\dot{\Psi})^2 \right] \tag{21}$$

and

$$\ddot{\Psi} + 3H\dot{\Psi} + U'_{\text{rad}}(\Psi) = 0 \tag{22}$$

where an overdot denotes the derivative $\frac{d}{dt}$, $H = \frac{\dot{a}}{a}$ is known as the Hubble parameter and the form of $U_{\text{rad}}(\Psi)$ is given in Eq. (19). To derive the above equations, we assume that the radion field ($\Psi(t)$) is homogeneous in space.

In order to solve the effective Friedmann equations, the potential energy of radion field is taken as very much greater than the kinetic energy (known as slow roll approximation) i.e.

$$U_{\text{rad}}(\Psi) \gg \frac{1}{2}(\dot{\Psi})^2.$$

With this approximation, Eqs. (21) and (22) are simplified to

$$H^2 = \frac{1}{3} U_{\text{rad}}(\Psi) \tag{23}$$

and

$$3H\dot{\Psi} + U'_{\text{rad}}(\Psi) = 0, \tag{24}$$

respectively. Substituting $H(t)$ from Eqs. (23)–(24) and using the explicit form of $U_{\text{rad}}(\Psi)$, we get the equation of motion for the radion field:

$$\frac{d\Psi}{dt} = -8v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6} \Psi (B\Psi^\sigma - 1) \tag{25}$$

where $B = \frac{1}{v_v f^\sigma} \left(v_h - \frac{\kappa v_h^2}{2\sqrt{3}} + \frac{\kappa v_v v_h}{2\sqrt{3}} \right)$. Equation (25) immediately leads to the dynamics of the radion field,

$$\Psi(t) = \frac{\Psi_0}{\left[B\Psi_0^\sigma - (B\Psi_0^\sigma - 1) \exp \left(-8\sigma v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6} (t - t_0) \right) \right]^{1/\sigma}}, \tag{26}$$

where Ψ_0 is the value of radion field ($\Psi(t)$) at $t = t_0$. Equation (26) clearly indicates that $\Psi(t)$ decreases with time. Comparison of Eqs. (20) and (26) clearly reveals that the radion field reaches its vacuum expectation value (VEV) asymptotically (within the slow roll approximation) at large time ($t \gg t_0$) i.e.

$$\Psi(t \gg t_0) = \left[\frac{v_v f^\sigma}{\left(v_h - \frac{\kappa v_h^2}{2\sqrt{3}} + \frac{\kappa v_v v_h}{2\sqrt{3}} \right)} \right]^{1/\sigma} = \langle \Psi \rangle. \tag{27}$$

This VEV of the radion field leads to the stabilized interbrane separation (between Planck and TeV branes),

$$k\pi < T(x) >= \frac{4k^2}{m_\Phi^2} \left[\ln \left(\frac{v_h}{v_v} \right) - \frac{\kappa v_v}{2\sqrt{3}} \left(\frac{v_h}{v_v} - 1 \right) \right] \tag{28}$$

where m_Φ^2 is given in Eq. (11).

Putting the solution of $\Psi(t)$ into Eq. (23) one gets, on integration, the evolution of the scale factor,

$$a(t) = C \exp \left[2v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} (g_1(t) - g_2(t)) \right], \tag{29}$$

where C is an integration constant and $g_1(t)$ has the following form:

$$\begin{aligned} g_1(t) = & -\frac{B\Psi_0^\sigma}{(B\Psi_0^\sigma - 1)} \frac{1}{16\sigma v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}}} \Psi_0^2 * 2F1 \\ & \times \left(1, 1, 2 + \frac{2}{\sigma}, \frac{B\Psi_0^\sigma}{B\Psi_0^\sigma - 1} \exp \right. \\ & \times \left. \left(8\sigma v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} (t - t_0) \right) \right) \\ & \exp \left(8\sigma v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} (t - t_0) \right) (B\Psi_0^\sigma - (B\Psi_0^\sigma - 1) \\ & \exp \left(-8\sigma v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} (t - t_0) \right) \right)^{-2/\sigma} \end{aligned} \tag{30}$$

where $2F1$ denotes the hypergeometric function. Similarly the form of $g_2(t)$ is given by

$$\begin{aligned} g_2(t) = & -\frac{\Psi_0^\sigma}{(B\Psi_0^\sigma - 1)} \frac{1}{16\sigma v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}}} \Psi_0^2 * 2F1 \\ & \times \left(1, 1, 1 + \frac{2}{\sigma}, \frac{B\Psi_0^\sigma}{B\Psi_0^\sigma - 1} \right. \\ & \times \exp \left(8\sigma v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} (t - t_0) \right) \Big) \\ & \times \exp \left(8\sigma v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} (t - t_0) \right) \\ & \times \left(B\Psi_0^\sigma - (B\Psi_0^\sigma - 1) \right. \\ & \times \left. \exp \left(-8\sigma v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} (t - t_0) \right) \right)^{1-2/\sigma}. \end{aligned} \tag{31}$$

It may be noticed from Eqs. (26) and (29) that, for $\alpha \rightarrow 0$, the solution of the radion field and Hubble parameter become $\Psi(t) = \Psi_0$ and $H(t) = 0$, respectively. It is expected because in the absence of the higher curvature term, $U_{\text{rad}}(\Psi)$ goes to zero and thus the radion field has no dynamics, by which in turn the evolution of the scale factor of the universe vanishes.

5 Beginning of inflation

After obtaining the solution of $a(t)$ (in Eq. (29)), we can now examine whether this form of the scale factor corresponds to an accelerating era of the early universe (i.e. $t \gtrsim t_0$) or not. In order to check this, we expand $a(t)$ in the form of Taylor series (about $t = t_0$) and retain the terms only up to first order in $t - t_0$:

$$a(t \gtrsim t_0) = a_0 \exp \left[2(B\Psi_0^\sigma - 1) \Psi_0^2 v_v \sqrt{\frac{5}{3\sqrt{3}} \frac{\alpha k^5}{M^6}} (t - t_0) \right] \tag{32}$$

where a_0 is the value of the scale factor at $t = t_0$; it is related to the integration constant C by

$$a_0 = C \exp \left[-\frac{\Psi_0^2}{8} \right].$$

It is evident from Eq. (32) that $a(t)$ corresponds to an exponential expansion at an early age of the universe, where t_0 specifies the onset of inflation. Moreover, the Hubble parameter ($H = \frac{\dot{a}}{a}$) depends on the higher curvature parameter α and for $\alpha \rightarrow 0$, $a(t) = a_0$. Thus the accelerating period of the early universe is triggered entirely due to the presence of a higher curvature term in the five dimensional bulk spacetime.

6 End of inflation

In the previous section, we show that the very early universe expands with an acceleration and this accelerating stage is termed the inflationary epoch. In this section, we check whether the acceleration of the scale factor has an end in a finite time or not.

In the case of inflation, $\ddot{a} > 0$. By relating the definition of inflation to the Hubble parameter, one readily obtains

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 > 0. \tag{33}$$

We now estimate the time interval which is consistent with this condition. Recall the slow roll equation (Eq. (23)),

$$\begin{aligned} H^2 &= \frac{1}{3} U_{\text{rad}}(\Psi) \\ &= \frac{20}{3\sqrt{3}} \frac{\alpha k^5}{M^6} v_v^2 \Psi^4 (B\Psi^\sigma - 1)^2. \end{aligned}$$

Differentiating both sides of this equation with respect to t , we get the time derivative of the Hubble parameter as follows:

$$\dot{H} = -\frac{160}{3\sqrt{3}} \frac{\alpha k^5}{M^6} v_v^2 \Psi^2 (B\Psi^\sigma - 1)^2 \tag{34}$$

where we use the expression of $\dot{\Psi}$ from Eq. (25). Plugging back the expressions of H^2 and \dot{H} into Eq. (33) one gets the following condition on the radion field:

$$\Psi > 2\sqrt{2} = \Psi_f = \Psi(t_f) \tag{35}$$

where t_f is the time when the radion field acquires the value $2\sqrt{2}$ (in Planck units). Equation (35) clearly indicates that the inflationary era of the universe continues as long as the radion field remains greater than $\Psi_f (= 2\sqrt{2})$. Correspondingly the duration of inflation (i.e. $t_f - t_0$) can be calculated from the solution of $\Psi(t)$ as follows:

$$\Psi_f^\epsilon = \frac{\Psi_0^\sigma}{\left[B\Psi_0^\sigma - (B\Psi_0^\sigma - 1) \exp\left(-8\sigma v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6} (t_f - t_0)\right) \right]}$$

Simplifying the above expression, we obtain

$$t_f - t_0 = \frac{1}{8\sigma v_v \sqrt{\frac{5}{3\sqrt{3}}} \frac{\alpha k^5}{M^6}} \ln \left[\frac{\Psi_f^\sigma (B\Psi_0^\sigma - 1)}{\Psi_0^\sigma (B\Psi_f^\sigma - 1)} \right]. \tag{36}$$

So inflation comes to an end in a finite time. In order to estimate the duration of inflation explicitly, one needs the initial value of the radion field (i.e. Ψ_0) which can be determined from the expression of the number of e-foldings, discussed in the next section.

7 Number of e-foldings and slow roll parameters

The total number of e-foldings (N_0) of the inflationary era is defined by

$$N_0 = \int_{t_0}^{t_f} H(t) dt. \tag{37}$$

Using the slow roll equation, the above expression is simplified to the form

$$N_0 = \int_{\Psi_0}^{\Psi_f} \sqrt{\frac{U_{\text{rad}}(\Psi)}{3}} \frac{1}{\dot{\Psi}} d\Psi. \tag{38}$$

Putting the explicit form of $U_{\text{rad}}(\Psi)$ (Eq. (19)) and the time derivative of $\Psi(t)$ (Eq. (25)) into the right hand side of Eq. (38) and integrating over Ψ , one obtains the final result of number of e-foldings to be given by

$$N_0 = \frac{1}{16} (\Psi_0^2 - \Psi_f^2). \tag{39}$$

We may define

$$N(\Psi) = N_0 - \int_{\Psi_0}^{\Psi} \sqrt{\frac{U_{\text{rad}}(\Psi)}{3}} \frac{1}{\dot{\Psi}} d\Psi,$$

the number of e-foldings remaining until the end of inflation when the inflaton field crosses the value $\Psi(t)$. Simplifying the above expression, one obtains

$$N(\Psi) = \frac{1}{16} (\Psi^2 - \Psi_f^2).$$

In order to test the broad inflationary paradigm as well as particular models against precision observations [1], it is crucial to calculate the slow roll parameters (ϵ_V and η_V), which are defined as follows:

$$\epsilon_V = \frac{1}{2} \left(\frac{U'_{\text{rad}}(\Psi)}{U_{\text{rad}}(\Psi)} \right)^2$$

and

$$\eta_V = \left(\frac{U''_{\text{rad}}(\Psi)}{U_{\text{rad}}(\Psi)} \right)$$

The slow roll condition requires that the parameters ϵ_V and η_V should be less than unity as long as the inflationary era continues. By using the form of inflaton potential $[U_{\text{rad}}(\Psi)$, in Eq. (19)], the above expressions can be simplified and turn out to be

$$\epsilon_V = \frac{8}{16N(\Psi) + \Psi_f^2} \tag{40}$$

and

$$\eta_V = \frac{6}{16N(\Psi) + \Psi_f^2}. \tag{41}$$

Using these expressions of the slow roll parameters, one determines the spectral index of the curvature perturbation (n_s) and the tensor to scalar ratio (r) in terms of N_* ($= \frac{1}{16} (\Psi_*^2 - \Psi_f^2)$), the number of e-foldings remaining until the end of inflation when the cosmological scales exit the horizon and Ψ_* is the corresponding value of the inflaton field) [31–33]:

$$n_s = \frac{16N_* - 28}{16N_* + 8} \tag{42}$$

and

$$r = \frac{128}{16N_* + 8}. \tag{43}$$

To derive Eq. (42) and Eq. (43), we use the value of $\Psi_f (= 2\sqrt{2})$, which has been obtained earlier (see Eq. (35)). From observational results (*PlanckTT + lowP + lensing*, 2015) [1] n_s and r are constrained to be $n_s = 0.968 \pm 0.006$ and $r < 0.14$, respectively. Using Eqs. (42) and (43), it can easily be shown that in order to have agreement between the theoretical and observational results, N_* should be equal to

Table 1 Theoretical and observational results of n_s and r

Parameters (n_s and r)	Theoretical results from the present model (for $N_* = 60$)	Observational results from PlanckTT + lowP + lensing 2015
n_s	0.963	0.968 ± 0.006
r	0.132	< 0.14

60. Putting this value of N_* into Eqs. (42) and (43), we obtain the following results for n_s and r :

$$n_s = 0.963,$$

$$r = 0.132.$$

At this stage it deserves mention that any non-minimally coupled theory and its conformally transformed version (the so-called Einstein frame) may be physically quite different. However, we want to emphasize that in the present context the inflationary solutions as well as the values of n_s and r are obtained not in the conformally related scalar–tensor version of the theory, but rather in the original $F(R)$ gravity model by using the inverse conformal transformation to get back the $F(R)$ model (see Eq. (16)). Furthermore the purpose of introducing the scalar–tensor version in Sect. 3.1 is to just extract an analytic solution of the original $F(R)$ gravity model (obtained in Eq. (16)). For a detailed discussion of this problem and its remedy we refer to the work of Capozziello, Nojiri, Odintsov and Troisi [63] (see also [64]).

At the pivot scale ($N(\Psi) = N_*$), ϵ_V and η_V acquire the values as 0.009 and 0.007, respectively.

In Table 1, we now summarize our results.

It is evident from Table 1 that the present model of five dimensional higher curvature gravity is not ruled out in terms of the values of n_s and r as per the observations of Planck [1].

The required value of N_* ($= 60$) can be achieved if Ψ_* is adjusted to the value $\Psi_* \simeq 31$ (in Planck units). Also requiring the total number of e-foldings of the inflationary era to be equal to 70 (i.e. $N_0 = 70$), we obtain the initial value of the inflaton field, $\Psi_0 = 33.5$ (in Planck units). With this value of Ψ_0 , duration of inflation ($t_f - t_0$) comes as $\sim 10^{-33}$ sec [or $10^{-10} (\text{GeV})^{-1}$; see Eq. (36)] if the higher curvature parameter α and κv_v are taken as

$$\alpha \sim \frac{1}{M^2} \tag{44}$$

where M is the five dimensional Planck scale and

$$\kappa v_v \sim 10^{-7},$$

respectively. Furthermore, the effective gravitational constant ($M_{(4)}$) is $\sim 10^{19}$ GeV for the estimated value of α presented in Eq. (44).

Once we find the initial (Ψ_0) and final (Ψ_f) values of the inflaton field, we can show the plots (Figs. 2 and 3) between the slow roll parameters and Ψ [by using Eqs. (40) and (41)].

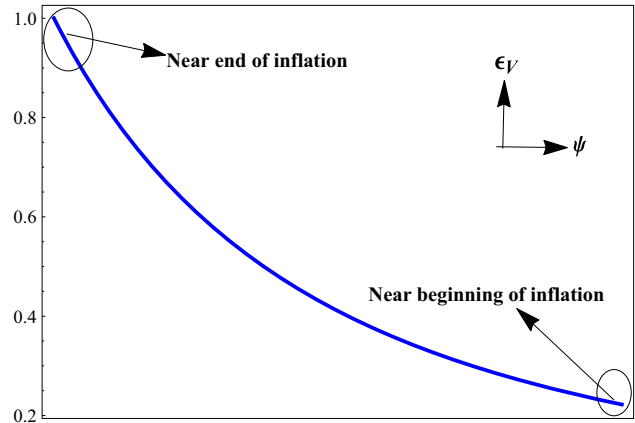


Fig. 2 ϵ_V vs. Ψ

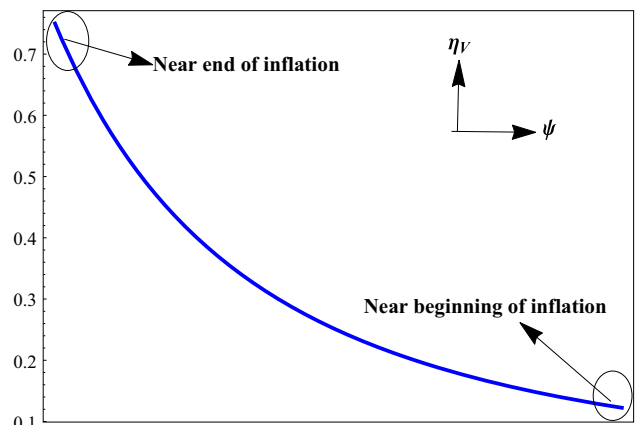


Fig. 3 η_V vs. Ψ

Figures 2 and 3 clearly demonstrate that as long as inflation continues, the two slow roll parameters (ϵ_V and η_V) remain less than unity. This behavior of ϵ_V and η_V is expected from the slow roll approximation. Furthermore, the values of ϵ_V and η_V increase with the evolution of universe during the inflationary epoch and at the end of inflation, and ϵ_V becomes 1, i.e. $\epsilon_V(\Psi_f) = 1$.

8 Comparison of solutions with and without slow roll approximation

In this section, we solve the radion field and Hubble parameter numerically from the complete form of the effective

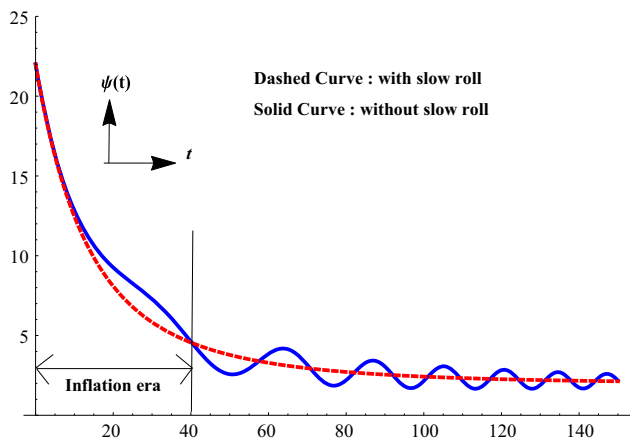


Fig. 4 $\Psi(t)$ vs. t with/without slow roll approximation for $m_{\text{rad}} = 10^{-4}$ (in Planck units)

Friedmann equations (Eqs. (21) and (22), without slow roll approximations). These numerical solutions are then compared with the solutions [in Eqs. (26) and (29)] obtained by solving the slow roll equations.

Equations (21) and (22) lead to the following equation of $\Psi(t)$:

$$\ddot{\Psi} + \sqrt{3[U_{\text{rad}}(\Psi) + \frac{1}{2}(\dot{\Psi})^2]}\dot{\Psi} + U'_{\text{rad}}(\Psi) = 0.$$

Using the form of $U_{\text{rad}}(\Psi)$, the above differential equation is solved numerically for $\Psi(t)$. The comparison between this numerical solution and the solution obtained in Eq. (26) is presented in Fig. 4.

Figure 4 demonstrates that the plotted result of $\Psi(t)$ based on solving the slow roll equations and the plotted result of $\Psi(t)$ based on solving the full Friedmann equations (in the presence of $\dot{\Psi}^2$ and $\ddot{\Psi}$) are almost the same during the inflation. But after the inflation the acceleration term of the inflaton (the term containing $\ddot{\Psi}$) starts to contribute and as a result the two solutions (with and without slow roll conditions) differ from each other. Moreover, in the slow roll approximation, $\Psi(t)$ does not exhibit an oscillatory phase at the end of inflation, but it tends to its minimum value asymptotically. Such an oscillatory character of $\Psi(t)$ occurs when the term $\ddot{\Psi}$ is taken into account in the equation of motion. Another point to mention is that even without the slow roll approximation, the oscillatory character of $\Psi(t)$ is allowed as long as the mass of inflaton field $m_{\text{rad}} > \frac{3}{2}H_{\text{end}} \sim 10^{-7}$ (in Planck units, where H_{end} is the Hubble parameter after the inflation) [62], otherwise $\Psi(t)$ decays continually (without any oscillation) and finally reaches zero asymptotically. This statement can be verified from the solution of inflaton field near the minimum of $U_{\text{rad}}(\Psi)$ (i.e., near at $\langle \Psi \rangle$), which takes the following form:

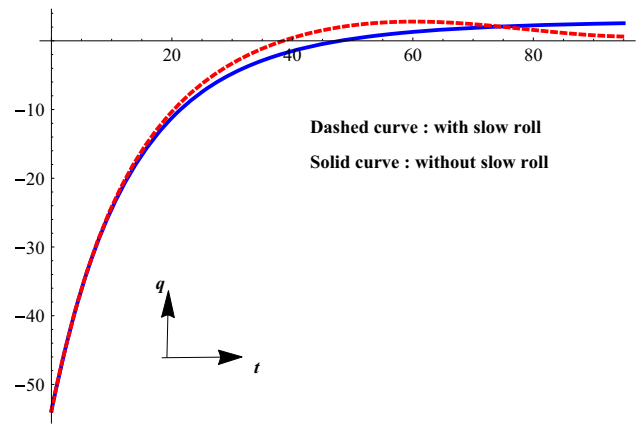


Fig. 5 $q(t)$ vs. t with/without slow roll approximation

$$\Psi(t) = \langle \Psi \rangle + \exp \left[\frac{1}{2} \left(-3H_{\text{end}} \pm \sqrt{9H_{\text{end}}^2 - 4m_{\text{rad}}^2} \right) t \right]. \tag{45}$$

The above expression clearly indicates that the oscillation of $\Psi(t)$ around its VEV is possible if the term inside the square root becomes negative, which immediately leads to the condition $m_{\text{rad}} > \frac{3}{2}H_{\text{end}}$. Furthermore, it may be noted that the oscillatory behavior of $\Psi(t)$ occurs in Fig. 4 (see the solid curve), because in that case, m_{rad} is taken as 10^{-4} (in Planck units), which satisfies the said criteria for oscillation.

Finally the numerical solution of Hubble parameter and correspondingly the deceleration parameter ($q(t) = -(\dot{H} + H^2)$) are also obtained from Eq. (21). The variation of $q(t)$ versus t (with/without slow roll approximation) is shown in Fig. 5.

Figure 5 clearly depicts that the duration of inflation predicted from the numerical solution of complete Friedmann equations is longer than that predicted from the solutions of the slow roll equations.

9 Summary and concluding remarks

In this work, we consider a five dimensional compactified warped geometry model with two 3-branes embedded within the spacetime. Due to the large curvature (\sim Planck scale) in the bulk, the spacetime is considered to be governed by a higher curvature expression like $F(R) = R + \alpha R^2$. Our visible universe is identified with the TeV scale brane, which emerges out of the four dimensional effective theory. On projecting the bulk gravity on the brane, the extra degrees of freedom of $R_{(5)}$ appear as a scalar field on the brane and this is known as the radion field. The potential term ($U_{\text{rad}}(\Psi)$) of the radion field is proportional to the higher curvature parameter α and goes to zero as $\alpha \rightarrow 0$. Thus it is clear that the

radion potential is generated entirely due to the presence of the higher curvature term in the five dimensional bulk space-time. The form of $U_{\text{rad}}(\Psi)$ [in Eq. (19)] indicates that the radion potential is stable as long as α is considered to be positive and the minimum of $U_{\text{rad}}(\Psi)$ is zero.

From the perspective of four dimensional effective theory, we examine the possibility of an “inflationary scenario” by taking the on-brane metric ansatz as a spatially flat FRW one. In the presence of the radion potential, $U_{\text{rad}}(\Psi)$, we determine the solutions [in Eqs. (26) and (29)] of the effective Friedmann equations by considering the potential energy of the radion field as very much greater than the kinetic energy (also known as the slow roll approximation). The solution of the scale factor corresponds to an accelerating expansion of the early universe and the rate of expansion depends on the parameter α . It may be mentioned that the radion field as well as the scale factor becomes constant as α goes to zero. Thus, it can be argued that due to the presence of the higher curvature term, the radion field has a certain dynamics which in turn triggers an exponential expansion of the universe at an early epoch. The expression of duration of inflation ($t_f - t_0$) is also obtained in Eq. (36), which reveals that the accelerating phase of the universe terminates within a finite time.

We determine the slow roll parameters (ϵ_V and η_V) and it is found that both ϵ_V and η_V remain less than unity as long as the inflation continues. The expressions of the slow roll parameters yield the spectral index of the curvature perturbation (n_s) and tensor to scalar ratio (r) in terms of N_* (number of e-foldings remaining until the end of inflation when the cosmological scales exit the horizon). For $N_* = 60$, n_s and r take the values of $n_s = 0.963$ and $r = 0.132$, which match the observational results based on the observations of Planck 2015 (see Table 1). Thus our considered model of five dimensional higher curvature gravity predicts the correct values of n_s and r as per the observations of Planck 2015. Moreover the duration of inflation turns out to be $\sim 10^{-33}$ s (or 10^{-10} (GeV) $^{-1}$) if the higher curvature parameter α and κv_b are taken as $\sim \frac{1}{M^2}$ (M is the five dimensional Planck scale) and $\sim 10^{-7}$, respectively.

Finally, we find the solution for the radion field and Hubble parameter numerically from the complete form of the Friedmann equations (without the slow roll approximations). During the inflation, these numerical solutions are almost the same as the solutions of the slow roll equations, as demonstrated in Figs. 4 and 5. Another important point to note is that in the slow roll approximation, $\Psi(t)$ does not exhibit an oscillatory phase at the end of inflation, but it tends to its minimum value asymptotically, while such an oscillatory behavior of the inflaton is indeed there if the slow roll approximation is relaxed [with $m_{\text{rad}} > \frac{3}{2}H_{\text{end}}$; see Eq. (45)], i.e., the acceleration term of $\Psi(t)$ in the equation of motion is not dropped, as depicted in Fig. 4.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP³.

References

1. Planck Collaboration (P.A.R. Ade (Cardiff U.) et al.); Planck 2015 results. XX. Constraints on inflation. *Astron. Astrophys.* **594**, A20 (2016). [arXiv:1502.02114](https://arxiv.org/abs/1502.02114)
2. N. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Lett. B* **429**, 263 (1998)
3. N. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Rev. D* **59**, 086004 (1999)
4. I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, G. Dvali, *Phys. Lett. B* **436**, 257 (1998)
5. P. Horava, E. Witten, *Nucl. Phys. B* **475**, 94 (1996)
6. P. Horava, E. Witten, *Nucl. Phys. B* **460**, 506 (1996)
7. L. Randall, R. Sundrum, *Phys. Rev. Lett.* **83**, 3370 (1999)
8. N. Kaloper, *Phys. Rev. D* **60**, 123506 (1999)
9. T. Nihei, *Phys. Lett. B* **465**, 81 (1999)
10. H.B. Kim, H.D. Kim, *Phys. Rev. D* **61**, 064003 (2000)
11. A.G. Cohen, D.B. Kaplan, *Phys. Lett. B* **470**, 52 (1999)
12. C.P. Burgess, L.E. Ibanez, F. Quevedo, *Phys. Lett. B* **447**, 257 (1999)
13. A. Chodos, *Phys. Lett. B* **471**, 119 (1999)
14. T. Gherghetta, M. Shaposhnikov, *Phys. Rev. Lett.* **85**, 240 (2000)
15. G.F. Giudice, R. Rattazzi, J.D. Wells, *Nucl. Phys. B* **544**, 3 (1999)
16. R. Martens, K. Koyama, *Living Rev. Relat.* **13**, 5 (2010)
17. W.D. Goldberger, M.B. Wise, *Phys. Rev. Lett.* **83**, 4922 (1999)
18. W.D. Goldberger, M.B. Wise, *Phys. Lett. B* **475**, 275–279 (2000)
19. C. Csaki, M.L. Graesser, G.D. Kribs, *Phys. Rev. D* **63**, 065002 (2001)
20. J. Lesgourgues, L. Sorbo, *Phys. Rev. D* **69**, 084010 (2004)
21. O. DeWolfe, D.Z. Freedman, S.S. Gubser, A. Karch, *Phys. Rev. D* **62**, 046008 (2000)
22. S. Das, D. Maity, S. SenGupta, *J. High Energy Phys.* **05**, 042 (2008)
23. A. Das, H. Mukherjee, T. Paul, S. SenGupta. [arXiv:1701.01571](https://arxiv.org/abs/1701.01571) [hep-th]
24. T. Paul, S. SenGupta, *Phys. Rev. D* **93**(8), 085035 (2016)
25. A. Das, T. Paul, S. SenGupta, [arXiv:1609.07787](https://arxiv.org/abs/1609.07787) [hep-ph]
26. D. Perkins, *Particle Astrophysics*, 1st edn. (Oxford University Press, Oxford, 2005)
27. G. Scott Watson, (2000). [arXiv:astro-ph/0005003](https://arxiv.org/abs/astro-ph/0005003)
28. A.H. Guth, *Phys. Rev. D* **23**, 347–356 (1981)
29. A. Linde, *Particle Physics and Inflationary Cosmology*, (Electronic version of the book from 1990) (2005). [arXiv:hep-th/0503203](https://arxiv.org/abs/hep-th/0503203)
30. W.H. Kinney, [arXiv:astro-ph/0301448](https://arxiv.org/abs/astro-ph/0301448) (2004)
31. D. Langlois, [arXiv:hep-th/0405053](https://arxiv.org/abs/hep-th/0405053) (2004)
32. S. Habib, A. Heinen, K. Heitmann, G. Jungman, *Phys. Rev. D* **71**, 043518 (2005)
33. A. Riotto, [arXiv:hep-ph/0210162](https://arxiv.org/abs/hep-ph/0210162) (2002)
34. N. Banerjee, S. Sen, *Phys. Rev. D* **57**, 4614 (1998)
35. J.D. Barrow, P. Saich, *Class. Quantum Gravity* **10**, 279 (1993)
36. J.D. Barrow, J.P. Mimoso, *Phys. Rev. D* **50**, 3746 (1994)
37. J.P. Mimoso, D. Wands, *Phys. Rev. D* **51**, 477 (1995)
38. C. Csaki, M. Graesser, L. Randall, J. Terning, *Phys. Rev. D* **62**, 045015 (2000)
39. P. Binetruy, C. Deffayet, D. Langlois, *Nucl. Phys. B* **565**, 269 (2000)
40. C. Csaki, M. Graesser, C. Kolda, J. Terning, *Phys. Lett. B* **462**, 34 (1999)

41. J.M. Cline, C. Grojean, G. Servant, Phys. Rev. Lett. **83**, 4245 (1999)
42. P. Kanti, I.I. Kogan, K.A. Olive, M. Pospelov, Phys. Lett. B **468**, 31 (1999)
43. D.J. Chung, K. Freese, Phys. Rev. D **61**, 023511 (2000)
44. S. Nojiri, S.D. Odintsov, JHEP **0007**, 049 (2000)
45. S. Nojiri, S.D. Odintsov, S. Ogushi, Phys. Rev. D **65**, 023521 (2002)
46. T.P. Sotiriou, V. Faraoni, Rev. Mod. Phys. **82**, 451497 (2010). [arXiv:0805.1726](#) [gr-qc]
47. A. De Felice, S. Tsujikawa, Living Rev. Relat. **13**, 3 (2010). [arXiv:1002.4928](#) [gr-qc]
48. A. Paliathanasis, Class. Quantum Gravity **33**(7), 075012 (2016). [arXiv:1512.03239](#) [gr-qc]
49. S. Nojiri, S.D. Odintsov, Phys. Rev. D **68**, 123512 (2003)
50. S. Nojiri, S.D. Odintsov, Int. J. Geom. Meth. Mod. Phys. **4**, 115–146 (2007). [arXiv:hep-th/0601213](#)
51. S. Nojiri, S.D. Odintsov, Phys. Rept. **505**, 59–144 (2011). [arXiv:1011.0544](#)
52. S. Nojiri, S.D. Odintsov, Phys. Lett. B **631** (2005). [arxiv:hep-th/0508049](#)
53. S. Nojiri, S.D. Odintsov, O.G. Gorbunova, J. Phys. A **39**, 6627 (2006). [arxiv:hep-th/0510183](#)
54. G. Cognola, E. Elizalde, S. Nojiri, S.D. Odintsov, S. Zerbini, Phys. Rev. D **73**, 084007 (2006)
55. C. Lanczos, Z. Phys. **73**, 147 (1932)
56. C. Lanczos, Ann. Math. **39**, 842 (1938)
57. D. Lovelock, J. Math. Phys. **12**, 498 (1971)
58. S. Capozziello, R. de Ritis, A.A. Marino, Class. Quantum Gravity **14**, 3243–3258 (1997). [arXiv:gr-qc/9612053](#) [gr-qc]
59. S. Bahamonde, S.D. Odintsov, V.K. Oikonomou, M. Wright, [arXiv:1603.05113](#) [gr-qc]
60. R. Catena, M. Pietroni, L. Scarabello, Phys. Rev. D **76**, 084039 (2007). [arXiv:astro-ph/0604492](#) [astro-ph]
61. A.A. Starobinsky, JETP Lett. **30**(11), 682–685 (1979)
62. A.R. Liddle, Phys. Rev. D **49**, 739–747 (1994)
63. S. Capozziello, S. Nojiri, S.D. Odintsov, A. Troisi, Phys. Lett. B **639**, 135 (2006)
64. D.J. Brooker, S.D. Odintsov, R.P. Woodard, Nucl. Phys. B **911**, 318 (2016)