Inflow-Outflow Infiltration Measurement Accuracy

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Abstract: Furrow infiltration and channel seepage are often measured with inflow-outflow measurements. Inaccuracy in the flow measurement will cause a larger uncertainty in the calculated infiltration. The infiltration rate determination uncertainty increases rapidly as the percent of the inflow that is infiltrated decreases. The effect of measurement uncertainty on infiltration measurements can be estimated so that the confidence interval of a mean or the actual infiltration variability level can be determined.

Introduction

Soil infiltration rate is a critical parameter in the design and evaluation of irrigation systems. A common method of measuring infiltration rate under graded surface irrigation is the inflow-outflow method, in which infiltration rate is calculated as the difference between measured water inflow and outflow rate from a furrow or border section. The technique is also used to determine water loss from conveyance channels. Inaccuracy inherent in flow measurement will cause uncertainty in the measured infiltration rate. This paper analyzes the sensitivity of measured infiltration to flow measurement uncertainty and indicates achievable infiltration measurement accuracy and procedures to maximize accuracy. The analysis provides engineers and soil scientists with the tools required to provide a statistical basis for the uncertainty in infiltration measurements.

Flow Measurement Uncertainty

Scatter in a set of data, such as the random deviations of measured values around the true value, is described by the variance V or its square root, the standard deviation s. The uncertainty of a measurement refers to the interval within which the true value lies with a certain confidence or probability P. Uncertainty limits E are calculated from the Student's t-statistic, based on the desired probability level and the degrees-of-freedom involved in determining s and on n, the number of measurements made as follows

$$E = \frac{ts}{\sqrt{n}} \quad (1)$$

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The uncertainty commonly used in flow measurement is the 95% probability confidence limits (ISO 1978). If a large number of samples are used to determine s for a measurement process $t = 2.0$ for $P = 95\%$.

A discussion and analysis of flow measurement error or uncertainty is given in Trout and Mackey (1988), ISO (1978), Bos (1976), and Bos et al. (1984). The largest uncertainty source in well-calibrated, well-constructed, and properly used flow measurement devices is the measurement of flow depth or head in flumes, weirs, and orifices, or the time reading in volumetric measurements. The relationship between the variance of a device head or time reading V_R and the variance of a flow measurement V_Q can be approximated by the first term of a Taylor series expansion (Kendall and Stuart 1963):

$$V_Q = \left(\frac{dQ}{dR}\right)^2 V_R \quad (2)$$

where Q = the flow rate; and R = the device head or time reading. Taking the square root of both sides gives

$$s_Q = \left|\frac{dQ}{dR}\right| s_R \quad (3)$$

where the subscripts are as defined for Eq. 2.

The derivative term is called the sensitivity of the device. The discharge relationship of many flow measurement devices can be approximated by a power curve equation:

$$Q = aR^u \quad (4)$$

where a and u = the discharge coefficient and exponent, respectively. Thus the sensitivity of these types of devices is

$$\frac{dQ}{dR} = auR^{u-1} \quad (5)$$

By solving Eq. 4 for R and inserting into Eq. 5, the sensitivity can be written in terms of Q

$$\frac{dQ}{dR} = ua^{(1/u)}Q^{[1-(1/u)]} \quad (6)$$

By solving Eq. 4 for a and inserting into Eq. 5, the sensitivity can be expressed as

$$\frac{dQ}{dR} = u\left(\frac{Q}{R}\right) \quad (7)$$

Standard deviations can be expressed in relative terms as a coefficient of variation CV, where $CV_Q = s_Q/Q$ and $CV_R = s_R/R$. Combining Eq. 7 and Eq. 3 yields

$$s_Q = u\left(\frac{Q}{R}\right)s_R = uQ \cdot CV_R \quad (8a)$$

or dividing both sides by Q

$$CV_Q = uCV_R \quad (8b)$$

INFILTRATION DETERMINATION UNCERTAINTY

Inflow-outflow measured infiltration is calculated from the difference of two flow measurements. Uncertainties due to random variations are combined by adding the variances

$$V_D = V_{Q1} + V_{Q2} - 2\ \mathrm{COV} \quad (9)$$

where V_D = the variance of the difference and thus of the infiltration determination; V_{Q1} = the variance of the inflow measurement; V_{Q2} = the variance of the outflow measurement; and COV = the covariance of the two measurements.

For inflow-outflow determinations, the random errors in the two flow measurements are assumed independent, so the covariance is zero. Consequently, the variance of the inflow-outflow determination is simply the sum of the measurement variances. In terms of standard deviations:

$$s_D = (s_{Q1}^2 + s_{Q2}^2)^{1/2} \quad (10)$$

where the subscripts are as used in Eq. 9. If the two flow measurement conditions (devices and reader) are identical, and the flow measurement uncertainty is constant, the two measurement standard deviations will be equal, and the infiltration determination standard deviation will be simply

$$s_D = \sqrt{2 \cdot s_Q^2} = 1.4s_Q \quad (11)$$

or about 40% greater than the measurement standard deviation.

However, flow measurement uncertainty is not generally constant because a primary source of uncertainty is reading uncertainty. Inserting Eq. 6 into Eq. 3 and the result into Eq. 10 yields

$$s_D = \{[u_1a_1^{(1/u_1)}Q_1^{(1-1/u_1)}]^2 s_{R1}^2 + [u_2a_2^{(1/u_2)}Q_2^{(1-1/u_2)}]^2 s_{R2}^2\}^{1/2} \quad (12)$$

where subscript 1 = the inflow measurement; and subscript 2 = the outflow measurement. If the two measurement methods are the same, $u_1 = u_2$, $a_1 = a_2$, and $s_{R1} = s_{R2}$, so that

$$s_D = ua^{(1/u)}[Q_1^{(2-2/u)} + Q_2^{(2-2/u)}]^{1/2} s_R \quad (13)$$

Infiltration, $Q_1 - Q_2$, can be expressed relative to the inflow

$$I_p = \frac{Q_1 - Q_2}{Q_1} \quad (14)$$

Substituting this relative infiltration I_p into Eq. 13:

$$s_D = ua^{(1/u)}Q_1^{(1-1/u)}\{1 + (1 - I_p)^{(2-2/u)}\}^{1/2} s_R$$

$$= \{1 + (1 - I_p)^{[2-(2/u)]}\}^{1/2} s_{Q1} \quad (15)$$

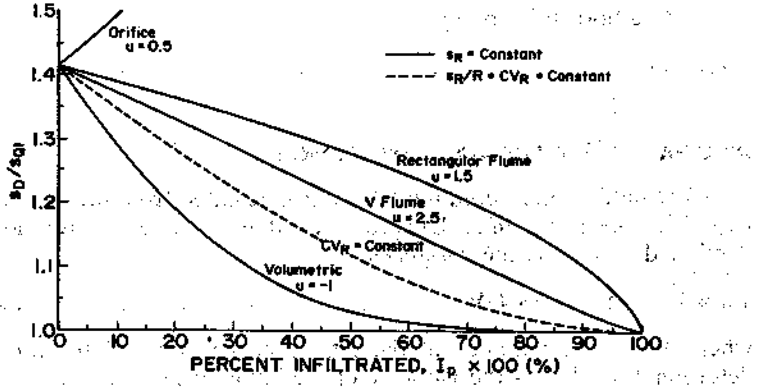

FIG. 1. Infiltration Measurement Standard Deviation s_D Relative to Inflow Measurement Standard Deviation s_{Q1} versus Percent Infiltrated for Four Types of Flow Measurement Devices

where $s_{Q1} = s_Q$ at the inflow rate. When s_R is constant, and the same device is used to measure inflows and outflows, the standard deviation of the flow rate difference and consequently the standard deviation of the infiltration determination can be calculated from the device discharge equation parameters, a and u, the relative amount infiltrated I_p, (percent/100), and either the device reading standard deviation s_R and inflow rate Q_1, or the device measurement standard deviation at the inflow rate s_{Q1}.

Fig. 1 shows how the infiltration determination standard deviation and thus uncertainty varies with the percent infiltrated for four types of measurement devices. For all except the orifice, the uncertainty decreases from $\sqrt{2} \cdot s_{Q1}$ at $I_p = 0$ to s_{Q1} at $I_p = 1$. The rate of decrease varies with the discharge equation exponent u. Because orifice head is very sensitive to flow, the orifice flow standard deviation increases rapidly with decreasing flow rate, and the infiltration standard deviation actually increases with I_p or as the outflow decreases. However, due to this sensitivity, s_{Q1} is usually lower for orifices than other devices, and they can provide accurate infiltration measurement within their narrow range. For the other three devices, s_D/s_{Q1} differs by as much as 20% at a given infiltration, depending upon the type of measurement device.

Flow measurement device head-reading uncertainty often increases with the reading due to increased flow velocity and surface turbulence (Bos et al. 1984). If the reading standard deviation is proportional to the reading, then the coefficient of variation CV_R is constant. Substituting Eq. 8a into Eq. 10 yields

$$s_D = (u_1^2 Q_1^2 CV_{R1}^2 + u_2^2 Q_2^2 CV_{R2}^2)^{1/2} \quad (16)$$

Using the same assumptions that led to Eq. 15 but with a constant CV_R gives

$$s_D = uQ_1(2 - 2I_p + I_p^2)^{1/2} CV_R = (2 - 2I_p + I_p^2)^{1/2} s_{Q1} \quad (17)$$

This relationship, which is also plotted in Fig. 1, does not depend on the device discharge exponent and nests between those for constant s_R.

Absolute uncertainty in infiltration measurements has limited practical application, since infiltration is expressed on a per unit furrow length or field area basis. The infiltration determination standard deviation can also be expressed relative to the infiltration, $Q_1 - Q_2$:

$$CV_D = \frac{s_D}{Q_1 - Q_2} = \frac{s_D}{I_p Q_1} \quad (18)$$

Substituting Eq. 18 into Eq. 15 and using the definition $CV_Q = s_Q/Q$ gives

$$CV_D = u\left(\frac{a}{Q_1}\right)^{(1/u)}\left(\frac{1}{I_p}\right)[1 + (1 - I_p)^{(2-2/u)}]^{1/2} s_R$$

$$= \left(\frac{1}{I_p}\right)[1 + (1 - I_p)^{(2-2/u)}]^{1/2} CV_{Q1} \quad (19)$$

Likewise when CV_R is constant, substituting Eq. 18 into Eq. 17 yields

$$CV_D = u\left(\frac{2}{I_p^2} - \frac{2}{I_p} + 1\right)^{1/2} CV_R = \left(\frac{2}{I_p^2} - \frac{2}{I_p} + 1\right)^{1/2} CV_Q \quad (20)$$

These equations, written in terms of standard deviations and coefficient of variations, can be converted to uncertainties E by Eq. 1:

$$E_{Dp} = \frac{t \cdot CV_D}{\sqrt{n}} \quad (21)$$

where E_{Dp} = the relative uncertainty, $E_D/(Q_1 - Q_2)$.

Inserting Eq. 19 into Eq. 21, and defining $E_{Q1p} = tCV_{Q1}/\sqrt{n}$, yields

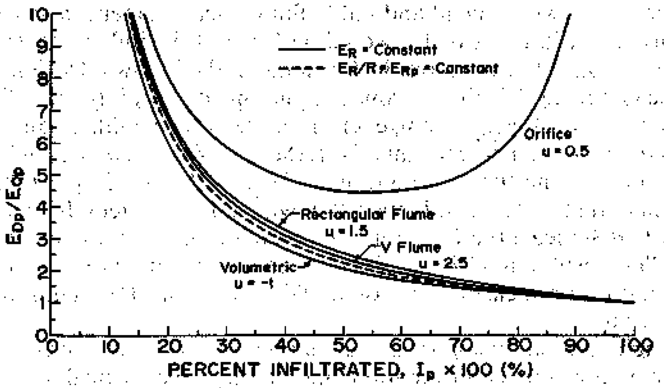

FIG. 2. Relative Infiltration Uncertainty E_{Dp} Divided by Relative Flow Measurement Uncertainty E_{Qp} versus Percent Infiltrated for Constant and Proportional Device Reading Uncertainty

$$E_{Dp} = \left(\frac{1}{I_p}\right) [1 + (1 - I_p)^{(2-2/u)}]^{1/2} E_{Q1p} \quad (22)$$

Similarly, when CV_R is constant, inserting Eq. 20 into Eq. 21 yields

$$E_{Dp} = \left(\frac{2}{I_p^2} - \frac{2}{I_p} + 1\right)^{1/2} E_{Qp} \quad (23)$$

where E_{Qp} = the relative uncertainty of the flow measurement.

Eqs. 22 and 23 for the four types of devices are plotted in Fig. 2. Note how rapidly the uncertainty increases with decreasing infiltration when expressed in relative terms. This factor overshadows differences between devices (except the orifice) and whether reading uncertainties are assumed constant or proportional. Fig. 2 also shows higher relative uncertainty for the constant-sized orifice, due to its narrow range.

ANALYSIS

Section Length

Infiltration measurements are sometimes made on fairly short furrow sections to reduce intake opportunity time differences within the measured section. This is important because infiltration rate changes with time, following the introduction of water. However, not only does the absolute infiltration determination uncertainty E_D increase up to 40% as the amount infiltrated decreases (Fig. 1), but uncertainty relative to the measurement E_{Dp} varies inversely with the portion infiltrated. The relative uncertainty is the most useful parameter because an infiltration measurement is physically useful only when given relative to the furrow length (or soil surface area) and the flow decrease is proportional to this length.

To illustrate the importance of section length, if a furrow is infiltrating 60% of its inflow over its entire length, the average infiltration determination uncertainty with a head and tail V-flume measurement ($u = 2.6$, E_r = constant), according to Eq. 15, would be 115% of the inflow measurement uncertainty E_{Q1p}. If an outflow 80% as large as the inflow (20% infiltration) were measured at 1/3 the furrow length, the uncertainty of infiltration would be 133% of E_{Q1p}. However, relative to the infiltration, the tail measured infiltration uncertainty is $1.15E_{Q1p}/0.6 = 1.9E_{Q1p}$; while that measured at 1/3 the furrow length is $1.33E_{Q1p}/0.2 = 6.6E_{Q1p}$ or 3.5 times larger than that determined over the whole furrow length. Trout and Mackey (1988) measured E_{Qp} values in the midrange of *V* furrow flumes of ±10%. This results in a 95% probability confidence interval for the whole furrow infiltration measurement of ±19% and for the 1/3 length measurement of ±66%.

Although carefully using measurement devices with low sensitivity will decrease infiltration determination uncertainty, the critical factor is to measure as large a flow decrease as possible. If making measurements over small portions of furrows, borders, or conveyance ditches is necessary, use accurate measurement procedures and be aware of the inherent uncertainty.

Systematic Errors

This analysis has assumed that all systematic errors in the flow measurement have been eliminated or taken into account. Often devices are not carefully enough calibrated or constructed to insure this, or they are consistently used improperly. Systematic error is added to random uncertainties to determine the total error.

Although systematic errors are difficult to assess, the affect on inflow-outflow infiltration measurements can be reduced by using the same device for both the inflow and outflow measurement. If the systematic flow measurement error is constant (does not vary with flow) then, by using the same device, the errors will cancel out, and the infiltration determination systematic error will be zero. If the systematic error is proportional to the flow rate and the same devices are used, then the infiltration error is proportional to the calculated infiltration. For example, a 5% systematic flow measurement error will cause a 5% infiltration measurement error. Consequently, as long as the same devices are used, the systematic infiltration measurement error will generally be no larger than the flow measurement systematic error.

Applications

Infiltration determinations are usually made for one of two purposes. The first is to estimate the infiltration value or relationship for a soil, a field, or a treatment. In this process, a second uncertainty is introduced due to the spacial variability inherent in infiltration. If there were no measurement uncertainty, the uncertainty of the average infiltration E_I, based on n infiltration determinations, would be

$$E_I = \frac{ts_I}{\sqrt{n}} \quad (24)$$

where s_I = the standard deviation of the infiltration; and t is as defined in Eq. 1. Because there is also uncertainty in each infiltration measurement used to calculate the average, the two variabilities must be combined:

$$E_I = t'\left(\frac{s_I^2}{n_I} + \frac{s_D^2}{n_D}\right)^{1/2} \quad (25)$$

where n_I = the number of furrows measured; n_D = the number of measurements per furrow; and t' = a weighted-average t-statistic value calculated as (Steel and Torrie 1960)

$$t' = \frac{W_I t_I + W_D t_D}{W_I + W_D} \quad (26a)$$

where $$W_I = \frac{s_I^2}{n_I} \quad (26b)$$

$$W_D = \frac{s_D^2}{n_D} \quad (26c)$$

$$t_I = t_{(n_I - 1)} \quad (26d)$$

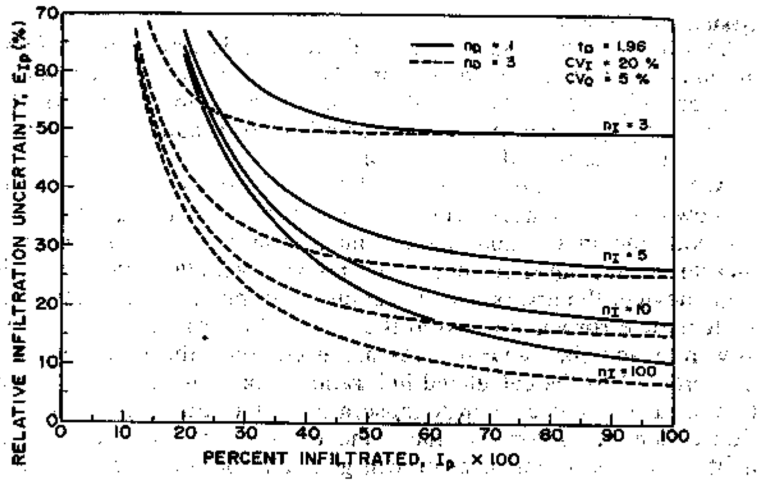

FIG. 3. Uncertainty (95% Level) of Average Infiltration Estimate with Varying Percents Infiltrated, Numbers of Furrows Measured n_I and One and Three Measurements per Furrow n_D Assuming, CV_I = 20% and CV_Q = 5%

$$t_D = t_{(n_D - 1)} \quad (26e)$$

Fig. 3 shows the importance of making multiple measurements to estimate the true infiltration, based on a predetermined CV_Q value of 5% and CV_I of the measured furrows of 20%. With the high furrow-to-furrow coefficient of variation, the number of furrows measured to establish the average is more important to the uncertainty of the mean than the number of measurements per furrow, especially if more than 30% of the inflow is infiltrated. Average furrow-to-furrow CV_I in southern Idaho was measured to be 25% (Trout and Mackey 1988b). Furrow flow measurement device CV_Q was measured between 2–8%, depending on the device and flow rate (Trout and Mackey 1988).

A second purpose for infiltration measurement is to determine the amount of spacial or temporal variability in the infiltration rate. For example, a series of adjacent furrows could be measured to determine the furrow-to-furrow water application variability due to inherent infiltration differences. In this case, the measurement uncertainty will increase the measured variability beyond the true variability, and thus, the measurement effects must be subtracted from the results. Representing variability as the standard deviation of a series of determinations

$$s_{IT} = (s_{IM}^2 - s_D^2)^{1/2} \quad (27)$$

where s_{IT} = the best estimate of the true variability; s_{IM} = the standard deviation of a series of single measurements; and s_D = the predetermined standard deviation of the measurement process.

For example, inflows and outflows from 50 adjacent furrows were measured with V-notch flumes. The average infiltration rate was 30 L/min of an average 45-L/min inflow (I_p = 67%), and the infiltration rate standard

deviation was 6 L/min (CV_I = 20%). The s_R of the flumes (a = 0.000543, u = 2.63) was measured to be 1.5 mm (Trout and Mackey 1988), so the infiltration determination standard deviation is, from Eq. 13

$$s_D = 2.63(0.000543^{1/2.63})[45^{(2 - \frac{2}{2.63})} + 15^{(2 - \frac{2}{2.63})}]^{1/2}1.5 = 2.7 \text{ L/min} \quad (28)$$

So, by Eq. 27:

$$s_{IT} = [(6 \text{ L/min})^2 - (2.7 \text{ L/min})^2]^{1/2} = 5.4 \text{ L/min} \quad (29)$$

or the true coefficient of variation is 18% rather than 20%.

Infiltration is usually described either as a base infiltration rate (or a rate at some arbitrary intake opportunity time) or, through integrating several rate measurements over time, by a cumulative infiltrated volume. If several measurements are made over time to establish an infiltrated volume (or an average rate), the uncertainty of the volume (or average) E_{DV} will be decreased. If each pair of inflow and outflow measurements are independent, the uncertainty will be reduced by a factor of $1/\sqrt{n}$, where n = the number of measurement pairs:

$$E_{DV} = \frac{E_D}{\sqrt{n}} \quad (30)$$

However, as device uncertainty measurements established (Trout and Mackey 1988), unless the measurement devices and reading personnel are actually changed for each reading, the measurements may not include all the random variability, and some systematic error might be introduced. Consequently, the best estimate of infiltrated volume uncertainty will lie somewhere between E_D and E_{DV}, depending upon the error sources that remain constant.

Conclusions

1. Inflow-outflow infiltration determination uncertainty varies with the flow measurement uncertainty and can be large.
2. Inflow-outflow infiltration determination uncertainty increases rapidly as the percent of inflow that is infiltrated decreases. Therefore, accurate infiltration measurement requires measuring long furrow sections in which much of the flow is infiltrated.
3. Using the same type of device to measure inflows and outflows will usually decrease the effects of any systematic errors.
4. Due to high spacial variability, measuring several furrows is critical to determining the average infiltration rate with confidence.
5. Measured infiltration variability can be corrected for measurement process uncertainty.

Appendix I. References

Bos, M. G., ed. (1976). *Discharge measurement structures*. International Institute for Land Reclamation and Improvement, Wageningen, The Netherlands.

Bos, M. G., Replogle, J. A., and Clemmens, A. J. (1984). *Flow measuring flumes for open channel systems*. John Wiley and Sons, New York, N.Y.

International Organization for Standardization (ISO). (1978). "Measurement of

fluid flow: estimation of uncertainty of a flow rate measurement." *International Standard ISO 5168, 1978(E)* (available from ANSI, New York, N.Y.).
Kendall, M. G., and Stuart, A. (1963). *The advanced theory of statistics*, 2nd Ed., Vol. 1, Hafner, New York, N.Y., 232.
Steele, R. G. D., and Torrie, J. J. (1960). *Principles and procedures of statistics*. McGraw-Hill, Inc., New York, N.Y., 81.
Trout, T. J., and Mackey, B. E. (1988). "Furrow flow measurement accuracy." *J. Irrig. and Drain. Engrg.*, ASCE, 114(2), 244–255.
Trout, T. J., and Mackey, B. E. (1988b). "Furrow inflow and infiltration variability." Trans. ASAE (In Press).

Appendix II. Notation

The following symbols are used in this paper:

a = measurement device discharge equation coefficient;
COV = covariance of two measurements;
CV = coefficient of variation = standard deviation/mean;
E = uncertainty;
E_{DP} = relative uncertainty in flow difference (infiltration);
E_{DV} = uncertainty of volumetric difference;
E_{QP} = relative uncertainty in flow measurement;
I_P = relative infiltration;
n = number of measurements;
Q = flow rate;
R = measurement device reading such as flow depth;
s = standard deviation = $\sqrt{V}$;
t = Student's t-statistic;
t' = weighted average t-statistic;
V = variance;
u = measurement device discharge equation exponent; and
W = s^2/n.

Subscripts

D = difference of two measurements;
I = infiltration;
IM = measured infiltration;
IT = true infiltration;
$Q1$ = inflow measurement;
$Q2$ = outflow measurement;
R = reading;
$R1$ = inflow device reading;
$R2$ = outflow device reading;
1 = inflow device or measurement; and
2 = outflow device or measurement.