Influence in a large society: Interplay between information dynamics and network structure

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Influence in a Large Society: Interplay Between Information Dynamics and Network Structure

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***Abstract*— Motivated by the recent emergence of large online social networks, we seek to understand the effects the underlying social network (graph) structure and the information dynamics have on the creation of *influence* of an individual. We examine a natural model for information dynamics under two important temporal scales: a *first impression* setting and a *long– term* or *equilibrated* setting. We obtain a characterization of relevant network structures under these temporal aspects, thereby allowing us to formalize the existence of influential agents. Specifically, we find that the existence of an influential agent corresponds to: (a) strictly positive information theoretic capacity over an infinite-sized noisy broadcast tree network in the first impression case, and (b) positive recurrent property of an appropriate (countable state space) Markov chain in the long-term case. As an application of our results, we evaluate the parameter space of the popular "small world" network model to identify when the network structure supports the existence of influential agents.**

I. Introduction

Social networks have been a key medium for information spreading since ages, and recently they are becoming increasingly important as online social communities of unprecedented size are being formed. This phenomenon has given rise to many important socio-economically relevant questions, such as what determines the likelihood of an individual joining a particular group [3] ?; how and when can one perform efficient search in social networks [19] ?; how and for what information acquisition should incentive-based approaches be used [13], [14] ? and what properties of a social network determine the feasibility of efficient information spreading (e.g., viral advertisement) or of reaching a global consensus?

In this work, we wish to address this last question. Specifically, we wish to identify qualitative structural properties of social networks that determine the existence of influential agent(s). Indeed, this structural characterization has to be strongly coupled with the dynamics of the information propagation. In the broad temporal spectrum of information propagation, there are two natural extreme aspects: the short-term or first impressions and the long-term or equilibrium. We therefore undertake the task of characterizing influence of an individual in terms of a structural property of networks under these two representative types of information dynamics.

Anecdotal evidence of the importance of leaving good first impressions is vast. Consider the following motivating situation: suppose a political candidate speaks in person for the very first time in front of a community. The audience will likely form a strong first opinion of the candidate based on the candidate's ability to persuade them during the speech. In turn, this first impression can quickly spread to other people through the social network. However, this setting will leave little chance for people to discuss the candidate's message at length. In that sense, the social impact of the candidate's political message is akin to broadcasting some information from a source successively through a network. Other natural examples where the influence arising from first impressions abound: advertisement of a product that yields to its quick and wide adoption (known as viral marketing), ad-injection in the peer-to-peer applications, fast rumor (or emergency information) spreading in a social community, etc.

In addition to the short-term or first impressions effect of information spreading, in many other situations the opinion is allowed to settle after individuals of a society communicate and exchange information over a longer time scale. Influence again plays a central role in many scenarios. Representative applications include developing an interest for a particular type of a television show, adoption of a certain lifestyle or a prevalence of a certain (un)healthy habit, and so on.

A. Related work

In order to capture the notion of influence in an effective manner for a variety of applications discussed above, it is important to devise a simple, useful framework to study information dynamics. This topic has been of great recent interest, and here we present only a small part of relevant related literature.

The algorithmic perspective and the approximations to the optimization problem of identifying a subset of influential nodes in a social network was studied in [10]. Subsequent work [18] generalized the results of [10]. Decentralized search in a (social) network was the subject of [11], [12], where, among other observations, a precise connection between the delivery time and the explicit values of the parameters of the underlying model was established. Cascades of information and their patterns where the influence is via a recommendation of one agent to another was the subject of [15]. A related line of work studying influence in a social network focuses on explaining empirical and/or experimental data, with appli-

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cation across multiple and wide-ranging disciplines including medicine [4], sociology [16], and economics [1].

The most relevant related work is by Golub and Jackson [9], who recently studied the process of learning in a social network under the naïve update model: this work models the effect of information propagation in a large network over a long time-scale. In this model, each agent takes a weighted average of her/his neighbors' opinions in forming her/his own. Under this model, Golub and Jackson defined and characterized the notion of "wisdom" in a society.

B. Our contributions

The primary contribution of this paper is a simple, effective model for characterizing the existence of influential agents in terms of the social network structure and information dynamics that itself captures the two extreme but representative temporal aspects.

Here, the short-term or the first-impression setting reflects the following natural information propagation: suppose an individual, call it v, wishes to disseminate a particular opinion on an important issue over the social network. To so do, s/he communicates the opinion to immediate neighbors (friends). They in turn communicate the opinion to their respective friends and so on. While propagating this information, an individual may change the opinion depending upon her/his belief on the topic. When only first impressions count, various individuals do not get a chance to interact further to reach a global consensus. In such a setup, we define the individual v to be influential if her/his opinion survives (information-theoretically) among the far away individuals.

This characterization of the existence of influence of an individual v becomes equivalent to the well studied reconstruction problem under broadcasting over a tree (e.g., [7], [8]). Using the results of [7], we obtain the following : if the neighborhood of the agent v in a large social network is *socially expanding*[1] then the agent v is influential; else if her/his neighborhood is *socially non-expanding* then v is not influential.

The other important temporal aspect to consider is that of the equilibrium or long term setting. This state of the social network can be viewed as the generalization of the first-impressions setting, equilibrated after a sufficiently long period of time. In this setting it is then natural to consider that the agents will have the opportunity to interact with *all* of their immediate neighbors when settling on their opinions. Here, we establish that the existence of influential individuals in such a setup is equivalent to the existence of positive recurrent states of an appropriate Markov chain. This characterization allows to interpret influence of individuals as well as a "degree of democracy" in the society.

C. Organization

The rest of the paper is organized as follows. In Section II, we formally introduce the above discussed model. Next, in Section III we state the main results. Detailed proofs are deferred to the longer version of this paper [6]. We present conclusions and future work in Section IV.

[1]The precise definitions of a socially (non)-expanding network are provided later.

II. MODEL

We begin by describing a natural model for dynamics of information spreading. Let $G_n = (V_n, E_n)$ be the graph given by the vertex set V_n, with $|V_n| = n$, and by the edge set E_n that together describe this social network: the elements of V_n correspond to the individuals (or agents), and the elements of E_n describe the connections in the social network between the respective individuals. Let $N_n(v)$ denote the neighborhood of vertex (agent) v in G_n. We are interested in the large-scale behavior of this social network, i.e., as $n \to \infty$. We assume throughout a simplistic model where the opinion is *binary* or $\{+1, -1\}$ valued or YES/NO. Next, we describe information dynamics of the opinions of individuals over a very short time scale (first impressions) or a very long time scale (equilibrium).

A. Information dynamics: First impression/short-term

Here, our interest is in understanding whether an agent or individual is influential on a short-term time scale. To this end, consider an individual $v \in V_n$ having an opinion ± 1. We want to understand influence of this agent, given the social network structure G_n.

On the short time scale, the opinion is propagated through the social network as follows. Initially, v communicates the opinion to her/his neighbors as–is. These neighbors communicate it further to their neighbors, who have not heard the opinion before from any of the other nodes, and so on. Here the restriction of propagating the opinion to neighbors who have not heard it before tries to capture the 'short time scale' or 'first impressions' effect – agents do not have enough time to discuss over the opinion, and hence agree or disagree only with the opinion they hear for the first time. When an agent hears an opinion from one of her/his neighbors, s/he will form her/his opinion to be the same as the heard opinion with probability $1 - p_a$, and will form the complement of the heard opinion with probability p_a. The first impressions restriction will mean that each agent forms an opinion once and retains it forever.

This information dynamics essentially leads to a propagation of opinion over a 'sub-tree' of G_n with node v as its root. Specifically, let $T_n(v, p_a)$ denote this subtree of G_n, and with v as its root. For simplicity, we will assume that $T_n(v, p_a)$ is generated by the breadth first search of G_n starting from v. Clearly, it has n total number of nodes. We will restrict our attention to social networks G_n with each node having a bounded degree – therefore, $T_n(v, p_a)$ has a bounded degree. The use of notation p_a in this representation is to capture the 'noisy' propagation of the original opinion of v along the tree. Initially, v has value ± 1. Each node, starting with v, sends its information (± 1) to its children successively. A node, upon receiving the information from its parent, retains it as-is with probability $1 - p_a$, and takes the opposite value with probability p_a. In summary, each agent pays attention to the first instance of the received opinion (a.k.a. the agent listens only once), which s/he hears from her/his parent. Note that we assume that the probability of alteration p_a is the same on every edge, and that the opinion propagation is independent across siblings and across levels.

The following auxiliary objects will be useful. Let $T_v^{n,k}$ be the depth k sub-tree of $T_n(v,p_a)$ from root v. By construction, $T_v^{n,k-1}$ is strictly contained in $T_v^{n,k}$. Let $L_v^{n,k} = T_v^{n,k} \backslash T_v^{n,k-1}$, i.e., $L_v^{n,k}$ is the set of leaf nodes at level k for $T_v^{n,k}$.

A.1. Influence of an individual

Now, we define when is an individual influential. Intuitively, if v is influential then nodes or individuals that are far away from v should still share the opinion originally held by v, or equivalently, they should be able to collectively reconstruct what was agent's v original opinion based on their received belief under information dynamics described above. Formally, consider the node v, the root of the tree $T_n(v,p_a)$. Let $R_v^{n,k}$ be the set of nodes in V_n that are at a distance at least $k+1$ away from v (i.e., not contained in the tree $T_v^{n,k}$). Let $X_{R_v^{n,k}}$ denote the (random) collection of the received opinions under the dynamics described above at nodes in $R_v^{n,k}$. A predictor of X_v, the opinion of v, based on $X_{R_v^{n,k}}$, is some function F that generates prediction of X_v (i.e., with the value in $\{-1,+1\}$) from $X_{R_v^{n,k}}$. The reconstruction probability, Q_F, is defined as

$$Q_F = \frac{1}{2} \sum_{i \in \{-1,+1\}} \mathbb{P}(F(X_{R_v^{n,k}}) = i | X_v = i);$$

where we assume equal prior uncertainty across all values of X_v. Define Q_k as the maximum of Q_F over the set of all possible predictors based on $X_{R_v^{n,k}}$. Clearly, $Q_k \geq 1/2$ since this lower bound is achieved through a trivial prediction function that predicts any one of the two values uniformly at random without using $X_{R_v^{n,k}}$. This suggests that if there is any information captured in $X_{R_v^{n,k}}$ about X_v, then Q_k should be strictly greater than $1/2$. Therefore, the following definition about v being influential follows naturally: v is influential, if there exists $k_n \to \infty$ as the social network size $n \to \infty$, such that

$$\liminf_{n \to \infty} Q_{k_n} \geq \frac{1+\delta}{2},$$

for some $\delta > 0$, where then δ indicates the improvement in the reconstruction probability relative to the uninformed case. If no such positive δ exists, the agent v is not influential.

B. Information dynamics: equilibrium/long-term

Recall that the graph $G_n = (V_n, E_n)$ describes the social network of n members. To determine the influence of an agent $v \in V_n$ in the equilibrium state, we consider a natural extension of the first-impressions setting by now assuming that the agents have enough time to listen to *all* of their neighbors' (friends') opinions in forming their own. In contrast to the first-impressions setting where the opinion of an agent is based on the first instance of the heard opinion and is subsequently set once and for all, here it is natural to consider that the opinion value can change over time. Let $x_v(t)$ denote the opinion of the agent v at time $t, t \geq 0$, for $1 \leq v \leq n$, and where the opinion $x_v(t) \in \{-1,+1\}$. The opinion update at time $t+1$ is then

$$x_v(t+1) = \begin{cases} x_w(t) \text{ with probability } p_n(v,w) \text{ for } w \neq v \\ x_v(t) \text{ with probability } p_n(v,v) \text{ for } w = v, \end{cases}$$

where $p_n(v,w)$ is the probability that the agent v will adopt the current opinion of neighbor w, whereby $\sum_{w \in N_n(v) \cup v} p_n(v,w) = 1$.

The expected evolution of opinions $\underline{x}_n(t) = (x_1(t), \ldots, x_n(t))$ is then

$$\mathbb{E}[x_v(t+1) | \underline{x}_n(t)] = \sum_{w \in N_n(v) \cup v} x_w(t) p_n(v,w).$$

Hence,

$$\mathbb{E}[\underline{x}_n(t+1)^T | \underline{x}_n(t)^T] = P_n \underline{x}_n(t)^T,$$

where P_n is the stochastic $n \times n$ matrix with $p_n(w,v)$ as the (w,v) entry, and where $(\cdot)^T$ denotes the transpose of the row vector. The matrix P_n is referred to as the influence matrix. Then

$$\mathbb{E}[\underline{x}_n(t+1)^T] = P_n \mathbb{E}[\underline{x}_n(t)^T].$$

By successively iterating,

$$\mathbb{E}[\underline{x}_n(t)^T] \quad = \quad P_n^t \mathbb{E}[\underline{x}_n(0)^T]. \tag{1}$$

We consider a connected social network so that each agent could eventually influence every other agent. As a result, in the underlying Markov chain representation $M(V_n, P_n)$ we assume that the transition matrix P_n is irreducible and aperiodic.

B.1. Influence of an individual

Under this set-up, we are interested in the persistence of the original opinion value after sufficiently many interactions (steps) have taken place. Intuitively, agent v is influential if after a long time period other agents in the network continue to pay attention to v's opinion.

By the standard theory of Markov chains, the underlying aperiodic and irreducible Markov chain $M(V_n, P_n)$ has a unique stationary distribution $\underline{\pi}_n = (\pi_n(1), \ldots, \pi_n(n))$, such that

$$P_n \underline{\pi}_n^T = \underline{\pi}_n^T,$$

where $\underline{\pi}_n^T$ corresponds to the transpose of row vector $\underline{\pi}_n$. Now due to uniqueness of stationary distribution of P_n, it follows that

$$\lim_{t \to \infty} P_n^t \quad = \quad P_n^\infty, \tag{2}$$

where P_n^∞ has all rows equal to $\underline{\pi}_n$.

Let $\underline{x}_n(0)$ be the initial set of values held by the members of the social network. From (1) and (2), it follows that

$$\begin{aligned} \lim_{t \to \infty} \mathbb{E}[\underline{x}_n(t)^T] \quad &= \quad \left(\underline{\pi}_n \underline{x}_n(0)^T\right) \mathbf{1}, \\ &\triangleq \quad y_n \mathbf{1}, \end{aligned} \tag{3}$$

where $\mathbf{1}$ is the vector of all 1s. Thus, as time $t \to \infty$, the average value of each agent goes to the same value y_n.

Since we seek to capture the long-term influence of the agent v in a large society (i.e., as the social network size $n \to \infty$), suppose that $x_v(0) = 1$ and $x_w(0) = -1$ for all $w \in V_n \setminus v$. We define y^* as $y^* = \liminf_{n\to\infty} y_n$. Then, the agent v is considered influential if

$$y^* > -1 \ .$$

For the assumed $\underline{x}_n(0)$, we have that $y_n = \pi_v - \sum_{i\in V_n\setminus v} \pi_i = 2\pi_v - 1$ from (3). Therefore, in the limit $n \to \infty$ of a large society, if π_v is strictly positive, the probability that any agent w holds the opinion value $+1$ is also strictly positive. That is, even if there is severe opposition to this opinion value, it is strictly possible that any given agent will adopt v's original opinion – thus capturing what influence is all about!

III. Main Results

In this section we state the main results. We first consider the short-term (or first-impression) setting, and we describe the characterization of the relevant network graph structures under which influential individuals exists under this temporal constraint. We apply these results to the case of a small world network model, and express the lack of/presence of influential agents in terms of the value of the model parameter. Then, we state the results regarding the characterization of the network graph structure that imply the existence of influential individuals under the long-term (or equilibrium) setting, itself viewed as the time-averaged version of the first-impressions setting. We contextualize this characterization as a "democratic property" of a society. Detailed proofs of these results are omitted due to space constraints and they will appear in the longer version of this work [6].

A. Information dynamics: first-impression/short-term

A.1. Some preliminaries

We start off with some useful definitions and notation.

Definition 1 (Socially expanding network): We say that G_n is socially expanding with respect to v if, for a fixed v, $\exists$ constants $c, c \in (1/2, 1)$ and $d, d > 0$, such that $\forall k, k \leq k^*(n, c)$,

$$\mathbb{E}\left[|N(S) \cap L_v^{n,k+1}| \;\middle|\; T_v^{n,k}\right] \geq (1 + d)|S|, \qquad (4)$$

where $S \subset L_v^{n,k}$, and $N(S)$ denotes the neighborhood set of S. Here , $k^*(n, c)$ is the largest k satisfying $T_v^{n,k} \leq n^c$.

In the above definition, the expectation in (4) is with respect to the probability distribution over the choice of graph – in case of a random graph model, this is over the choice of randomly generated graph; in case of a deterministic graph, it becomes a deterministic condition.

Note that the above definition is a somewhat more constrained condition than the traditional expansion property [5]. It nonetheless captures the essence of what it means to expand: the innovation in new connections by freshly introduced nodes should be non-negligible, until at least a large enough subgraph, which is itself captured by the constant c. Another reason to introduce c is that under the breadth-first search construction of a tree based on the graph G_n, and when considering $n \to \infty$, connections to previously unseen nodes are eventually exhausted and the graph cannot be expanding forever. However, to maintain reconstruction and hence influence it is in fact sufficient to have a large enough subgraph be socially expanding. Based on this theoretical foundation, we provide a concrete example in the next section where we prove that the well-studied and general model of small-world networks [11] contains influential agents for non-trivial ranges of the probability of alteration p_a. Another useful definition for the broadcasting tree model is the following.

Definition 2 (socially non-expanding network): We say that G_n is socially non-expanding (has a polynomial growth) with respect to the node v if the total number of nodes up to the level k from the root v, i.e., the number of nodes in $T_v^{n,k}$, scales as $O(k^a)$ for some positive constant $a \geq 1$.

A.2. Result

We now state the result. The proofs are a somewhat direct adaption of techniques from [7]. Again, the details are in [6]. In essence, we establish a dichotomy of the (non-) existence of influential agents: if the graph is socially expanding then influential agents exists, if graph is socially non-expanding then influential agents do not exist.

Theorem 1: Consider a social network $G_n = (V_n, E_n)$ and an individual $v \in V_n$. Let v start with an opinion in $\{+1, -1\}$. Consider the short-term information dynamics of opinion of v as described above with alteration probability $p_a > 0$. Then, in the limit $n \to \infty$, the following holds:

(a) *If the graph G_n is socially non-expanding with respect to the node v, then v is not influential.*
(b) *If the graph G_n is socially expanding with respect to the node v, then v is influential as long as $p_a \leq \gamma$, for some $\gamma > 0$.*

A.3. An application: small world model

For many social networks, the popular small world network model is considered appropriate. To this end, recall the small-world network model over $n = M^2$ nodes in 2 dimensions [2] with parameter $\alpha > 0$ [11]: (a) nodes are organized in an $M \times M$ 2-dimensional lattice with each node having 4 immediate neighbors (with the exception of the nodes on the boundary); (b) each node, say v, has an additional so-called long range contact, say u, chosen randomly and independently with probability proportional to $\frac{1}{d(v,u)^\alpha}$, where $d(v, u)$ is the ℓ_1 (or manhattan) distance between the nodes v and u.

Consider a social network G_n with $n \to \infty$ exhibiting the small-world property. For α small (i.e., close to 0), the random contact is roughly uniform and hence most nodes indeed have a long-range contact (and the network has a small diameter). For α large (i.e., going to ∞), nodes have almost exclusively their random contacts within a short range.

Theorem 2: If $\alpha = 0$, then the underlying graph G_n is socially expanding. Hence, as $n \to \infty$ the social network G_n contains influential agents. If, however $\alpha \to \infty$ (or

[2] We consider a 2-dimensional lattice, the extension to higher dimensions follows immediately.

$\Omega(\log n)$), then the underlying graph G_n is socially non-expanding. Hence, as $n \to \infty$ the social network contains no influential agents.

The proof of theorem is in [6]. These observations have the following socio-economic implications for large on-line communities with a small-world network property [2]: even as the on-line communities grow very large, the influence of a singled-out member does not vanish with respect to members that are far away, provided certain underlying structural properties are satisfied. These observations could play a critical role in designing marketing strategies that target commanding members of social communities.

B. Information dynamics: long-term/equilibrium

B.1. Some preliminaries

Before stating the main result for the long-term setting, we recall the following useful result from the standard Markov chain theory. For a positive recurrent Markov chain defined on a countably infinite state space, the stationary distribution π is unique, and is allocated to a finite subset Y of the state space V, which we call stationary support. Foster-Lyapunov criterion [17] can be used to establish positive recurrence by using a so-called negative drift from $V \backslash Y$ into Y.

B.2. Result

Here we state the result, where we show that the existence of influential agents is equivalent to the positive-recurrent property of the underlying Markov chain. The proof is in [6].

Theorem 3: Consider a social network $G_n = (V_n, E_n)$ with the influence matrix P_n. Consider the long-term information dynamics of opinions as described above via the irreducible and aperiodic Markov chain $M(V_n, P_n)$. Let $G = (V, E)$ be limiting graph of sequence G_n and P be the limiting influence matrix of ${P_n}^3$. Here V is a countable state space. Then, the following holds.

(a) *If the Markov chain $M(V, P)$ is positive recurrent, then there exists an agent v that is influential.*
(b) *If the Markov chain $M(V, P)$ is null-recurrent or transient, then no agent is influential.*

B.3. Implications

An interpretation of the above results is that an agent is influential if and only if the majority of a large society believes her/him. Moreover, since the absence of individual influence is equivalent to wisdom as defined in [9], a consequence of the above is that for the näive update model proposed in [9], positive recurrence implies lack of wisdom.

One social implication is regarding a degree of democracy. We say that the society is "reasonably democratic", in the sense that the concordance of the agents implies that eventually only reasonable and unimposing opinions can survive. Null recurrence of the underlying Markov chain can be interpreted as follows: everyone in the society is influential (infinite number of influential agents) but it takes infinitely long for individual opinions to be widely accepted. This situation then describes an "overly democratic" society, in which too many, possibly conflicting, opinions matter for them to propagate in finite time. Under the transience where no one has any influence on opinion of others, the implication is that nobody cares what others believe, and nobody is cared about. This setting then results in an "anarchic" society in which each agent forms her/his own opinion as s/he pleases without caring about what others think.

[3] We assume that the notion of convergence is well defined.

IV. CONCLUDING REMARKS

In this work we addressed the question of what kind of effect the underlying graphical structure and the information propagation have on the existence of influential agents in large social networks. In order to answer this question, we considered archetypical (short and long) time-scales of propagated opinion. While the focus of this paper was on the notion of influence, future work involves establishing the connection between other important social features – such as privacy and anonymity – and an appropriate graphical model that describes a large social network.

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REFERENCES

[1] L. A. Adamic, O. Buyukkokten, and E. Adar. A social network caught in the web. *First Monday*, 8(6), 2003.
[2] Z. Anwar, W. Yurcik, V. Pandey, A. Shankar, I. Gupta, and R. H. Campbell. Leveraging 'social-network' infrastructure to improve peer-to-peer overlay performance: results from orkut. Available online at http://arxiv.org/abs/cs/0509095.
[3] L. Backstrom, D. Huttenlocher, J. Kleinberg, and X. Lan. Group formation in large social networks: membership, growth, and evolution. In *SIGKDD*, 2006.
[4] N. A. Christakis and J. H. Fowler. The spread of obesity in a large social network over 32 years. *New England Journal of Medicine*, 357(4):370–379, 2007.
[5] F. R. K. Chung. *Spectral Graph Theory*. American Mathematical Society, 1997.
[6] L. Dolecek and D. Shah. Characterizing influence in large-scale networks. Available at http://web.mit/edu/dolecek/www/influence.pdf.
[7] W. Evans, C. Kenyon, Y. Peres, and L. J. Schulman. Broadcasting on trees and the ising model. *Annals of Applied Probability*, 10(2):410–433, May 2000.
[8] A. Gerschenfeld and A. Montanari. Reconstruction for models on random graphs. In *STOC*, 2007.
[9] B. Golub and M. Jackson. Naive learning in social networks: convergence, influence and the wisdom of crowds. Working paper, 2007.
[10] D. Kempe, J. Kleinberg, and E. Tardos. Maximizing the spread of influence through a social network. In *SIGKDD*, 2003.
[11] J. Kleinberg. Small-world phenomena and the dynamics of information. In *NIPS*, 2001.
[12] J. Kleinberg. Complex networks and decentralized search algorithms. In *ICM*, 2006.
[13] J. Kleinberg. Social networks, incentives and search. In *SIGIR*, 2006.
[14] J. Kleinberg and P.Raghavan. Query incentive networks. In *FOCS*, 2005.
[15] J. Leskovec, A. Singh, and J. Kleinberg. Patterns of influence in a recommendation network. In *PAKDD*, 2006.
[16] S. D. McClurg, D. P. A, and J. Mendez. The social agenda: The influence of social networks and context on personal issue agendas. Ann. Mtng of Amer. Pol. Science Assoc. 2004.
[17] S. Meyn and R. Tweedie. *Markov Chains and Stochastic Stability*. Springer-Verlag, 1993.
[18] E. Mossel and S. Roch. On the submodularity of influence in social networks. In *STOC*, 2007.
[19] B. Yu and M. P. Singh. Searching social networks. In *AAMAS*, 2003.