J. Inst. Eng. India Ser. A (June 2017) 98(1-2):201–210
DOI 10.1007/s40030-017-0188-0

ARTICLE OF PROFESSIONAL INTERESTS

Influence of Distributed Dead Loads on Vehicle Position for Maximum Moment in Simply Supported Bridges

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Received: 23 August 2016 / Accepted: 20 April 2017 / Published online: 8 May 2017

Abstract Usually, the design moments in the simply supported bridges are obtained as the sum of moments due to dead loads and live load where the live load moments are calculated using the rolling load concept neglecting the effect of dead loads. For the simply supported bridges, uniformly distributed dead load produces maximum moment at mid-span while the absolute maximum bending moment due to multi-axel vehicles occur under a wheel which usually do not lie at mid-span. Since, the location of absolute maximum bending moment due to multi-axel vehicle do not coincide with the location of maximum moment due to dead loads occurring at mid-span, the design moment may not be obtained by simply superimposing the effect of dead load and live load. Moreover, in case of Class-A and Class-70R wheeled vehicular live loads, which consists of several axels, the number of axels to be considered over the bridge of given span and their location is tedious to find out and needs several trials. The aim of the present study is to find the number of wheels for Class-A and Class-70R wheeled vehicles and their precise location to produce absolute maximum moment in the bridge considering the effect of dead loads and impact factor. Finally, in order to enable the designers, the design moments due to Class-70R wheeled and Class-A loading have been presented in tabular form for the spans from 10 to 50 m.

Keywords IRC loading · Critical load · Maximum bending moment · CSiBridge · Beam-line model

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Introduction

Design of bridges for flexure requires determination of maximum bending moment in the deck. For the aseismic design of bridges for gravity loads, traditionally, dead load and live load in conjunction with impact load are considered for the design of the bridge superstructure [1]. For the elastic design of bridge, usually the design moment is obtained by superimposing the maximum moments due to dead load and live load effects. In the concrete bridges, a major portion of loads is due to dead loads which include self-weight of bridge superstructure, railing load, footpath, wearing course, water/electricity/telephone lines etc. and it is considered uniformly distributed throughout the span [2]. For the design of highway bridges, IRC code [3] specifies two types of live loads namely Class-A and Class-70R/Class-AA. For the two-lane Indian highway bridges, IRC code recommends that the bridges should be designed Class-70R/Class-AA tracked and wheeled vehicles placed on one lane and the bridge need to be checked for Class-A loading on both the lanes. Absolute maximum bending moment due to vehicular live load is usually calculated using the rolling concept [4]. For the IRC Class-70R tracked vehicle, whose wheel contact length is large, but shorter than medium span bridges, the load is assumed uniformly distributed over the wheel contact area. For the uniformly distributed load maximum moment occurs at mid-span, therefore, IRC Class-70R tracked vehicle is placed symmetrically at mid-span to produce maximum moment. Since, the maximum bending moment due to Class-70R tracked vehicle as well as due to self-weight occur at mid span, design moment is simply obtained by superimposing the effect of dead load and live load. However, in case of wheeled vehicular loads consisting of several axels, absolute maximum bending moment in the simply supported bridge occurs under one of the wheel which usually not occurs at mid-span and, therefore, the moments due to dead loads (at mid-span) and due to live loads (under a wheel) cannot be simply added to find design moment. Moreover, in case of Class70-R/Class-A wheeled load, which consists of several axels, the number of axels to be considered over the bridge of given span and their location is tedious to find out and needs several trials to determine the exact number of wheels that should be considered. Shipman examined the effect of lane load on the position of AASHTO HL-93 loading to develop maximum bending moment in the simply supported bridges and developed a simple expression to calculate the position of critical load resulting in maximum moment; however, he did not incorporate the effect of dead load in locating the critical axle [5]. The aim of the present study is to find the number of wheels and their location to produce absolute maximum moment in the bridge considering the effect of dead load and dynamic impact factor used by IRC [3] for live loads. Finally, in order to enable the designers, the design moments due to Class-70R wheeled and Class-A loading, critical load position from rolling load concept and present approach and shift distance of critical load have been presented in tabular form for simply supported spans ranging from 10 to 50 m.

Identification of Wheel Loads Producing Maximum Moment

For larger spans, generally all the vehicular axels together produce maximum moment in the bridge, however, for the small span bridges, especially for spans less than the distance between extreme axels, all the axels may not lay on the bridge, and even for spans longer than the distance between the extreme axels, sometimes do not develop maximum moment. For instance, in case of IRC Class-70R wheeled vehicle, the distance between first axle and last axle is (excluding bumper and front clearances) measures 13.4 m, but even for 17.5 m long bridge span, maximum moment is developed when last six wheels are over the span and first wheel is on approach slab. In general, the axel(s) to be considered beyond the bridge either may be on front side or on rear side and it depends on span of bridge and the magnitude of load on front and rear axles. In order to facilitate the designers for manual design, a study has been made to identity the loads to be considered over the span producing maximum bending moment in the simply supported bridges of span ranging from 10 to 50 m. The loads to be considered over the span which develop maximum moment in simply supported bridge are referred as 'active axels'. The CSiBridge beam-line modelling method has been used to determine active axels in IRC Class-70R wheeled and Class-A loading, these vehicles were gradually moved over the span at the interval of 0.1 m in longitudinal direction for all spans ranging from 10 to 50 m. Tables 1 and 2 show active axles for IRC Class-70R wheeled and Class-A loading respectively over the length for bridge span ranging from 10 to 50 m.

IRC Class-70R Wheeled Vehicle Loading

The types of bending moment by considering various load applied to the bridge have been calculated and shown in Tables 1 and 2. Figure 1 shows the load arrangements based on vehicle movement at a particular direction.

IRC Class-A Wheeled Vehicle Loading

Figure 2 shows the load arrangement based on vehicle movement for Class-A wheeled vehicles.

Determination of Design Moment using Conventional Approach

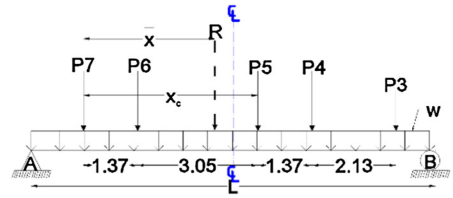
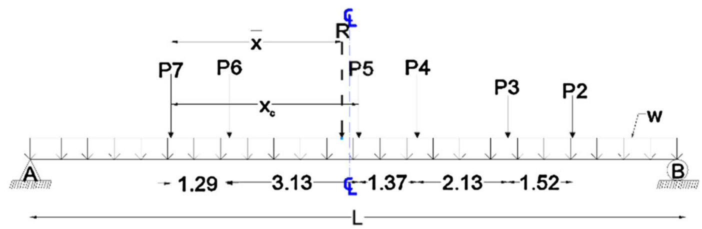
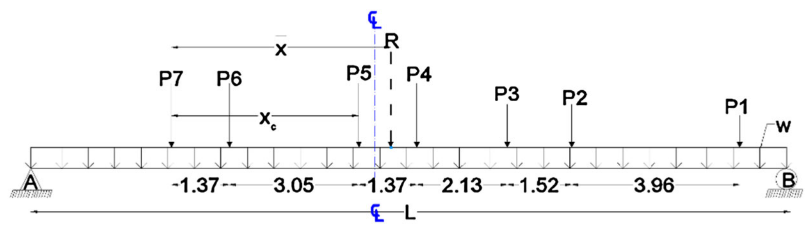
In the conventional approach, design moments are obtained by superposing the moments due to dead loads at mid-span and maximum moment due to vehicular loads using the rolling load concept.

For the IRC Class-70R wheeled and Class-A loading, the wheel contact area is considered small and, therefore, wheel loads are considered as concentrated point loads acting at the center of gravity of actual wheels as shown in Figs. 1 and 2. Since the live loads plying over the bridge deck may occupy any location on the lane, for the safe design of bridge there is a need to determine maximum bending moment due to vehicular live loads. The longitudinal position of vehicle resulting maximum longitudinal bending moment in the bridge may be determined as follows.

Let the loading system consisting loads $P_1 \ldots P_i \ldots P_n$ cross the bridge from left to right. Considering the reference point for the live load at the last load of the loading system over the bridge i.e. P_n, and assuming that the ith load and the resultant of the loading system R are located at distance of x_i and $\bar{x}$ respectively from the reference point. From the rolling load concept, maximum bending moment in a simply supported beam should occur under one of the concentrated load, referred as critical load here onwards. Let the load P_i be the critical load (P_c) under which maximum moment will occur and it is located at distance $x_c (=x_i)$ from reference point and at distance 's_0' from the mid-span of bridge as shown in Fig. 3.

From the Fig. 3 the distance of resultant R from mid-span 'z' may be determined as $(x_i - \bar{x} - s_0)$ and the reaction at support B, R_B may be calculated by considering the equilibrium of moment at support A as,

Table 1 IRC Class-70R wheeled vehicle axle arrangement for span 10–50 m

<table>
<tr><th>Span range (L), m</th><th>Case no.</th><th>Wheel loads to be considered over span for Max. BM (active axels)</th><th>Loading to be considered for IRC Class 70-R wheeled vehicle</th></tr>
<tr><td>10–10.4</td><td>70R-1</td><td>P3, P4, P5*, P6, P7
(R = 800 kN)</td><td>
</td></tr>
<tr><td>10.5–17.5</td><td>70R-2</td><td>P2, P3, P4, P5*, P6, P7
(R = 920 kN)</td><td>
</td></tr>
<tr><td>17.6–50</td><td>70R-3</td><td>P1, P2, P3, P4, P5*, P6, P7
(R = 1000 kN)</td><td>
</td></tr>
</table>

*Indicates the critical wheel under which maximum moment occurs

$$\sum M_A = 0 = R \times \left(\frac{L}{2} - z\right) - R_B \times L = 0;$$

Substituting the value of z as $z = (x_i - \bar{x} - s_0)$ in above expression,

$$R_B = R\left(\frac{1}{2} - \frac{z}{L}\right) = R\left(\frac{1}{2} - \frac{[(x_c - \bar{x}) - s_0]}{L}\right)$$

$$R_B = R\left(\frac{1}{2} - \frac{[(x_c - \bar{x}) - s_0]}{L}\right)$$

Bending moment at the location of critical load (P_c) may be calculated as,

$$M_i = R_B \times \left(\frac{L}{2} - s_0\right) - P_1 \times (x_1 - x_i) - P_2 \times (x_2 - x_i) \cdots - P_{i-1} \times (x_{i-1} - x_i)$$

Substituting value of R_A

$$M_i = \left[R\left(\frac{1}{2} - \frac{[(x_c - \bar{x}) - s_0]}{L}\right)\right] \times \left(\frac{L}{2} - s\right) - P_1 \times (x_1 - x_i) - P_2 \times (x_2 - x_i) \cdots - P_{i-1} \times (x_{i-1} - x_i)$$

$$M_i = \frac{RL}{4} - \frac{R[(x_c - \bar{x}) - s_0]}{2} - \frac{Rs_0}{2} + \frac{Rs_0[(x_c - \bar{x}) - s_0]}{L} - P_1 \times (x_1 - x_i) - P_2 \times (x_2 - x_i) \cdots - P_{i-1} \times (x_{i-1} - x_i)$$

For maximum moment under critical load, $(dM_i/ds_0) = 0$

$$\frac{dM_i}{ds_0} = 0 - R\frac{[0-1]}{2} + 0 - \frac{R}{2} + \frac{R[(x_c - \bar{x}) - 2s_0]}{L} - 0 \cdots - 0$$

or, $\frac{dM_i}{ds_0} = [R(x_c - \bar{x})]/L - 2Rs_0/L$

$$\frac{d^2M_i}{ds^2} = 0 - 2R/L = -2R/L$$

Table 2 IRC Class-A wheeled vehicle axle arrangement for span 10–50 m

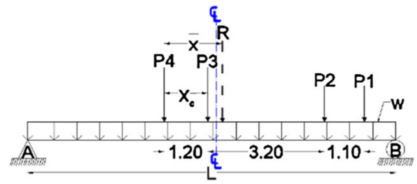
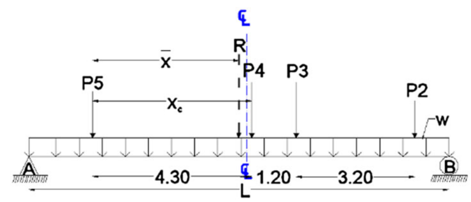
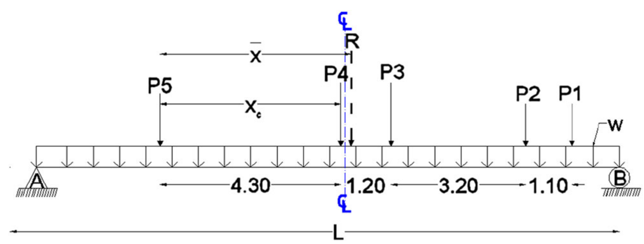
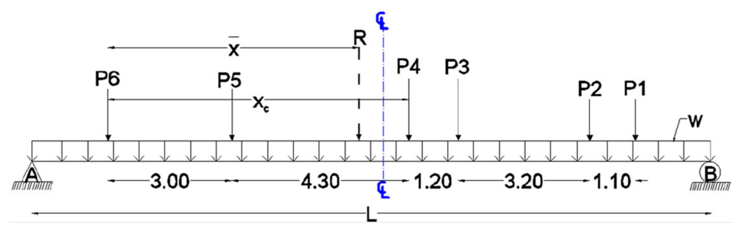
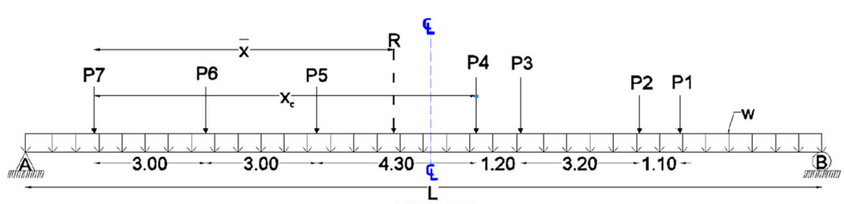
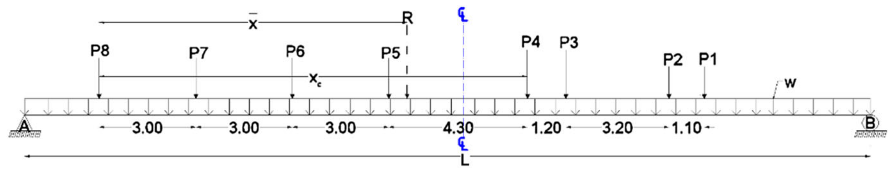
Span range (L), m	Case no.	Wheel loads to be considered over span for Max. BM (active axels)	Wheel load arrangement for maximum moment
10–10.2	*A*-1	*P*1, *P*2, *P*3*, *P*4 ($R = 282$ kN)	
10.3–10.9	*A*-2	*P*2, *P*3, *P*4*, *P*5 ($R = 323$ kN)	
11–14.2	*A*-3	*P*1, *P*2, *P*3, *P*4*, *P*5 ($R = 349$ kN)	
14.3–18.9	*A*-4	*P*1, *P*2, *P*3, *P*4*, *P*5, *P*6 ($R = 418$ kN)	
19–23.6	*A*-5	*P*1, *P*2, *P*3, *P*4*,*P*5, *P*6, *P*7 ($R = 486$ kN)	

Table 2 continued

Span range (L), m	Case no.	Wheel loads to be considered over span for Max. BM (active axels)	Wheel load arrangement for maximum moment
23.7–50	*A*-6	*P*1, *P*2, *P*3, *P*4*, *P*5, *P*6, *P*7, *P*8 ($R = 554$ kN)	

*Indicates the critical wheel under which maximum moment occurs

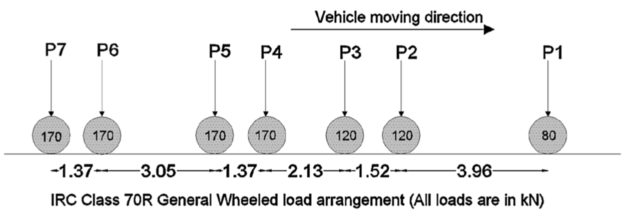


Fig. 1 IRC Class-70R wheeled vehicle: *line* diagram

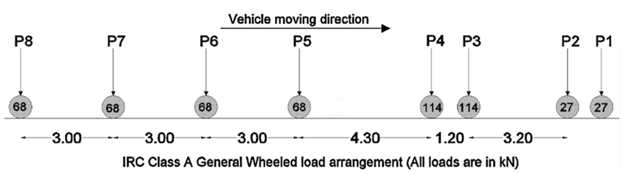


Fig. 2 IRC Class-A wheeled vehicle: *line* diagram

Since R and L are always positive, thus d^2M/ds^2 will be always negative, hence maximum moment may be obtained as,

$$\frac{dM_i}{ds_0} = [R(x_c - \bar{x})]/L - 2Rs_0/L = 0$$

or, $s_0 = \frac{(x_c - \bar{x})}{2} = z.$

The above expression verifies the rolling load concept, which states that 'in simply supported beams, maximum bending moment occurs under one of the concentrated load (critical load), when the loads are placed such that the critical load and the resultant of the loading system are equidistant from the beam's centerline'. For a given loading system, critical load may be identified mathematically. In order to determine the critical load, first calculate the position of resultant of loads to be considered on a given span (as shown in Tables 1, 2) and calculate the total loads acting on both sides of this resultant. The wheel load nearer to resultant located towards the higher load intensity side with respect to resultant will be the critical load. For instance, for a bridge of 20 m span, the sum of the loads on left and right side of the resultant for Class-70R loading (case 70R-3) is 510 and 490 kN respectively. Since the total load on left side of resultant is more than the sum of the loads on right side, the load nearer to resultant towards left side i.e. P_5 will be the critical load.

Maximum moment at the location of critical load may be determined as,

$$M_{LL} = [R_B \times (L/2 - s_0) - P_1 \times (x_1 - x_i) \\ - P_2(x_2 - x_i) \cdots P_{i-1}(x_{i-1} - x_i)]$$

If the span of the bridge is L and the dead load on the bridge due to bridge self-weight, railing load, footpath, wearing course, water/electricity/telephone lines is 'w' then maximum bending moment produced at mid-span may be calculated as:

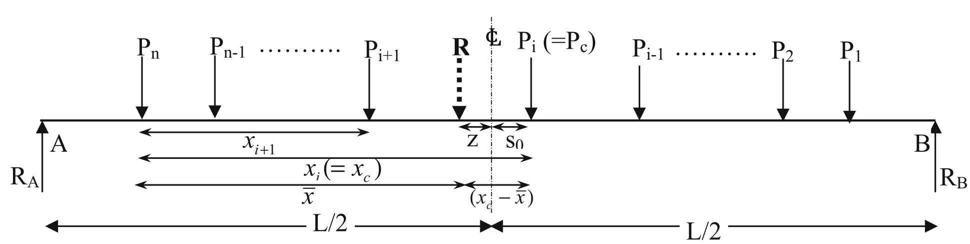
$$M_{DL} = \frac{wL^2}{8}$$

And the total moment may be calculated as,

$$M_{DL+LL} = M_{DL} + M_{LL}$$

$$M_{DL+LL} = wL^2/8 + [R_B \times (L/2 - s_0) - P_1 \times (x_1 - x_i) \\ - P_2(x_2 - x_i) \cdots P_{i-1}(x_{i-1} - x_i)]$$

Fig. 3 Placement of a general wheeled vehicle over simply supported bridge to determine maximum moment using conventional approach



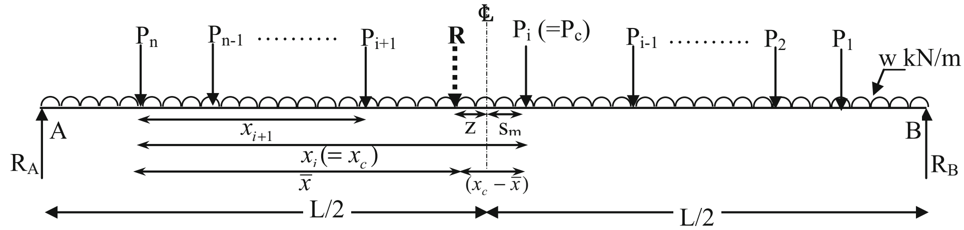


Fig. 4 Placement of a general wheeled vehicle over simply supported bridge to determine maximum moment using proposed approach

Determination of Design Moment using Proposed Approach

The bridge with loading system shown in Fig. 4 (with dead load of w kN m^{-1}) is considered in proposed approach. If the resultant of the loading system is located at distance 'z' from the mid-span, the reaction at support B, R_B may be calculated by considering the equilibrium of moment at support A as, $\sum M_A = 0 = R \times \left(\frac{L}{2} - z\right) + wL \times \left(\frac{L}{2}\right) - R_B \times L = 0;$

Let for this combined loading system, the critical load P_c is located at distance 's_m' from the mid-span of bridge. The distance of resultant R from mid-span 'z' may be determined as $(x_i - \bar{x} - s_m)$.

Substituting the value of z in above expression,

$$R_B = R\left(\frac{1}{2} - \frac{z}{L}\right) + \frac{wL}{2} = R\left(\frac{1}{2} - \frac{[(x_c - \bar{x}) - s_m]}{L}\right) + \frac{wL}{2}$$

$$R_B = R\left(\frac{1}{2} - \frac{[(x_c - \bar{x}) - s_m]}{L}\right) + \frac{wL}{2}$$

Bending moment at the location of critical load (P_c) may be calculated as,

$$M_i = R_B \times \left(\frac{L}{2} - s_m\right) - \frac{w}{2} \times \left(\frac{L}{2} - s_m\right)^2 - P_1 \times (x_1 - x_i) - P_2 \times (x_2 - x_i) \cdots - P_{i-1} \times (x_{i-1} - x_i)$$

Substituting value of R_A

$$M_i = \left[R\left(\frac{1}{2} - \frac{[(x_c - \bar{x}) - s_m]}{L}\right) + \frac{wL}{2}\right] \times \left(\frac{L}{2} - s_m\right) - \frac{w}{2} \times \left(\frac{L}{2} - s_m\right)^2 - P_1 \times (x_1 - x_i) - P_2 \times (x_2 - x_i) \cdots - P_{i-1} \times (x_{i-1} - x_i)$$

$$M_i = \frac{RL}{4} - R\frac{[(x_c - \bar{x}) - s_m]}{2} + \frac{wL^2}{4} - \frac{Rs_m}{2} + \frac{Rs_m[(x_c - \bar{x}) - s_m]}{L} - \frac{wLs_m}{2} - \frac{w}{2} \times \left(\frac{L^2}{4} + s_m^2 - Ls_m\right) - P_1 \times (x_1 - x_i) - P_2 \times (x_2 - x_i) \cdots - P_{i-1} \times (x_{i-1} - x_i)$$

For maximum moment under critical load, $(dM_i/ds) = 0$

$$\frac{dM_i}{ds_m} = 0 - R\frac{[0-1]}{2} + 0 - \frac{R}{2} + \frac{R[(x_c - \bar{x}) - 2s_m]}{L} - \frac{wL}{2} - \frac{w}{2} \times (0 + 2s_m - L) - 0 \cdots - 0 = 0$$

or, $\frac{dM_i}{ds_m} = \frac{[R(x_c - \bar{x})]}{L} - \frac{2Rs_m}{L} - ws_m$

$$\frac{d^2M_i}{ds_m^2} = 0 - 2R/L - w = -(2R/L + w)$$

Since R, L and w are always positive, thus d^2M/ds_m^2 will be always negative, hence maximum moment is obtained,

$$\frac{dM_i}{ds_m} = \frac{[R(x_c - \bar{x})]}{L} - \frac{2Rs_m}{L} - ws_m = 0$$

Or, $s_m = \frac{[R(x_c - \bar{x})]/L}{2R/L + w} = \frac{R(x_c - \bar{x})}{2R + wL} = \frac{(x_c - \bar{x})}{2} \times \left[\frac{R}{R + wL/2}\right].$

The above expression for s may be re-written after including the impact factor 'I' as,

$$s_m = \frac{(x_c - \bar{x})}{2} \times \left[\frac{(1+I)R}{(1+I)R + wL/2}\right] = \frac{(x_c - \bar{x})}{2} \times m$$

$$where,\ m = \left[\frac{(1+I)R}{(1+I)R + wL/2}\right]$$

Here, 's_m' is the distance of the critical load P_c from mid-span to obtain maximum moment and 'm' is the modification factor to incorporate the dead load effect and impact factor in determining the position of critical load producing absolute maximum longitudinal bending moment in the deck.

Thus it may be observed that the position of critical load significantly depends on magnitude of total dead loads and live loads including dynamic impact effect in addition to distance between critical load and resultant of loads. The shift in the position of critical load with respect to conventionally calculated critical load position, denoted as 'δ', may be calculated as

$$\delta = \frac{(x_i - \bar{x})}{2} - \frac{(x_i - \bar{x})}{2} \times m = \frac{(x_i - \bar{x})}{2}[1 - m]$$

Substituting the value of 'm' in above expression

$$\delta = \frac{(x_i - \bar{x})}{2}\left[1 - \frac{(1+I)R}{(1+I)R + wL/2}\right] = \frac{(x_i - \bar{x})}{2}\left[\frac{wL/2}{(1+I)R + wL/2}\right]$$

As a special case, if self-weight of bridge (wL) is small compared to sum of moving loads (R) on the bridge span i.e.$wL << R$, δ tends to zero, consequently, the exact position of critical load almost coincides with position of critical load calculated using the conventional approach.

So via proposed approach, moment due to vehicular load may be calculated as

$$M_{LL} = [R_B \times (L/2 - s_m) - P_1 \times (x_1 - x_i) - P_2(x_2 - x_i) \cdots P_{i-1}(x_{i-1} - x_i)]$$

Moments due to dead loads at the location of maximum moment i.e. at critical load location may be calculated as

$$M_{DL} = (wL/2)(L/2 - s_m) - \left[w(L/2 - s_m)^2\right]/2$$

Thus the total moment becomes

$$M_{DL+LL} = M_{DL} + M_{LL}$$

$$M_{DL+LL} = (wL/2)(L/2 - s_m) - \left[w(L/2 - s_m)^2\right]/2 + (wL/2)(L/2 - s_m) - \left[w(L/2 - s_m)^2\right]/2$$

Critical Load Positions for IRC Class-70R and Class-A Loadings

Since, the number of wheels to be placed on bridge is governed by bridge span, depending on number of wheels to be considered over the bridge, the bridge spans between 10 and 50 m have been divided into six span ranges (A-1 to A-6) for Class-A loading and three span ranges (70R-1 to 70R-3) for Class 70R loading. Figure 5a, b show the location of critical load (with respect to center line of bridge) for producing maximum bending moment in the bridge due to Class-A and Class-70R loadings respectively for all the nine span ranges using the conventional and modified approaches. From the Fig. 5a, b, it may be observed that for Class-A as well as for Class-70R loading, the critical load positions calculated using the modified approach are closer to mid-span compared to those calculated using the conventional approach for all the span ranges. It may also be observed from these figures that for a span range if the span of bridge increases, critical load position calculated using the modified approach moves toward the mid-span, however, it remains constant in the conventional approach.

Design Moments for IRC Class-70R and Class-A Loading

In India, design of highway bridges is based on the live loads specified by IRC (IRC6:2014) [3] which involves placement of IRC Class-A and Class-70R (tracked and wheeled) loading to produce maximum moment in the span. The IRC Class-70R tracked vehicle comprises of two wheels and they may be placed easily to produce maximum moment in longitudinal direction, however, Class-A and Class-70R wheeled vehicles consist of several wheels and,

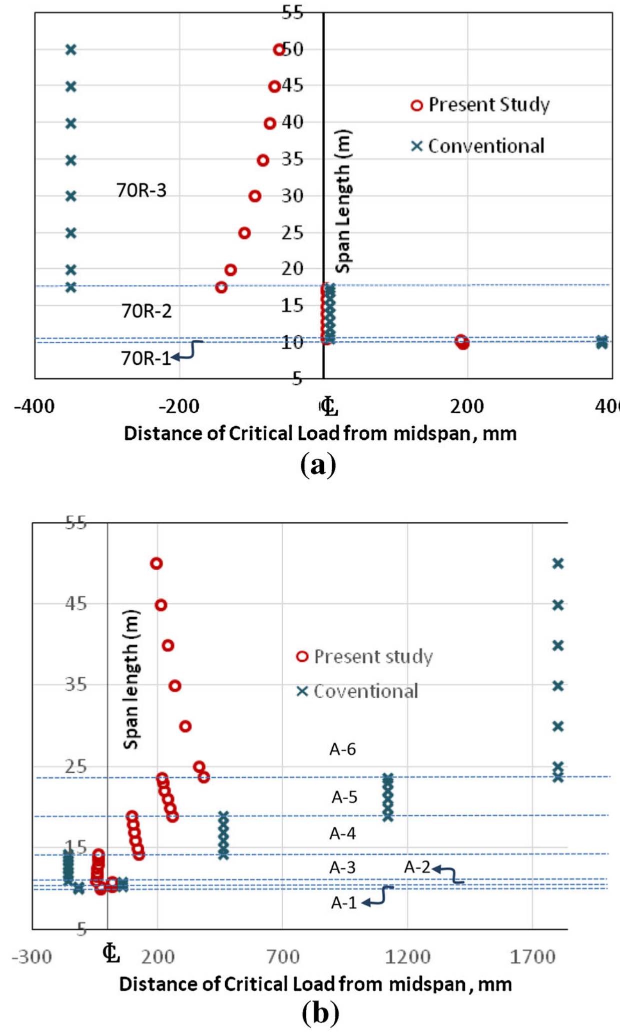


Fig. 5 Comparison of critical load position by conventional approach and proposed approach. **a** For IRC Class 70-R wheeled vehicle. **b** For IRC Class-A wheeled vehicle

therefore, it becomes cumbersome to place them, to develop maximum moment. In order to facilitate designers, the position of Class-A and Class-70R wheeled vehicles producing maximum moment has been determined by carrying out beam-line analysis using the CsiBridge. The distance of the critical load from mid-span based on conventional approach (s_0) and using the proposed methodology (s_m) and their difference have been presented in Tables 3 and 4 for Class-70R and Class-A wheeled vehicle respectively.

Maximum Bending Moment (BM), caused by IRC Class-A and Class-70R wheeled vehicles on two lane simply supported concrete bridges have been computed using proposed approach, where the critical load position has been determined including the effect of dead loads & impact factor and also via conventional approach. For homogeneity, the dead loads on the bridges (including its

Table 3 Critical load position and bending moments for Class-70R wheeled loading

Span, m	Case no.	Distance of critical load from mid-span using conventional method (s_0)	Modification factor, m	Distance of critical load from mid-span using proposed method (s_m)	Shift in position of critical load ($s_0 - s_m$)	Moment at critical load location including DL effect due to kNm		
						Live load (M_{LL})	Dead load (M_{DL})	Total load (M_{DL+LL})
10.0	70R-1	0.386	0.500	0.193	0.307	1309	2496	3806
10.1			0.498	0.192	0.306	1334	2547	3881
10.2			0.495	0.191	0.304	1359	2597	3956
10.3			0.493	0.190	0.303	1384	2649	4033
10.4			0.490	0.189	0.301	1409	2700	4109
10.5	70R-2	0.008	0.523	0.004	0.519	1441	2756	4197
11.0			0.511	0.004	0.507	1584	3025	4609
12.0			0.489	0.004	0.485	1872	3600	5472
13.0			0.467	0.004	0.463	2137	4225	6362
14.0			0.446	0.004	0.442	2398	4900	7298
15.0			0.427	0.003	0.424	2656	5625	8281
16.0			0.409	0.003	0.406	2912	6400	9312
17.0			0.393	0.003	0.39	3165	7225	10,390
17.5			0.405	−0.143	0.548	3291	7656	10,948
17.6	70R-3	−0.352	0.404	−0.142	0.546	3313	7742	11,055
20.0			0.370	−0.130	0.500	3967	9998	13,966
25.0			0.314	−0.111	0.425	5303	15,624	20,927
30.0			0.273	−0.096	0.369	6615	22,499	29,114
35.0			0.241	−0.085	0.326	7912	30,624	38,537
40.0			0.215	−0.076	0.291	9199	39,999	49,199
45.0			0.195	−0.069	0.264	10,479	50,625	61,103
50.0			0.178	−0.063	0.241	11,753	62,500	74,253

self-weight) is considered uniform for all the spans and it is assumed as 200 kN m^{-1} and impact factor for live loads is taken as per IRC6 [3]. The maximum bending moments for concrete bridges of spans ranging between 10 and 50 m due to assumed dead loads of 200 kN m^{-1} and in conjunction with IRC Class-70R and Class-A wheeled vehicle have been presented in Tables 3 and 4 respectively.

Numerical Example

For a 25 m span simply supported two-lane bridge, determine the design bending moment due to IRC Class-A loading. Take superimposed dead load including its self weight as 200 kN m^{-1}.

It may be observed from Table 1 that for the span 25 m the critical load case will be A-6, consequently, all the eight axles are to be placed on the bridge for finding out maximum longitudinal bending moment in the bridge. Thus the resultant of the loading system (R) will be R = 27×2 + 114×2 + 68×4 = 554 kN. If the location of load P_8 is considered as reference point, then the distance of resultant R i.e. CG of loading system from the reference point may be determined as,

$$\bar{x} = \frac{\sum P_i x_i}{\sum P_1} = \frac{27 \times (18.8 + 17.7) + 114 \times (14.5 + 13.3) + 68 \times (9 + 6 + 3))}{27 \times 2 + 114 \times 2 + 68 \times 4} = 9.7089\,\text{m}$$

The critical load may be identified by comparing the sum of the loads on both side of resultant.

Sum of Loads towards the left of C.G of loads = $P_5 + P_6 + P_7 + P_8 = 68 + 68 + 68 + 68 = 272\,\text{kN}$

Sum of Loads towards the right of C.G of loads = $P_4 + P_3 + P_2 + P_1 = 114 + 114 + 27 + 27 = 282\,\text{kN}$

Since, the sum of the loads towards right of C.G of the loading system is greater than sum of loads towards the left of C.G of loads, the first load towards right i.e. P_4 will be the critical load.

$$\rightarrow x_c = x_7 + x_6 + x_5 + x_4 = 3 + 3 + 3 + 4.3 = 13.3\,\text{m}$$

Table 4 Critical load position and bending moments for Class-A loading

Span, m	Case no.	Distance of critical load from mid-span using conventional method (s_0)	Modification factor, m	Distance of critical load from mid-span using proposed method (s_m)	Shift in position of critical load ($s_0 - s_m$)	Moment at critical load location including DL effect due to kNm		
						Live load (M_{LL})	Dead load (M_{DL})	Total load (M_{DL+LL})
10.0	A-1	−0.117	0.265	−0.031	−0.086	1372	2500	3872
10.1			0.263	−0.031	−0.086	1388	2550	3939
10.2			0.261	−0.031	−0.086	1405	2601	4005
10.3	A-2	0.057	0.286	0.016	0.041	1424	2652	4076
10.5			0.281	0.016	0.041	1461	2756	4217
10.7			0.277	0.016	0.041	1498	2862	4360
10.9			0.273	0.016	0.041	1535	2970	4506
11.0	A-3	−0.159	0.287	−0.046	−0.113	1555	3025	4579
11.5			0.277	−0.044	−0.115	1655	3306	4961
12.0			0.267	−0.042	−0.117	1755	3600	5355
12.5			0.258	−0.041	−0.118	1854	3906	5761
13.0			0.250	−0.040	−0.119	1953	4225	6178
13.5			0.242	−0.038	−0.120	2061	4556	6617
14.0			0.234	−0.037	−0.121	2169	4900	7069
14.2			0.232	−0.037	−0.122	2213	5041	7254
14.3	A-4	0.460	0.263	0.121	0.338	2228	5111	7339
15.0			0.253	0.116	0.342	2409	5624	8032
16.0			0.239	0.110	0.350	2597	6399	8996
17.0			0.227	0.105	0.355	2827	7224	10,051
18.0			0.216	0.099	0.361	3055	8099	11,154
18.9			0.207	0.095	0.365	3260	8929	12,189
19.0	A-5	1.116	0.232	0.259	0.857	3245	9018	12,263
20.0			0.222	0.248	0.868	3508	9994	13,502
21.0			0.213	0.237	0.879	3770	11,019	14,790
22.0			0.204	0.228	0.888	4031	12,095	16,126
23.0			0.196	0.219	0.897	4291	13,220	17,511
23.6			0.192	0.214	0.902	4446	13,919	18,365
23.7	A-6	1.796	0.212	0.381	1.415	4404	14,028	18,432
25.0			0.202	0.364	1.432	4786	15,612	20,398
30.0			0.172	0.309	1.487	6243	22,490	28,734
35.0			0.149	0.268	1.528	7686	30,618	38,303
40.0			0.132	0.237	1.559	9117	39,994	49,111
45.0			0.118	0.212	1.584	10,537	50,621	61,158
50.0			0.108	0.193	1.603	12,041	62,496	74,537

As per the IRC6:2014, impact factor (I) for 25 m Concrete bridge subjected to Class-A loading is, $I = 4.5/[6 + 25] = 0.1451$.

Conventional Approach

Neglecting the effect of dead load, the distance of the critical load (i.e. P_4) from the mid-span 's_0' may be calculated as

$$s_0 = \frac{(x_c - \bar{x})}{2} = \frac{(13.3 - 9.709)}{2} = 1.796\,\text{m}$$

Now, the distance of the critical load P_4 from right support will be $12.5 - 1.796 = 10.704$ m

Taking summation of bending moments at left support A to find R_B

$$\sum M_{R_A} = R_B \times 25 - R \times (25 - 10.704 - (9.091 - 5.5)) = 0$$
$$R_B = [554 \times 10.704]/25 = 237.211\,\text{kN}$$

Now the moment due to vehicular load may be calculated as

$$M_{LL} = [2 \times (1 + I)] \times [R_B \times 10.704 - P_3 \times 1.2 - P_2 \times 4.4 - P_1 \times 5.5]$$

$$M_{LL} = [2 \times (1 + I)] \times [237.211 \times 10.704 - 114 \times 1.2 - 27 \times 4.4 - 27 \times 5.5]$$

$$M_{LL} = [2 \times (1 + I)] \times 2135.125 = 4270.25 \times 1.145 = 4889.43\,\mathrm{kN\,m}$$

Moments due to Gravity and super imposed load at mid span: $M_{DL} = WL^2/8 = [200 \times 25^2]/2.2 = 15625$ kN m

$$M_{LL} = 4889.43 + 15625 = 20514.43\,\mathrm{kN\,m}$$

Proposed Approach

The modification factor 'm' may be calculated as,

$$m = \left[\frac{R(1+I)}{R(1+I) + wL/2}\right] = \frac{554 \times (1 + 0.1451)}{554 \times (1 + 0.1451) + 200 \times 25/2} = 0.202$$

Hence, the distance of the critical load from the mid-span of the bridge including the effect of uniformly distributed dead load may be evaluated as,

$$s_m = s_0 \times m = 1.796 \times 0.202 = 0.3628\,\mathrm{m}$$

Since, s_m is positive, the critical load P_4 should be placed towards right with respect to mid-span,

Taking summation of bending moments at left support A to find R_B

$$\sum M_{R_A} = R_B \times L - R \times (L/2 + s_m - (x_c - \bar{x})) = 0$$

$$R_B = 554 \times [25/2 + 0.3628 - (13.30 - 9.709)]/25 = 205.463\,\mathrm{kN}$$

The bending moment at critical load position due to dead load and live load may be calculated as,

$$M_{LL} = [2 \times (1 + I)] \times [R_B \times (L/2 - s_m) - P_3 \times 1.2 - P_2 \times 4.4 - P_1 \times 5.5]$$

$$M_{LL} = [2 \times (1 + 0.1451)] \times [205.463 \times 12.137 - 114 \times 1.2 - 27 \times 4.4 - 27 \times 5.5] = 4785.61\,\mathrm{kN\,m}$$

Moments due to Dead loads at the location of critical load may be calculated as, $M_{DL} = (wL/2)(L/2 - s_m) - \left[w(L/2 - s_m)^2\right]/2 = (200 \times 25 /2)(25/2 - 0.3628) - \left[200 \times (25/2 - 0.3628)^2\right]/2 = 15611.83\,\mathrm{kM\,m}$

Hence, total bending moment due to dead loads and live load may be calculated as,

$$M_{LL+DL} = 4785.61 + 15611.83 = 20397.44\,\mathrm{kN\,m}$$

Thus it may be observed that keeping the critical load at conventionally calculated distance of 1.796 m from mid-span it overestimates the moments by 115 kN compared to proposed modified methodology, where the critical load is to be kept at 0.363 m from the mid-span. Also upon checking the case where wheel load was placed directly above the centerline of the bridge to incorporate maximum dead load effect, it was found that this case gives bending moments lower than those calculated by placing the load at critical location i.e. at 0.363 m from mid-span.

Conclusions

Present study helps in eradicating the need of rigorous trial and error to determine the number of wheels to be placed on a particular span bridge to produce maximum bending moment, which can save valuable engineering decision making time. Moreover, to facilitate the designers a simple and accurate method for determining the critical position of axels and maximum bending moment in the simply supported bridges due to IRC Class-A and Class-70R wheeled vehicles, has been developed and the results have been provided for bridges having span between 10 to 50 m for the ready reference to the designers. Based on detailed study, it has been observed that as the span length increases, critical load approaches to mid-span. Via a typical numerical example for 25 m span bridge subjected to Class-A loading, it has been demonstrated that the conventional approach is quite conservative and overestimates the maximum values of moments that are occurring in the bridge. The proposed approach is recommended to identify the critical load position and moment calculations which is exact in nature. Proposed approach is quite easily programmable for quicker use with any design software.

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