

INFLUENCE OF OHMIC HEATING ON ADVECTION-DOMINATED ACCRETION FLOWS

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ABSTRACT

Advection-dominated, high-temperature, quasi-spherical accretion flow onto a compact object, recently considered by a number of authors, assumes that the dissipation of turbulent energy of the flow heats the ions and that the dissipated energy is advected inward. It is suggested that the efficiency of conversion of accretion energy to radiation can be very much smaller than unity. However, it is likely that the flows have an equipartition magnetic field with the result that dissipation of magnetic energy at a rate comparable to that for the turbulence must occur by ohmic heating. We argue that this heating occurs as a result of plasma instabilities and that the relevant instabilities are current driven in response to the strong electric fields parallel to the magnetic field. We argue further that these instabilities heat predominantly the electrons. We conclude that the efficiency of conversion of accretion energy to radiation can be much smaller than unity only for the unlikely condition that the ohmic heating of the electrons is negligible.

Subject headings: accretion, accretion disks — galaxies: active — magnetic fields — plasmas — stars: magnetic fields — X-rays: stars

1. INTRODUCTION

Advection-dominated accretion flows have been intensely studied during the past several years (for example, Narayan & Yi 1994, 1995; Abramowicz et al. 1995; Nakamura et al. 1996; Chakrabarti 1996). The basic dynamical equations for accretion disks including the advection of entropy were first discussed by Paczyński & Bisnovaty-Kogan (1981) and Muchotrzeb & Paczyński (1982). In contrast with the widely applied theory of thin accretion disks of Shakura (1973) and Shakura & Sunyaev (1973), in which the disk material cools efficiently by local radiation of viscously generated energy, the advection-dominated accretion flows of Narayan and Yi *assume* that the viscous dissipation heats the ions, a constant fraction f of this dissipated energy is advected inward, and the fraction $1 - f$ is locally radiated. The further assumption that the energy exchange between ions and electrons is by Coulomb scattering leads to conditions with the ion temperature T_i much larger than the electron temperature T_e , so that the cooling is inefficient. (Esin et al. 1996 treat advection-dominated accretion flows assuming $T_i = T_e$.) The *radiative efficiency*, the power output in radiation divided by $\dot{M}c^2$ (with \dot{M} being the mass accretion rate), is found to be very small compared with unity. The advection-dominated accretion flows tend to be quasi-spherical and optically thin (except for cyclotron radiation, as discussed below), with radial inflow speed $v_r \approx -\alpha v_K$, azimuthal speed $v_\phi \approx \text{const } v_K \ll v_K$, and ion thermal speed $c_{ei} \approx \text{const } v_K \sim v_K$ (Narayan & Yi 1995), where $v_K \equiv (GM/r)^{1/2}$ is the Kepler speed and α is the dimensionless viscosity parameter of Shakura (1973), usually assumed to be in the range 10^{-3} –1.

In §2 we discuss magnetized accretion flows and the importance of ohmic dissipation in addition to the earlier considered viscous dissipation. We argue that the ohmic heating is due to plasma instabilities that heat the electrons. In

§3 we treat a model for the radial variation of electron and ion temperatures assuming that a fraction g of the dissipated energy goes into heating the electrons and a fraction $(1 - g)$ goes into heating the ions. The electrons cool by bremsstrahlung and cyclotron radiation and exchange energy with ions by Coulomb collisions. In §4 we discuss conclusions of this work.

2. ACCRETION FLOWS WITH B FIELD

In quasi-spherical accretion onto a compact object of mass M of Schwarzschild radius $r_s \equiv 2GM/c^2$ (for a black hole), the accreting matter is likely to be permeated by a magnetic field $\mathbf{B}(\mathbf{r}, t)$. Typically, the accreting matter is ionized and consequently highly conducting, with the result that the magnetic field is frozen into the flow. One result of this is that $|B_r| \propto r^{-2}$. Thus, the magnetic energy density varies as $\mathcal{E}_{\text{mag}} = B^2/8\pi \propto r^{-4}$. On the other hand, the kinetic energy density varies as $\mathcal{E}_{\text{kin}} = \rho v^2/2 \propto r^{-5/2}$. Thus, one can expect that equipartition between magnetic and kinetic energy densities occurs in the flow at a large distance $r = r_{\text{equi}} \gg r_s$ (Shvartsman 1971) and that it is maintained for smaller r . Further accretion for $r < r_{\text{equi}}$ is possible only if magnetic flux is destroyed by reconnection and the magnetic energy \mathcal{E}_{mag} is dissipated. The dissipation of magnetic energy was first taken into account by Bisnovaty-Kogan & Ruzmaikin (1974), who showed that accretion for conditions of equipartition ($\mathcal{E}_{\text{mag}} \sim \mathcal{E}_{\text{kin}}$) is accompanied by the dissipation of magnetic energy into heat with entropy s (per unit mass) production rate $\rho T(ds/dr) = -3B^2/(16\pi r)$. We point out that the ohmic dissipation of the magnetic energy is an important, possibly dominant, heating process in advection-dominated accretion flows with $\mathcal{E}_{\text{mag}} \sim \mathcal{E}_{\text{kin}}$. In this regard, note that although Narayan & Yi (1995) assume an equipartition magnetic field, they do not consider the ohmic heating.

The basic equations for accretion flows with $\mathcal{E}_{\text{mag}} \sim \mathcal{E}_{\text{kin}}$ are

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) \mathbf{v} = -\frac{1}{\rho} \nabla p + \mathbf{g} + \frac{1}{\rho c} \mathbf{J} \times \mathbf{B} + \nu_m \nabla^2 \mathbf{v}, \quad (1a)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta_m \nabla^2 \mathbf{B}, \quad (1b)$$

where $\mathbf{v}(\mathbf{r}, t)$ is the flow velocity, $p(\mathbf{r}, t)$ the pressure, $\mathbf{g} = -\nabla(GM/|\mathbf{r}|)$ the gravitational acceleration, ν_m the *microscopic* kinematic viscosity coefficient, and η_m the *microscopic* magnetic diffusivity.

It is well known that the microscopic classical transport coefficients ν_m and η_m are much too small to influence directly the macroscopic flow \mathbf{v} and magnetic field \mathbf{B} evolution. For example, for conditions pertinent to a flow onto a massive black hole, $n \sim 10^{12} \text{ cm}^{-3}$, $T_i \sim 10^{12} \text{ K}$, and $B \sim 10^4 \text{ G}$ for $r \sim r_s$, the Reynolds number for the flow $\text{Re}_v = r|\mathbf{v}|/\nu_m \sim 10^{24}$, where $\nu_m \sim r_{gi}^2/\tau_{ii}$ is the viscosity appropriate for a tangled magnetic field (Braginskii 1965; Paczyński 1978), and where $r_{gi} \sim 10^2 \text{ cm}$ is the ion gyroradius, $\tau_{ii} \sim 10^6 \text{ s}$ is the ion-ion Coulomb scattering time, and $\omega_{ci}\tau_{ii} \gg 1$ with $\omega_{ci} \sim 10^8/\text{s}$ is the ion cyclotron frequency. (Under some conditions, it is possible that ν_m is larger than r_{gi}^2/τ_{ii} , as discussed by Subramanian, Becker, & Kafatos 1996.) The magnetic Reynolds number $\text{Re}_B = r|\mathbf{v}|/\eta_m \sim 10^{27}$, where $\eta_m = c^2/(4\pi\sigma_s)$, with σ_s being the Spitzer conductivity.

It was proposed by Shakura (1973) that accretion flows are in general turbulent and that roughly equations (1a) and (1b) should be taken with turbulent transport coefficients ν_t and η_t replacing the microscopic coefficients, and with $\mathbf{v} \rightarrow \bar{\mathbf{v}}$ and $\mathbf{B} \rightarrow \bar{\mathbf{B}}$ interpreted as *mean fields*. The turbulent viscosity has a crucial role in thin Keplerian disks, where it provides a mechanism for the outward transport of angular momentum. According to Shakura (1973), $\nu_t = \alpha_{ci} H$, where $\alpha = \text{const}$ is the above-mentioned dimensionless viscosity parameter, c_{si} is the ion sound speed, and H is the half-thickness of the disk, which is the *outer scale* of the turbulence. Note that for an advection-dominated accretion flow, $H \sim r$. The shear stress in a magnetized accretion flow, which causes outflow of the angular momentum, appears in large part to be due to magnetic stress (Eardley & Lightman 1975; Brandenburg et al. 1995; Hawley, Gammie, & Balbus 1995). Bisnovaty-Kogan & Ruzmaikin (1976) argued that $\eta_t \sim \nu_t$. The turbulent diffusivity will have a crucial role in dissipating the magnetic energy in advection-dominated flows. In addition to ν_t and η_t , there will be a turbulent transport coefficient α_h (with units of cm s^{-1}) associated with the helicity of the turbulence in a rotating accretion flow (see, for example, Ruzmaikin, Shukurov, & Sokoloff 1988).

Neglecting for the moment the possible difference between T_e and T_i and the radiative energy losses, energy conservation for the accretion flow can be expressed in terms of the mean fields as

$$\rho T \frac{ds}{dt} = \frac{1}{2} \rho \nu_t \left(\bar{v}_{i,j} + \bar{v}_{j,i} - \frac{2}{3} \delta_{ij} \bar{v}_{k,k} \right)^2 + \frac{1}{4\pi} \eta_t (\nabla \times \bar{\mathbf{B}})^2, \quad (2)$$

where s is the entropy per unit mass. The first term on the right-hand side of equation (2) represents the viscous dissipation or heating of the plasma, and the second term the ohmic dissipation. The two terms are of comparable magnitude for an accretion flow with $\mathcal{E}_{\text{mag}} \sim \mathcal{E}_{\text{kin}}$ and $\nu_t \sim \eta_t$.

However, equation (2) says nothing about the *actual* microscopic dissipation of energy in the plasma. Rather, it expresses the loss of energy from the outer scale ($\sim r$ or H if $H \ll r$) of the flow \mathbf{v} and from the \mathbf{B} field by the nonlinear processes implicit in equations (1a) and (1b) and the presumed Kolmogorov cascade of this energy to smaller scale eddies and field structures of the flow. The turbulence may be characterized by wavenumber-frequency ensemble-averaged spectra $\langle v_{k\omega}^2 \rangle$ and $\langle B_{k\omega}^2 \rangle$, where the wavenumber ranges from the small value corresponding to the mentioned outer scale $k_{\text{min}} \sim r^{-1}$ to some much larger value $k_{\text{max}} \gg k_{\text{min}}$. The conventional Kolmogorov description has a dissipation scale corresponding to $k_{\text{max}} \sim (\text{Re})^{3/4} k_{\text{min}}$, which corresponds to an unphysically small length scale using either Re_v or Re_B . Thus, the actual dissipation must be due to plasma instabilities.

The relevant plasma instabilities are probably current driven in response to the large mean electric field, $\bar{\mathbf{E}} = -\bar{\mathbf{v}} \times \bar{\mathbf{B}}/c - \alpha_h \bar{\mathbf{B}}/c + \eta_t \nabla \times \bar{\mathbf{B}}/c$, which in general has a significant component parallel to $\bar{\mathbf{B}}$. It is unclear to us why current-driven instabilities resulting from E_{\parallel} were not considered by Begelman & Chiueh (1988). The typical electric field $|E_{\parallel}| \sim 10^6 \text{ V cm}^{-1}$ (for $r \sim r_s$) is much larger than the Dreicer electric field for electron runaway (Parail & Pogutse 1965), $E_D = 4\pi e^3 (n_e/kT_e) \ln \Lambda \sim 10^{-4} \text{ V cm}^{-1}$ for $T_e \sim 10^9 \text{ K}$, where n_e is the electron density. Thus, the electrons will run-away. An electron becomes relativistic in a distance of travel of about 1 cm, which is comparable to the electron gyroradius. The drift speed of the electrons parallel to $\bar{\mathbf{B}}$ will be sufficient to give rise to streaming instability (Parail & Pogutse 1965). Streaming instability will occur if the electron drift velocity is larger than the ion thermal speed. In contrast with the ions, the travel distance for a proton to become relativistic is about 10^3 cm . However, acceleration of protons parallel to the magnetic field is strongly suppressed by scattering by magnetic fluctuations (Alfvén waves) with wavelengths of the order of the proton gyroradius, which are generated by the proton streaming (Kulsrud & Pearce 1969). For these reasons, we believe that most of the free energy driving the instability goes into heating the electrons. However, we also consider the case in which a fraction g of the dissipated energy goes into heating the electrons and $(1 - g)$ goes into heating the ions. We illustrate the behavior in this case with the following simple model.

3. MODEL

We generalize equation (2) by taking into account that (1) T_i and T_e may differ with energy exchange between ions and electrons by Coulomb collisions, (2) the ohmic plus viscous dissipation heats electrons and ions as discussed below, and (3) the main energy loss is from optically thin bremsstrahlung and optically thick cyclotron emission. Note that the thickness of

the flow H/r is not restricted. Note also that, in contrast with Narayan & Yi (1995), no assumption is made that a constant fraction f of the dissipated energy is advected inward. Hence,

$$\frac{3}{2} \frac{dT_i}{dt} - \frac{T_i}{\rho} \frac{d\rho}{dt} = (1-g)\mathcal{H} - \nu_{ie}(T_i - T_e), \quad (3a)$$

$$\frac{3}{2} \frac{dT_e}{dt} - \frac{T_e}{\rho} \frac{d\rho}{dt} = g\mathcal{H} - \mathcal{C}_{\text{brem}} - \mathcal{C}_{\text{cyc}} + \nu_{ie}(T_i - T_e), \quad (3b)$$

where $g \leq 1$ is the fraction of the ohmic plus viscous dissipation that goes into heating the electrons. We assume $g = \text{const}$, which we view as more physically plausible than the assumption that $f = \text{const}$ of Narayan and Yi. For simplicity of the formulae we assume $T_i < m_e c^2$ and $T_e < m_e c^2$, where T_i and T_e are measured in ergs. Here,

$$\nu_{ie} \approx \frac{4(2\pi)^{1/2} n e^4}{m_i m_e} \left(\frac{T_e}{m_e} + \frac{T_i}{m_i} \right)^{-3/2} \ln \Lambda$$

is the ion-electron energy exchange rate with $\ln \Lambda = \mathcal{O}(20)$ being the Coulomb logarithm (Spitzer 1940); $\mathcal{H} \approx (9/4)m_i \alpha (c_{\text{si}}/v_{\text{K}})^2 v_{\text{K}}^3 \mathcal{F}/r$ is the heating rate per ion, with $\mathcal{F} = 1 - (r_{\text{S}}/r)^{1/2}$; $\mathcal{C}_{\text{brem}} \approx n \sigma_{\text{T}} \alpha_f m_e c^3 (T_e/m_e c^2)^{1/2}$ is the bremsstrahlung cooling rate per electron, with n being the electron or ion density, σ_{T} the Thomson cross section, and α_f the fine-structure constant; and $\mathcal{C}_{\text{cyc}} \approx T_e \omega_{\text{ce}}^3 \mathcal{M}_c^3 / (8\pi^3 n c^2 r)$ is the self-absorbed cyclotron radiation cooling rate per electron, with $\mathcal{M}_c \gg 1$ being the cutoff harmonic number of the cyclotron radiation below which the radiation is self-absorbed (Trubnikov 1958). For $\mathcal{M}_c \gg (2/9)\mu \gg 1$, with $\mu \equiv m_e c^2 / T_e$, Trubnikov's analysis gives $\mathcal{M}_c \approx (2\mu/9)[1 + \ln(\mathcal{D})/\mu]^3$, where $\mathcal{D} \approx \omega_{\text{pe}}^2 r / (c \omega_{\text{ce}} \mathcal{M}_c)$, with ω_{pe} and ω_{ce} being the electron plasma and cyclotron frequencies, respectively. Trubnikov's expression for \mathcal{C}_{cyc} is similar to that of Narayan & Yi (1995).

It is useful to rewrite equations (3a) and (3b) in dimensionless form. Note that $d/dt = v_r(d/dr)$, with $v_r = -(3/2)\alpha \hat{T}_i v_{\text{K}}$, and that $H/r = \hat{T}_i^{1/2}$, the number density of electrons or ions $n = M / (6\pi \alpha m_i r^2 \hat{T}_i^{3/2} v_{\text{K}})$, the mass density $\rho = n m_i$, and the magnetic field $B = [2M v_{\text{K}} / (3\alpha r^2 \hat{T}_i^{3/2})]^{1/2}$, where $\hat{T}_i \equiv T_i / T_v$, with $T_v \equiv GMm_i/r$ being the virial temperature. We also normalize the electron temperature with the same T_v , $\hat{T}_e \equiv T_e / T_v$. Equations (3a) and (3b) become

$$\frac{d\hat{T}_i}{d\hat{r}} = -(1-g)\hat{\mathcal{H}} + \hat{A}(\hat{T}_i - \hat{T}_e), \quad (4a)$$

$$\begin{aligned} \frac{d\hat{T}_e}{d\hat{r}} = & -[(2+\zeta)g - \zeta]\hat{\mathcal{H}} + \hat{\mathcal{C}}_{\text{brem}} \\ & + \hat{\mathcal{C}}_{\text{cyc}} - (2+\zeta)\hat{A}(\hat{T}_i - \hat{T}_e), \end{aligned} \quad (4b)$$

where $\hat{r} \equiv r/r_{\text{S}}$, with r_{S} being the Schwarzschild radius, $\zeta \equiv \hat{T}_e / \hat{T}_i$, and

$$\hat{A} \approx \frac{8}{9\pi^{1/2}\alpha^2} \left(\frac{m_e}{m_i} \right) \left(\frac{\dot{M}c^2}{L_E} \right) \left(\hat{T}_i + \frac{m_i}{m_e} \hat{T}_e \right)^{-3/2} \frac{\hat{r}^{1/2}}{\hat{T}_i^{5/2}} \ln \Lambda, \quad (4c)$$

$$\hat{\mathcal{H}} \approx \frac{\mathcal{F}}{2\hat{r}}, \quad (4d)$$

$$\hat{\mathcal{C}}_{\text{brem}} \approx \frac{2^{1/2} 8\alpha_f}{27\alpha^2} \left(\frac{m_e}{m_i} \right)^{1/2} \left(\frac{\dot{M}c^2}{L_E} \right) \frac{\hat{T}_e^{1/2}}{\hat{r}^{1/2} \hat{T}_i^{5/2}}, \quad (4e)$$

$$\hat{\mathcal{C}}_{\text{cyc}} \approx \frac{1}{9\pi^2 2^{1/4} \alpha^{3/2}} \left[\left(\frac{m_i}{m_e} \right)^3 \left(\frac{r_e}{r_{\text{S}}} \right) \left(\frac{\dot{M}c^2}{L_E} \right) \right]^{1/2} \left(\frac{\hat{T}_e}{\hat{T}_i^{7/4}} \right) \frac{\mathcal{M}_c^3}{\hat{r}^{11/4}}, \quad (4f)$$

where $L_E \equiv 4\pi GMm_i c / \sigma_{\text{T}}$ is the Eddington luminosity, and $r_e \equiv e^2 / (m_e c^2)$ is the classical radius of the electron. The terms $d\hat{T}_i/d\hat{r}$ and $d\hat{T}_e/d\hat{r}$ in equations (4a)–(4f) describe the advection of energy by the flow. Apart from the cyclotron cooling, the different terms depend only on α and $\dot{M}c^2/L_E$. The cyclotron cooling is relatively more important for accretion onto a stellar mass object than for accretion onto a massive black hole. The assumed condition for optically thin bremsstrahlung radiation requires $(\dot{M}c^2/\alpha L_E)\hat{r}^{-1/2} < 1$ for $\hat{T}_i = \mathcal{O}(1)$.

We have solved equations (4a)–(4f) starting from different given “initial” values of \hat{T}_i and \hat{T}_e at large $\hat{r} = 10^3$, different accretion rates $\dot{M}c^2 = (0.01-1)L_E$, different values of α , and different values of $g = 0-1$, and integrating inward. For the accretion rates where advection-dominated flows are suggested to occur (Narayan & Yi 1995), $\dot{M}c^2 \leq 0.1L_E$ for $\alpha = 0.1$, we find that the scaled ion temperature \hat{T}_i remains almost constant, whereas the scaled electron temperature \hat{T}_e decreases rapidly as \hat{r} decreases from 10^3 . In this limit, the Coulomb energy exchange between ions and electrons is negligible. The advection terms on the left-hand side of equation (3b) are also negligible. Consequently, the ohmic heating of the electrons $g\mathcal{H}$ goes into radiation, mainly cyclotron radiation; that is, $g\mathcal{H} \approx \mathcal{C}_{\text{cyc}}$. The total radiation is the volume integral of $g\mathcal{H}n$, which gives $gGM\dot{M}/(2r_i)$, where r_i is the inner radius of the flow. Thus, the radiative efficiency is reduced by a factor of g from that of a thin disk with $\hat{T}_i = \hat{T}_e \ll 1$, which is the volume integral of $\mathcal{H}n$. This efficiency can be very small compared with unity only if g is very small compared with unity.

4. CONCLUSIONS

This work considers magnetized advection-dominated accretion flows where the magnetic field is in equipartition with the turbulent motions of the flow (Shvartsman 1971). The magnetic energy density of the flow must be dissipated by ohmic heating with a rate comparable to that of the viscous dissipation (Bisnovatyi-Kogan & Ruzmaikin 1974). We argue that the ohmic and viscous dissipation must occur as a result of plasma instabilities. Further, we argue that the instabilities are likely to be current driven in response to the electric field (associated with the turbulent motion), which has a significant component parallel to the magnetic field. These instabilities are likely to heat mainly the electrons. We have analyzed a model for the radial variation of the electron and ion temperatures assuming that a constant fraction g of the viscous plus ohmic heating goes into heating the electrons and that a fraction $(1-g)$ goes into heating the ions. In contrast with Narayan & Yi (1995), we do not assume that a constant fraction f of the dissipated energy is advected inward by the flow. The electrons cool by bremsstrahlung and cyclotron radiation and exchange energy with the ions by Coulomb collisions. At large accretion rates \dot{M} , Coulomb collisions act to give $T_i \approx T_e$, high radiative efficiency, and geometrically thin, optically thick disk accretion. For small accretion rates, where advection-dominated accretion flows are suggested to occur, and only Coulomb energy exchange between ions and electrons, a regime of optically thin accretion flows with a large difference between ion and electron temperatures ($T_e \ll T_i$)

exists (Shapiro, Lightman, & Eardley 1976). Here, we emphasize that the accretion flow properties depend critically on the ohmic heating of the electrons. For small accretion rates where the electron temperature is much less than the ion temperature, we show that the ohmic heating of the electrons gives a radiative efficiency that is reduced by a factor of g from that for a thin disk. Thus, the tiny radiative efficiencies ($<10^{-3}$) found by Narayan & Yi (1995) correspond to tiny values of g , which are unlikely for the reasons discussed in § 2.

Plasma instabilities due to electron-ion streaming (for electron drift velocity larger than the ion thermal speed) may greatly enhance the energy exchange between ions and elec-

trons. In this case the two-temperature regime disappears, the ion and electron temperatures collapse to small values, $\hat{T}_{i,e} \ll 1$, and the disk is geometrically thin, that is, advection-dominated accretion flows do not occur (Fabian & Rees 1995).

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