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coated with positive-type photoresist (AZ-1350) and exposed to mercury light for 1 minute after prebaking at 90°C for 20 minutes. Then, the lasers were operated at 20°C, 5 mW/facet for 20 minutes. After this, they were syringed in the acetone solution, with ultrasonic vibration, for 3 minutes. The photoresists coated on three lasers were completely dissolved, but in two lasers, no matter what, only part of the photoresist coated at the emitting area could not be dissolved. Note that remaining photoresist was observed on either, but not both, mirrors of the laser. The actinic absorbance of the photoresist decays suddenly in light with a wavelength above 0.5 μm, so 0.8 μm radiation from a laser is transparent to the photoresist. We have certified that photoresist baked at 150°C for 20 minutes could be dissolved completely in the acetone solution, but not 180°C for 20 minutes. Thus, we concluded that a temperature rise of about 150°C did take place at the mirror surface at least in two lasers. Similar results have been presented by O. Matsuda *et al.* at the 26th Nat. Conv. Japan. Soc. Appl. Phys., Spring 1979 (in Japanese).

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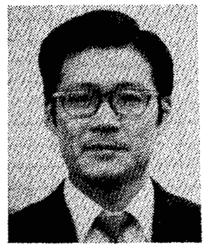

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Influence of Optical Feedback on Laser Frequency Spectrum and Threshold Conditions

JENS HENRIK OSMUNDSEN AND NIELS GADE

***Abstract*–The steady state behavior of the external cavity operated laser has been analyzed, taking into account multiple reflections.**

The effect of optical feedback is included in the phase- and gain-conditions by a factor which is shown to have a simple geometrical representation. From this representation it is easily seen how the laser frequency spectrum and the threshold gain depend on external parameters such as distance to the reflection point and the amount of optical feedback.

Furthermore, by inserting a variable attenuator in the external cavity and measuring the threshold current versus transmittance we have simultaneously determined the photon lifetime and the absolute amount of optical feedback. For the laser considered we found the photon lifetime τ_p = 1.55 ps.

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I. Introduction

It is well known that external optical feedback strongly affects the properties of semiconductor lasers. The reflected light causes variations in, e.g., the threshold gain, output power, and output spectrum [1], [2].

Most of the models dealing with these problems neglect multiple reflections in the external cavity and simply incorporate the optical feedback by adding a time delayed feedback term to the standard laser equation [3], [4].

The purpose of the present work has been to analyze the influence of optical feedback on the laser frequency spectrum and on the threshold gain, taking into account multiple reflections.

A complex feedback parameter is introduced and the phase

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and gain conditions are derived in Section II-A. Two cases appear to be qualitatively different, namely the cases where the external reflection coefficient is smaller than or larger than the reflection coefficient of the laser mirror facing the external cavity. These cases are treated in Section II-B and II-C, respectively. The influence of optical feedback on the threshold current is analyzed in Section II-D. Finally in Section III, we describe the experimental determination of the photon lifetime.

II. Theory

A. Basic Equations

A model for the external cavity operated laser is shown in Fig. 1. Here r_2 and r_3 are amplitude reflection coefficients and d and L are the lengths of the laser diode and the external cavity, respectively.

In order to account for multiple reflections in the external cavity the boundary value problem for the electrical fields in the compound cavity has to be solved. This has been done in [1], where the threshold condition for laser oscillations is given by the secular equation

$$e^{(g-\alpha)d} = \frac{1 + r_2 r_3 e^{-j\phi_o}}{r_2 e^{-j\phi_1}[r_2 + r_3 e^{-j\phi_o}]} \tag{1}$$

where the parameters are

- g Linear gain per unit length
- α Absorption coefficient per unit length
- d Laser diode cavity length
- r_2 Laser cavity reflection coefficient. For numerical calculations we use $r_2 = 0.565$ [1]
- r_3 External reflection coefficient

and $\phi_o = \omega\tau$, where τ is the round trip time in the external cavity and ω is the angular frequency of the laser light. Furthermore, $\phi_1 = (\omega/\omega_D)2\pi$, where $\omega_D = 2\pi f_d = \pi c_g/d$ is the angular frequency mode spacing and c_g is the group velocity.

We introduce the complex feedback parameter

$$z = \frac{1 + r_2 r_3 e^{-j\phi_o}}{1 + (r_3/r_2)\, e^{-j\phi_o}} = e^{G + j\phi} \tag{2}$$

where $G = \ln |z|$ and $\phi = \operatorname{Arg} z$.

In the following we shall for simplicity assume r_3 to be real. The case of complex r_3 could be dealt with by including the phase in ϕ_o. The threshold condition (1) can then be written

$$e^{(g-\alpha)d} = r_2^{-2} e^{j\phi_1} z. \tag{3}$$

z describes the change in phase and threshold gain due to optical feedback. The effective reflection coefficient of the Fabry-Perot cavity defined by the laser mirror and the external cavity can then be written as [5]

$$r_{\text{eff}} = r_2 z^{-1}. \tag{4}$$

From (3) the phase and gain conditions become

$$\phi_1 = p \cdot 2\pi - \phi \qquad \text{(phase condition)} \tag{5a}$$

$$g = \alpha + \frac{1}{d}\ln\frac{1}{r_2^2} + \frac{1}{d}G \qquad \text{(gain condition)} \tag{5b}$$

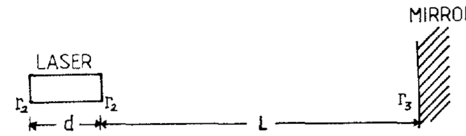
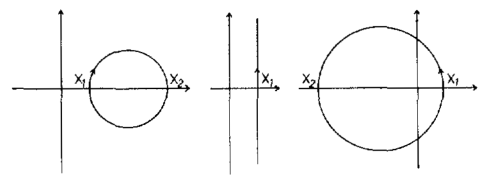


Fig. 1. Schematic representation of the external cavity operated laser.



(a) (b) (c)

Fig. 2. Loci of $z(\phi_o)$ in the complex plane. (a) $r_3 < r_2$, (b) $r_3 = r_2$, (c) $r_3 > r_2$. Arrows indicate direction of revolution for increasing ϕ_o.

where p is an integer. In the case of no optical feedback ($r_3 = 0$), (5) reduce to the well-known expressions [1]

$$\phi_1 = p \cdot 2\pi \Longleftrightarrow \omega = p \cdot \omega_D \tag{6a}$$

$$g = \alpha + \frac{1}{d}\ln\frac{1}{r_2^2}. \tag{6b}$$

In the phase condition (5a) the additional term ϕ accounts for the influence of optical feedback, while the term $(1/d)G$ in the gain condition (5b) represents the excess required gain for the laser to oscillate.

The parameter z is a periodic function of ϕ_o with period 2π. The locus of $z(\phi_o)$ as ϕ_o varies is a circle in the complex plane (see Fig. 2) with center

$$(x_o, y_o) = \left(\frac{r_2^2(1 - r_3^2)}{r_2^2 - r_3^2}, 0\right) \tag{7a}$$

and radius

$$R = \frac{r_2 r_3 (1 - r_2^2)}{|r_2^2 - r_3^2|}. \tag{7b}$$

The intersections with the x axis are

$$x_1 = \frac{r_2(1 + r_2 r_3)}{r_2 + r_3} \quad \text{for } \phi_o = 0 \pmod{2\pi} \tag{8a}$$

$$x_2 = \frac{r_2(1 - r_2 r_3)}{r_2 - r_3} \quad \text{for } \phi_o = \pi \pmod{2\pi}. \tag{8b}$$

The required excess gain G due to optical feedback lies between $G_{\min} = \ln x_1$ and $G_{\max} = \ln |x_2|$, depending on the phase of the reflected field.

We have $G = G_{\min}$ for $\phi_o = 0 \pmod{2\pi}$, i.e., when the reflected field is in phase with the emitted field. It is seen from Fig. 3 that $G_{\min}$ decreases monotonically for r_3 increasing from 0 to 1.

For the reflected field in counter phase with the emitted field, $\phi_o = \pi \pmod{2\pi}$, we have $G = G_{\max}$. For r_3 approaching r_2, $G_{\max}$ increases to infinity. This is to be expected since the effective reflection coefficient r_{eff} equals 0 for $r_2 = r_3$ and $\phi_o = \pi \pmod{2\pi}$, i.e., the external cavity acts as a Fabry-Perot cavity with 100 percent transmission.

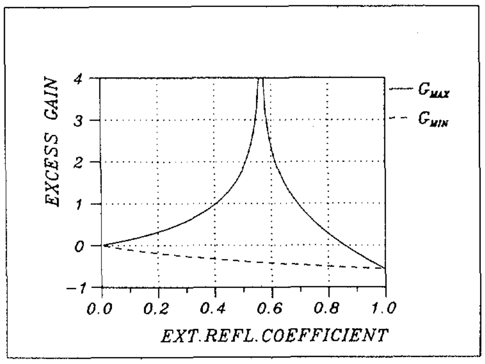


Fig. 3. Upper and lower bounds on G as a function of the optical feedback.

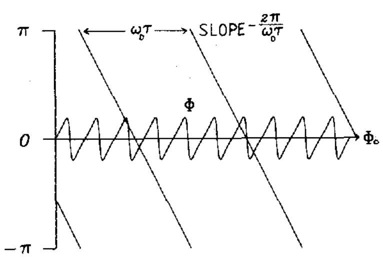


Fig. 4. Phase condition for $r_3 = 0.4$.

For $r_3 = 1$ the threshold gain becomes $g = \alpha + 1/d \ln 1/r_2$ in both cases $\phi_o = 0 \pmod{2\pi}$ and $\phi_o = \pi \pmod{2\pi}$. This is to be expected since $|r_{\text{eff}}| = 1$, independently of the reflectivity of the mirror facing the external cavity.

B. Small Reflections ($r_3 < r_2$)

We now consider the phase condition in the case $r_3 < r_2$. Using $\phi_1 = (\omega/\omega_D)\, 2\pi$ (5a) can be written

$$\phi = -\frac{\omega}{\omega_D} 2\pi + p \cdot 2\pi = -\frac{2\pi}{\omega_D \tau} \phi_o + p \cdot 2\pi. \tag{9}$$

The graphical interpretation of (9) is shown in Fig. 4.

It is seen that solutions to the phase condition will occur in groups centered around the angular frequencies given by

$$\frac{2\pi}{\omega_D \tau} \phi_o = p \cdot 2\pi \Longleftrightarrow \omega = p\omega_D \tag{10}$$

i.e., the longitudinal frequencies of the laser without external cavity. The frequency spacing between the solutions within each longitudinal mode group will be slightly less than $f = 1/2\tau$.

The argument ϕ varies periodically between $\pm\phi_{\max}$, where $\phi_{\max}$ is given by (see Fig. 2)

$$\sin \phi_{\max} = \frac{R}{x_o} = \frac{r_3(1 - r_2^2)}{r_2(1 - r_3^2)}. \tag{11}$$

$\phi_{\max}$ is a measure of the maximum achievable frequency shift. From (9) this maximum frequency shift is

$$\Delta\omega_{\max} = \frac{\phi_{\max}}{2\pi} \omega_D. \tag{12}$$

It is seen from Fig. 2 that $\phi_{\max} < \pi/2$ for $r_3 < r_2$. This means that solutions will only occur in the intervals

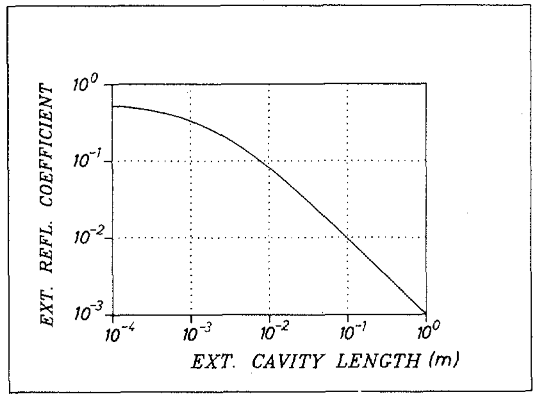


Fig. 5. Condition for single mode operation within each longitudinal mode group.

$$I_p = \left[p\omega_D - \frac{\omega_D}{4},\ p\omega_D + \frac{\omega_D}{4}\right]. \tag{13}$$

From Fig. 4 it is easily seen how the solutions to the phase condition are affected by the amount of optical feedback. Reducing r_3 causes a decrease of $\phi_{\max}$, which again causes a reduction of the number of solutions to the phase condition.

The distance to the external reflection point also affects the number of solutions. For decreasing distance the slope of the straight lines becomes steeper (more negative), causing a decrease in the number of solutions.

In general, the phase condition has multiple solutions, but for certain combinations of the external cavity length L and the external reflection coefficient r_3, there is only one solution to the phase condition present within each interval I_p, i.e., each mode group consists of only one mode. This is ensured if the slope $-2\pi/\omega_D\tau$ of the straight line exceeds the minimum slope of the ϕ-curve.

The maximum value $r_{\max}$, for which there is only one solution in I_p, is given by (see Fig. 5)

$$r_{\max} = \frac{1 + r_2^2 + f_d\tau(1 - r_2^2) - \sqrt{(1 + r_2^2 + f_d\tau(1 - r_2^2))^2 - 4r_2^2}}{2r_2}. \tag{14}$$

As long as $r_{\max} \ll 1$, the values determined by (14) are in good agreement with [2], in which a first order model is used, i.e., multiple reflections have been neglected.

In Fig. 6 is shown the variation of ϕ and G versus ϕ_o. From this figure and Fig. 4 it is seen that only solutions corresponding to intersection points near $\phi_o = 0 \pmod{2\pi}$ will have low-threshold gains. Therefore, each longitudinal mode group in the laser emission spectrum will consist of a number of modes separated by $f \lesssim 1/\tau$ and with increasing intensity as the wavelength corresponding to a longitudinal mode of the free running laser is approached (see Fig. 7).

The allowed frequencies have to satisfy the analytical expression

$$p\omega_D - \omega = \frac{\omega_D}{2\pi} \text{Arctan}\left[\frac{r_3(1 - r_2^2)\sin \omega\tau}{r_2(1 + r_3^2) + r_3(1 + r_2^2)\cos \omega\tau}\right]. \tag{15}$$

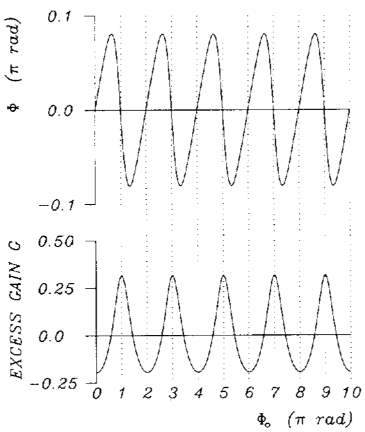


Fig. 6. Variation of $G = \ln |z|$ and phase ϕ of the complex feedback parameter z ($r_3 = 0.4$).

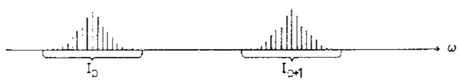


Fig. 7. Laser emission spectrum consisting of longitudinal mode groups. The modes within each group are separated by $f \lesssim 1/\tau$. There are no solutions to the phase condition in the intermediate interval.

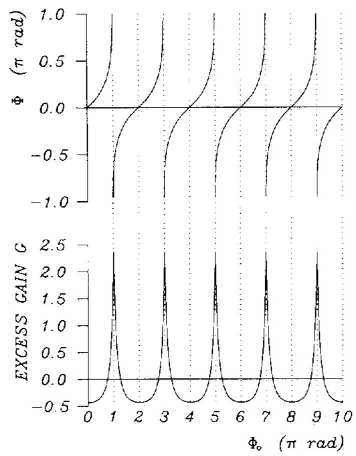


Fig. 8. Variation of $G = \ln |z|$ and the phase ϕ of the complex feedback parameter z ($r_3 = 0.6$).

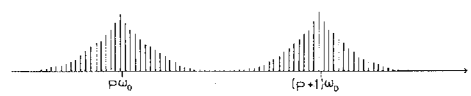


Fig. 9. Laser emission spectrum for $r_3 > r_2$. The solutions to the phase condition occur all over the frequency axis with spacing $f \lesssim 1/\tau$.

For small reflections ($r_3 \ll 1$) the first order approximation becomes

$$p\omega_D - \omega = \frac{\omega_D}{2\pi} \frac{r_3(1 - r_2^2)}{r_2} \sin \omega\tau. \tag{16}$$

This is the same expression for the frequency shift caused by external reflections as given in [3] and [4], where first order models have been used. This approximation is only good in the case of small reflections. For large amounts of optical feedback, multiple reflections cannot be neglected, and (5a) has to be used in determining the allowed frequencies.

C. Large Reflections ($r_3 > r_2$)

In the case $r_3 > r_2$, the argument ϕ is no longer restricted to the interval $-\pi/2 < \phi < \pi/2$, but will vary in the whole range $-\pi < \phi \leqslant \pi$. Solutions to the phase condition will thus occur all over the frequency axis.

In Fig. 8 is shown the variation of the argument ϕ and the excess required gain G versus ϕ_o.

The phase condition is fulfilled for every period of ϕ_o, but only solutions corresponding to intersection points near the ϕ_o axis, i.e., angular frequencies near $p \cdot \omega_D$, have low-threshold gains. Therefore, the laser emission spectrum will consist of longitudinal mode groups which no longer are clearly separated. The mode spacing is $f \lesssim 1/\tau$, and as in the previous case, the intensity of the modes increases as the wavelength corresponding to a longitudinal mode of the free running laser is approached (Fig. 9).

D. Threshold Current Reduction

The threshold gains are calculated from (5b) for the frequencies satisfying (5a). The onset of laser oscillations will take place at the frequency ω_o with lowest threshold gain.

Assuming a linear relationship between gain and current

$$g = \beta I \tag{17}$$

leads to the following expression for the threshold current.

$$I_{th} = I_o \left\{ 1 + f_d \tau_p \ln \left[r_2^2 \frac{1 + r_2 r_3 (r_2 r_3 + 2 \cos \omega_o \tau)}{r_2^2 + r_3^2 + 2 r_2 r_3 \cos \omega_o \tau} \right] \right\} \tag{18}$$

where I_o is the threshold current for the free running laser and τ_p is the photon lifetime,

$$\tau_p = \frac{1}{\beta I_o c_g} \tag{19}$$

In the first order approximation ($r_3 \ll 1$) (18) reduces to

$$I_{th} = I_o \left(1 - 2 f_d \tau_p \frac{r_3 (1 - r_2^2)}{r_2} \cos \omega_o \tau \right) \tag{20}$$

which is consistent with earlier derived expressions based on models neglecting multiple reflections [4].

Measuring I_{th} versus the external reflection coefficient makes it possible to determine the photon lifetime τ_p.

III. Experimental

In order to measure the threshold current versus the amount of optical feedback, we inserted a variable attenuator in the external cavity. The variable attenuator was of type NRC 925 B, and the laser diode used was a Hitachi CSP-laser.

The threshold current was measured versus transmittance T of the variable attenuator. The corresponding reflection coefficient therefore becomes $r_3 = kT$ where k is a coupling parameter including external mirror reflectivity and coupling between laser and external cavity.

We used a large external cavity length (31 cm) and could therefore assume the laser to oscillate at $\phi_o = 0 \pmod{2\pi}$. In this case (18) becomes

$$I_{th} = I_o \left(1 + 2 f_d \tau_p \ln \left[r_2 \frac{1 + r_2 kT}{r_2 + kT} \right] \right). \tag{21}$$

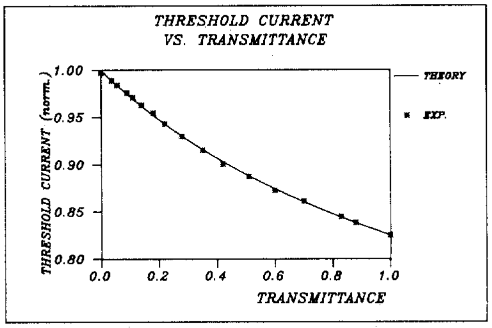


Fig. 10. Normalized threshold current versus transmittance of variable attenuator. The solid curve is calculated from (21) using the optimized values of τ_p and k.

Using the measured value of the mode spacing f_d = 125 GHz the best fit to the experimental points was found for τ_p = 1.55 ps and k = 0.65 (see Fig. 10). Within an approximate 90 percent confidence region τ_p was determined to $\tau_p = 1.55 \pm 0.15$ ps.

It should be noticed that the threshold current reduction is proportional to the product $\tau_p \cdot k$ to first order in r_3. This means that in order to determine τ_p and k separately, the amount of optical feedback should be as large as possible. Therefore, a better coupling between laser and external cavity will lead to a better determination of τ_p.

Using the measured value of the diode cavity length d = 320 μm, the value of τ_p corresponds to an absorption coefficient $\alpha = 45$ cm^{-1}. This result is consistent with α-values used elsewhere [6].

Conclusion

We have analyzed the steady state behavior of the external cavity operated laser taking into account multiple reflections. In this analysis a complex feedback parameter has been introduced.

From the geometrical representation of this parameter it has been shown how external optical feedback influences the laser frequency spectrum and threshold gain. A condition for single mode operation within each longitudinal mode group has been given. Also, analytical expressions have been given for frequency shift and threshold current reduction due to optical feedback, and it has been shown that these expressions in the first order approximation reduce to earlier derived expressions based on models neglecting multiple reflections.

Furthermore, by measuring the threshold current versus the transmittance of a variable attenuator in the external cavity, the photon lifetime and the absolute amount of optical feedback have simultaneously been determined. The result of the measurement is in good agreement with expectations, and it shows that this method is a very convenient way of determining the photon lifetime τ_p without making any assumption on the coupling efficiency between laser and external cavity.

Acknowledgment

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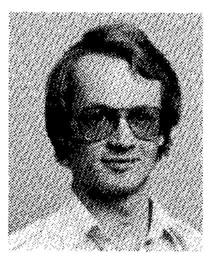

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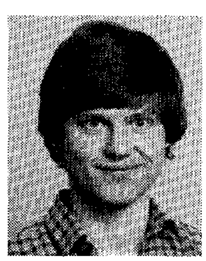
He is now with the Electromagnetics Institute, Technical University of Denmark, where he is working on properties and applications of external-cavity operated lasers.



Niels Gade was born on March 14, 1958 in Aarhus, Denmark. He received the M.Sc. degree in electrical engineering in January 1982 from the Technical University of Denmark, Lyngby, Denmark, where he is now working on a Ph.D. degree at the Electromagnetics Institute. His current interests are in the field of semiconductor lasers, in particular ps pulse generated, mode locking, and external cavity operation.