Influence of signal stationarity on digital stochastic measurement implementation

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Abstract—**Digital stochastic measurement method implementation is considered from signal stationarity point of view. It is shown how conceptual block diagram for digital stochastic measurement of one Fourier coefficient can be simplified and used for developing conceptual block diagram for measuring predefined set of signal harmonics. Hardware of digital stochastic measurement block interfaced to recording block. made for recording stationary signals is used also for recording EEG signal as an example of non-stationary signals. The results of simulation and experimental verification of the method implementations are presented, and the main numerical results are: experiment error of 2.50E-03V, simulation error of 1.34E-04V and theory uncertainty of 2,83E-03V, thus confirming theoretical calculation of standard measurement uncertainty of the method.**

Keywords-digital measurement; dither; signal stationarity; measurement uncertainty

I. INTRODUCTION

All signals can be divided into either stationary or non-stationary categories. Non-stationary signals are not constant in their statistical parameters over time (i.e. its amplitude distribution and standard deviation are not the same over time). Stationary signals are constant in their statistical parameters over time. Stationary signals further can be divided into deterministic and random signals. Random signals are unpredictable in their frequency content and their amplitude level, but they still have relatively uniform statistical characteristics over time.[1-2]

Development of DSM (Digital Stochastic Measurement) method is described in [3-6]. Concept of digital stochastic measurement compared with typical digital measurement is shown at Fig. 1. The outputs of digital measurement are digital values in time domain. Each digital value is actually digitized value of appropriate analog sample from the input and that is well known classical approach of digital measurement – sample by sample. Instead of such approach, the outputs of digital stochastic measurement are Fourier coefficients a_i and b_i. Each Fourier coefficients is the function of all analog samples from the input over the measurement subinterval. Hence, this method is not based on "sample by sample" approach, but it is an interval-based method.

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II. METHOD IMPLEMENTATION FOR STATIONARY SIGNALS

Over the measurement interval *T*, signal *s*, from viewpoint of Fourier analysis can be interpolated as:

$$s(t) = \frac{a_0}{2} + \sum_{n=1}^{M} a_n \cos n\omega_0 t + \sum_{n=1}^{M} b_n \sin n\omega_0 t \qquad (1)$$

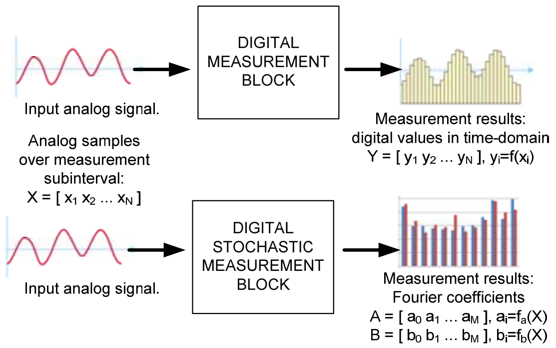

Figure 1. Digital measurement versus digital stochastic measurement.

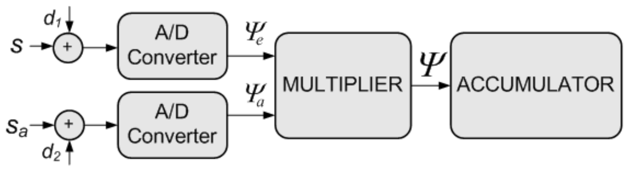

Figure 2. Conceptual block diagram for digital stochastic measurement of one Fourier coefficient.

a_i are cosine Fourier coefficients, b_i are sine Fourier coefficients and *M* is the last index of the coefficients (interpolation of the signal is more accurate for greater M).

The conceptual block diagram of digital stochastic measurement of one Fourier coefficient is given at Fig. 2. Auxiliary signal s_a is a dithered base (cosine or sine) function. If *R* is input range of analog-to-digital (A/D) converter from Fig. 2, then $s_a = R\cos k\omega_0 t$, for measuring *kth* cosine Fourier coefficient and $s_a = R\sin k\omega_0 t$ for measuring *kth* sine Fourier coefficient. d_1 and d_2 are generated dithering signals and they satisfy the following conditions that limit their amplitude and define their probability density function:

$$p(d_i) = \frac{1}{\Delta_i}, \quad 0 \le |d_i| \le \frac{\Delta_i}{2}, \, i = 1, 2 \qquad (2)$$

Sampled values of signal s and auxiliary signal s_a at every time instant within the measurement interval T are Ψ_e and Ψ_a, respectively. The measured value Ψ (multiplier output) differs from the input signals' product by the measurement error e, which includes effect of quantization within A/D converter and the introduced dither:

$$\Psi = \Psi_e \cdot \Psi_a = s \cdot s_a + e \qquad (3)$$

The two terms in (3) are statistically independent, and average is the sum of their average values. The average value of the second term in (3) is zero, as shown in [5] and does not affect the average value of the expected output over the measurement period. A finite input range of $\pm R$ of digital stochastic measurement block defines the boundary of the average noise integration. Therefore the remaining term in the average value is:

$$\overline{\Psi} = \frac{1}{T} \int_0^T s \cdot s_a dt \qquad (4)$$

On the other side, for N digital samples of the measured signal over the interval T, the average value is:

$$\overline{\Psi} = \frac{1}{N} \sum_{k=1}^{N} \Psi_k \qquad (5)$$

Summing of samples during the measurement subinterval is done by the accumulator and this sum is the output of the accumulator (Fig. 2). This output can be processed by microprocessor which divide the accumulator output by the number of samples N, and also calculates each sine (or cosine) component of the kth harmonic of the output (subscripts $sink$ and $cosk$ indicates that kth sine and kth cosine Fourier coefficient is measured):

$$a_k = 2\overline{\Psi}_{\cos k} / R, \; b_k = 2\overline{\Psi}_{\sin k} / R \qquad (6)$$

According to [6-9], the standard measurement uncertainty u is limited by:

$$u(\overline{\Psi}) \le \frac{R \cdot \Delta_1}{2\sqrt{2 \cdot N}} \qquad (7)$$

According to (6) and (7) standard measurement uncertainty of any Fourier coefficient measured by this method is limited by:

$$u(a_k) = u(b_k) \le \Delta_1 / \sqrt{2N} \qquad (8)$$

The quantum Δ_1 is defined by the A/D converter resolution, and the number of samples N can be chosen according to the necessary measurement speed and the required accuracy.

The conceptual block diagram from Fig. 2 can be implemented as in Fig. 3 having Ψ_a digital values stored in the memory thus resulting in elimination of second A/D converter from Fig. 2. If the system should measure DC component and N_h harmonics this structure requires $2N_h+1$ multipliers and $2Nh+1$ accumulators. At first sight, block diagram from Fig. 3 seems to require complex hardware structure but its hardware implementation can be relatively simple. These multipliers and accumulators can be implemented by field-programmable gate array (FPGA) structure which finally calculates Fourier coefficients, while microprocessor interfaces this block with recording block (as it is done in [6-9]). Block diagram of hardware implementation is given at Fig. 4. Pseudostochastic dither signal can be generated by FPGA chip, analog adder is required for performing addition of dither, and finally interface to recording block can be implemented by microprocessor too.

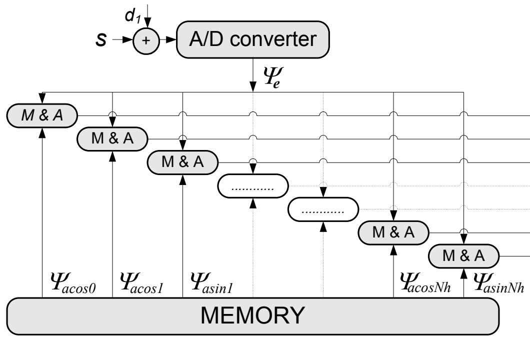

Figure 3. Conceptual block diagram for measuring predefined set of signal harmonics. Each element marked with M&A is consisted of one multiplier and one accumulator.

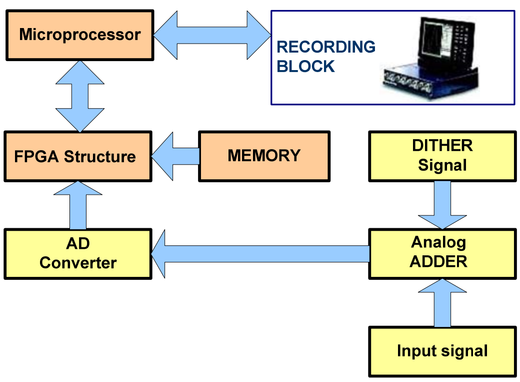

Figure 4. Hardware block diagram of digital stochastic measurement block interfaced to recording block.

III. Method Implementation for Non-stationary Signals

Fourier Transform is adequate for processing stationary signals. Hence, at first sight it can be concluded that digital stochastic measurement is convenient only for measuring harmonics of stationary signals. But, it can be also used for measuring non-stationary signals, by modifying the implementation according to Short Time Fourier Transform (STFT) approach.

First modification of the method is to divide the measurement interval T into short subintervals. The subintervals are chosen such that each one by itself can be considered a windowed sample of a stationary signal. The duration of the subintervals has to be determined either by having some a priori information about the signal or by examining its local characteristics. Also, it is possible to make the subintervals of equal or different duration, depending on the signal and the application.

In general, the STFT is a two-dimensional, time-frequency function, and the resolution of the STFT on the time axis depends on the duration T of the subinterval. Modification in PC software should be made for representing the results. Now, the result is two-dimensional STFT function represented by the spectrogram with time and frequency axes. The narrower the subinterval, the better the time resolution. When choosing a short-duration subinterval, a wider-band window is obtained. The wider the window in the frequency domain, the larger the spectral leakage and hence the deterioration of the frequency resolution[10]. In very highly non-stationary signals, such as the speech signal, short, constant-duration subintervals should be used. In Electroencephalography (EEG) signal measurement, the signal may consist of "stationary" subintervals with very wide duration range[10]. Hence, segmentation should be carefully made, because a priori fixed-duration subintervals may be inadequate.

It should be also noted that standard STFT approach is based on calculating Fourier coefficients after all analog samples over a signal segment are digitized by A/D converter. Instead of waiting for digitalization of all analog samples before beginning calculations, digital stochastic measurement method simultaneously perform digitalization of the samples and calculations of Fourier coefficients.

IV. Simulation and Experimental Results

One example of digital stochastic measurement of stationary signal is the calibrator voltage measurement described in [11]. Root Mean square (RMS) of the calibrator sinusoidal voltage is 1,0V (HP3490A is the referent instrument used for measuring RMS of the voltage), and frequency is 50Hz. Measurement interval T is equal to signal period of 20ms, and 200 measurements are done.

DSM is implemented by 1-channel instrument (Fig. 5), based on Analog Devices AD7663AST A/D converter, FPGA chip CY39K100, and external memory M29040B. The instrument measured 15 harmonics' amplitudes. Results of these measurements is given at Table I, showing that measurement error is less than measurement uncertainty limit(8).

TABLE I. Results of measuring calibrator sinusoidal voltage. RMS is 1V.

Average of first harmonic (V)	1,0002
Number of measurements	200
Experiment error (V)	0,0002
Theory uncertainty (V)	0,0013

TABLE II. Non-stationary signal measurement example: DSM block properties in simulation and experiments sets.

Number of simulations and experiments	100
A/D converter	Resolution: m_1=6 bits Input range: ±R and R=2.5V Sampling frequency: f_{adc} = 15625 Hz
Measurement interval	[0,T] and T = 20ms
Fundamental frequency	$f_0 = 1/T$ = 50Hz
Digital dithered base functions	Stored in memory in 64-bit floating point resolution but passed to the multiplier in 8-bit resolution, thus faithfully simulating an A/D converter with properties: Resolution: m_2 = 8 bits Range: ±R and R=2.5V Sampling frequency: f_{adc} = 15625 Hz

TABLE III. Non-stationary signal measurement example: average error per harmonic compared versus theory maximum for measurement uncertainty.

Experiment error (V)	2.50E-03
Simulation error (V)	1.34E-04
Theory uncertainty (V)	2,83E-03

FPGA chip CY39K100 is programmed with a very-high-speed integrated circuits hardware description language (VHDL) program. The VHDL program is consisted of 4 processes (P1, P2, P3 and P4) which execute simultaneously. Process P1 receives 6-bit digital values from A/D converter. Process P2 is the main process, and all the mathematical calculations are implemented by this process. Process P3 has the task to send the results of process P2 to the microprocessor. Process P4 waits for request from the microprocessor, and when the request comes in P4 activates the process P3.

Figure 5. Prototype DSM instrument.

Basic technical characteristics of FPGA chip CY39K100 are:

- high packing density,
  - up to 200K logic gates,
  - up to 3072 macrocell,
  - up to 428 IO pins,
- up to 480k bits of SRAM memory,
- maximum working speed of 233 MHz,
- 3.3 V, 2.5 V, 1.8 V and 1.5 V at IO lines
- low consumption of 5 mA in "standby" mode,
- programmable delay on all IO pins,
- "in-system" reprogrammable chip
- "carry-chain" logic for fast arithmetic operations.

One example of digital stochastic measurement of non-stationary signal is the measurement of EEG signal produced by EEG calibrator (Fig. 6). In preparatory real measurement (which was a typical digital measurement), EEG signal was stored 256 samples per second (S/s). For obtaining smooth input for simulation and experiment, these 256 S/s records were transformed into 3,840 S/s data. This was achieved by 1) calculating Fourier coefficients by Discrete Fourier Transform (DFT) for original (256 S/s) records and 2) calculating 3,840 S/s data by using IDFT with previously calculated Fourier coefficients. Each sample of measured signal is stored as 64-bit floating point value in simulation lookup-table.

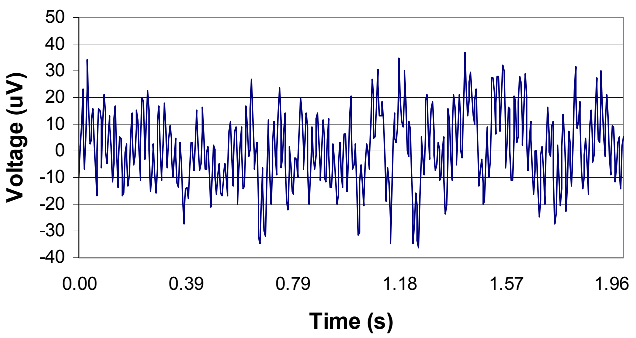

Figure 6. 2 seconds of EEG signal. This signal was used for both simulation and experiment input.

For obtaining correct experimental results, comparable with theory and simulation, each experiment included measuring the same EEG signal. Of course, this repeatability of EEG signals could not be achieved with a humane subject and "live" measurement for each experiment. Therefore, the source of EEG signal in experimental measurements was not a humane subject, but an artificial source of EEG signal was made. This source was made by development board with a programmable system-on-chip (PSoC) CY8C27843, using an embedded 8-bit digital-to-analog (D/A) converter, 16-bit counter and lookup table.

The DSM instrument was configured according to data presented in Table II. 100 computer simulations and 100 experiments are done, and results are given in Table III. Results showed well adjustment with the limits calculated by (8). The experimental error when noise is not added is above the simulation error and this exception can be explained with the resolution of the D/A converter used for generating input signal, and with the ambient noise interfered with the interface between input signal generator and DSM block.

V. Conclusion

Digital stochastic measurement method implementation for stationary signals is described and compared to the. method implementation for non-stationary signals. One example of measuring stationary signals is presented and experimental results are given. Also, simulation and experimental results for measuring non-stationary signal are presented. Both results confirm theory calculations of measurement uncertainty. Future development of the method implementations should be directed on improvement of used A/D converter and FPGA chip for further improvement of accuracy. Also, acquisition software should be improved by visualization of STFT diagrams for non-stationary signals measurement.

References

[1] M.B. Priestley, Non-linear and Non-stationary Time Series Analysis, Academic Press, 1988.

[2] W. J. Fitzgerald, R. L. Smith, A. T. Walde , P. C. Young, Nonlinear and Nonstationary Signal Processing, Cambridge University Press; 1st ed, 2001.

[3] V. Vujičić, S. Milovančev, M. Pešaljević, D. Pejić and I. Županski, "Low frequency stochastic true RMS instrument," IEEE Trans. Instrum. Meas., vol. 48, pp.467-470, Apr. 1999.

[4] D. Pejic, V. Vujicic, "Accuracy limit of high-precision stochastic Watt-hour meter," IEEE Trans. Instrum. Meas., vol. 49, pp. 617-620, June 2000.

[5] V. Vujičić, "Generalized low frequency stochastic true RMS instrument", IEEE Trans. Instrum. Meas., vol. 50, pp. 1089-1092, Oct. 2001.

[6] B. Santrač, M. A. Sokola, Z. Mitrović, Ivan Županski and V. Vujičić, "A Novel Method for Stochastic Measurement of Harmonics at Low Signal-to-Noise Ratio", IEEE Trans. Instrum. Meas., vol. 58, pp. 3434-3441, Oct. 2009.

[7] V. Pjevalica, and V. Vujičić, "Further Generalization of the Low-Frequency True-RMS instrument", IEEE Trans. Instrum. Meas., vol. 59, pp. 736-744, Marh 2010.

[8] V. Vujičić, I. Županski, Z. Mitrović, M. A. Sokola, " Measurement in a point versus measurement over an interval," in XIX IMEKO World Congress Fundamental and Applied Metrology, September 6-11, 2009, Lisbon, Portugal.

[9] Sovilj P. M., Milovančev S. S., Vujičić V.: Digital Stochastic Measurement of a Nonstationary Signal With an Example of EEG Signal Measurement, Instrumentation and Measurement IEEE Transactions on, Vol. 60 - issue 9, pp. 3230-3232, 2011.

[10] A. Cohen, "Biomedical Signals: Origin and Dynamic Characteristics; Frequency-Domain Analysis", in Biomedical Engineering Handbook, Vol. I, J.D. Bronzino, 2nd ed., New York: CRC Press LLC, 2000.

[11] N. Pjevalica, Merenja u elektrodistributivnoj mreži u frekvencijskom domenu, PhD thesis, FTN Novi Sad, 2007.