College of Engineering
Virginia Polytechnic Institute and State University
Blacksburg, Virginia 24061

VPI E-80-26 September, 1980

INFLUENCE OF STRESS INTERACTION ON THE BEHAVIOR OF OFF-AXIS UNIDIRECTIONAL COMPOSITES

Marek-Jerzy Pindera[1]

Carl T. Herakovich[2]

Department of Engineering Science and Mechanics

Interim Report 21
The NASA-Virginia Tech Composites Program

NASA Cooperative Agreement NCCI-15

Prepared for: Materials Applications Branch
National Aeronautics & Space Administration
Langley Research Center
Hampton, Virginia 23665

[1]Graduate Student - Virginia Tech

[2]Professor of Engineering Science and Mechanics - Virginia Tech

Presented at the Second USA-USSR Symposium on Fracture of Composite Materials, 3-5 September 1980

ACKNOWLEDGEMENT

This work was supported by the NASA-Virginia Tech Composites Program NASA CA NCCI-15. This support is gratefully acknowledged.

ABSTRACT

The influence of combined states of stress on the shear response along material principal directions in off-axis, unidirectional, composite coupons is examined for systems whose matrix material obeys the von Mises yield condition. Such analysis is motivated by trends in experimental data observed for at least two composite systems that indicate deviation from pure shear behavior along material directions for various off-axis configurations.

The yield function for plane stress of a transversely isotropic composite lamina consisting of stiff, linearly elastic fibers and a von Mises matrix material is formulated in terms of Hill's elastic stress concentration factors and a single plastic constraint parameter. The above are subsequently evaluated on the basis of observed average lamina and constituent response for the Avco 5505 boron-epoxy system. It is shown that inclusion of residual stresses in the yield function together with the incorporation of Dubey and Hillier's concept of generalized yield stress for anisotropic media in the constitutive equation correctly predicts the trends observed in experiments. The incorporation of the strong axial stress interaction necessary to predict the correct trends in the shear response is directly traced to the high residual axial stresses in the matrix induced during fabrication of the composite.

TABLE OF CONTENTS

LIST OF FIGURES

INTRODUCTION

The use of the off-axis tensile specimen for determination and/or verification of elastic properties and fracture strength of unidirectional composite materials is well documented, eq. Sih [1]. As most composite materials exhibit varying degrees of nonlinear response prior to failure in an off-axis tension test, this specimen would also appear well suited for verification of the various assumptions in a number of nonlinear theories recently proposed in the literature, as well as some of their consequences. An indirect outgrowth of certain assumptions employed in one such theory, for instance, is a recently proposed method for determination of nonlinear lamina shear stress-strain response on the basis of the 10° off-axis tension test [2].

The nonlinear response of unidirectional composites is a complex phenomenon which can be caused by a number of diverse mechanisms such as nonlinear constituent behavior, damage accumulation through fiber or matrix cracking or debonding, fiber rotation or any combination of the above to mention the better known causes. The extension of results of the uniaxial test along principal material directions to constitutive models for multiaxial loading situations is not always straightforward due to the possibility of various stress interactions taking place at the microlevel. Foy [3] for example demonstrated the influence of the transverse normal stress component on the shear strain response (and vice versa) along the material axes for composites with stiff, linearly elastic fibers and ductile matrices using a finite element, micro-mechanical analysis. He showed that for combined normal shear loading in a fixed ratio yielding initiated sooner and the extent of inelastic behavior was more pronounced than for either normal or shear stress acting alone (Fig. 1).

A number of macromechanical constitutive models have been proposed in the literature [4-8] to describe the nonlinear response of lamina with stiff, elastic fibers and significantly more compliant and often ductile matrices. The early attempts [4,5] disregarded the possibility of stress interaction in the nonlinear range during combined loadings, while in the later formulations this effect was taken into account, often in an heuristic manner. One such model due to Hashin et al [6] is reminiscent of the total deformation theory of plasticity in that the stress interaction is accounted for through a loading function, based upon an invariant formulation, which is used to determine the different functional forms of the various strain components. The loading function is limited to quadratic terms in transverse normal and shear stresses hus eliminating the possibility of the axial normal stress influence on the transverse and shear strain behavior.

Data generated by Cole and Pipes [9] are employed in this paper to re-examine and study the extent of stress interaction in off-axis unidirectional boron-epoxy coupons with the aid of Hashin's model and a micromechanical approach utilizing a yield function concept based on Hill's method [10] for determining the elastic stress concentration factors. The results of the micromechanical analysis are subsequently employed in Dubey and Hillier's generalized yield stress formulation [11] to study the nonlinear shear stress-strain response along the principal material directions in off-axis tension coupons. Several fiber orientations are considered including the 10° coupon which has been proposed for the

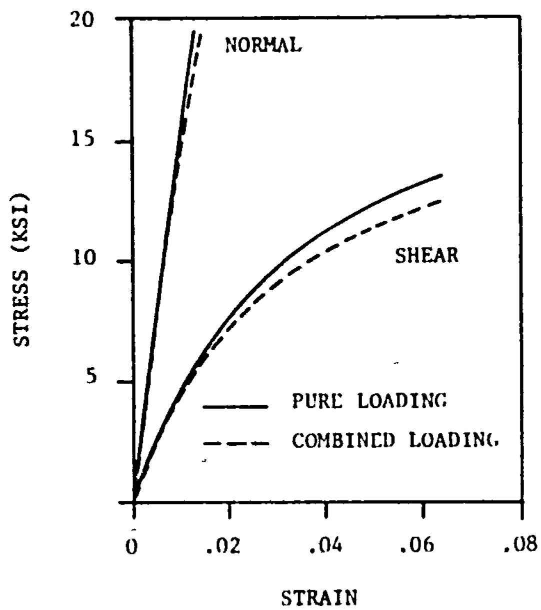

FIG. 1. STRESS-STRAIN CURVES FOR UNIDIRECTIONAL BORON/EPOXY (50% FIBER VOLUME FRACTION) UNDER COMBINED LOADS. NORMAL:SHEAR LOAD RATIO IS 8:3, REF. 3.

determination of pure shear response.

The influence of stress interaction on the shear response in the off-axis tensile tests of Cole and Pipes is shown in Fig. 2 along with the pure shear response obtained from unidirectional tubes. These experimental results show that the extent of inelastic behavior is not a monotonic function of fiber orientation. The inelastic shear strains (for a specified shear stress) decrease in magnitude as the fiber angle is increased from 15° to 30°, but as the fiber angle is increased further to 45° the inelastic strains increase. (Similar behavior has been observed by the authors for the graphite-polyimide off-axis coupons.) One of the goals of this paper is to provide an explanation for such behavior.

In the first part of the paper the model of Hashin et al is briefly outlined. It is then used to predict the shear stress-strain response along material axes using unidirectional data taken from Ref. [9]. An empirical observation is made that modification of Hashin's model by incorporation of an axial normal stress interaction term significantly improves correlation with the experimental data. Based on the above results, micromechanical formulation of the yield function for the composite in the presence of stiff fibers is employed along with Dubey's generalized yield stress concept to examine the effect of the σ_{11} stress component on yielding in the matrix phase and its consequences on the nonlinear shear response along material directions. It is shown that incorporation of the presence of residual curing stresses in the composite leads to the correct prediction of trends in the experimental shear stress-strain data of Cole and Pipes.

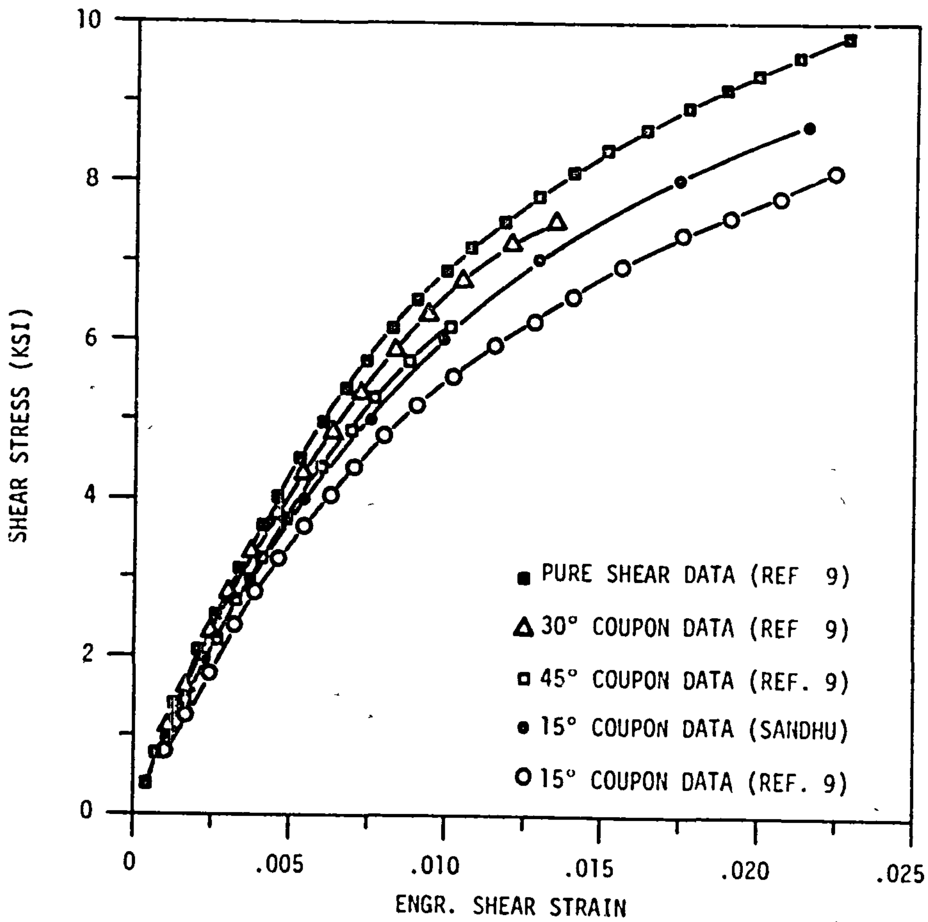

FIG. 2. EXPERIMENTAL SHEAR STRESS-STRAIN RESPONSE ALONG MATERIAL PRINCIPAL DIRECTIONS FOR VARIOUS OFF-AXIS TENSION COUPONS. AVCO 5505 BORON-EPOXY SYSTEM.

STRESS INTERACTION MODEL

Following classical concepts of deformation theory of plasticity Hashin et al [6] assume that the inelastic part of strain can be expressed in the following manner.

$$\varepsilon^P_{ij} = \sigma_{ij} f_{ij}(L) \qquad \text{(no summation)} \qquad (1)$$

where L is some general quadratic loading function of the stresses. Since composites which employ stiff fibers such as boron or graphite behave linearly (or nearly so) under loads applied in the fiber direction, the plastic response is assumed to be limited to transverse and shear strains. Furthermore, the effect of the stress in the fiber direction on the remaining strain components is neglected and thus Eq. (1) is expressible as follows:

$$\begin{aligned} \varepsilon^P_{12} &= \frac{\sigma_{12}}{2G_{12}} f_{12}\ (\alpha^2\sigma_{22} + \beta\sigma^2_{12}) \\ \varepsilon^P_{22} &= \frac{\sigma_{22}}{E_{22}} f_{22}\ (\alpha^2\sigma_{22} + \beta\sigma^2_{12}) \end{aligned} \qquad (2)$$

where the interaction term $\sigma_{12}\sigma_{22}$ has been excluded on the basis of material symmetry arguments and only strains relevant to laminate analysis retained. The functional form of $f_{22}(L)$ and $f_{12}(L)$ is determined from uniaxial tension and torsion tests along material directions. It is noted that no explicit mention is made of the yield function and its relationship to the loading function in this formulation despite apparent separation of the total strain into elastic and inelastic or plastic portions. Combining plastic and elastic portions of strain, the relevant total strain components become:

$$\epsilon_{11} = \frac{\sigma_{11}}{E_{11}} - \frac{\nu_{12}}{E_{11}} \sigma_{22}$$

$$\epsilon_{22} = \frac{-\nu_{12}}{E_{11}} \sigma_{11} + \frac{\sigma_{22}}{E_{22}} + \frac{\sigma_{22}}{E_{22}} \left[\left(\frac{\sigma_{22}}{\sigma_y}\right)^2 + \left(\frac{\sigma_{12}}{\tau_y}\right)^2 \right]^{\frac{M-1}{2}} \qquad (3)$$

$$\epsilon_{12} = \frac{\sigma_{12}}{2G_{12}} + \frac{\sigma_{12}}{2G_{12}} \left[\left(\frac{\sigma_{22}}{\sigma_y}\right)^2 + \left(\frac{\sigma_{12}}{\tau_y}\right)^2 \right]^{\frac{N-1}{2}}$$

where the constants M, N, σ_y, and τ_y are obtained from the Ramberg-Osgood approximation of uniaxial tests.

THEORETICAL-EXPERIMENTAL-CORRELATION

Data presented by Cole and Pipes on the Avco 5505 boron-epoxy system have been employed to determine the Ramberg-Osgood parameters in Hashin's model. These are given in the appendix. Predicted shear response in the material coordinate system for the various off-axis fiber orientations is plotted in Fig. 3. While Hashin's model predicts monotonically decreasing response with increasing off-axis angle, the experimental data points to a reversal around 30° off-axis angle (Fig. 2).

The concept of stress interaction in an elastoplastic material can be illustrated by considering the shear stress-strain response along axes inclined at some angle with respect to the loading direction of an isotropic tensile specimen. The hypothetical isotropic material is taken to have the same shear properties as the considered boron-epoxy system for illustration purposes. Using the J_2 isotropic theory (in this case the flow and deformation approaches are coincident) the shear response along the inclined axes is given by:

$$\varepsilon_{12} = \frac{\sigma_{12}}{2G_{12}} + \frac{\sigma_{12}}{2G_{12}} \left[\frac{2}{3} + \frac{1}{3} (\tan^2\theta + \cot^2\theta) \right]^{\frac{N-1}{2}} \left(\frac{\sigma_{12}}{\tau_y}\right)^{N-1} \qquad (4)$$

with yield governed by:

$$\sigma_{12}^2 \left[\frac{2}{3} + \frac{1}{3} (\tan^2\theta + \cot^2\theta) \right] = S^2, \; 0<\theta<90° \qquad (5)$$

(or alternatively, $\sigma_{12} = \frac{\sqrt{3}}{2} S \sin 2\theta$) where S is the yield stress in in pure shear. Eq. (4), plotted in Fig. 4, predicts monotonically increasing shear response along rotated axes with increasing angle between 0° and 45° with reversal occuring at 45°. This follows from the symmetry

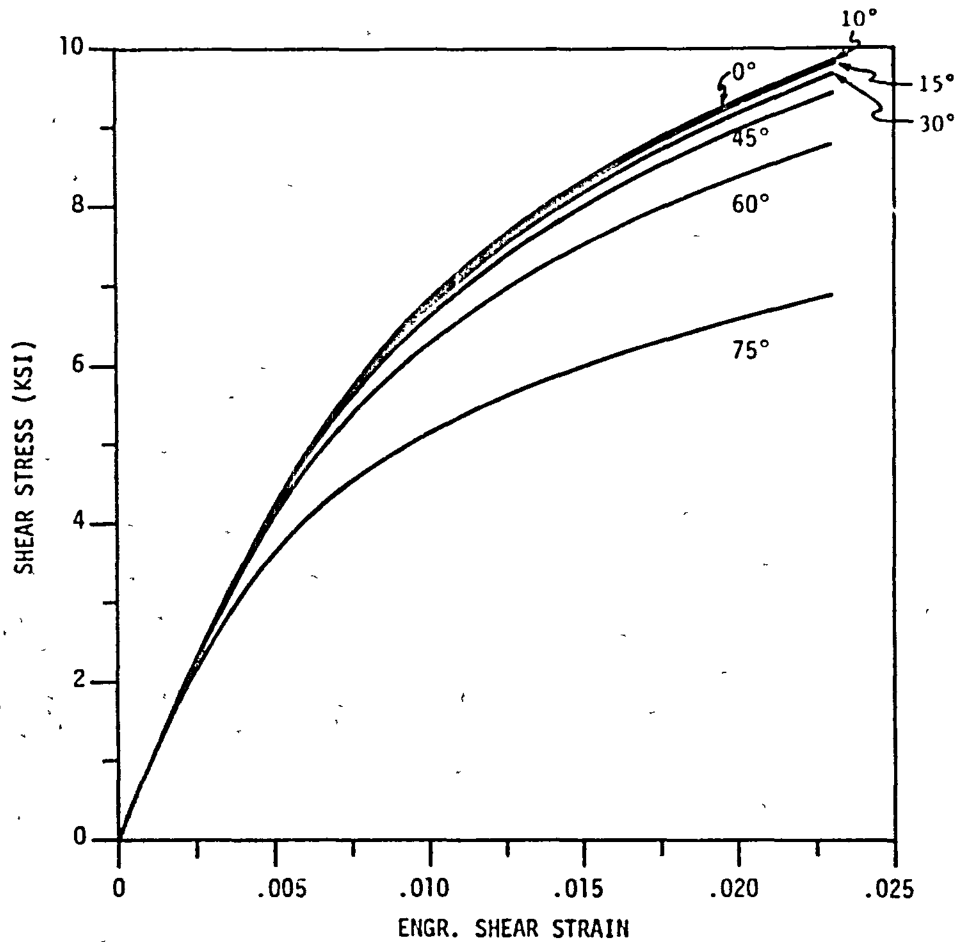

FIG. 3. PREDICTED SHEAR STRESS-STRAIN RESPONSE ALONG MATERIAL PRINCIPAL DIRECTIONS FOR VARIOUS OFF-AXIS TENSION COUPONS. HASHIN ET AL MODEL. AVCO 5505 BORON-EPOXY SYSTEM.

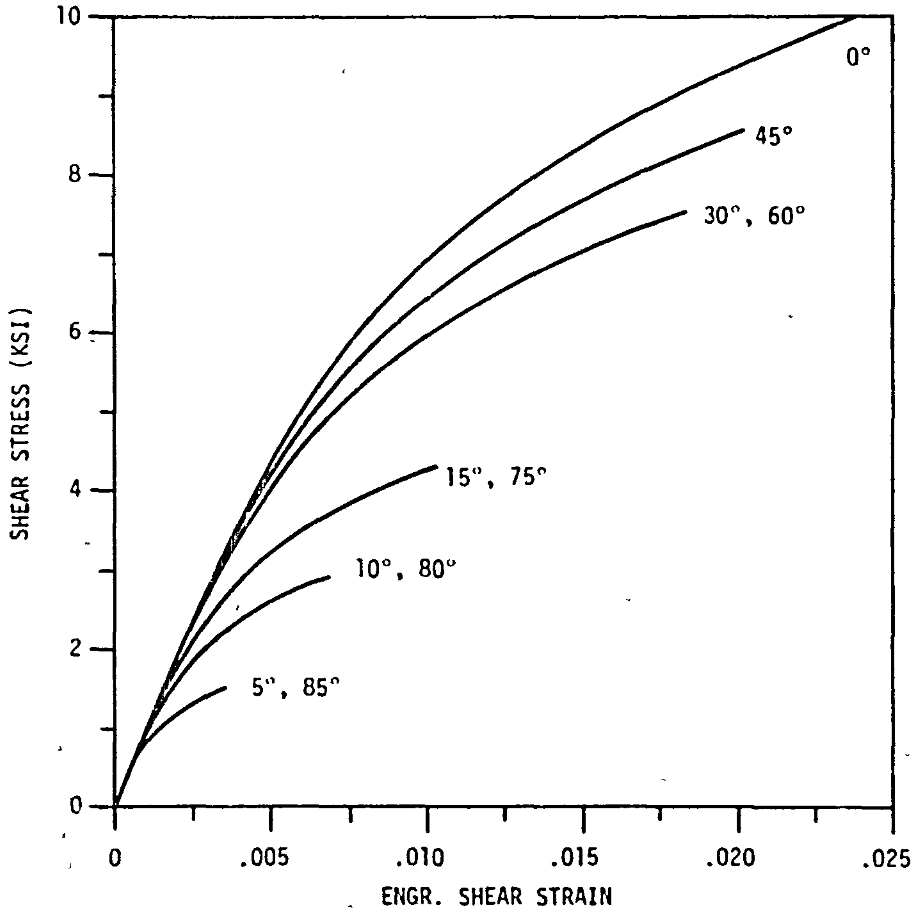

FIG. 4. PREDICTED SHEAR STRESS-STRAIN RESPONSE ALONG ROTATED AXES FOR AN EQUIVALENT VON MISES ISOTROPIC COUPON IN TENSION. BASED ON PURE 5505 BORON-EPOXY SYSTEM.

of the yield function (Eq. 5) about the 45° angle. For comparison purposes Hashin's theory (Fig. 3) yields the following relation for the shear response along material axes inclined at the same angle as the reference axes in the hypothetical isotropic specimen

$$\epsilon_{12} = \frac{\sigma_{12}}{2G_{12}} + \frac{\sigma_{12}}{2G_{12}} \left[1 + \left(\frac{\tau_y}{\sigma_y}\right)^2 \tan^2\theta\right]^{\frac{N-1}{2}} \left(\frac{\sigma_{12}}{\tau_y}\right)^{N-1} \qquad (6)$$

The absence of the resolved shear stress reversal in the above model can be directly traced to the absence of the axial stress (σ_{11}) interaction term.

MICROMECHANICAL CONSIDERATIONS

Assuming that the matrix can be modelled as a von Mises solid that controls the nonlinear response of the composite, the yield function of the lamina can be determined from the knowledge of the stress concentration factors at yield. We take the view that these stress concentration factors are related to the average elastic concentration factors through a single plastic constraint parameter. This appears to be partially substantiated by various finite-element, micromechanical studies, c.f. Dvorak and Rao [12], which have revealed that although yielding usually starts at the fiber/matrix interface, the stress vector remains nearly radial under proportional loading. The yield zone on the other hand spreads rapidly throughout the entire matrix with increasing external tractions.

The elastic stress concentration factors can be determined from the knowledge of the average lamina as well as constituent response following Hill's outline [10]. The relationship between average matrix and external stresses at yield, including residual stresses, is then assumed to be expressible as follows:

$$\begin{Bmatrix} \bar{\sigma}_{11m} \\ \bar{\sigma}_{22m} \\ \bar{\sigma}_{12m} \end{Bmatrix} = K_p \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix} \begin{Bmatrix} \bar{\sigma}_{11} \\ \bar{\sigma}_{22} \\ \bar{\sigma}_{12} \end{Bmatrix} + \begin{Bmatrix} \bar{\sigma}_{11mr} \\ \bar{\sigma}_{22mr} \\ 0 \end{Bmatrix} \qquad (7)$$

where subscript m refers to matrix, r residual and no subscript external stresses. Also, $\bar{\sigma}_{33m} = \bar{\sigma}_{33mr}$ where $\bar{\sigma}_{22mr} = \bar{\sigma}_{33mr} = \frac{\bar{\sigma}_{11mr}}{A}$, and the elastic stress concentration factors $B_{11},\ldots\ldots,B_{66}$ are given in the appendix. Here, K_p is the previously mentioned plastic constraint factor to be determined from experimental data, and the single scale factor A is a consequence of transverse isotropy of the composite.

Employing the above formulation, von Mises yield function for the transversely isotropic lamina in plane stress becomes:

$$\frac{1}{3}(B_{11}^2-B_{11}B_{21}+B_{21}^2)\bar{\sigma}_{11}^2K_p^2+\frac{1}{3}(B_{12}^2-B_{12}B_{22}+B_{22}^2)\bar{\sigma}_{22}^2K_p^2$$
$$+\frac{1}{3}(2B_{11}B_{12}+2B_{21}B_{22}-B_{11}B_{22}-B_{21}B_{12})\bar{\sigma}_{11}\bar{\sigma}_{22}K_p^2$$
$$+\frac{1}{3}(1-\frac{1}{A})\bar{\sigma}_{11mr}(2B_{11}-B_{21})\bar{\sigma}_{11}K_p+\frac{1}{3}(1-\frac{1}{A})\bar{\sigma}_{11mr}(2B_{12}-B_{22})\bar{\sigma}_{22}K_p$$
$$+B_{66}^2\bar{\sigma}_{12}^2K_p^2=K^2-\frac{1}{3}[(1-\frac{1}{A})\bar{\sigma}_{11mr}]^2 \quad (8)$$

where K is the yield shear stress of the matrix material.

The importance of the residual stresses cannot be overemphasized. Employing a single inclusion composite cylinder model and temperature dependent material properties for this particular system, the average curing stresses have been estimated to be:

$$\begin{Bmatrix}\bar{\sigma}_{11mr}\\ \bar{\sigma}_{22mr}\\ \bar{\sigma}_{33mr}\end{Bmatrix}=2786\begin{Bmatrix}1\\ 1/3\\ 1/3\end{Bmatrix}\text{ psi} \quad (9)$$

Although time dependent response has not been considered due to insufficient data, the reduction in residual stresses due to viscoelastic response will be partially off-set by the use of a single inclusion composite cylinder model which generally underestimates induced stresses.

The above yield function (Eq. 8) is plotted in Fig. 5 in terms of the resolved shear stress along the material axes and the off-axis angle.

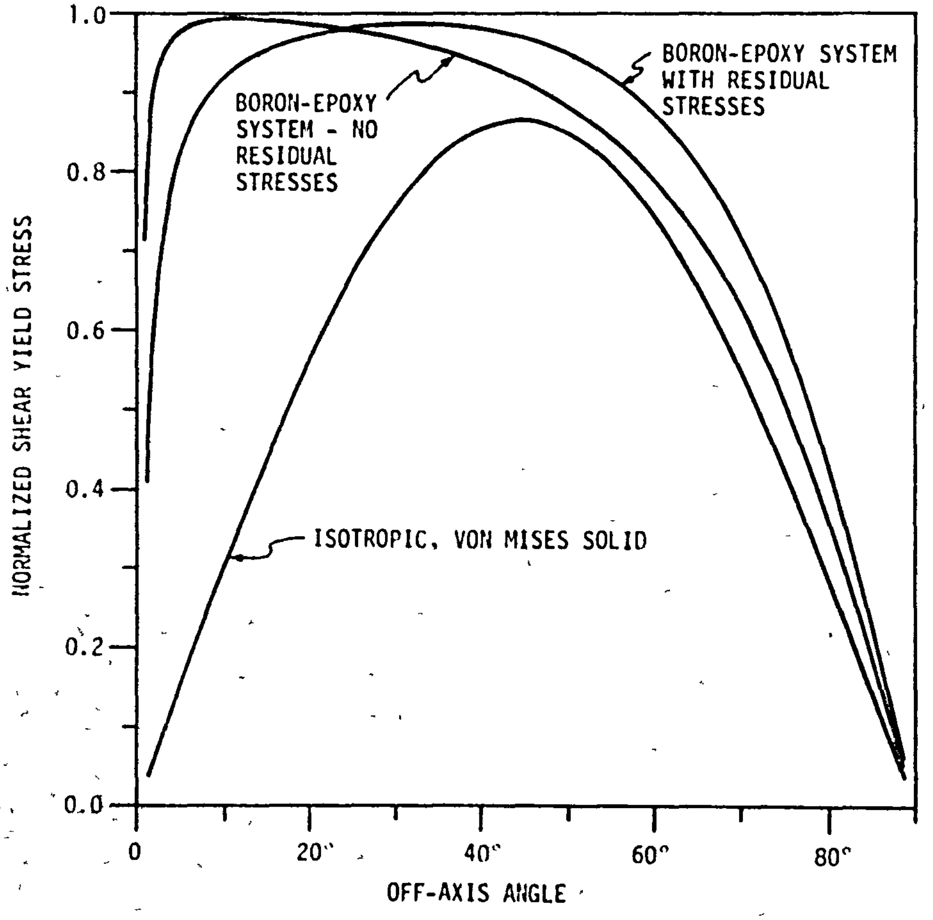

FIG. 5. YIELD SHEAR STRESS OF AVCO BORON-EPOXY SYSTEM ALONG THE MATERIAL PRINCIPAL DIRECTIONS OF AN OFF-AXIS TENSION COUPON AS A FUNCTION OF THE FIBER ORIENTATION.

Also plotted are the yield functions of an equivalent isotropic material and a transversely isotropic lamina with no residual stresses. It is clearly seen that the reversal of the shear stress-strain response evident in Cole's and Pipes' data can be explained on the basis of initial yielding in the matrix. To do this we formulate a generalized yield stress function according to Dubey's and Hillier's suggestion as follows:

$$\bar{\sigma} = C_{ij}\sigma_{ij} + \sqrt{D_{ijk\ell}\sigma_{ij}\sigma_{k\ell}}$$

and assume that the plastic shear strain is a function of $\bar{\sigma}$. The coefficients C_{ij}, $D_{ijk\ell}$ are determined from the yield function given by Eq. (8) and the pure shear response. These are given in the appendix. The resulting shear stress-strain curves along material axes for the various off-axis angles are presented in Fig. 6 indicating the extent of stress interaction. It is seen that the use of the 10° off-axis specimen for determination of pure shear response can lead to noticeable deviations for a material system that can be modelled by the assumed set of governing equations.

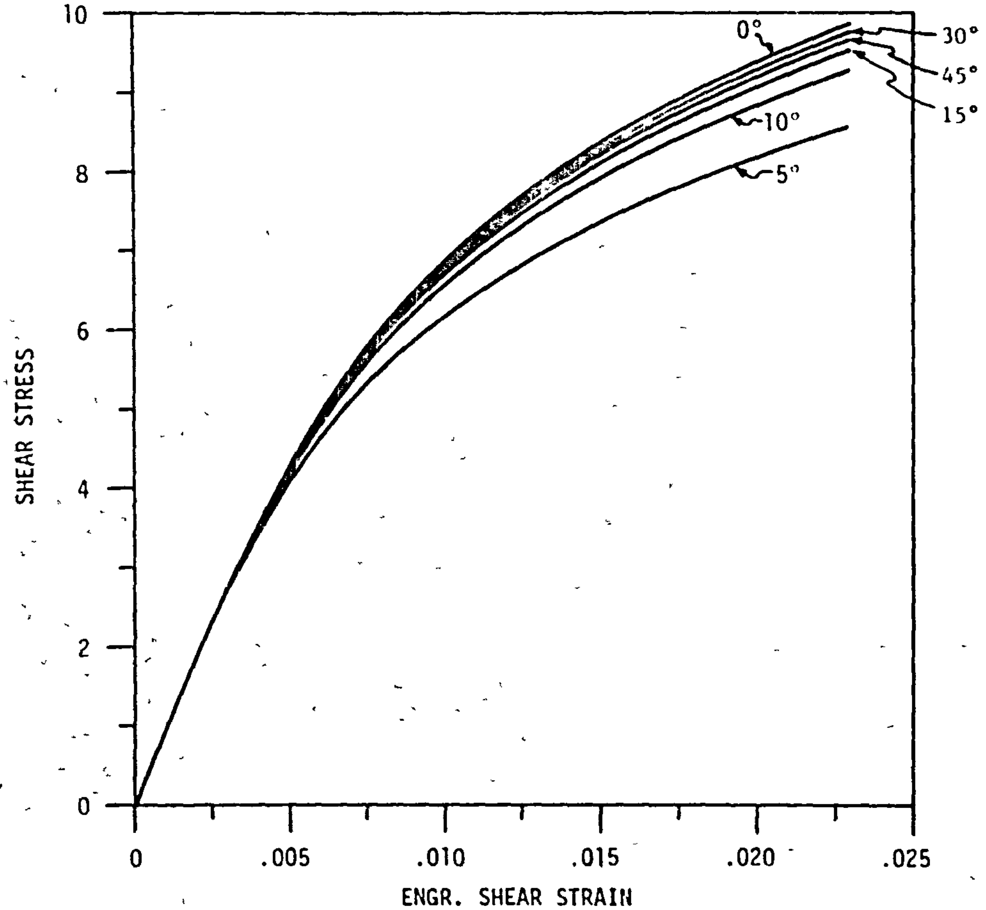

FIG. 6. PREDICTED SHEAR STRESS-STRAIN RESPONSE ALONG MATERIAL PRINCIPAL DIRECTIONS FOR VARIOUS OFF-AXIS TENSION COUPONS BASED ON DUBEY AND HILLIER'S GENERALIZED YIELD STRESS CONCEPT AND MICRO-MECHANICAL CONSIDERATIONS INCLUDING RESIDUAL STRESSES. AVCO 5505 BORON-EPOXY SYSTEM.

CONCLUSIONS

The deviations from pure shear response along material axes in an off-axes tension test for the Avco 5505 boron-epoxy system have been explained to a large extent on the basis of micromechanical formulation of a yield surface and a generalized yield stress concept. The model is applicable to systems with stiff, linearly elastic fibers and ductile matrices whose response can be approximated by that of a von Mises solid. The trends evident in the data of Cole and Pipes for the shear response along the material principal directions are traced to the strong axial stress ($\bar{\sigma}_{11}$) interaction which in turn follows from relatively significant residual stresses. The model predicts that in the absence of these stresses the influence of axial stress on nonlinear shear stress-strain response for low off-axis fiber orientations is only significant in the 0°-5° off-axis range. When the residual stresses are included, however, this influence is noticeable up to approximately 15°. This is interesting in view of the recently proposed test method for determination of pure nonlinear shear stress-strain response on basis of the 10° off-axis tension test.

REFERENCES

1. Sih, G. C., "Fracture Mechanics of Composite Materials," Fracture of Composite Materials, Sih and Tamuzs, editors, Sijthoff and Noordhoff, 1979, p. 111.

2. Chamis, C. C., Sinclair, J. H., "10° Off-Axis Tensile Test for Shear Properties in Fiber Composites," Experimental Mechanics, Vol. 17, No. 9, Sept. 1977, p. 339.

3. Foye, R. L., "Theoretical Post-Yielding Behavior of Composite Laminates, Part I - Inelastic Micromechanics," J.C.M. Vol. 7, April, 1973, p. 178.

4. Petit, P. H. and Waddoups, M. E., "A Method for Predicting the Nonlinear Behavior of Laminated Composites," J.C.M., Vol. 3, January, 1969, p. 2.

5. Hahn, H. T. and Tsai, S. W., "Nonlinear Elastic Behavior of Unidirectional Composite Laminae," J.C.M., Vol. 7, January, 1973, p. 102.

6. Hashin, Z., Bagchi, D., Rosen, B. W., "Nonlinear Behavior of Fiber Composite Laminates," NASA CR-2313, April, 1974.

7. Sandhu, R. S., "Nonlinear Behavior of Unidirectional and Angle-Ply Laminates," J. Aircraft, Vol. 13, No. 2.

8. Jones, R. M. and Morgan, M. S., "Analysis of Nonlinear Stress-Strain Behavior of Fiber-Reinforced Composite Materials," AIAA Journal, Vol. 15, No. 12, December, 1977.

9. Cole, B. and Pipes, R., "Filamentary Composite Laminates Subjected to Biaxial Stress Fields," AFFDL-TR-73-115, 1974.

10. Hill, R., "Elastic Properties of Reinforced Solids: Some Theoretical Principles," J. Mech. Phys. Solids, 1963, Vol. 11, pp. 357-372.

11. Dubey, R., Hillier, M. J., "Yield Criteria and the Bauschinger Effect for a Plastic Solid," Transactions of the ASME, Paper No. 71-Met-P.

12. Dvorak, G. J., Rao, M. S. M., "Axisymmetric Plasticity Theory of Fibrous Composites," Int. J. Engr. Sci., Vol. 14, pp. 361-373.

APPENDIX

i) Ramberg-Osgood parameters for the Avco 5505 Boron-Epoxy system

M	σ_y(psi)	N	τ_y(psi)
3.434	26,155.2	4.430	9,593.9

ii) Elastic stress concentration factors $B_{11},\ldots.B_{66}$

$$B_{11} = \frac{1}{v_m}\left[\frac{E_m}{(E_f-E_m)^2-(\nu_f E_m-\nu_m E_f)^2}\right]\left[\frac{(E_f-E_m)(E_f-E_{11})}{E_{11}} - \frac{(\nu_f E_m-\nu_m E_f)(\nu_f E_{11}-\nu_{12}E_f)}{E_{11}}\right]$$

$$B_{12} = \frac{1}{v_m}\left[\frac{E_m}{(E_f-E_m)^2-(\nu_f E_m-\nu_m E_f^{2})}\right]\left[\frac{(E_f-E_m)(\nu_f E_{11}-\nu_{12}E_f)}{E_{11}} - \frac{(\nu_f E_m-\nu_m E_f)(E_f-E_{22})}{E_{22}}\right]$$

$$B_{21} = \frac{1}{v_m}\left[\frac{E_m}{(E_f-E_m)^2-(\nu_f E_m-\nu_m E_f)^2}\right]\left[\frac{-(\nu_f E_m-\nu_m E_f)(E_f-E_{11})}{E_{11}} + \frac{(E_f-E_m)(\nu_f E_{11}-\nu_{12}E_f)}{E_{11}}\right]$$

$$B_{22} = \frac{1}{v_m}\left[\frac{E_m}{(E_f-E_m)^2-(\nu_f E_m-\nu_m E_f)^2}\right]\left[\frac{-(\nu_f E_m-\nu_m E_f)(\nu_f E_{11}-\nu_{12}E_f)}{E_{11}} + \frac{(E_f-E_m)(E_f-E_{22})}{E_{22}}\right]$$

$$B_{66} = \frac{1}{v_m}\left[\left(\frac{G_f-G_{12}}{G_f-G_m}\right)\left(\frac{G_m}{G_{12}}\right)\right]$$

where

$G_m = 0.19 \times 10^6$ psi

$E_m = 0.49 \times 10^6$ psi

$\nu_m = 0.31$

$\nu_m = 0.50$

$G_f = 24.0 \times 10^6$ psi

$E_f = 58.0 \times 10^6$ psi

$\nu_f = 0.20$

$G_{12} = 0.88 \times 10^6$ psi

$E_{11} = 30.1 \times 10^6$ psi

$E_{22} = 2.87 \times 10^6$ psi

$\nu_{12} = 0.225$

iii) Determination of generalized yield stress. From experiment

$\varepsilon_{12}^P = \frac{\sigma_{12}}{2G_{12}} \left(\frac{\sigma_{12}}{\tau_y}\right)^{N-1}$. Also $\bar{\sigma} = \sqrt{2D_{66}}\, \sigma_{12}$ in contracted notation. Thus

$\varepsilon_{12}^P = \frac{\sigma_{12}}{2G_{12}} \left(\frac{\bar{\sigma}}{\tau_y\sqrt{2D_{66}}}\right)^{N-1}$

From longitudinal tension and compression we obtain: $C_1 = \xi \sqrt{D_{11}}$ in contracted notation, where

$$\xi = \frac{(X_c - X_t)}{(X_c + X_t)}$$

X_t, X_c being the magnitudes of yield stress in tension and compression respectively. Similarly in transverse tension and compression

$$C_2 = \eta \sqrt{D_{22}},$$

where

$$\eta = \frac{(Y_c - Y_t)}{(Y_c + Y_t)}$$

From the invariance of $\bar{\sigma}$ we also have:

$$\sqrt{D_{11}} = \left(\frac{S}{X_t}\right) \frac{\sqrt{2D_{66}}}{(1+\xi)}$$

and

$$\sqrt{D_{22}} = \left(\frac{S}{Y_t}\right) \frac{\sqrt{2D_{66}}}{(1+\eta)}$$

where S is the yield stress in pure shear. Thus the function $\dfrac{\bar{\sigma}}{\tau_y \sqrt{2D_{66}}}$ becomes:

$$\frac{\bar{\sigma}}{\tau_y \sqrt{2D_{66}}} = \frac{1}{\tau_y} \left[\left(\frac{S}{X_t}\right) \frac{\xi}{(1+\xi)} \bar{\sigma}_{11} + \left(\frac{S}{Y_t}\right) \frac{\eta}{(1+\eta)} \bar{\sigma}_{22} + \sqrt{\left(\frac{S}{X_t}\right)^2 \frac{\bar{\sigma}_{11}^2}{(1+\xi)^2} + \left(\frac{S}{Y_T}\right)^2 \frac{\bar{\sigma}_{22}^2}{(1+\eta)^2} + \bar{\sigma}_{12}^2} \right]$$

The parameters S, X_t, Y_t, ξ and η have been evaluated from the yield function given by Eq. (8) and the experimental data. The plastic constraint factor K_p has been determined on the basis of yielding (i.e. deviation from linearity) in transverse tension.

End of Document