

# Influence of the axial position of the external load on the mechanical behavior of the bolted lap joint with different materials

Peng He<sup>\*</sup>, Zhiqiang Fan, Xiangyu Yu, and Zhansheng Liu

School of Energy Science and Engineering, Harbin Institute of Technology, Harbin 150001, PR China

Received: 23 January 2019 / Accepted: 10 August 2019

**Abstract.** The axial mechanical behaviors of the bolted joint are a primary concern of the bolted joint structures. The axial load position has obvious influence on the axial mechanical behaviors of the bolted joint with different materials, while it can not be considered by the analytical method yet. The objective of this work is to fill this gap. With the pressure cone assumption, the axial resilience of the stress zones is derived, and the detailed formulation of the load introduction factor is proposed. The mechanical behavior of the bolted joint subjected to external axial load is formulated and compared with finite element analysis. Results show that the distance between load planes has apparent influence on the axial mechanical behaviors of the bolted joint. In particular for the bolted joint with different materials, the axial positions of the external load have obvious effect on the mechanical behavior. Comparisons with the finite element results indicate that with the proposed formulation of load introduction factor, the analytical formulations could predict the mechanical behavior of the bolted joint with different materials accurately. The proposed formulation of load introduction factor in this work could be an effective supplement of the heavy duty bolt design guideline.

**Keywords:** Bolted joint / load introduction factor / bolt load / clamping force / separation load

## 1 Introduction

The mechanical behavior of tensile bolted joint in service, which includes the additional bolt load, the residual clamping force and the critical work load, is the primary concern in the design and analysis of the bolted joint structures. The axial position of the external load in the clamped parts has obvious influence on the mechanical behavior of the bolted joint [1,2]. The analytical formulations for the mechanical behaviors of the bolted joint, which is widely applied in the early design stage of bolted joints in engineering, still can not take the effect of axial position of the external load into consideration easily, especially for the bolted joint with different materials [2]. This work is motivated to fill this gap by extending the analytical formulations in [2] for the axial mechanical behavior of the bolted lap joint with different materials with considering the influence of the axial position of the external axial load.

At present, the finite element method, experimental test and analytical formulations are the main methods to predict the axial mechanical behaviors of the bolted joint.

The finite element method is widely adopted to investigate the stiffness of the components [3–5] and the axial mechanical behavior of the bolted joint [6–8]. It could accurately predict the mechanical behavior of the specific bolted joint configurations, while much time is required for the modeling and simulation work. In particularly, the nonlinear contact problem in the bolted joint requires much experience for the finite element simulation. The experimental test could obtain the mechanical behavior of the bolted joint with the best accuracy and with a higher cost [6–8]. For its convenience, the analytical formulations for the mechanical behavior of bolted joint [1,2,9], still play an important role in the early stage of the design of the critical bolted joints. The accurate analytical formulations could reduce much design iterations of the bolted joint. Although there are various configurations of bolted joints in engineering, the single bolted joint could be released from the complete joints and the total mechanical behavior of the joints could be obtained as the superposition of each single bolted joint [2]. As a result, the analytical formulations for the axial mechanical behavior are generally established based on the single bolted joint model [1,2,10]. This work is intended to extend the analytical formulations in [2] for the mechanical behaviors of a single bolted lap joint with different materials.

<sup>\*</sup> e-mail: [hepeyfy@foxmail.com](mailto:hepeyfy@foxmail.com)

The axial mechanical behaviors of the bolted joint depend on the axial stiffness of the components. The axial stiffness of the element on the bolt and the nut can be formulated by elasticity principles [1,2], and was improved by introducing empirical coefficients [11]. The axial stiffness of the clamped parts in bolted joint depends on the contact pressure distribution on the interface. Many researches put emphasis on formulating the stiffness of the clamped parts with the assumptions of the constant [12], third order polynomial function [13] or fourth order polynomial function [14] for the stress distribution on the section of the pressure cone. The influences of the radius of the clamped parts [15] and the material difference [16] on the stiffness of the clamped parts are considered in simple formulations. The stiffness formulations of the components are essential to predict the mechanical behavior of the bolted joint. In this work, the formulations of the stiffness of the bolt [2] and the clamped parts [14] are applied to formulate the axial mechanical behaviors of the bolted joint with different materials.

In addition to the stiffness of the components in the bolted joint, the mechanical behavior is also affected by the load patterns. The mechanical behavior of the bolted joint subjected to the bolt preload is different from that of the external load case [17]. The radial position and the axial position of the external load both have influence on the mechanical behavior of the bolted joint. The radial position of the axial load would lead to the nonlinear mechanical behavior of the bolted joint [17–19], and it is beyond the capability of the analytical formulations. However, the influence of the axial position of the external load could be considered in the analytical formulations. The bolt head loading case and the interface loading case are modeled by different mathematical formulations [1]. In practical engineering, the external axial load is transmitted into the bolted joint at a certain axial position within the clamped parts. In order to account for the effect of the axial position of the external load, the load plane within the clamped parts is generally assumed [1,2], and its axial position within the clamped parts could be determined for some typical bolted joints directly [20] or by a systematic method [2]. With the axial position of the external load plane, the effect of the axial position of the external load on the mechanical behaviors could be considered in the analytical formulations by the loading plane factor [1] or the load introduction factor [2]. The loading plane factor in [1], which equals to the ratio of the distance between load planes to the total grip length, is only valid for the sleeve case of bolted joint. In the recent version of the guideline for heavily loaded bolted joint, the general form of the load introduction factor was defined as the ratio of the axial resilience of the stress zone between the load planes to that of the total clamped parts [2]. Reference [2] suggested that the load introduction factor could be obtained directly or be interpolated from the values of six configurations. However, there is not a detailed formulation of the load introduction factor. Moreover, the values of the load introduction factor were only valid for the components in the bolted joint having the same material. It can be seen that the detailed formulation of the load introduction factor, especially for the case of the bolted joint with the

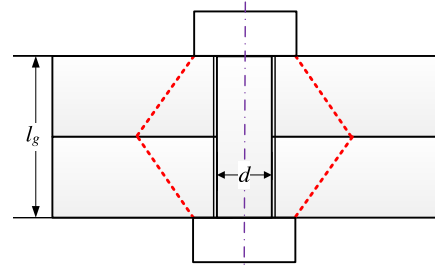


Fig. 1. Simplified model of a single bolted joint.

clamped parts having different materials, is still missing. It becomes an obstacle for the analytical formulation to take the effect of the external axial load position into consideration. As a result, this work is inspired to establish a detailed formulation of the load introduction factor, with which the mechanical behaviors of the bolted joint with different materials could be predicted by the analytical formulations.

The basic mechanical formulations of the components in the bolted joint are given in Section 2. Section 3 presents the derivation of the axial resilience of the stress zone between the load planes and the total resilience of the clamped parts. The finite element model to validate the proposed formulations, is introduced in Section 4. In Section 5, the proposed formulations in this work are validated and the axial mechanical behaviors for the bolted joint are investigated. Finally, the conclusions of this work are given in Section 6.

## 2 The mechanical behaviors of the components in a single bolted joint

A single bolted lap joint consists of the bolt, the nut and the clamped parts. The thread in the bolted joint plays an important role on the axial mechanical behavior of the bolted joint [21]. In order to focus on the influence of the positions where the external load applied, the bolt is simplified as a cylinder with two identical heads, as depicted in Figure 1.

Under the pretension of the bolt, the region with the compress stress in the clamped parts expands radially. The shape of the stressed zone in the clamped parts is complex, while it could be equal to a truncated pressure cone in the mechanical analysis [2], as illustrated by the dashed red line in Figure 1. The axial mechanical behaviors of the bolted joint depend on the axial resilience of the simplified bolt and the stressed zone in the clamped parts.

### 2.1 The axial resilience of the bolt

The axial resilience of the simplified bolt is the sum of resilience of the elements on the bolt:

$$\delta_b = 2\delta_{sk} + \delta_g. \quad (1)$$

Here  $\delta_{sk}$  represents the contribution of the bolt head on the axial resilience,  $\delta_g$  is the axial resilience of the simplified

cylinder with the grip length. According to VDI 2230 [2],  $\delta_{sk} = d/(2E_b A_b)$ ,  $\delta_g = L/(E_b A_b)$ . Here  $d$  is the diameter of the simplified cylinder of the bolt,  $E_b$  is the Young's modulus of the bolt material,  $A_b$  is the section area of the simplified cylinder in the bolt,  $L$  denotes the total grip length of the bolted joint.

### 2.2 The axial resilience of the clamped parts

According to the pressure cone assumption, the outer radius of the stress zone can be written as:

$$r_e = r_h + z \tan \alpha \quad (2)$$

where  $r_h$  is the radius of the bolt head,  $\alpha$  is the angle of the equivalent pressure cone. The cone angle depends on the configuration of the bolted joint, an empirical formulation of the tangent of the cone angle was given in VDI [2], and improved by considering the difference of the thickness of the clamped parts [22].

The total axial resilience of the clamped parts is equal to that of the pressure cone within the clamped parts. When the outer radius of the clamped parts is greater than the maximum radius of the pressure cone, which is the case studied in this work, the total axial resilience of the pressure cone can be obtained based on the average strain and the axial stress distribution on the section of the pressure cone [14]. The axial stress distribution on the section of the pressure cone can be modeled by a fourth order polynomial function, in which the five coefficients can be obtained by the force balance equation and the boundary conditions of the axial stress on the section of the pressure cone [14,23]. With the assumption of the average normal strain of the pressure cone section, the axial resilience of the whole pressure cone can be derived by the integration of the axial strain distribution function [14]:

$$\delta_c = \frac{\ln \left[ \frac{L_n \tan \varphi + r_h - r_H}{L_n \tan \varphi + r_h + 3r_H} \frac{r_h + 3r_H}{r_h - r_H} \right]}{2\pi r_H E_c \tan \alpha} \quad (3)$$

When external axial load is applied at the bolt heads or at the interface between the clamped parts, the mechanical behaviors of the bolted joint could be analyzed based on the resilience of bolt and the clamped parts [1]. While the external load is generally applied at some axial position between the bolt head and the interface between the clamped parts, the load plane is assumed to consider the axial positions where the external load applied [1].

### 2.3 The axial mechanical behaviors of the bolted joint with the loading plane

Bolted joints in engineering are of various configurations, while the single bolted joint could be released from the complete joint [2]. In order to account for the axial position of the external load, the imaginary load planes are generally introduced within the stress region and assumed flat during the deformation. As depicted in Figure 2, the axial positions of the load planes in each clamped parts are defined by the distance  $L_{p1}$ , and  $L_{p2}$ , which indicate the

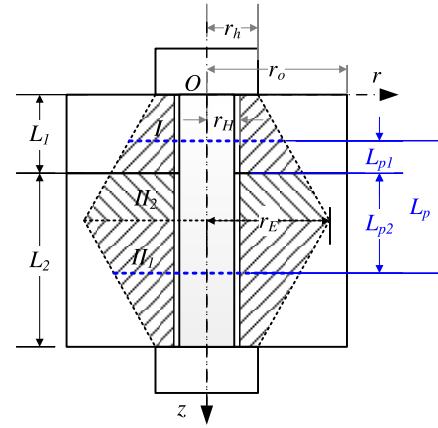


Fig. 2. Bolted joint with the load planes of the external load.

distance of the load planes to the interface between the load planes. The total distance between the load planes is  $L_p = L_{p1} + L_{p2}$ . The pressure cone in each clamped part is divided into two portions by the load plane. One is between the bolt head and the load plane, and the other is between the load plane and the interface. The two portions of stress zones have different contributions to the mechanical behaviors of the bolted joint. The load introduction factor is introduced to account for the effect of the external load axial position.

A simple form of the load introduction factor is defined as the ratio of the distance between the load planes to the total grip length of the bolted joint [1], while it is only valid for the sleeve case of the bolted joint. A general form of the load introduction factor is defined as the ratio of the axial resilience of the stress zones between the load planes to that of the total clamped parts [2]:

$$n = \frac{\delta_{lp}}{\delta_c} \quad (4)$$

Here  $\delta_{lp}$  is the axial resilience of the stress zone between the load planes,  $\delta_c$  is the total axial resilience of the clamped parts.

With the load introduction factor, the load factor for the case of arbitrary external load positions on the bolted joint becomes [1,2]:

$$\Phi_{kn} = n \frac{\delta_c}{\delta_b + \delta_c} = \frac{\delta_{lp}}{\delta_b + \delta_c} \quad (5)$$

The mechanical behaviors of the bolted joint could be obtained with the load factor. The variations of the bolt load and the clamping force with the external load can be obtained:

$$\Delta F_b = \Phi_{kn} F_E \quad (6)$$

$$\Delta F_c = (1 - \Phi_{kn}) F_E \quad (7)$$

Here  $\Delta F_b$  is the variation of the bolt load,  $\Delta F_c$  represents the variation of the clamping force. The critical external load, under which the interface between the

clamped parts separates, can be obtained:

$$F_{cri} = \frac{F_E}{1 - \Phi_{kn}}. \quad (8)$$

Once the load introduction factor is determined, the load factor of the bolted joint under the external load could be obtained by (5), and finally the mechanical behaviors of the bolted joint could be got by (6)–(8). The difficulty in this procedure is the value of the load introduction factor. In the bolt design guideline [2], although the definition of the load introduction was provided, there was not a detailed formulation of the load introduction factor. The load introduction factor value was suggested to be obtained directly or to interpolate the values in the table of six bolted joint configurations. Moreover, the values in the table were on the basis that the bolted joint having the same material [2]. All these factors make it difficult to consider the effect of the axial position of the external load in predicting the mechanical behaviors by the analytical formulations. As a result, the emphasis of this work is to extend the basic definition of the load introduction factor in [2] to a detailed analytical formulation. According to (4), it is clear that the load introduction factor depends on the axial resilience of the stress zone between the load planes and the clamped parts. With the recent advance of the mechanical research on the bolted joint [14], the analytical formulations of the resilience of the different portions could be derived.

### 3 The axial resilience of the clamped parts

According to the shape of the pressure cone in the clamped parts, the bolted joint can be classified into the full contact, the truncated pressure cone and the sleeve. The truncated pressure cone case and the sleeve case have been formulated in detail in VDI [1,2], respectively. Therefore, the full contact case is focused in this study. There is a full pressure cone in the clamped parts for the full contact case. The axial resilience of the pressure cone within the clamped parts could be obtained based on the assumption of the average compression on the section of the pressure cone [14].

As depicted in Figure 2, the thickness of the clamped parts are  $L_1$  and  $L_2$  respectively. There is a portion of pressure cone in the upper part. The stress zone in the lower part could be divided into two portions. With the fourth order polynomial distribution of the axial stress in the radial direction on the section of the pressure cone [14], the axial resilience of each portion of the stress zone can be obtained:

$$\delta_I = \frac{1}{2\pi r_H E_u \tan \alpha} \ln \left[ \frac{L_1 \tan \alpha + r_h - r_H}{L_1 \tan \alpha + r_h + 3r_H} \frac{r_h + 3r_H}{r_h - r_H} \right] \quad (9)$$

$$\delta_{II2} = \frac{1}{2\pi r_H E_d \tan \alpha} \times \ln \left[ \frac{((L_1 + L_2)/2) \tan \alpha + r_h - r_H}{((L_1 + L_2)/2) \tan \alpha + r_h + 3r_H} \frac{L_1 \tan \alpha + r_h + 3r_H}{L_1 \tan \alpha + r_h - r_H} \right] \quad (10)$$

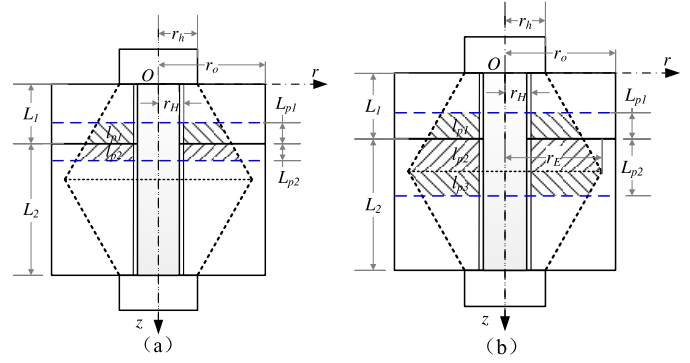


Fig. 3. Two cases of the load planes within the pressure cone.

$$\delta_{II1} = \frac{1}{2\pi r_H E_d \tan \alpha} \times \ln \left[ \frac{((L_1 + L_2)/2) \tan \alpha + r_h - r_H}{((L_1 + L_2)/2) \tan \alpha + r_h + 3r_H} \frac{r_h + 3r_H}{r_h - r_H} \right]. \quad (11)$$

The axial resilience of the lower clamped part is the sum of the two portions:

$$\delta_d = \delta_{II1} + \delta_{II2}. \quad (12)$$

And the total axial resilience of the stress zone in the clamped parts can be obtained:

$$\delta_c = \delta_I + \delta_d. \quad (13)$$

The resilience of the portion of the stress zone between the load planes depends on the locations of the load planes. The axial locations of the load planes within the clamped parts in the full contact bolted joint can be classified into two cases.

#### 3.1 Case (a)

As illustrated in Figure 3a, in the first case, one of the load plane locates at the upper part with a distance  $L_{p1}$  to the interface, while the distance  $L_{p2}$  of the other load plane to the interface is less than  $(L_2 - L_1)/2$ .

The stress zone between the load planes is divided in to two portions by the interface between the clamped parts, as denoted as  $l_{p1}$  and  $l_{p2}$  in Figure 3a. The axial resilience of each portion could be obtained by the similar method in [14]:

$$\delta_{lp1} = \frac{1}{2\pi r_H E_u \tan \alpha} \times \ln \left[ \frac{L_1 \tan \alpha + r_h - r_H}{L_1 \tan \alpha + r_h + 3r_H} \frac{(L_1 - L_{p1}) \tan \alpha + r_h + 3r_H}{(L_1 - L_{p1}) \tan \alpha + r_h - r_H} \right] \quad (14)$$

$$\delta_{lp2} = \frac{1}{2\pi r_H E_d \tan \alpha} \times \ln \left[ \frac{(L_1 + L_{p2}) \tan \alpha + r_h - r_H}{(L_1 + L_{p2}) \tan \alpha + r_h + r_H} \frac{L_1 \tan \alpha + r_h + r_H}{L_1 \tan \alpha + r_h - r_H} \right]. \quad (15)$$

Then the axial resilience of the stress zone between the load planes for this case is:

$$\delta_{lp} = \delta_{lp1} + \delta_{lp2}. \quad (16)$$

### 3.2 Case (b)

In case of the distance of the load plane in the lower clamped part to the interface  $L_{p2}$  is greater than  $(L_2 - L_1)/2$ , the formulation of the axial resilience between the load planes is different from the first case. There are three portions of stress zone between the load planes, as denoted as  $l_{p1}$ ,  $l_{p2}$  and  $l_{p3}$  in Figure 3a. The axial resilience of  $l_{p1}$  is given in (17), and the resilience of the other two portions could be formulated by the similar method.

$$\delta_{lp2} = \frac{1}{2\pi r_H E_d \tan \alpha} \times \ln \left[ \frac{((L_1 + L_2)/2) \tan \alpha + r_h - r_H}{((L_1 + L_2)/2) \tan \alpha + r_h + 3r_H} \frac{L_1 \tan \alpha + r_h + 3r_H}{L_1 \tan \alpha + r_h - r_H} \right] \quad (17)$$

$$\delta_{lp3} = \frac{1}{2\pi r_H E_d \tan \alpha} \times \ln \left[ \frac{((L_1 + L_2)/2) \tan \alpha + r_h - r_H}{((L_1 + L_2)/2) \tan \alpha + r_h + 3r_H} \frac{(L_2 - L_{p2}) \tan \alpha + r_h + 3r_H}{(L_2 - L_{p2}) \tan \alpha + r_h - r_H} \right]. \quad (18)$$

The axial resilience of the stress zone between the load planes is the sum of the resilience of the three portions:

$$\delta_{lp} = \delta_{lp1} + \delta_{lp2} + \delta_{lp3}. \quad (19)$$

With the axial resilience of the stress zone between the load planes, the load introduction factor could be determined analytically by (4) and the load factor could be obtained by (5). The variations of the bolt load and the clamping force could be obtained by (6) and (7), and finally the critical external load can be obtained by (8).

Different from the guideline of bolted joint [2], with the analytical formulations of the axial resilience of the stress zone portions in different cases, the value of the load introduction factor could be obtained, and the mechanical behaviors of the bolted joint under different external load cases could be predicted by the analytical formulations. This work could be an effective supplement of the bolted joint guideline. Moreover, all the value of the load introduction factor in reference [2] should only be valid for the bolted joint with the same material. This limitation is released in this work. As can be seen in the equations of the resilience of the stress zones, such as (9), (10), (11), (14), (15), (17) and (18), the Young's modulus of the material of each clamped part is considered in the equation. The proposed formulation of the load introduction factor is able to account for the bolted joints with the clamped parts having different materials, which extends the application

**Table 1.** Parameters applied in the finite element study.

Parameter	Value	Unit
Bolt nominal diameter $d$	10	mm
Radius of bolt head $r_h$	8	mm
Thickness of bolt head	6.4	mm
Radius of bolt hole $r_H$	5.25	mm
Outer radius of the clamped parts $r_o$	14	mm
The thickness of the clamped parts $L_1 + L_2$ ( $L_1, L_2$ )	20 (10, 10)	mm
The Young's modulus of the bolt $E_b$	200	GPa
The Young's modulus of the clamped parts ( $E_{c1}, E_{c2}$ )	100, 200, 300	GPa
The preload of bolt	10 000	N

range of the analytical formulation for the axial mechanical behaviors of the bolted joint.

## 4 Finite element analysis

A finite element model of a M10 single bolted joint is established and analyzed to validate the formulations of the axial mechanical behaviors. The dimensions of the bolt are determined according to ISO 4014: 1999, and the single bolted joint is simplified as Figure 1. The grip length and the outer radius of the clamped parts are varied to cover the three cases in Section 3. The parameters of the simplified model applied in the finite element analysis are listed in Table 1.

As the bolt and the clamped parts are axisymmetric about the axis of the bolt, the axisymmetric model can save the computational cost without losing the accuracy of the results. The axisymmetric finite element model built in the commercial package Abaqus is given in Figure 4. The axisymmetric constrain is set at the nodes on the axis of the bolt. The node at the outer radius of the bottom surface of the bolt head is axially fixed in the finite element study. There are three interfaces in the finite element model: the interface between the clamped parts, the interfaces between the bolt head and the clamped parts. All these interfaces are set as surface to surface contact. The frictionless tangential behavior and the hard normal behavior of the contact properties are selected. The bolt load model in Abaqus is adopted to simulate the pretension of bolt. In the step when the external axial load is introduced, the bolt load is set as 'fix at current length', which could calculate the bolt load and the deformation of the bolt as a result of the external load. The external load is applied through the reference points, which are coupled with the load planes [24].

The finite element is mesh by the axisymmetric stress element CAX4R, the size of the edges of the element is about 0.2 mm. The results are compared with a model with doubled mesh density, results show that the errors of the

bolt load and the clamping force are less than  $1E-3\%$ , and the error of the axial displacement of the bolted joint under external load is less than  $0.5\%$ , which indicate that the mesh density of this model could predict the mechanical behaviors of the bolted joint precisely.

The typical stress distributions of the bolted lap joint under bolt pretension and external load are given in Figure 5. Under the pretension of the bolt, there is a cone region in the clamped part. With the application of external load in certain axial position, the stress of the portion between the load planes gradually releases, and finally becomes zero after the separation of the interface between the clamped parts.

The variations of the bolt load and the clamping force, the axial displacement of the joint could be obtained from the finite element results. The normal contact force on the interfaces between the bolt head and the clamped parts could be obtained as the bolt load, while the clamping force is obtained directly from the normal contact force on the interface between clamped parts. The axial displacement of

the joint is obtained by the combination of the axial displacements of four points on the load planes (denoted as 1, 2, 3 and 4 in Fig. 4):

$$u = \frac{u_1 + u_2}{2} - \frac{u_3 + u_4}{2}. \tag{20}$$

### 5 Results and discussion

The mechanical behaviors of the single bolted lap joint could be analyzed by the theoretical equations and by the finite element methods. The variations of the bolt load, the clamping force and the axial displacement with the external load are obtained theoretically and compared with the finite element results.

The outer radius of the clamped parts is set as  $r_o = 14$  mm, and the total grip length is  $L = 20$  mm with the two clamped parts having the same thickness. The tangent of the cone angle is set as  $0.4326$  for the full contact case according to the empirical formulation in [2].

The dimensionless values of the distance between the load planes and the axial positions of the load planes are defined in the following study. The distance between the load planes is normalized by the total grip length of the bolted joint:

$$L'_p = \frac{L_p}{L}, L'_{p1} = \frac{2L_{p1}}{L}, L'_{p2} = \frac{2L_{p2}}{L}. \tag{21}$$

And the distances of the load planes to the interface between the clamped parts are normalized by half of the total grip length:

$$L'_{p1} = \frac{2L_{p1}}{L}, L'_{p2} = \frac{2L_{p2}}{L}. \tag{22}$$

The mechanical behaviors of the cases of the bolted joint with the same material and with different materials are to be studied.

#### 5.1 The joint with the same material

For the bolted joint with the same material, the effect of the distance between the load planes is investigated at first.

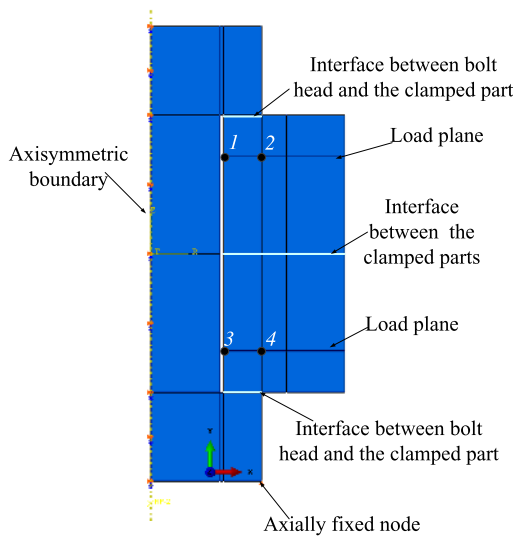


Fig. 4. Finite element model of the bolted joint.

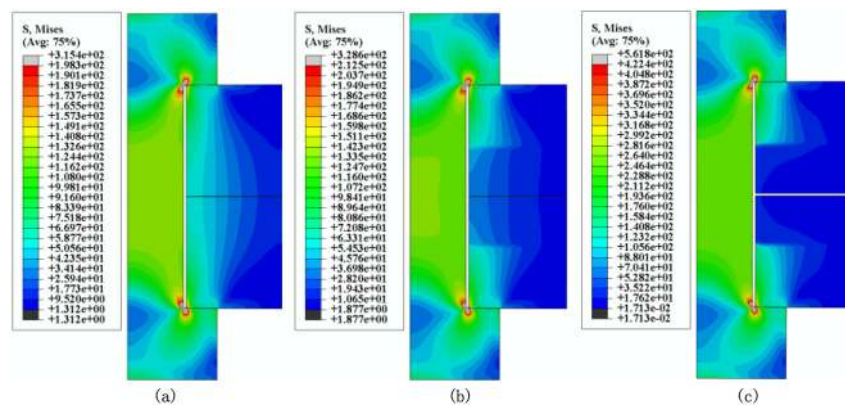
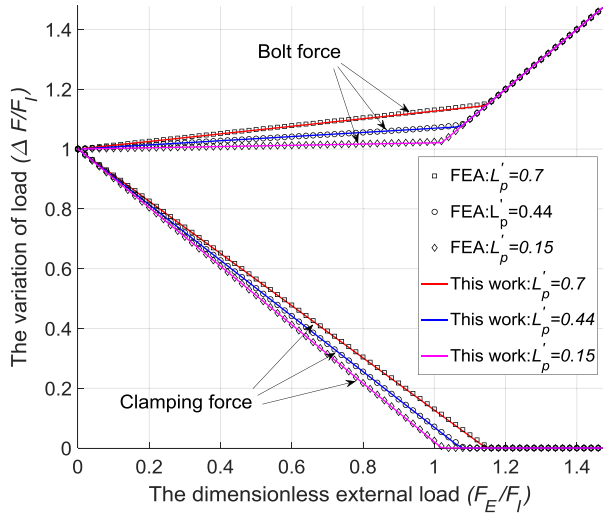
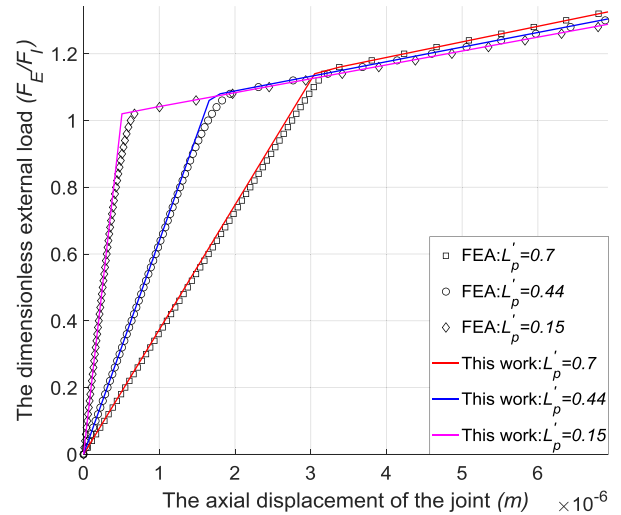


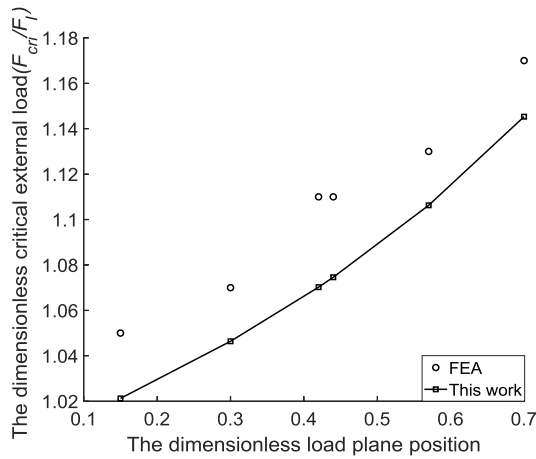
Fig. 5. Stress distribution of the bolted joint under the axial external axial load: (a) the axial external load is zero, (b) the external load is 6000 N, (c) the external load is 20 000 N.



**Fig. 6.** Variations of the bolt load and the clamping force with the distance between load planes.



**Fig. 8.** Variations of the axial displacements with external load for different distances between load planes.



**Fig. 7.** Variation of the separation load with the distance between load planes.

In this case, the load planes are located evenly at each side of the interface, and the distance between the load planes is set as 0.7, 0.44 and 0.15.

As shown in Figure 6, the variations of the bolt load and the clamping force with the external load for different external load positions are compared. Under the same external load, with the increase of the distance between the load planes, the variation of the bolt load increases while the variation of the clamping force decrease. The results predicted by the formulations are consistent with the finite element results, which indicate that the proposed formulations of the full contact case are capable to predict the variations of the loads with the external load accurately.

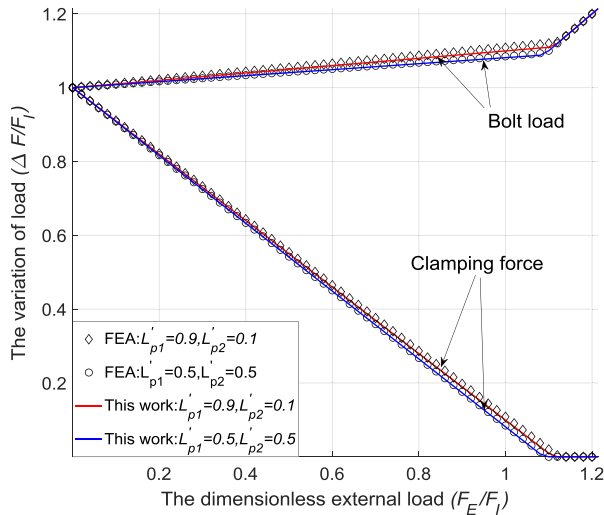
The variations of the critical external load are obtained and compared with the finite element results, as depicted in Figure 7. The critical external load increases with the distance between the load planes. The critical external loads predicted by this work are less than the finite element results, which results from the fact that the local

mechanical behaviors of the bolt and the clamped parts are not considered in the theoretical equations, which is derived based on the average mechanical behaviors on the sections of bolt and the pressure cone. Nevertheless, the maximum error of the critical external load predicted by this work is 3.58%, which is of adequate accuracy for engineering.

The axial displacements of the bolted joint for the three cases are given in Figure 8. The axial displacement varies linearly with the external load. There is an apparent decrease of the gradient of the load-displacement curve at the critical external load. The external load position has a significant influence on the load-displacement gradient when the external load is less than the critical load. The increase of the distance of the load plane to the interface will result in the decrease of the load-displacement gradient. The results predicted by this work are close to the finite element results, which indicate that the formulations in this work are capable to describe the axial deformation of the bolted joint accurately.

When the distance between the load planes is kept constant as 0.5, the influence of the axial positions of the load planes within the clamped parts is studied. As shown in Figure 9, the variations of the bolt load and the clamping force with the external load are plotted for the two cases with different positions of the load planes. In this case, the axial position of the load planes has little effect on the variations of the loads. The comparison with the finite element results indicates that the analytical formulations in this work could account for the effect of axial positions of load planes well.

For the bolted joint with the same material, the distance between the load planes has apparent influence on the axial mechanical behavior of the bolted joint, while effect of the axial position for the load planes with constant distance is negligible. The comparison with the finite element results validates the accuracy of the analytical formulations in this work.



**Fig. 9.** Variations of the axial displacements with external load for different axial positions of load planes.

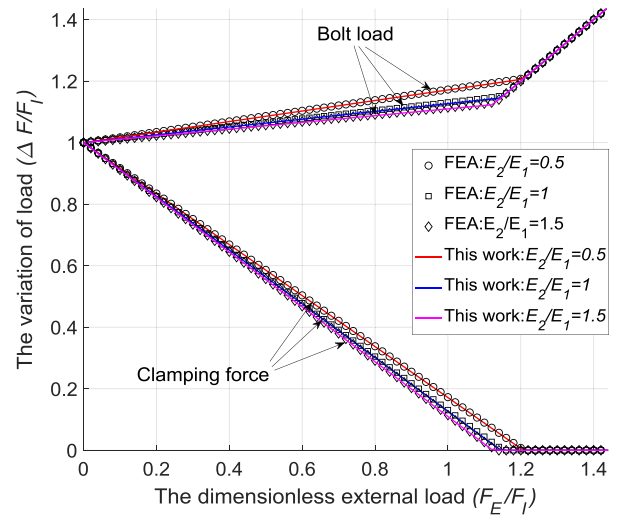
## 5.2 The bolted joint with different materials

It is common for a bolted joint in engineering having different materials. According to the basic assumption in [2], the analytical formulations for the mechanical behaviors of the bolted joint are not able to predict the mechanical behaviors of the bolted joint with different materials. Since the Young's modulus is considered in the formulation of the axial resilience, it is convenient to predict the axial mechanical behavior of the bolted joint with different materials by the formulations in this work. The theoretical results are compared with the finite element results. The model with the same geometrical parameters of the above case is adopted in this study. The bolt and one of the clamped parts have the same material with the Young's modulus as  $E_1 = 200$  GPa, while the Young's modulus of the other clamped part is set as 100 GPa, 200 GPa and 300 GPa in this study.

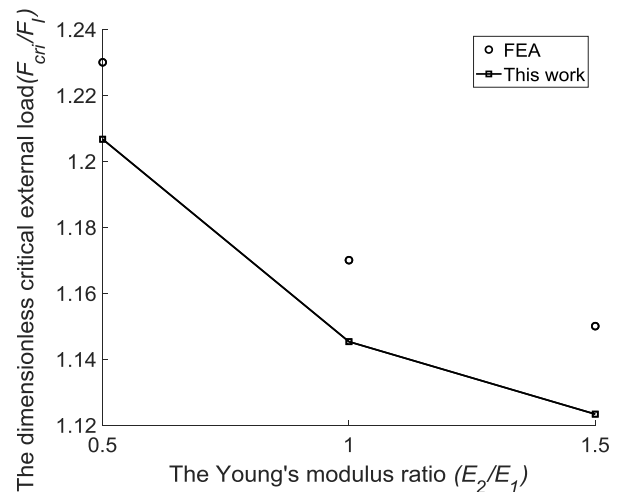
The variations of the bolt load and the clamping force for different Young's modulus combinations are plotted in Figure 10. Apparently, the difference of the Young's modulus of the clamped parts has influence on the variations of the bolt load and the clamping force. The comparison with finite element results indicates that the formulations in this work are capable to predict the variations of the loads with external load accurately.

The comparison of the critical external load predicted theoretically with that of the finite element results is given in Figure 11. The critical external load predicted by the formulations is less than the finite element results with the maximum difference is 2.32%. This work could predict the critical external load precisely for the bolted joint with different materials.

With the three combinations of the Young's modulus of the clamped parts, the axial displacements of the bolted joint subjected to the axial external load are given in Figure 12. Although there is a slight discrepancy around the critical external load, this work could predict the axial displacement of this case accurately.



**Fig. 10.** Comparisons of the variations of the bolt load and the clamping force with the external load for the different combination of materials of the clamped parts.

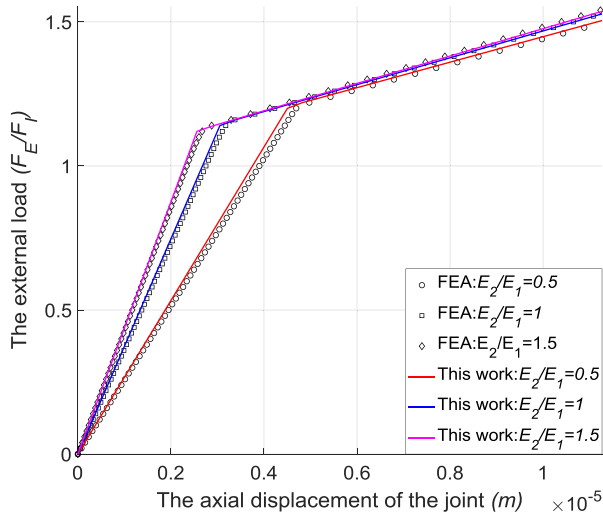


**Fig. 11.** Comparisons of the critical external load with the finite element analysis for the different combination of materials of the clamped parts.

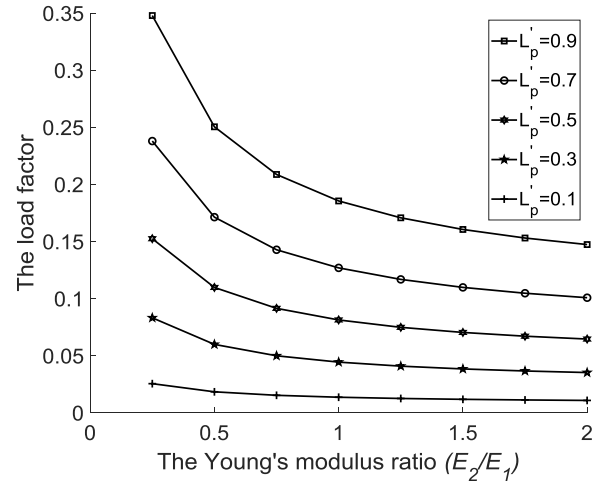
The comparisons with the finite element results indicate that this work could describe the mechanical behaviors of the bolted joint with different material properties accurately. Therefore, the influence of the Young's modulus ratio on the load factor for various load plane positions is studied by the formulations in this work.

In the first instance, the load planes are located evenly at each side of the interface in the study. According to the formulation of the load introduction factor, it is clear that once the locations of the load planes are determined, the load introduction factor will keep constant. However, due to the difference of the Young's modulus, the load factor decreases with the ratio of the Young's modulus, as plotted in Figure 13. The influence of the Young's modulus is different for the different positions of the load planes.

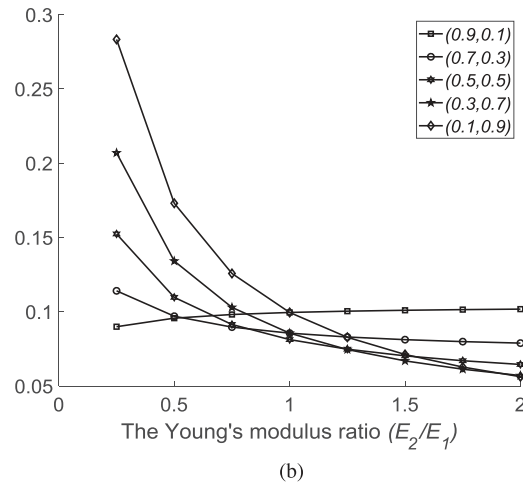
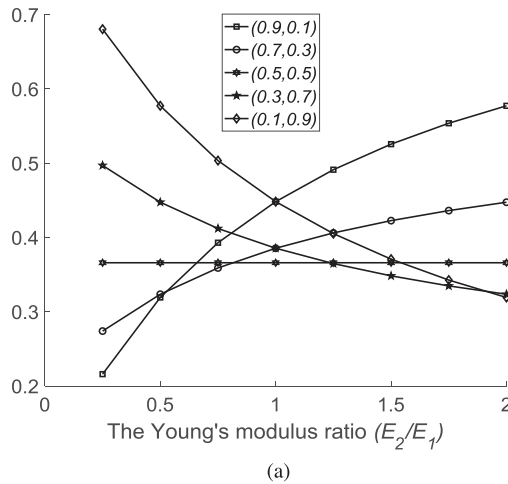




**Fig. 12.** Comparisons of the variations of the axial displacements with the external load for the different combination of materials of the clamped parts.



**Fig. 13.** Variations of the load factor for different cases of the material combination with the load planes locate evenly at two sides of the interface.



**Fig. 14.** Variations of the factor with the Young's modulus ratio for different axial positions while constant distance of the load planes: (a) the load introduction factor, (b) the load factor.

For the other case, the distance between the load planes is kept constant while the positions of the load planes vary in the axial direction. The influence of the Young's modulus ratio on the load introduction factor can be seen in Figure 14a. For the different combinations of the load plane positions, the effects of the Young's modulus on the load introduction factor are different. The variation of the load introduction factor with the Young's modulus ratio will affect the load factor. As depicted in Figure 14b, the variation of the load factor with the Young's modulus ratio is similar to that of the load introduction factor. Since the load factor depends on the total axial resilience of the clamped parts as well, the influence of the Young's modulus ratio on the load factor is a bit different from that on the load introduction factor.

It is obvious that for the bolted joint with different materials, both the distance between the load planes and the axial positions of the external load have influence on the mechanical behavior of the bolted joint. The proposed

formulations in this work are capable to consider the effects of the external load positions on the behaviors of the bolted joint with different materials accurately. The research work in this work could be an effective supplement for the bolted joint design guidelines [2].

It should note that in this work, only the axial position of the external load is considered. The radial position of the external load generally leads to nonlinear behavior of the bolted joint. Especially the coupling effect of the external tension, the moment and the shear load would make the mechanical behavior of the bolted joint more complex, and these effects will be taken into consideration in the future study.

## 6 Conclusions

The effect of the axial external load on the mechanical behavior of the bolted joint with different materials is

studied in this work. The axial resilience of the portions of the stress zone within the clamped parts divided by the external load planes and the interfaces are derived, with which the load introduction factor are obtained. The axial mechanical behaviors of the single bolted lap joint subjected to the axial external load with arbitrary loading positions are formulated. A finite element model the bolted joint is established and studied to validate the theoretical formulations. The following conclusions could be drawn by this study:

- The formulations in this work are capable to predict the variations of the bolt load and the clamping force, the critical external load and the axial displacement with the external load precisely, which could be an effective extension of the analytical formulations for the mechanical behaviors of the bolted joint [2].
- For the bolted joint with the clamped parts having the same material, the distance between the external load planes has apparent influence on the axial mechanical behavior, while the axial positions of the external load has little effect on the mechanical behavior of the bolted joint.
- Both the distance between the load planes and the axial positions of the external load have obvious influence on the axial mechanical behavior of the bolted joint with different materials. The difference of the Young's modulus of the clamped parts will result in the different variations of the load introduction factor and the load factor for different external loading positions. It should be taken in to consideration in the investigation of the mechanical behaviors of the bolted joint. The proposed formulation in this work could consider the influence of the different material properties on the mechanical behaviors accurately, which extends the valid range of the analytical formulations in bolted joint design guidelines [2].

## Nomenclature

$A_b$	The area of the section of the bolt
$A_c$	The area of the section of the clamped part
$E_c$	The Young's modulus of the clamped part
$E_u, E_d$	The Young's modulus of the upper and lower clamped part respectively
$E_1, E_2$	The Young's modulus of the components in the bolted joint
$F_E$	The external load
$F_{cri}$	The critical external load
$L$	The total grip length of the bolted joint
$L_n$	The thickness of the clamped part
$L_1, L_2$	The thickness of the upper and lower clamped parts respectively
$L_{p1}, L_{p2}$	The distance of the load planes to the interface
$L'_{p1}, L'_{p2}$	The dimensionless distance of the load planes to the interface
$d$	The diameter of the bolt
$n$	The load introduction factor

$r_0$	The outer radius of the clamped part
$r_H$	The radius of the bolt hole
$r_h$	The radius of the bolt head
$r_E$	The maximum radius of the pressure cone
$r_e$	The radius of the pressure cone
$u$	The axial displacement of the joint under the axial external load
$u_1, u_2, u_3, u_4$	The axial displacements of the four nodes in the finite element model
$z$	The variable in the axial direction
$\Phi_{kn}$	The load factor with considering the load introduction factor
$\Delta F_b$	The variation of the bolt load
$\Delta F_c$	The variation of the clamping force
$\Delta_c$	The axial displacement of the clamped part under bolt pretension
$\alpha$	The angle of the pressure cone
$\beta$	The bolted joint configuration factor
$\delta_b, \delta_c$	The axial resilience of the bolt and the clamped part
$\delta_g$	The axial resilience of the cylinder of the bolt
$\delta_{ip}$	The axial resilience of the stress zone between the load planes
$\delta_{ipn}$	The axial resilience of the portions of the stress zone between the load planes
$\delta_{In}, \delta_{In}$	The axial resilience of the portions of the clamped parts divided by the load planes and the interface
$\sigma$	The pressure on the section of the pressure cone

*Acknowledgements.* The authors are appreciated for the funding support from the National Natural Science Foundation of China (grant No. 11602071).

## References

- [1] H. Bickford, John, Introduction to the Design and Behavior of Bolted Joints: Non-Gasketed Joints, 4th ed. CRC Press, New York, 2007
- [2] VDI 2230 Bratt 1, Systematic calculation of high duty bolted joints. Joints with one cylindrical bolt, VDI, 2003
- [3] R. Sethuraman, T. Sasi Kumar, Finite element based member stiffness evaluation of axisymmetric bolted joints, J. Mech. Des. **131**, 011012 (2009)
- [4] T. Naruse, Y. Shibutani, Equivalent stiffness evaluations of clamped plates in bolted joints under loading, J. Solid Mech. Mater. Eng. **4**, 1791–1805 (2010)
- [5] T. Naruse, Y. Shibutani, Higher accurate estimation of axial and bending stiffnesses of plates clamped by bolts, J. Solid Mech. Mater. Eng. **6**, 397–406 (2012)
- [6] M. Couchaux, M. Hjjaj, I. Ryan, A. Bureau, Tensile resistances of bolted circular flange connections, Eng. Struct. **171**, 817–841 (2018)
- [7] H. Deng, X. Song, Z. Chen, P. Fu, J. Dong, Experiment and design methodology of a double-layered flange connection in axial loads, Eng. Struct. **175**, 436–456 (2018)

- [8] Z. Xiong, X. Guo, Y. Luo, H. Xu, Numerical analysis of aluminium alloy gusset joints subjected to bending moment and axial force, *Eng. Struct.* **152**, 1–13 (2017)
- [9] F. Meisami, M. Moavenian, A. Afsharfard, Nonlinear behavior of single bolted flange joints: a novel analytical model, *Eng. Struct.* **173**, 908–917 (2018)
- [10] European Committee for Standardization (CEN), En 1993-1-8-Eurocode 3- Design of Steel Structures- Part 1-8: Design of Joints, vol. 2, no. 2005, 2005
- [11] F. Alkatan, P. Stephan, A. Daidie, J. Guillot, Equivalent axial stiffness of various components in bolted joints subjected to axial loading, *Finite Elem. Anal. Des.* **43**, 589–598 (2007)
- [12] T.F. Lehnhoff, M.L. McKay, Member stiffness and contact pressure distribution of bolted joints, *J. Mech. Des.* **116**, 550–557 (1994)
- [13] N. Haidar, S. Obeed, M. Jawad, Mathematical representation of bolted-joint stiffness: a new suggested model, *J. Mech. Sci. Technol.* **25**, 2827–2834 (2011)
- [14] S.A. Nassar, A. Abboud, An improved stiffness model for bolted joints, *J. Mech. Des.* **131**, 121001 (2009)
- [15] N.L. Pedersen, P. Pedersen, On prestress stiffness analysis of bolt-plate contact assemblies, *Arch. Appl. Mech.* **78**, 75–88 (2008)
- [16] N.L. Pedersen, P. Pedersen, Stiffness analysis and improvement of bolt-plate contact assemblies, *Mech. Based Des. Struct. Mach.* **36**, 47–66 (2008)
- [17] J.G. Williams, R.E. Anley, D.H. Nash, T.G.F. Gray, Analysis of externally loaded bolted joints: analytical, computational and experimental study, *Int. J. Press. Vessel. Pip.* **86**, 420–427 (2009)
- [18] I.R. Grosse, L.D. Mitchell, Nonlinear axial stiffness characteristics of axisymmetric bolted joints, *J. Mech. Des.* **112**, 442 (1990)
- [19] N.L. Pedersen, P. Pedersen, Bolt-plate contact assemblies with prestress and external loads: solved with super element technique, *Comput. Struct.* **87**, 1374–1383 (2009)
- [20] G. Meyer, D. Strelow, Simple diagrams aid in analyzing force in bolted joints, *Assem. Eng.* **15**, 28–33 (1972)
- [21] T.F. Lehnhoff, B.A. Bunyard, Effects of bolt threads on the stiffness of bolted joints, *J. Press. Vessel Technol.* **123**, 161 (2001)
- [22] T. Naruse, Axial and bending stiffnesses of clamped plates of bolted joints based on accurate equivalent cone, *Trans. Jpn. Soc. Mech. Eng. Ser. A* **76**, 1234–1240 (2010)
- [23] N. Motosh, Determination of joint stiffness in bolted connections, *J. Eng. Ind.* **98**, 858–861 (1976)
- [24] ABAQUS, User's Manual, Version 6.14, 2014

**Cite this article as:** P. He, Z. Fan, X. Yu, Z. Liu, Influence of the axial position of the external load on the mechanical behavior of the bolted lap joint with different materials, *Mechanics & Industry* **20**, 623 (2019)