

# Influence of van der Waals and Casimir Forces on Electrostatic Torsional Actuators

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**Abstract**—The influence of van der Waals (vdW) and Casimir forces on the stability of the electrostatic torsional nanoelectromechanical systems (NEMS) actuators is analyzed in the paper. With the consideration of vdW and Casimir effects, the dependence of the critical tilting angle and pull-in voltage on the sizes of structure is investigated. And the influence of vdW torque is compared with that of Casimir torque. The modified coefficients of vdW and Casimir torques on the pull-in voltage are, respectively, calculated. When the gap is sufficiently small, pull-in can still take place with arbitrary small angle perturbation because of the action of vdW and Casimir torques even if there is not electrostatic torque. And the critical pull-in gaps for two cases are, respectively, derived. [1222]

**Index Terms**—Casimir, electrostatic torsional actuator, M/NEMS, pull-in, van der Waals (vdW).

## I. INTRODUCTION

THERE are many kinds of microelectromechanical systems (MEMS) or nanoelectromechanical systems (NEMS) actuated by electrostatic forces, which include parallel plate actuators as well as torsional actuators, etc. An important parameter of electrostatic actuators is the pull-in voltage. In equilibrium, the electrostatic force/torque and the restoring force/torque are equal, which result in a stable condition of the actuator. As the voltage increases, the electrostatic force/torque increases and eventually exceeds the restoring force/torque, results in pull-in phenomena. The critical voltage is accordingly defined as pull-in voltage. To a parallel plate actuator, a critical gap between two plates can be defined. However, to a torsional actuator, in contrast to parallel plate actuator, one plate tilts when the voltage is applied, and a tilting angle  $\varphi$  appears between the two plates. So there exists not only a critical gap similar to that of parallel plate actuator, but also a critical tilting angle.

The pull-in analyses of the electrostatic torsional microactuators have been extensively reported in many literatures [1]–[14], and the analytical models have been derived for the calculation of the pull-in voltage and tilting angle. Degani *et al.* [1]–[3] proposed a pull-in polynomial algebraic equation for the pull-in angle and pull-in voltage of electrostatic actuator.

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The dependence of pull-in parameters on the design and properties of the actuator are also discussed. Farmer *et al.* [4]–[6] developed an angle-based design approach for rectangular and round double-gimbaled electrostatic torsional actuators, and presented the comparison between the results of theoretical calculation and those of numerical simulation and experiment. Plötz *et al.* [7]–[10] studied an electrostatic torsional RF MEMS switch with low actuation voltage via experiment, theory, and simulation approaches. The design and stability analysis of the RF switch are also presented. The electrostatic torsional mirror is investigated from experiment and theory by [11]–[14]. However, researches above on the electrostatic torsional actuators did not consider van der Waals (vdW) and Casimir effects, which can be neglected in MEMS, but play important roles in NEMS [15], [16].

An important feature of Casimir effect is that even though it is quantum in nature, it predicts a force between macroscopic bodies [16], [17]. This makes the Casimir force relevant to MEMS and NEMS. The Casimir force fundamentally influences the performance and yield of NEMS devices [17], [18]. Of course, the Casimir force can also be put to good use. Capasso *et al.* have shown how the force can be used to actuate the MEMS devices [19]. VdW force has been reported to be associated with Casimir force [16], [20], [22]. Stiction caused by vdW effect in MEMS is studied and a theoretical model is presented by van Spengen, Puers, and De Wolf [15]. Zhao *et al.* have done some researches in MEMS for years [22]–[25]. Lin and Zhao [23] studied the dynamic behavior of nanoscale electrostatic parallel plate RF switches with the consideration of the vdW effects. They found that the nanoscale electrostatic parallel plate RF switch exhibits a bifurcation from the case excluding an equilibrium point to the case including two equilibrium points as the geometrical dimensions of the device are altered [23], and adhesion [22] can be induced by vdW force, when the ratio of the gap to the length is sufficiently small, the detachment length is determined for a given gap of the parallel plate RF switches.

In this paper, the simplified one-degree of freedom (1DOF) model is applied to study the inherent instability of the electrostatic torsional NEMS actuator, and the influence of vdW and Casimir torques is investigated. Several kinds of driving torques and an elastic restoring torque are derived in Section II. In Section III, the dimensionless equilibrium equations are derived. Then the influence of vdW and Casimir torques on critical tilting angles and pull-in voltages is, respectively, compared and analyzed in details. In addition, the critical gaps under the actions of vdW and Casimir torques are, respectively, derived when there is not the electrostatic torque.

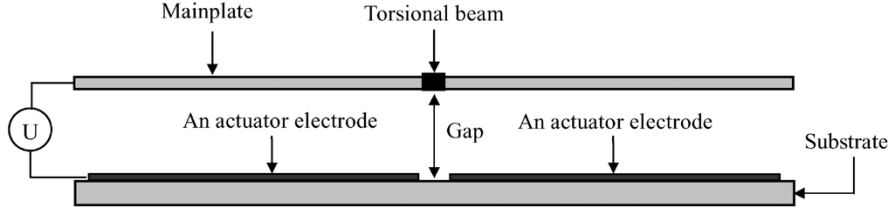


Fig. 1. Schematic side view of the torsional actuator.

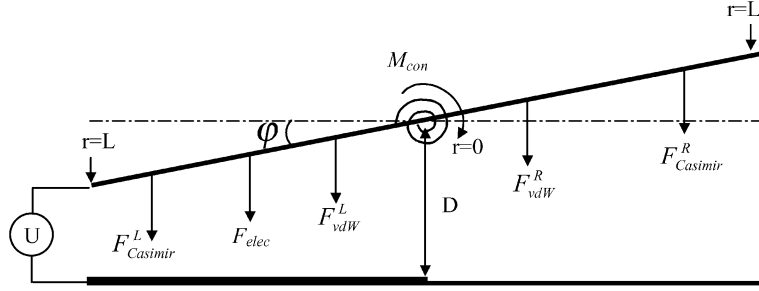


Fig. 2. 1DOF model of torsional actuator with vdW and Casimir forces.

## II. THEORETICAL MODEL

The Schematic of an electrostatic torsional actuator is shown in Fig. 1. In a first-order approximation, the mainplate of the device is considered to be a tiltable rigid body. Hence, the model has one degree of freedom, and the angle of torsion  $\varphi$  is the only variable (shown as Fig. 2). When a voltage between one of the actuator electrodes and the mainplate is applied, an electrostatic attracting force between them will produce an electrostatic torque, and under the action of the torque, the mainplate will tilt. The electrostatic torque is expressed as [7]

$$M_{elec} = \frac{1}{2} \varepsilon w U^2 \cdot \frac{1}{\sin^2 \varphi} \left[ \frac{D}{D - L \sin \varphi} - 1 + \log \left( \frac{D - L \sin \varphi}{D} \right) \right] \quad (1)$$

where  $\varepsilon$  denotes the electric permittivity,  $U$  the applied voltage,  $D$  the gap between torsion axis and bottom electrodes, and  $L$ ,  $w$ , and  $\varphi$  are, respectively, the half length, width, and tilting angle of the mainplate.

Besides the electrostatic force, vdW and Casimir forces also play important roles when the sizes of structure are sufficiently small. When two plates of torsional actuator keep parallel, i.e.,  $\varphi = 0$ , the vdW and Casimir forces at the left of fixed axis equal to their counterparts at the right. The resultant torque of forces at both sides to fixed axis is zero (Fig. 2). When the mainplate of actuator tilts, the vdW and Casimir forces at one side of fixed axis will be not equal to their counterparts at another side, and the resultants of vdW and Casimir torques are not zero.

When the mainplate rotates anticlockwise an angle  $\varphi$ , the vdW differential forces acting on parallel differential plates with width  $w$  and infinitesimal length  $dr$  at the both sides of torsional beam are, respectively

$$\begin{aligned} dF_{vdW}^L &= \frac{Aw}{6\pi} \cdot \frac{dr}{(D - r \sin \varphi)^3}, \\ dF_{vdW}^R &= \frac{Aw}{6\pi} \cdot \frac{dr}{(D + r \sin \varphi)^3} \end{aligned} \quad (2)$$

where  $A = \pi^2 C \rho^2$  is the Hamaker constant, which lies in the range  $(0.4 - 4) \times 10^{-19}$  J. The torque of these vdW differential forces to fixed torsional beam is

$$M_{vdW} = \int_0^L r \cdot (dF_{vdW}^L - dF_{vdW}^R) = \frac{Aw}{12\pi} \cdot \frac{1}{\sin^2 \varphi} \left[ \frac{D + 2L \sin \varphi}{(D + L \sin \varphi)^2} + \frac{-D + 2L \sin \varphi}{(-D + L \sin \varphi)^2} \right] \quad (3)$$

In the same way, Casimir torque can be derived as follows:

$$M_{Casimir} = \frac{\pi^2 \hbar c w}{1440} \cdot \frac{1}{\sin^2 \varphi} \left[ \frac{-D + 3L \sin \varphi}{(D - L \sin \varphi)^3} + \frac{D + 3L \sin \varphi}{(D + L \sin \varphi)^3} \right] \quad (4)$$

where  $\hbar$  is the Planck's constant divided by  $2\pi$ , which is equal to  $1.055 \times 10^{-34}$  Js, and  $c$  is the speed of light and equals to  $2.998 \times 10^8$  ms<sup>-1</sup>.

Due to  $D/L \ll 1$ , the tilting angle is small, and an approximation is used, i.e.,  $\sin \varphi \approx \varphi$ . Therefore, the maximum torsional angle is  $\varphi_0 \approx \sin \varphi_0 = D/L$  [7]. A dimensionless quantity  $d$  can be introduced as  $d = D/L$ , so  $\varphi_0 = d$ . The physical meaning of  $d$  is the ratio of the gap between two plates to the half-length of mainplate. Furthermore, by introducing the normalized tilting angle  $\gamma = \varphi/\varphi_0$ , where the value of  $\gamma$  is evidently in the range of  $[0, 1)$ , we rewrite the (1), (3), (4) as

$$M_{elec} = \frac{1}{2} \frac{\varepsilon w L^2 U^2}{D^2} \cdot \frac{1}{\gamma^2} \left[ \frac{\gamma}{1 - \gamma} + \log(1 - \gamma) \right] \quad (5)$$

$$M_{vdW} = \frac{Aw L^2}{3\pi D^3} \frac{\gamma}{(1 - \gamma^2)^2} \quad (6)$$

$$M_{Casimir} = \frac{\pi^2 \hbar c w L^2}{90 D^4} \frac{\gamma}{(1 - \gamma^2)^3} \quad (7)$$

When the mainplate rotates around the torsional beam, a restoring torque will be produced. The torque is caused by elastic restoring force of the beam, which can be simplified as a

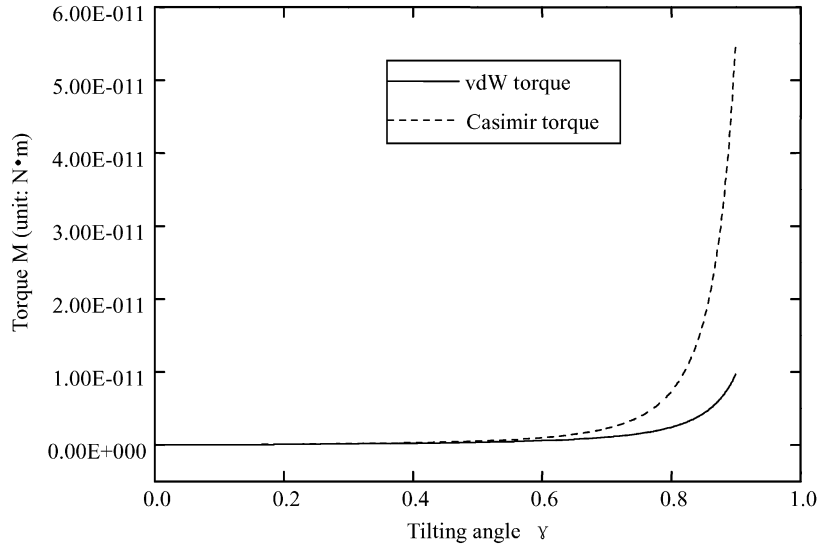


Fig. 3. Comparison between vdW torque and Casimir torque when the dimensionless gap is specified ( $d = 3 \times 10^{-3}$ ).

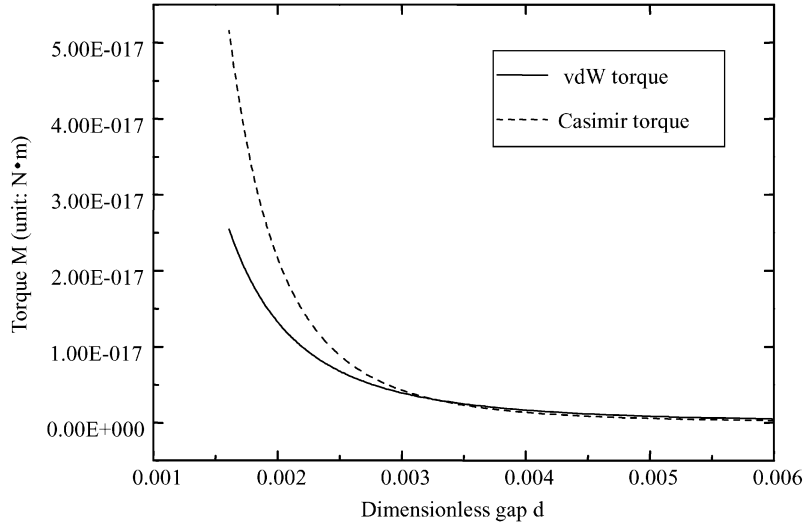


Fig. 4. Comparison between vdW torque and Casimir torque when the normalized tilting angle is specified ( $\gamma = 1.03 \times 10^{-5}$ ).

flexure spring with torsional stiffness  $K$ . Then elastic restoring torque of the torsional beam can be expressed as [7]

$$M_{con} = K\varphi = 2\frac{GJ_{\rho}D}{lL}\gamma \quad (8)$$

where  $G, J_{\rho}, l$ , respectively, denotes the shear modulus, the area moment of inertia and the length of torsional beam springs. The area moment of inertia  $J_{\rho}$  of the rectangular cross section is expressed as [12]

$$J_{\rho} = \begin{cases} ab^3 \left[ \frac{1}{3} - 0.21\frac{b}{a} \left( 1 - \frac{b^4}{12a^4} \right) \right] & (b \leq a) \\ a^3b \left[ \frac{1}{3} - 0.21\frac{a}{b} \left( 1 - \frac{a^4}{12b^4} \right) \right] & (b \geq a) \end{cases} \quad (9)$$

where  $a$  and  $b$  are the thickness and the width of torsional beam, respectively.

### III. STABILITY ANALYSIS OF ACTUATORS

Before the stability analysis, we first discuss vdW and Casimir forces. VdW and Casimir forces are both attractions between uncharged surfaces. They have close association. Lifshitz [20] compared them in 1956, and the dominant ranges of both forces for two thick films are given in experiments [21]. However, for torsional actuators, the vdW and Casimir torques dependent on not only separation between two surfaces but also tilting angle. By using (6) and (7), we compare, respectively, these two kinds of torques on the condition of specified dimensionless gap and normalized tilting angle (Figs. 3–4). It is not difficult to see that the difference of two dimensionless torques exists when the separation gap is in nano-scales, and it gets larger with the decrement of the gap. But the difference is not so sensitive to tilting angle when a small dimensionless gap is given. It is negligible unless the tilting angle is large enough. So we will study the influence of two types of torques

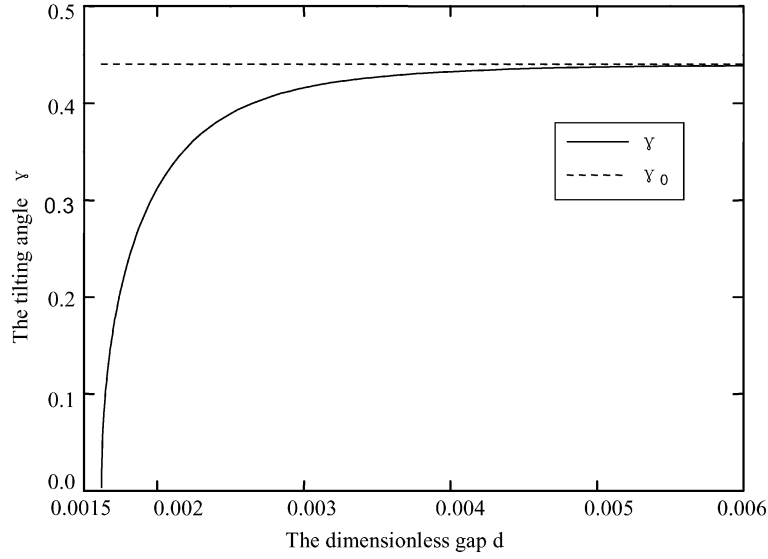


Fig. 5. Variation of the critical tilting angle with the dimensionless gap.

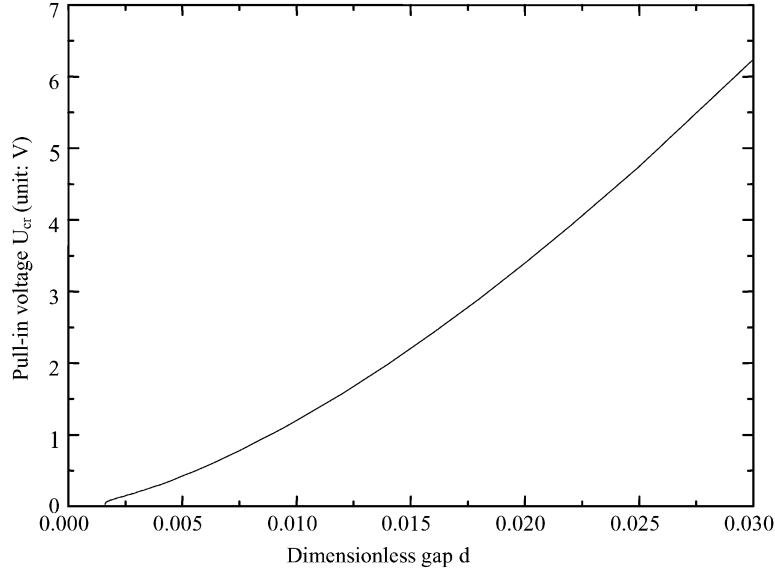


Fig. 6. Variation of the pull-in voltage with the dimensionless gap.

on torsional actuators in two sections, and compare critical tilting angles and pull-in voltages in two cases to continuously variational gaps.

#### A. vdW Torque

In this section, we mainly consider the influence of vdW force on the stability of torsional actuator. If the inertial and damping effects are neglected, the resultant of electrostatic and vdW torques reaches a balance with the elastic restoring torque of beam, i.e.,

$$M_{elec} + M_{vdW} - M_{con} = 0. \quad (10)$$

Substitute (5), (6), and (8) into (10), a dimensionless equilibrium equation can be derived

$$-\gamma + \mu \frac{1}{\gamma^2} \left[ \frac{\gamma}{1-\gamma} + \log(1-\gamma) \right] + \beta \frac{\gamma}{(1-\gamma^2)^2} = 0 \quad (11)$$

where  $\mu = (\epsilon w U^2)/(2Kd^3)$ ,  $\beta = (Aw)/(3\pi K L d^4)$ . Two dimensionless quantities denote, respectively, the ratios of the electrostatic and the vdW torques to the restoring torque.

Then the applied voltage can be expressed as the function of  $\gamma$

$$U(\gamma) = \left\{ \frac{2Kd^3}{\epsilon w} \cdot \frac{\gamma^3}{\frac{\gamma}{1-\gamma} + \log(1-\gamma)} \left[ 1 - \frac{\beta}{(1-\gamma^2)^2} \right] \right\}^{\frac{1}{2}}. \quad (12)$$

The electrostatic and vdW torques are dependent on the structural size and applied voltage. When the size of structure is specified, the resultant of electrostatic and vdW torques increases with the increase of applied voltage. When the voltage is low, the dimensionless equilibrium (11) has two nonzero real roots, where one is stable and the other unstable. When the voltage increases and reaches a certain value, the two roots coincide. The real root is defined as critical tilting angle  $\gamma_{cr}$ ,

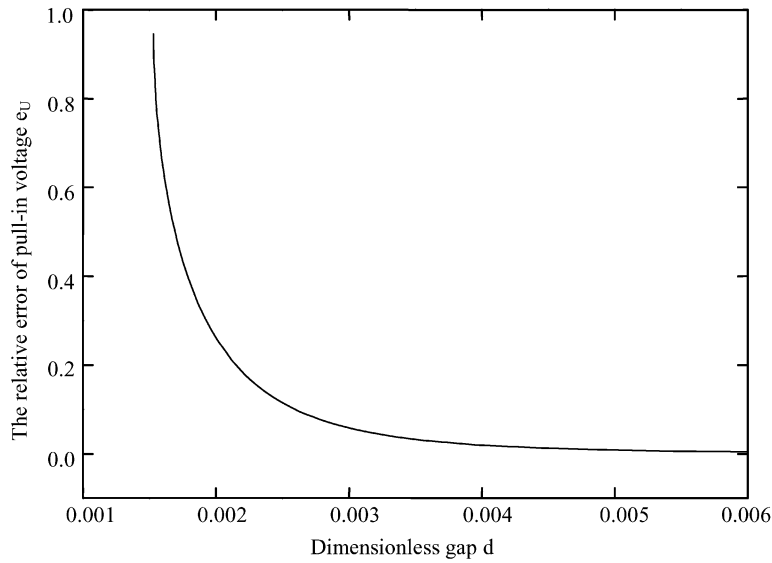


Fig. 7. Variation of the relative error of pull-in voltage with the dimensionless gap.

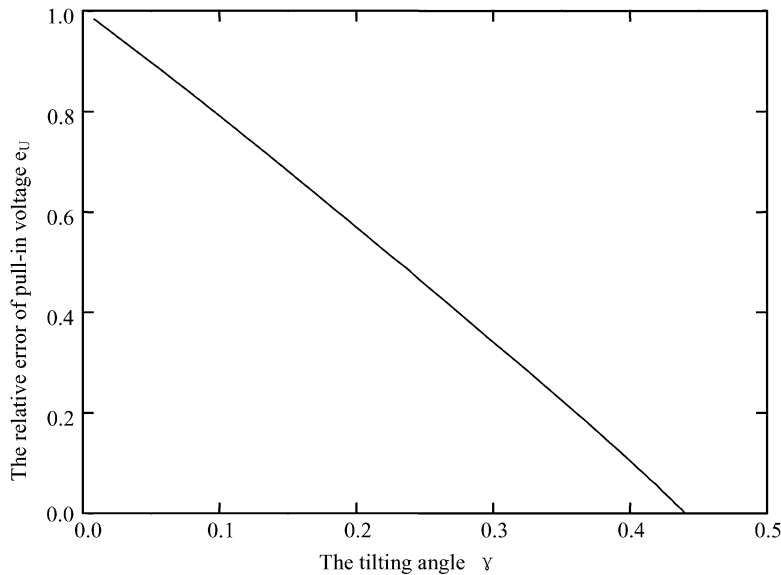


Fig. 8. Variation of the relative error of pull-in voltage with the normalized tilting angle.

and the voltage as pull-in voltage. Beyond the voltage, the mainplate will snap down. For the actuator with given sizes, the critical tilting angle  $\gamma_{cr}$  can be derived by calculating equation  $dU/d\gamma = 0$  [12]. Substitute  $\gamma_{cr}$  into (12), and the pull-in voltage  $U_{cr}$  can be obtained.

However, the values of critical tilting angle and pull-in voltage are in fact dependent on the sizes of structures. In this paper, we mainly consider the variation of the gap  $D$  between torsional beam and bottom electrodes. When other sizes are given, the variations of  $\gamma_{cr}$  and  $U_{cr}$  with the dimensionless gap  $d$  are, respectively, shown in Figs. 5 and 6. It can be seen from the Fig. 5 that the difference of critical tilting angles is distinct for two cases that one is with vdW force and the other without. With vdW force, the critical tilting angle decreases with the decrement of the gap, and the trend becomes considerably obvious when the dimensionless gap is less than about 0.005. Without vdW force, the critical tilting angle  $\gamma_{cr}$  is indepen-

dent of the structure parameters, and keeps a constant, i.e.,  $\gamma_{cr} \equiv 0.4404$  [7], [12]. And the corresponding pull-in voltage  $V_{cr}$  can be obtained by substituting  $\gamma_{cr} \equiv 0.4404$  and  $\beta = 0$  into (12), [7], [12].

To study the influence of vdW torque on pull-in voltages, we compare the pull-in voltage  $U_{cr}$  with  $V_{cr}$ . The pull-in voltage  $U_{cr}$  is always smaller than  $V_{cr}$ . The relative error of two kinds of pull-in voltage is calculated by

$$e_U = \frac{U_{cr} - V_{cr}}{U_{cr}}. \tag{13}$$

The variations of the relative error with dimensionless gap and critical tilting angle are, respectively, shown in Figs. 7 and 8. It can be seen from two figures that the relative error is not negligible unless the dimensionless gap is more than about 0.003 or the tilting angle reaches 0.4404. In fact, the vdW force can never be exactly zero, and it plays an important role when the sizes are sufficiently small, so it can not be neglected in the de-

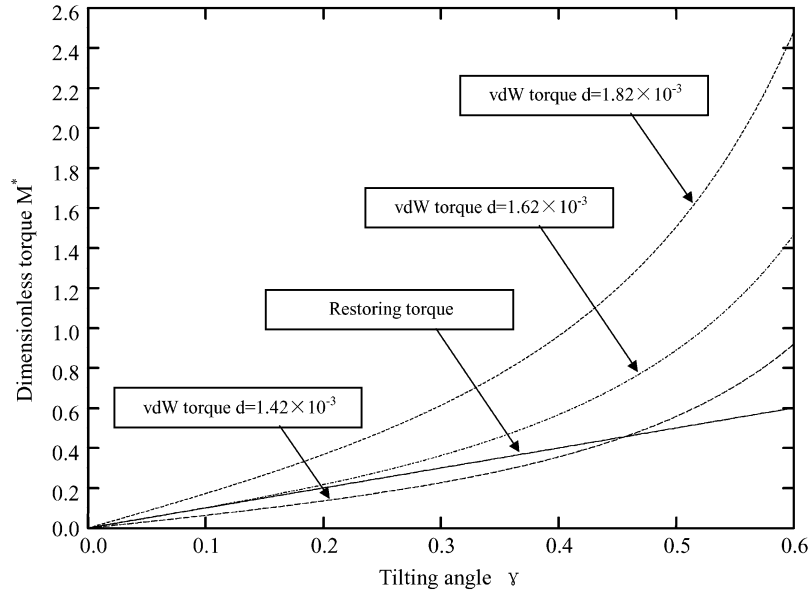


Fig. 9. Variation of the dimensionless vdW torque and restoring torque with the tilting angle for different gaps.

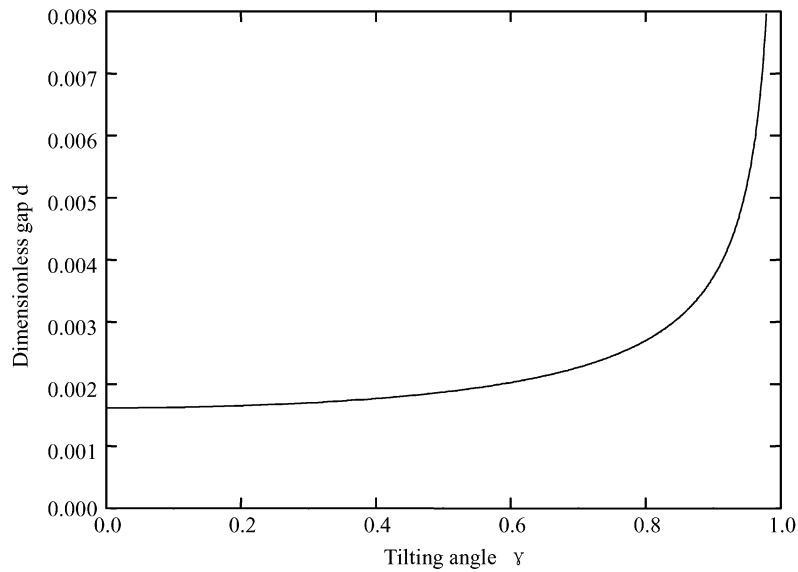


Fig. 10. Variation of the gap with the tilting angle when there is not electrostatic torque.

sign of torsional NEMS actuator. We introduce a parameter to modify pull-in voltage. It can be defined by the ratio of  $U_{cr}$  to  $V_{cr}$  as follows:

$$\Omega_{vdW} = \frac{U_{cr}}{V_{cr}} = \left[ 1 - \frac{\beta}{(1 - \gamma^2)^2} \right]^{\frac{1}{2}}. \quad (14)$$

We analyze the stability of the actuator without the electrostatic torque. Let  $U = 0$ , the dimensionless equilibrium (11) is reduced to (15)

$$-\gamma + \beta \frac{\gamma}{(1 - \gamma^2)^2} = 0. \quad (15)$$

If two plates of actuator keep parallel, the normalized tilting angle  $\gamma$  equals to zero and the system is stable. But if a small angle perturbation is given, it is possible for pull-in to take place. The angle perturbation is exactly dependent on the gap between

two plates. Compare the dimensionless vdW torque and elastic restoring torque for different gaps, the variations of two torques with normalized tilting angles are shown in Fig. 9. It can be seen that the vdW torque and elastic restoring torque have two equilibrium points for sufficiently small gap, where one is stable zero point and the other is unstable point. Correspondingly, (15) has a zero and a nonzero real roots. When the value of small angle perturbation is equal to or beyond the nonzero real root, the pull-in will happen. The actuators with different gaps have different critical values of small angle perturbations. The relation of dimensionless gaps and critical angle perturbations is shown in Fig. 10.

When the gap reaches a certain value, two equilibrium points coincide near the zero. This gap is defined as the critical gap. For gaps below the critical value, the mainplate snaps down for

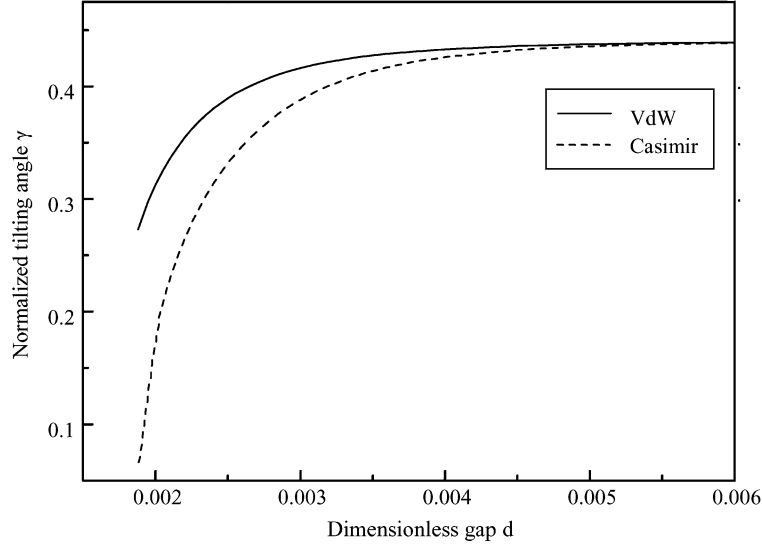


Fig. 11. Comparison of critical tilting angle with the vdW effect and that with the Casimir effect.

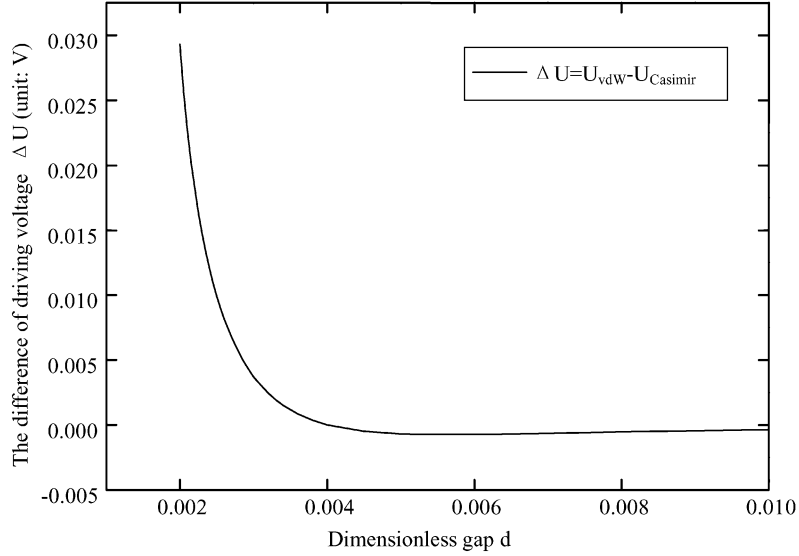


Fig. 12. Difference of pull-in voltages between the case with the consideration of vdW torque and the case with consideration of Casimir torque.

arbitrary small angle perturbation. The critical gap can be calculated by

$$d_{cr} = \lim_{\gamma \rightarrow 0} \left[ \frac{Aw}{3\pi KL} \cdot \frac{1}{(1-\gamma^2)^2} \right]^{\frac{1}{4}} = \left[ \frac{Aw}{3\pi KL} \right]^{\frac{1}{4}}. \quad (16)$$

### B. Casimir Torque

In this section, we discuss the influence of Casimir torque on the stability of actuators. We simply replace vdW torque with Casimir torque in (10). The dimensionless equilibrium equation can be written as

$$-\gamma + \mu \frac{1}{\gamma^2} \left[ \frac{\gamma}{1-\gamma} + \log(1-\gamma) \right] + \alpha \frac{\gamma}{(1-\gamma^2)^3} = 0 \quad (17)$$

where  $\alpha = (\pi^2 \hbar c w) / (90 K L^2 d^5)$ .

Accordingly the applied voltage is derived as

$$U(\gamma) = \left\{ \frac{2Kd^3}{\epsilon w} \cdot \frac{\gamma^3}{\frac{\gamma}{1-\gamma} + \log(1-\gamma)} \left[ 1 - \frac{\alpha}{(1-\gamma^2)^3} \right] \right\}^{\frac{1}{2}}. \quad (18)$$

When the sizes of actuators are given, the critical tilting angle  $\gamma_{cr}$  can be derived by the calculation of the equation  $dU/d\gamma = 0$ . Substitute  $\gamma_{cr}$  into (18), pull-in voltage  $U_{cr}$  can be obtained. Similar to the discussion of Section III-A, we can give the curves to illustrate the variations of critical tilting angles and pull-in voltages with dimensionless gaps. And we compare them to their counterparts in Section III-A. It can be seen from Fig. 11 that the critical tilting angle with Casimir torque is smaller than that with vdW torque. The difference of angles in two cases is distinct only when the gap is close to nano-scales, though both

TABLE I  
PARAMETERS OF NUMERICAL CALCULATION

Class	Parameter		
	Items	Symbol	Value
Physical constant	Hamaker constant	$A$	$(0.4\sim 4)\times 10^{-19}$ J
	Planck's constant divided by $2\pi$	$\hbar$	$1.055\times 10^{-34}$ Js
	Speed of light	$c$	$2.998\times 10^8$ ms <sup>-1</sup>
	Shear modulus of polysilicon	$G$	$6.6\times 10^{10}$ Pa
The movable plate	Length	$L$	$10^{-4}$ m
	Width	$w$	$10^{-4}$ m
Torsional beam	Length	$l$	$6.5\times 10^{-5}$ m
	Width	$B$	$1.55\times 10^{-6}$ m
	Thickness	$a$	$1.5\times 10^{-6}$ m
The gap	Dimensionless gap	$d$	$(0\sim 3)\times 10^{-2}$

of them have identical tendency. The difference of critical voltages in two cases is shown in Fig. 12. The pull-in voltage with the consideration of Casimir torque is lower than that with consideration of vdW torque though the difference of both is negligible in quantity. So the influence of Casimir torque on the pull-in voltage is accordingly greater. A modified parameter of the pull-in voltage with Casimir torque can be derived as

$$\Omega_{\text{Casimir}} = \left[ 1 - \frac{\alpha}{(1 - \gamma^2)^3} \right]^{\frac{1}{2}}. \quad (19)$$

Without applied voltage, the torsional actuator can still snap-down when small angle perturbation is given. The angle perturbation is subject to the gap between two plates. The critical gap under Casimir torque can be calculated as

$$d'_{\text{cr}} = \left[ \frac{\pi^2 \hbar c w}{90 K L^2} \right]^{\frac{1}{5}}. \quad (20)$$

When the gap reaches  $d'_{\text{cr}}$ , for an arbitrarily small disturbance, the pull-in will take place.

#### IV. CONCLUSION

The dimensionless equilibrium equations of electrostatic torsional actuators are presented with the consideration of the vdW and Casimir forces in the paper. Because the influence of vdW and Casimir torques, the inherent instability of the actuators is dependent on the scales of structures. The critical tilting angle of mainplate approximately keeps constant only in micron or larger scales, but it is not constant when the gap between two

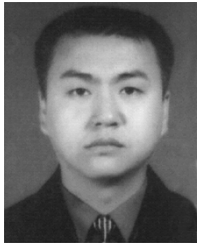
plates is in nano-scales. The tilting angle evidently decreases with the decrement of the gap. The critical pull-in voltage has large relative error for sufficiently small gap if vdW and Casimir torques are neglected. The modified coefficients of vdW torque and Casimir torque to pull-in voltage are presented. Without the electrostatic torque, the pull-in can still occur with small angle perturbation, and the critical gaps are, respectively, derived for vdW and Casimir torques. By comparison of vdW and Casimir torques as well as their corresponding critical tilting angles and pull-in voltages, we draw the conclusion that the influence of Casimir torque on torsional NEMS actuators is more notable than that of vdW torque.

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