DE GRUYTER OPEN

Int. J. of Applied Mechanics and Engineering, 2017, vol.22, No.3, pp.717-737
DOI: 10.1515/ijame-2017-0045

INFLUENCE OF WALL POROSITY AND SURFACES ROUGHNESS ON THE STEADY PERFORMANCE OF AN EXTERNALLY PRESSURIZED HYDROSTATIC CONICAL BEARING LUBRICATED BY A RABINOWITSCH FLUID

A. WALICKA*, E. WALICKI, P. JURCZAK and J. FALICKI
University of Zielona Góra, Faculty of Mechanical Engineering
ul. Szafrana 4, 65-516 Zielona Góra, POLAND
E-mails: A.Walicka@ijame.uz.zgora.pl; E.Walicki@ijame.uz.zgora.pl
P.Jurczak@ibem.uz.zgora.pl; J.Falicki@ibem.uz.zgora.pl

In the paper, the influence of both the bearing surfaces roughness as well as porosity of one bearing surface on the pressure distribution and load-carrying capacity of a curvilinear, externally pressurized, thrust bearing is discussed. The equations of motion of a pseudo-plastic Rabinowitsch fluid are used to derive the Reynolds equation. After general considerations on the flow in a bearing clearance and in a porous layer using the Morgan-Cameron approximation and Christensen theory of hydrodynamic lubrication with rough bearing surfaces the modified Reynolds equation is obtained. The analytical solution is presented; as a result one obtains the formulae expressing the pressure distribution and load-carrying capacity. Thrust radial and conical bearings, externally pressurized, are considered as numerical examples.

Key words: pseudo-plastic fluid, Rabinowitsch model, curvilinear and conical thrust bearings, porous layer, Christensen roughness.

1. Introduction

The reduction of energy consumption and the increase of the efficiency are, nowadays, important parameters in the design of machine elements, in particular slide bearings. Efficiency is also strongly dependent on lubricant formulation and its rheological behaviour (non-Newtonian fluids).

Viscosity of lubricating oils predominantly decreases with an increase of temperature. This viscosity increases with the additives concentration and it is relatively independent of temperature and usually exhibits a non-linear relation between the shear stress and the rate of shear in shear flow. There is no generally acceptable theory taking into account the flow behaviour of non-Newtonian lubricants. Studies have been done on fluid film lubrication employing several models such as micropolar (see e.g.: Walicka, [1]) couple-stress (Walicki and Walicka [2]), mixture (Khonsari and Dai [3]), viscoplastic (Lipscomb and Denn [4]; Dorier and Tichy [5]), pseudo-plastic (Wada and Hayashi [6]; Swamy *et al*. [7]; Rajalingham *et al*. [8]). Naturally, this list is not complete and given only to present the possibility of mathematical modelling. A more complete list may be found in (Walicka [9]; Walicki [10]).

The contact between rough surfaces of two bodies has a strong influence on the phenomena of friction, wear and lubrication, as well as the heat and electricity conduction. In general, the structure of most surfaces appears to be random on a small scale. In recent years, a considerable amount of tribology research has been exactly devoted to the study of the effect of surface roughness or geometric imperfections on hydrodynamic lubrication because the bearings surfaces, in practice, are all rough and the height of the roughness asperities may have the same order as the mean bearing clearance. Under these conditions, the surface roughness

* To whom correspondence should be addressed

affects the bearing performance considerably. The work in this area has mainly been confined to impermeable surfaces. The well-established stochastic theory of hydrodynamic lubrication of rough surfaces developed by Christensen [11] forms the basis of this paper. In a series of works (Lin [12, 13]; Bujurke *et al*. [14]; Prakash and Tiwari [15]; Walicka [16, 17]; Walicka and Walicki [18, 19]) this model was applied to the study of the surface roughness of various geometrical configurations.

Porous bearings have been widely used in industry for a long time. Basing on the Darcy model, Morgan and Cameron [20] first presented theoretical research on these bearings. To get a better insight into the effect of surface roughness in porous bearings, Prakash and Tiwari [21] developed a stochastic theory of hydrodynamic lubrication of rough surfaces proposed by Christensen [11].

The modified Reynolds equation (Gurujan and Prakash [22]) applicable to two types of directional roughness structure were used by Walicka and Walicki [18, 19] to find bearing parameters for the squeeze film between two curvilinear surfaces.

In recent years, tribologists have done a great deal of work on pseudo-plastic lubricants; the viscosity of these kinds of lubricants displays a non-linear relationship between the shear stress and the shear strain rate. There are many known formulae to model this relationship. One of the first was power-series development and in consequence polynomials were suggested. The polynomial given by Kraemer and Wiliamson [23] which was later independently proposed by Rabinowitsch [24] should be mentioned here. In the sixties of the past century Rotem and Shinnar [25] have returned to the polynomial representation proposing their own model similar to this one of Rabinowitsch.

Theoretical considerations and some ranges of experiments carried out by Wada and Hayashi [6] indicated the good usefulness of the Rabinowitsch fluid to modelling various lubrication problems. These problems have been analyzed by many investigators such as journal bearings by Wada and Hayashi [6], Rajalingham *et al*. [8], Sharma *et al.* [26]. Swamy *et al.* [7], hydrostatic thrust bearing by a Singh *et al.* [27], squeeze film bearings by Hashimoto and Wada [28], Lin [29], Lin *et al.* [30]. More general lubrication problems include the hybrid bearings modelled by two generally non-coaxial surfaces of revolution which can work simultaneously as journal and/or thrust bearings. Some theoretical considerations about these bearings may be found in the works given by Walicka *et al.* [31, 32], Ratajczak *et al.* [33], Walicka and Walicki [34]; these authors considered both externally pressurized bearings with and without rotational inertia and squeeze film bearings lubricated by a Rotem-Shinnar fluid. From the results of all the papers referred to above, it follows that the pseudo-plastic lubricant properties affect the bearing performance significantly.

Of late the use of non-Newtonian fluids as lubricants in porous bearings has gained importance in modern industry. From many studies, the works by Walicka [16], which contains considerations on the inertia effects in rough porous squeeze film bearings with visco-plastic Shulman's type lubricant, Walicka and Jurczak [35], who considered pressure distribution in squeeze film bearing lubricated by Vočadlo visco-plastic lubricant should be mentioned.

In this paper, the Rabinowitsch fluid model is used to describe the pseudo-plastic behaviour of a lubricant in a conical thrust bearing, externally pressurized, with rough surfaces and one porous wall considered as a porous matrix. The modified Reynolds equation is derived and its general solution for the curvilinear thrust bearing is presented. The analysis is based on the assumption that the porous matrix consists of a system of capillaries of very small radii which allows a generalization of the Darcy law and use of the Morgan-Cameron approximation for the flow in a porous layer. According to the Christensen stochastic model [11], different forms of Reynolds equations are derived to take account of various types of surface roughness. Analytical solutions for the film pressure are presented for the longitudinal and circumferential roughness patterns.

2. Derivation of the Reynolds equation for the Rabinowitsch fluid

It may be assumed that lubricating oils, with a viscosity index improver added, exhibit the same characteristics as pseudo-plastic fluids. Rotem and Shinnar [25] proposed a method for expressing empirically the relation between the stress and the shear rate as

$$\frac{d\gamma}{dt}=\frac{\tau}{\mu}\left(1+\sum_{i=1}^{n}k_i\tau^{2i}\right). \tag{2.1}$$

Retaining only the first order term $(i=1)$ the above equation reduces to the following form

$$\mu\frac{d\gamma}{dt}=\tau+k\tau^3. \tag{2.2}$$

well known as a Rabinowitsch model. The three-dimensional notation of Eq.(2.2) may be expressed as (Walicka [9])

$$\mu\boldsymbol{A}_1=\boldsymbol{\Lambda}\left(1+k\Lambda^2\right) \qquad \text{where} \qquad \Lambda=\left[\frac{1}{2}\mathrm{tr}\left(\boldsymbol{\Lambda}^2\right)\right]^{\frac{1}{2}} \tag{2.3}$$

is the magnitude of the second-order shear stress tensor $\boldsymbol{\Lambda}$, but $\boldsymbol{A}_1$ is the first Rivlin-Ericksen kinematic tensor.

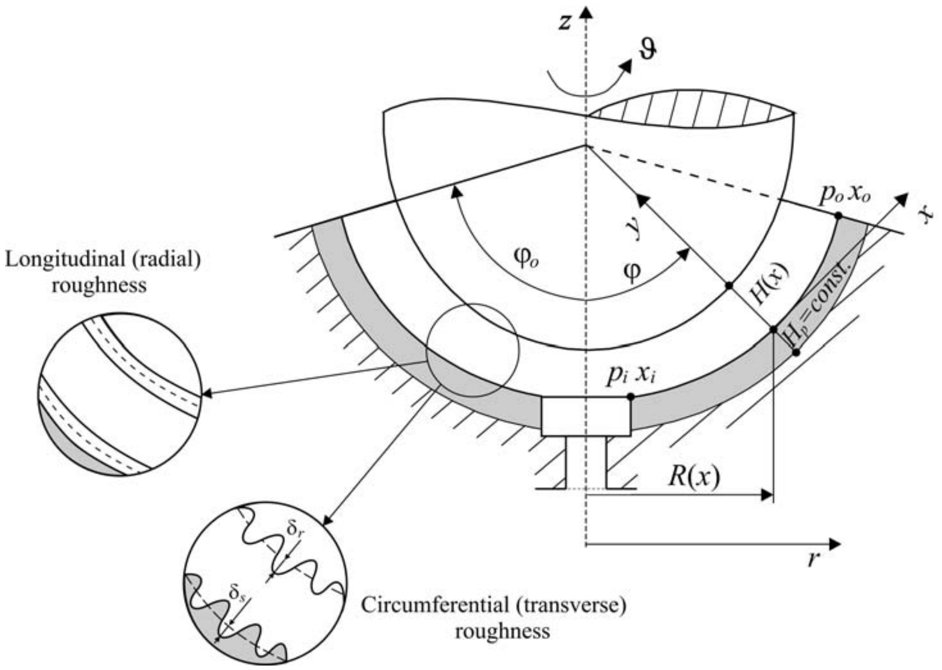

Fig.1. Geometry of a curvilinear thrust bearing.

Let us consider a thrust bearing with a curvilinear profile of the working surfaces shown in Fig.1. The upper bound of a porous layer is described by the function $R(x)$ which denotes the radius of this bound. The nominal bearing clearance thickness is given by the function $h(x)$, while the porous layer thickness is given by $H_p=\text{const.}$

The expression for the film thickness is considered to be made up of two parts.

$$H=h(x)+h_s(x,\vartheta,\xi) \tag{2.4}$$

where $h(x)$ represents the nominal smooth part of the film geometry, while $h_s=\delta_r+\delta_s$ denotes the random part resulting from the surface roughness asperities measured from the nominal level, ξ describes a random variable which characterizes the definite roughness arrangement. An intrinsic curvilinear orthogonal coordinate system x, ϑ, y linked with the upper surface of a porous layer is also presented in Fig.1.

Taking into account the considerations of the works (Walicka [9]; Walicki [10]) one may present the equation of continuity and the equations of motion of a Rabinowitsch fluid for axial symmetry in the form

$$\frac{1}{R}\frac{\partial\left(R\upsilon_x\right)}{\partial x}+\frac{\partial\upsilon_y}{\partial y}=0\,, \tag{2.5}$$

$$\frac{\partial\Lambda_{xy}}{\partial y}=\frac{\partial p}{\partial x}, \qquad \frac{\partial p}{\partial y}=0\,. \tag{2.6}$$

The constitutive Eq.(2.3)$_1$ takes the form

$$\mu\frac{\partial\upsilon_x}{\partial y}=\Lambda_{xy}+k\Lambda_{xy}^3\,. \tag{2.7}$$

The problem statement is complete after specification of boundary conditions. These conditions for the velocity components are stated as follows

$$\upsilon_x\left(x,0\right)=0\,, \qquad \upsilon_x\left(x,H\right)=0\,, \tag{2.8}$$

$$\upsilon_y\left(x,0\right)=V_H\,, \qquad \upsilon_y\left(x,H\right)=0\,. \tag{2.9}$$

Solving the equations of motion (2.5), (2.6) and taking into account the constitutive equation (2.7) one obtains the Reynolds equation [detailed solution may be found in works (Walicka [9]; Walicki [10])]

$$\frac{1}{R}\frac{\partial}{\partial x}RH^3\left[\frac{\partial p}{\partial x}+\frac{3}{20}kH^2\left(\frac{\partial p}{\partial x}\right)^3\right]=-12\mu V_H \tag{2.10}$$

for a lubricating pseudo-plastic Rabinowitsch fluid. If $k=0$, the above equation is identical to the Reynolds equation for a Newtonian lubricant.

3. Modified Reynolds equation for a bearing with a porous pad

To solve Eq.(2.10) let us study the flow of an R-R-S fluid in the porous layer. Assume that this layer consists of a system of capillaries with an average radius r_c and porosity φ_p. Let the porous layer be homogeneous and isotropic and let the flow within the layer satisfy the modified Darcy's law. Thus one has [10, 36]

$$\begin{aligned}\overline{\upsilon}_x&=\frac{\Phi_p}{\mu}\left(-\frac{\partial\overline{p}}{\partial x}\right)+\frac{\Phi_p}{\mu}\frac{kr_c^2}{6}\left(-\frac{\partial\overline{p}}{\partial x}\right)^3,\\ \overline{\upsilon}_y&=\frac{\Phi_p}{\mu}\left(-\frac{\partial\overline{p}}{\partial y}\right)+\frac{\Phi_p}{\mu}\frac{kr_c^2}{6}\left(-\frac{\partial\overline{p}}{\partial y}\right)^3,\end{aligned} \tag{3.1}$$

where $\overline{\upsilon}_x$, $\overline{\upsilon}_y$ are velocity components in the porous layer and

$$\Phi_p = \frac{\varphi_p r_c^2}{8}, \tag{3.2}$$

is permeability of the porous layer but φ_p is the coefficient of porosity.

Since the cross velocity component $\overline{\upsilon}_y$ must be continuous at the porous wall-fluid film interface and must be equal to V_H, we have then – by virtue of Eqs (2.10) and (3.1) – the following form of the modified Reynolds equation

$$\frac{1}{R}\frac{\partial}{\partial x} RH^3 \left[\frac{\partial p}{\partial x} + \frac{3}{20} kH^2 \left(\frac{\partial p}{\partial x} \right)^3 \right] = -12\Phi_p \left\{ \left(-\frac{\partial \overline{p}}{\partial y} \right) + \frac{kr_c^2}{6} \left(-\frac{\partial \overline{p}}{\partial y} \right)^3 \right\} \Bigg|_{y=0}. \tag{3.3}$$

Using the Morgan-Cameron approximation (Morgan and Cameron [20]) one obtains

$$\left\{ \left(\frac{\partial \overline{p}}{\partial y} \right) + \frac{kr_c^2}{6} \left(\frac{\partial \overline{p}}{\partial y} \right)^3 \right\} \Bigg|_{y=0} = -\frac{H_p}{R}\frac{\partial}{\partial x} R \left\{ \left(\frac{\partial p}{\partial x} \right) + \frac{kr_c^2}{6} \left(\frac{\partial p}{\partial x} \right)^3 \right\}. \tag{3.4}$$

When formula (3.4) is inserted into Eq.(3.3) the modified Reynolds equation takes the form

$$\frac{1}{R}\frac{\partial}{\partial x} R \left[\left(H^3 + \frac{3}{2}\varphi_p r_c^2 H_p \right) \frac{\partial p}{\partial x} + \frac{3k}{20} \left(H^5 + \frac{5}{3}\varphi_p r_c^4 H_p \right) \left(\frac{\partial p}{\partial x} \right)^3 \right] = 0. \tag{3.5}$$

If the film thickness is regarded as a random quantity, a height distribution function must be associated. Many real bearing surfaces show a roughness height distribution which is closely Gaussian, at least up to three standard deviations. From a practical point of view, the Gaussian distribution is rather inconvenient and therefore a polynomial form of its approximation is chosen. Following Christensen ([11], [37], [38]) such a probability density function is

$$f(h_s) = \begin{cases} \dfrac{35}{32c^7}\left(c^2 - h_s^2\right)^3, & -c \le h_s \le +c \\ 0, & \text{elsewhere} \end{cases} \tag{3.6}$$

where c is the half total range of the random film thickness variable. The function terminates at $c = \pm 3\sigma$, where σ is the standard deviation.

Inserting expected values in Eq.(3.5), we get the general form of the stochastic Reynolds equation

$$\frac{1}{R}\frac{\partial}{\partial x} \left(E\left\{ R \left[\left(H^3 + \frac{3}{2}\varphi_p r_c^2 H_p \right) \frac{\partial p}{\partial x} + \frac{3k}{20} \left(H^5 + \frac{5}{3}\varphi_p r_c^4 H_p \right) \left(\frac{\partial p}{\partial x} \right)^3 \right] \right\} \right) = 0 \tag{3.7}$$

where $E(\bullet)$ is the expectancy operator defined by

$$E(\bullet)=\int_{-c}^{+c}(\bullet)f(h_s)dh_s \tag{3.8}$$

The problem is now reduced to devising means of evaluating the left-hand side of Eq.(3.7) subject to the specific model of roughness.

The calculation of the mean film pressure distribution would require the evaluation of the expected value of various film thickness functions.
The forms of the distribution function described by Eq.(3.8) are given in (Walicka [17, 36]).

4. General solution to the modified Reynolds equation

In the present study, two types of roughness structure are of interest (see: Fig.1): the longitudinal (radial) one-dimensional roughness pattern, having the form of long narrow ridges and valleys running in the x direction, and the circumferential (transverse) one-dimensional roughness pattern, having the form of long narrow ridges and valleys running in the ϑ direction (Walicka [16, 36], Walicka and Walicki[18, 19]).
For the longitudinal one-dimensional roughness

$$H=h(x)+h_s(\vartheta,\xi) \tag{4.1}$$

the stochastic Reynolds equation is

$$\frac{1}{R}\frac{\partial}{\partial x}\left(R\left\{\left[E\left(H^3\right)+\frac{3}{2}\varphi_p r_c^2 H_p\right]\frac{\partial(Ep)}{\partial x}+\frac{3k}{20}\left[E\left(H^5\right)+\frac{5}{3}\varphi_p r_c^4 H_p\right]\left[\frac{\partial(Ep)}{\partial x}\right]^3\right\}\right)=0, \tag{4.2}$$

but for the circumferential one-dimensional roughness

$$H=h(x)+h_s(x,\xi) \tag{4.3}$$

the stochastic Reynolds equation is

$$\begin{gathered}\frac{1}{R}\frac{\partial}{\partial x}\left(R\left\{\left[\frac{1}{E\left(H^{-3}\right)}+\frac{3}{2}\varphi_p r_c^2 H_p\right]\frac{\partial(Ep)}{\partial x}+\right.\right.\\ \left.\left.+\frac{3k}{20}\left[\frac{1}{E\left(H^{-5}\right)}+\frac{5}{3}\varphi_p r_c^4 H_p\right]\left[\frac{\partial(Ep)}{\partial x}\right]^3\right\}\right)=0.\end{gathered} \tag{4.4}$$

Note that both Eqs (4.2) and (4.4) may be presented in one common form as follows

$$\frac{1}{R}\frac{\partial}{\partial x}\left(R\left\{\left[H_j^{(3)}+\frac{3}{2}\varphi_p r_c^2 H_p\right]\frac{\partial(Ep)}{\partial x}+\frac{3k}{20}\left[H_j^{(5)}+\frac{5}{3}\varphi_p r_c^4 H_p\right]\left[\frac{\partial(Ep)}{\partial x}\right]^3\right\}\right)=0, \tag{4.5}$$

where

$$H_j^{(3)} = \begin{cases} E\left(H^3\right) & \text{for} \quad j = l, \\ \dfrac{1}{E\left(H^{-3}\right)} & \text{for} \quad j = c, \end{cases} \qquad H_j^{(5)} = \begin{cases} E\left(H^5\right) & \text{for} \quad j = l, \\ \dfrac{1}{E\left(H^{-5}\right)} & \text{for} \quad j = c; \end{cases}$$

the case $j = l$ refers to the longitudinal one-dimensional roughness, but the case $j = c$ – to the circumferential one-dimensional roughness.

Consider the case of the R-R-S fluid of frequent occurrence for which the factor $k\Lambda_{xy}^2 < 1$; the value of this factor indicates that the solutions to the Reynolds equation (4.5) may be searched in a form of the sum [31, 32, 40]

$$Ep = Ep^{(0)} + Ep^{(1)}. \tag{4.6}$$

Assuming that $Ep^{(1)} << Ep^{(0)}$ and substituting Eq.(4.6) into Eq.(4.5), we arrive at two linearized equations, the first one

$$\frac{1}{R}\frac{\partial}{\partial x}\left\{ R\left[H_j^{(3)} + \frac{3}{2}\varphi_p r_c^2 H_p \right] \frac{\partial\left(Ep^{(0)}\right)}{\partial x} \right\} = 0, \tag{4.7}$$

and the other

$$\begin{gathered} \frac{1}{R}\frac{\partial}{\partial x}\left\{ R\left[H_j^{(3)} + \frac{3}{2}\varphi_p r_c^2 H_p \right] \frac{\partial\left(Ep^{(1)}\right)}{\partial x} \right\} = \\ = -\frac{3k}{20}\frac{1}{R}\frac{\partial}{\partial x}\left\{ R\left[H_j^{(5)} + \frac{5}{3}\varphi_p r_c^4 H_p \right] \left[\frac{\partial\left(Ep^{(0)}\right)}{\partial x} \right]^3 \right\}. \end{gathered} \tag{4.8}$$

The boundary conditions for pressure are

$$Ep^{(0)}\left(x_i\right) = p_i, \qquad Ep^{(0)}\left(x_o\right) = p_o, \qquad Ep^{(1)}\left(x_i\right) = Ep^{(1)}\left(x_o\right) = 0. \tag{4.9}$$

The solution of Eqs (4.7) and (4.8) is given as follows

$$\begin{gathered} Ep(x) = -\frac{3kC^3}{20}G(x) + \\ + \frac{\left[A(x) - A_o\right]\left(p_i + \dfrac{3kC^3}{20}G_i\right)}{A_i - A_o} - \frac{\left[A(x) - A_i\right]\left(p_o + \dfrac{3kC^3}{20}G_o\right)}{A_i - A_o} \end{gathered} \tag{4.10}$$

where

$$A(x)=\int\frac{dx}{R\left[H_j^{(3)}+\frac{3}{2}\varphi_p r_c^2 H_p\right]},\qquad A_i=A(x_i),\qquad A_o=A(x_o),$$

$$C=\frac{p_i-p_o}{A_i-A_o},\tag{4.11}$$

$$G(x)=\int\frac{\left[H_j^{(5)}+\frac{5}{3}\varphi_p r_c^4 H_p\right]dx}{R^3\left[H_j^{(3)}+\frac{3}{2}\varphi_p r_c^2 H_p\right]^4},\qquad G_i=G(x_i),\qquad G_o=G(x_o).$$

The load-carrying capacity is defined by

$$N=\pi R_i^2 p_i+2\pi\int_{x_i}^{x_o}EpR\cos\varphi dx;\tag{4.12}$$

the sense of angle φ arises from Fig.1.

All the above equations concern to general case of a curvilinear bearing. Note that the substitution

$$R(x)=sx\qquad\text{where}\qquad 0<s\le 1,$$

introduced into essential equations of this section, gives us the equations suitable for conical bearings (see Fig.5); for $s=1$ there are the equations suitable for radial bearings (see Fig.2).

The calculation of the mean film pressure distribution would require the calculation of the expected value for various film thicknesses. For the distribution function given by Eq.(3.7) we have (Walicka [17, 36])

$$E(H)=h,\qquad E\left(H^3\right)=h^3\left(1+\frac{1}{3}Y^2\right),\qquad E\left(H^5\right)=h^5\left(1+\frac{10}{9}Y^2+\frac{5}{33}Y^4\right),$$

$$E\left(H^{-1}\right)=\frac{1}{h}\left[1+\sum_{n=1}^{\infty}\frac{105\,Y^{2n}}{(2n+1)(2n+3)(2n+5)(2n+7)}\right],$$

$$E\left(H^{-3}\right)=\frac{1}{h^3}\left[1+\sum_{n=1}^{\infty}\frac{105(n+1)Y^{2n}}{(2n+3)(2n+5)(2n+7)}\right],\tag{4.13}$$

$$E\left(H^{-5}\right)=\frac{1}{h^5}\left[1+\sum_{n=1}^{\infty}\frac{35(n+1)(n+2)Y^{2n}}{2(2n+5)(2n+7)}\right],\qquad Y=\frac{c}{h}.$$

In what follows, we will consider two cases of the bearing geometry, namely: a radial thrust bearing and conical bearing, both externally pressurized.

5. Radial externally pressurized thrust bearing

An externally pressurized radial thrust bearing with a central inlet pocket is modelled by two parallel disks (Fig.2).

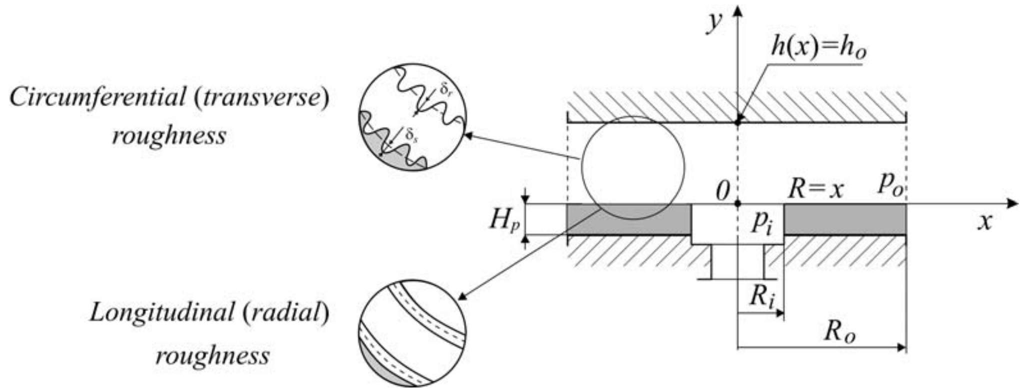

Fig.2. Radial externally pressurized thrust bearing.

Introducing the following nondimensional parameters

$$\tilde{x}=\frac{x}{R_o}, \quad x=R, \quad \tilde{R}=\frac{R}{R_o}, \quad \tilde{h}=\frac{h}{h_o}=1, \quad \varepsilon=\frac{x_i}{x_o}, \quad c^*=\frac{c}{h_o}, \quad K_p=\frac{r_c}{h_o},$$

$$\tilde{H}_p=\frac{\varphi_p H_p}{h_o}, \quad \delta=\frac{p_i}{p_o}, \quad \tilde{p}=\frac{Ep}{p_o}, \quad \tilde{N}=\frac{N}{\pi R_o^2 p_o}-1, \quad \lambda=k\left(\frac{p_o h_o}{x_o}\right)^2, \tag{5.1}$$

we will obtain the following formulae for the dimensionless pressure distribution and load-carrying capacity for the externally pressurized radial thrust bearing lubricated by the Rabinowitsch type lubricant

$$\tilde{p}=1+\left(\frac{1}{\tilde{x}^2}-1\right)P_j+\left[\delta-1-\left(\frac{1}{\varepsilon^2}-1\right)P_j\right]\frac{\ln\tilde{x}}{\ln\varepsilon}, \tag{5.2}$$

$$\tilde{N}=-2P_j\ln\varepsilon-\left[\delta-1-\left(\frac{1}{\varepsilon^2}-1\right)P_j\right]\frac{1-\varepsilon^2}{2\ln\varepsilon}, \tag{5.3}$$

where

$$P_j=\frac{3\lambda}{40}\frac{M_j^{(5)}}{M_j^{(3)}}\left(\frac{\delta-1}{\ln\varepsilon}\right)^3, \qquad M_j^{(3)}=\tilde{H}_j^{(3)}+\frac{3}{2}K_p^2\tilde{H}_p, \qquad M_j^{(5)}=\tilde{H}_j^{(5)}+\frac{5}{3}K_p^4\tilde{H}_p,$$

$$\tilde{H}_j^{(3)} = \begin{cases} \tilde{h}^3\left[1+\frac{1}{3}\left(\frac{c^*}{\tilde{h}}\right)^2\right] = 1+\frac{\left(c^*\right)^2}{3} & \text{for} \quad j=l \\ \left(\frac{1}{\tilde{h}^3}\left[1+\frac{2}{3}\left(\frac{c^*}{\tilde{h}}\right)^2\right]\right)^{-1} \approx 1-\frac{2}{3}\left(c^*\right)^2 & \text{for} \quad j=c \end{cases}$$

$$\tilde{H}_j^{(5)} = \begin{cases} \tilde{h}^5\left[1+\frac{10}{9}\left(\frac{c^*}{\tilde{h}}\right)^2\right] = 1+\frac{10}{9}\left(c^*\right)^2 & \text{for} \quad j=l \\ \left(\frac{1}{\tilde{h}^5}\left[1+\frac{5}{3}\left(\frac{c^*}{\tilde{h}}\right)^2\right]\right)^{-1} \approx 1-\frac{5}{3}\left(c^*\right)^2 & \text{for} \quad j=c \end{cases} \tag{5.4}$$

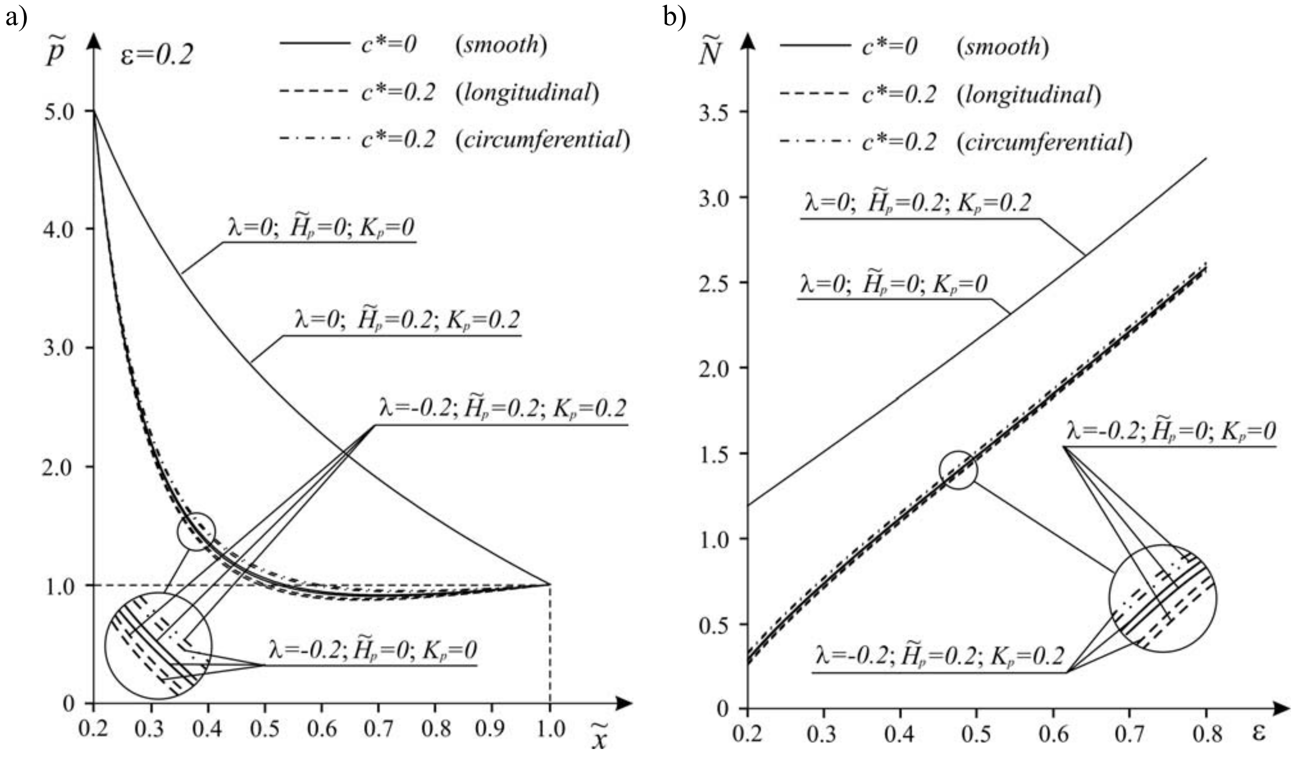

Fig.3. Nondimensional mechanical parameters for the radial rough bearing for $\lambda = -0.2$ and $\lambda = 0$: (a) pressure distribution; (b) load-carrying capacity.

Figures 3a and 4a present the dimensionless pressure distributions $\tilde{p}$ as functions of the radial coordinate $\tilde{x}$ for a definite value of the inlet pocket ratio $\varepsilon = 0.2$ and for definite values of the dimensionless coefficient of pseudo-plasticity λ and definite values of the dimensionless parameters K_p and $\tilde{H}_p$ characteristic for the bearing wall porosity. The pressure $\tilde{p}$ is presented for two kinds of surface roughness:

longitudinal $(j = l)$ and circumferential $(j = c)$ and for two cases of the bearing wall porosity: non-porous $(K_p = 0, \tilde{H}_p = 0)$ and porous $(K_p = \tilde{H}_p = 0.2)$.

A comparison with the case of Newtonian lubricants $(\lambda = 0)$ generally shows that the dilatants effects $(\lambda < 0)$ significantly decrease the film pressure, but the pseudo-plastic effects $(\lambda > 0)$ significantly increase the film pressure [10, 28-34].

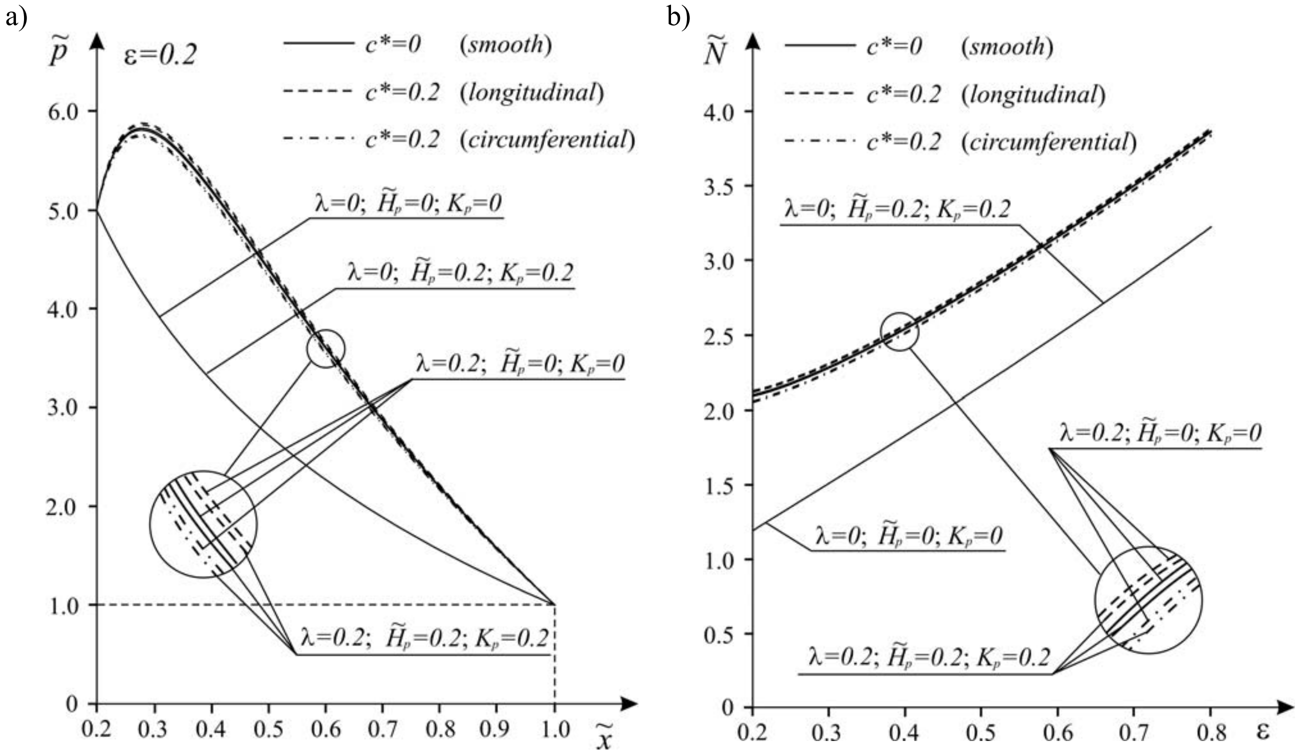

Fig.4. Nondimensional mechanical parameters for the radial rough bearing for $\lambda = 0$ and $\lambda = 0.2$: (a) pressure distribution; (b) load-carrying capacity.

In comparison with the case of smooth bearing surfaces (solid lines in Figs 3a-4a) it may be concluded that the influence of the surface roughness is small but twofold [10, 12-15, 18-22]; for $\lambda < 0$ the longitudinal roughness causes little decreases in pressure values, but the circumferential roughness causes little increases in pressure values (Fig.3a); for $\lambda > 0$ the changes in pressure run over in contrast.

The wall porosity little influencing the pressure distribution causes its increase for $\lambda < 0$ and its decrease for $\lambda > 0$ with respect to the pressure in the case of impermeable bearing walls.

Figures 3b and 4b present the dimensionless load-carrying capacity $\tilde{N}$ as a function of the inlet pocket ratio ε. The load-capacities are similarly induced by rheological and geometrical parameters as the pressure $\tilde{p}$.

6. Conical externally pressurized thrust bearing

An externally pressurized conical thrust bearing with a central inlet pocket is presented in Fig.5. For this bearing there are the following dimensional geometric relations

$$H = h(x) + h_s, \qquad h(x) = h_i + \delta_c(x - x_i), \qquad \delta_c = \frac{h_o - h_i}{x_o - x_i},$$
$$R(x) = x\sin(\alpha - \delta_c) \approx x\sin\alpha, \qquad \text{because} \qquad \delta_c / \alpha \ll 1. \tag{6.1}$$

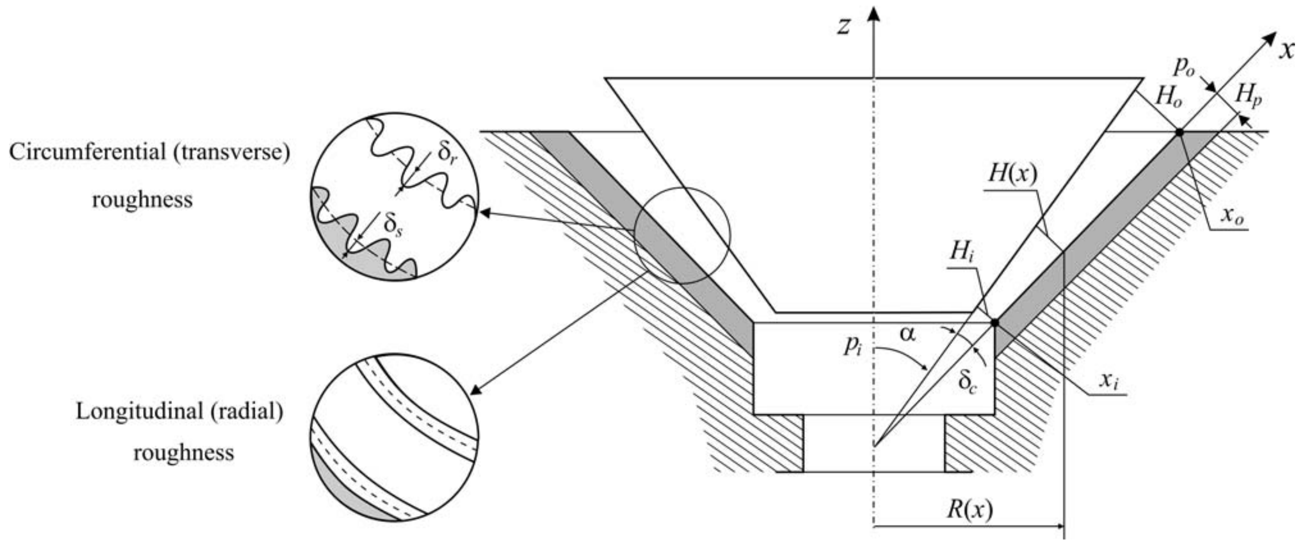

Fig.5. Conical externally pressurized thrust bearing.

Introducing the following nondimensional parameters

$$\tilde{x} = \frac{x}{x_o} = \frac{R}{R_o}, \qquad \tilde{h} = \frac{h}{h_o} = \alpha_o \tilde{x} + \beta_o = u, \qquad \varepsilon = \frac{x_i}{x_o}, \qquad \varepsilon_h = \frac{h_i}{h_o},$$

$$\alpha_o = \frac{\delta_c x_o}{h_o} = \frac{1 - \varepsilon_h}{1 - \varepsilon}, \qquad \beta_o = \varepsilon_h - \alpha_o \varepsilon, \qquad \delta = \frac{p_i}{p_o}, \qquad \lambda = k\left(\frac{p_o h_o}{x_o}\right)^2, \tag{6.2}$$

$$K_p = \frac{r_c}{h_o}, \qquad \tilde{H}_p = \frac{\varphi_p H_p}{h_o}, \qquad \tilde{p} = \frac{Ep}{p_o}, \qquad \tilde{N} = \frac{N}{\pi R_o^2 p_o} - 1,$$

we will obtain formulae for the dimensionless pressure distribution and load-carrying capacity for the externally pressurized conical thrust bearing lubricated by the Rabinowitsch type lubricant

$$\tilde{p}(u) = -\frac{3\alpha_o^2\lambda}{20}\tilde{C}^3\tilde{G}(u) + \frac{\tilde{A}(u) - \tilde{A}_o}{\tilde{A}_i - \tilde{A}_o}\left(\delta + \frac{3\alpha_o^2\lambda}{20}\tilde{C}^3\tilde{G}_i\right) - \frac{\tilde{A}(u) - \tilde{A}_i}{\tilde{A}_i - \tilde{A}_o}\left(1 + \frac{3\alpha_o^2\lambda}{20}\tilde{C}^3\tilde{G}_o\right), \tag{6.3}$$

$$\tilde{N}(\varepsilon) = \frac{3\lambda\tilde{C}^3}{20}\left(\tilde{G}_o^{(N)} - \tilde{G}_i^{(N)}\right) - \frac{\left(\delta + \frac{3\alpha_o^2\lambda}{20}\tilde{C}^3\tilde{G}_i\right) - \left(1 + \frac{3\alpha_o^2\lambda}{20}\tilde{C}^3\tilde{G}_o\right)}{\left(\tilde{A}_i - \tilde{A}_o\right)\alpha_o^2}\left(\tilde{A}_o^{(N)} - \tilde{A}_i^{(N)}\right) \tag{6.4}$$

where

$$\tilde{A}_i = \tilde{A}(u_i), \qquad \tilde{A}_o = \tilde{A}(u_o), \qquad \tilde{G}_i = \tilde{G}(u_i), \qquad \tilde{G}_o = \tilde{G}(u_o),$$

$$\tilde{A}_i^{(N)} = \tilde{A}^{(N)}(u_i), \qquad \tilde{A}_o^{(N)} = \tilde{A}^{(N)}(u_o), \qquad \tilde{G}_i^{(N)} = \tilde{G}^{(N)}(u_i), \qquad \tilde{G}_o^{(N)} = \tilde{G}^{(N)}(u_o),$$

$$u_i = \alpha_o \varepsilon + \beta_o, \qquad u_o = \alpha_o + \beta_o;$$

functions $\tilde{A}(u)$, $\tilde{G}(u)$ and $\tilde{A}^{(N)}(u)$, $\tilde{G}^{(N)}(u)$ are given in the Appendix.

Figures 6a-9a present the dimensionless pressure distributions $\tilde{p}$ as a function of the conical coordinate $\tilde{x}$ for definite values of the inlet pocket ratio $\varepsilon = 0.2$ and clearance divergence ratio $\varepsilon_h = 0.5$. The pressure $\tilde{p}$ is presented for definite values of the dimensionless coefficient of pseudo-plasticity λ, for two kinds of surface roughness: longitudinal and circumferential and for two cases of the bearing wall porosity: non-porous $\left(\tilde{H}_p = K_p = 0\right)$ and porous $\left(\tilde{H}_p = K_p = 0.2\right)$.

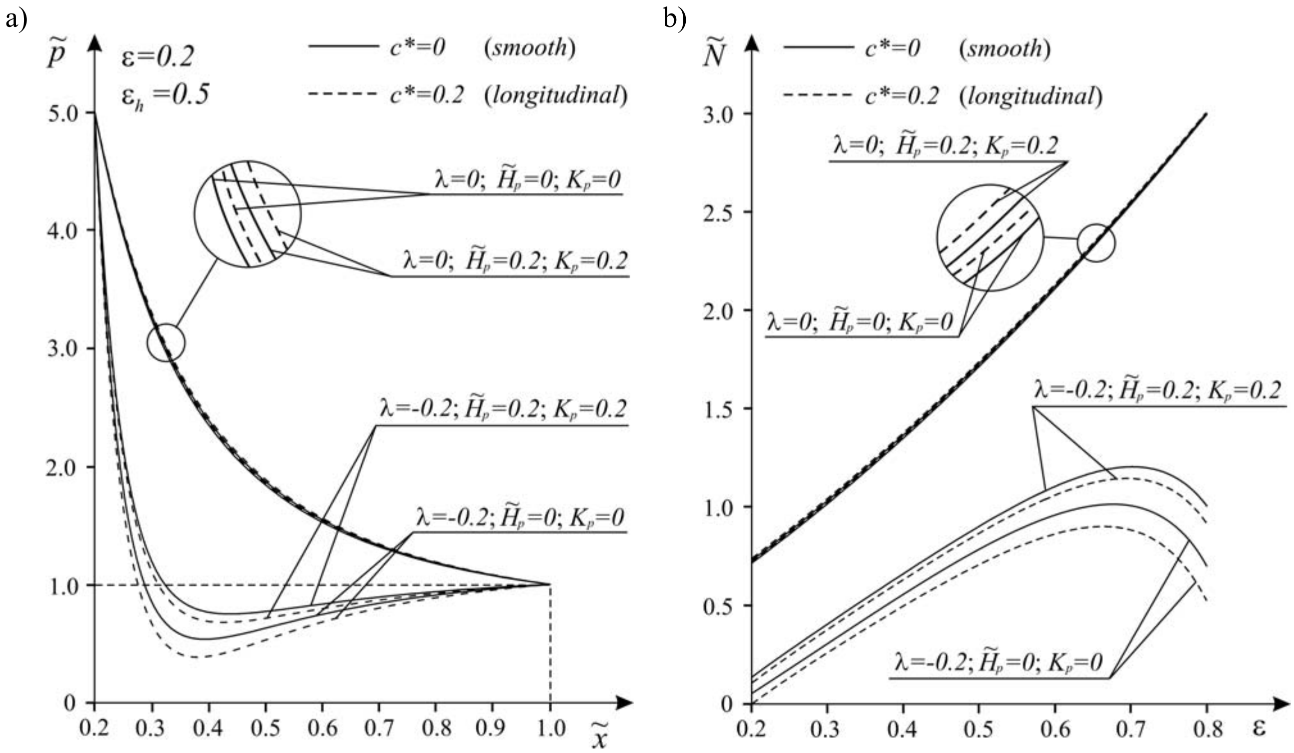

Fig.6. Nondimensional mechanical parameters for the conical rough bearing with longitudinal roughness for $\lambda = -0.2$ and $\lambda = 0$: (a) pressure distribution; (b) load-carrying capacity.

A comparison with the case of Newtonian lubricants $(\lambda = 0)$ shows also that the dilatant effects $(\lambda < 0)$ significantly decrease the film pressure, but the pseudo-plastic effects $(\lambda > 0)$ significantly increase the film pressure.

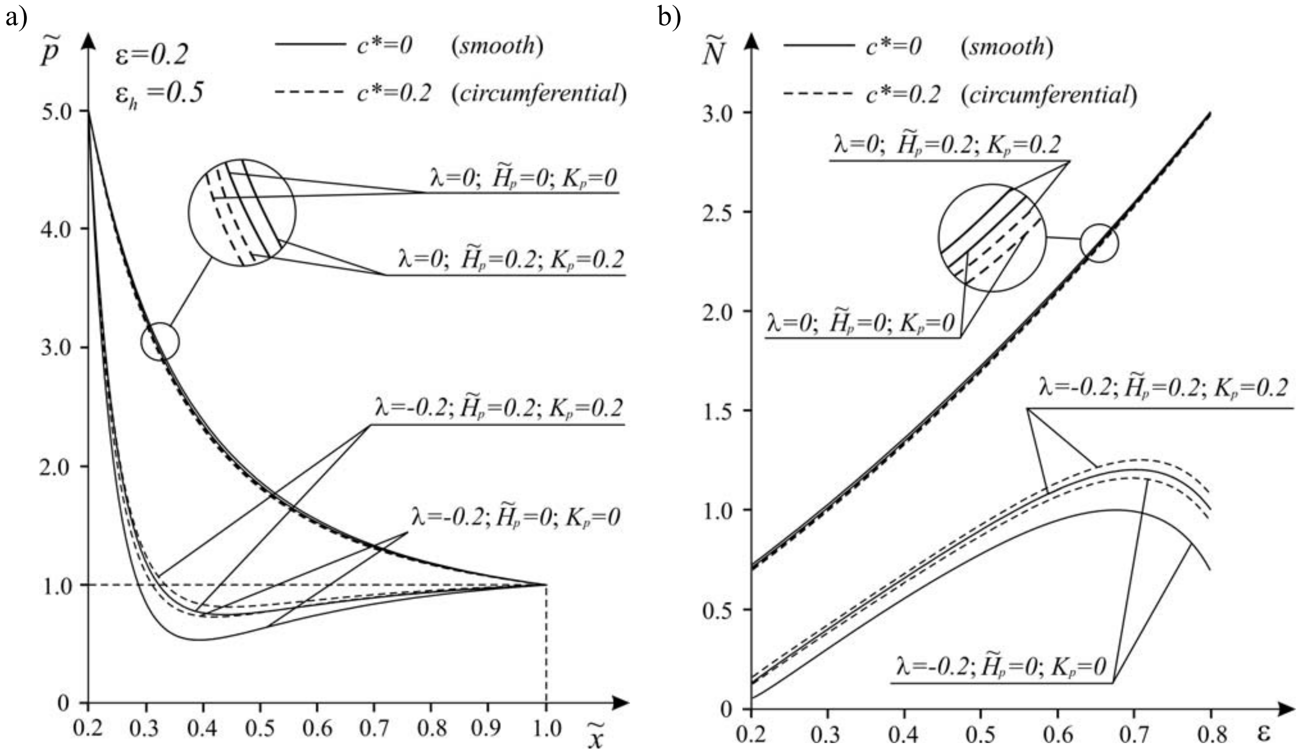

Fig.7. Nondimensional mechanical parameters for the conical rough bearing with circumferential roughness for $\lambda = -0.2$ and $\lambda = 0$: (a) pressure distribution; (b) load-carrying capacity.

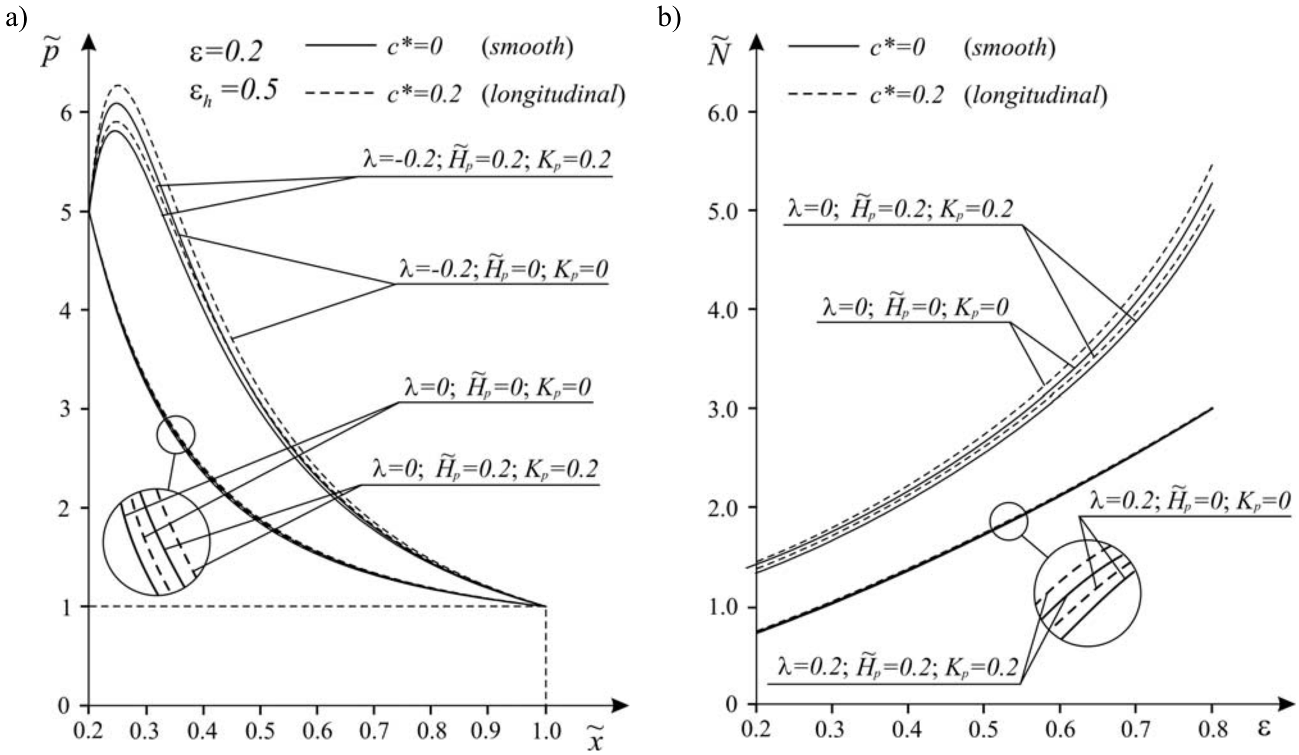

Fig.8. Nondimensional mechanical parameters for the conical rough bearing with longitudinal roughness for $\lambda = 0$ and $\lambda = 0.2$: (a) pressure distribution; (b) load-carrying capacity.

In comparison with the case of smooth bearing surfaces (solid lines in Figs 6a-9a) it may be concluded that the influence of the surface roughness is expressive here but also twofold [10, 12-15, 18-22]; for $\lambda < 0$ the longitudinal roughness causes clear decreases in pressure values, but the circumferential roughness causes clear increases in pressure values; for $\lambda > 0$ the changes in pressure run over in contrast.

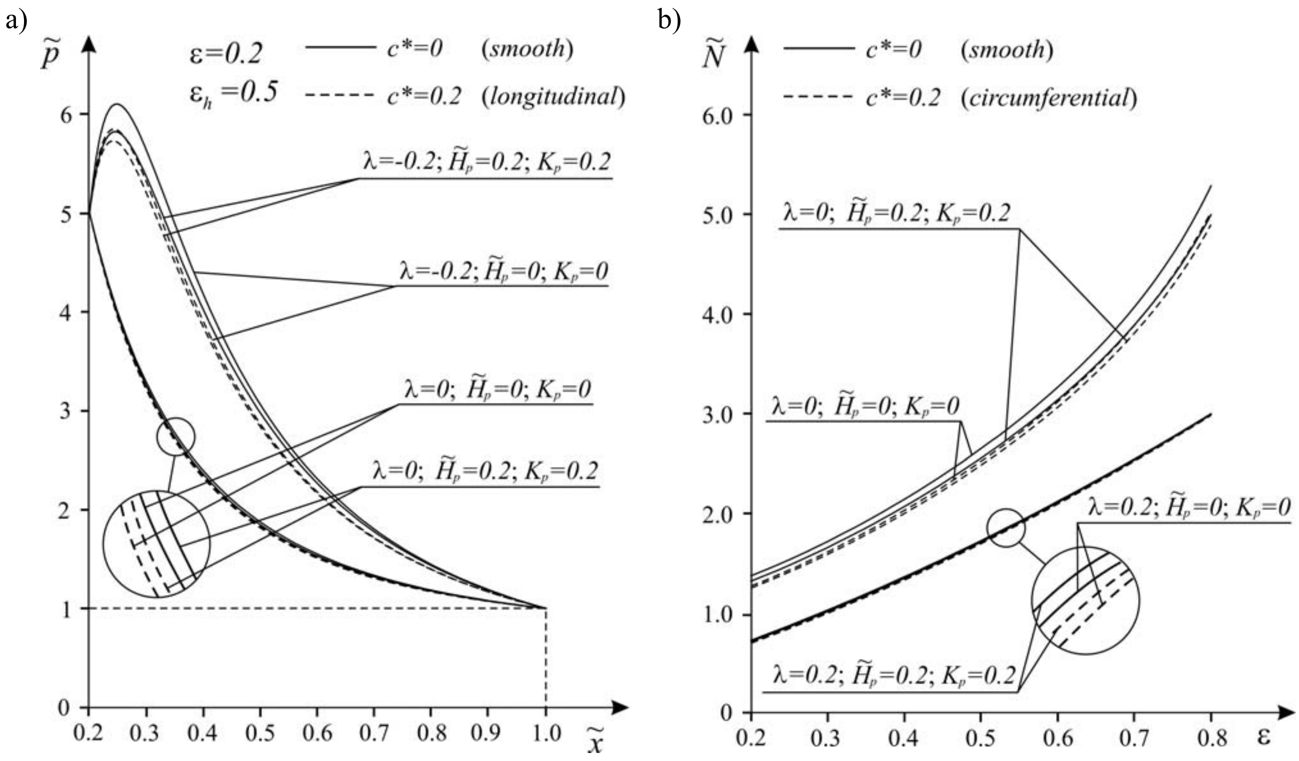

Fig.9. Nondimensional mechanical parameters for the conical rough bearing with circumferential roughness for $\lambda = 0$ and $\lambda = 0.2$: (a) pressure distribution; (b) load-carrying capacity.

The wall porosity also little influencing the pressure distribution causes its increase for $\lambda < 0$ and its decrease for $\lambda > 0$ in respect of the pressure in case of impermeable bearing walls.

Figures 6b-9b present the dimensionless load-carrying capacity $\tilde{N}$ as a function of the inlet pocket ratio ε versus the clearance divergence ratio $\varepsilon_h = 0.5$. The load-capacities are similarly influenced by rheological and geometrical parameters as the pressure $\tilde{p}$, but these influences are more visible for $\lambda < 0$.

7. Conclusions

The modified Reynolds equation for a Rabinowitsch type of pseudo-plastic lubricants flowing in a clearance of a thrust curvilinear bearing with rough surfaces is derived. A porous layer adheres to one bearing surface. Applying the Morgan-Cameron approximation to the Darcy flow of the Rabinowitsch lubricant in a porous layer the new modification of the Reynolds equation is introduced. According to the Christensen stochastic model of roughness the final form of the Reynolds equation is derived. As a result of solving this equation general formulae for pressure distributions and load-carrying capacity are obtained.

In the paper, two geometrically similar bearings are presented as practical examples: the radial bearing and conical thrust bearing both externally pressurized. It follows from the solutions of the Reynolds equation for these bearings, detailed calculations and their graphics presentations that both magnitudes (pressure distribution and load-carrying capacity) depend on the signs of the rheological parameters k and

λ. With the decrease of the λ values both these magnitudes increase in respect of their values for Newtonian lubricants and this increase is more visible for the conical bearing.

The bearing surface roughness effect is twofold: for $\lambda < 0$ the longitudinal roughness causes decreases in the pressure and load-carrying capacity values, but the circumferential roughness causes increases in the pressure and load-carrying capacity values; for $\lambda > 0$ the changes of these magnitudes run over in contrast.
The wall porosity causes the little increases for $\lambda < 0$ and little decreases for $\lambda > 0$ both these mechanical magnitudes.

Nomenclature

$\boldsymbol{A}_1$ – the first Rivlin-Ericksen kinematic tensor
c – maximum asperity deviation
c^* – nondimensional roughness parameter
$E(\bullet)$ – expectancy operator
$f(h_s)$ – probability density distribution function
$h(x)$ – nominal film thickness
$h_s(x,\vartheta,\xi)$ – random deviation of film thickness
H – film thickness
H_p – porous pad thickness
k, k_i – pseudo-plasticity coefficients
N – load-carrying capacity
p – pressure
r – radius
$R, R(x)$ – local radius of the lower bearing surface
υ_x, υ_y – velocity components
x, y – orthogonal coordinate
ε – inlet pocket ratio
ε_h – clearance divergence ratio
ϑ – angular coordinate
μ – coefficient of viscosity
ξ – random variable
ρ – fluid density

Appendix

The function $\tilde{A}(u)$ is given as follows

$$\tilde{A}(u) = \int \frac{du}{(u-\beta_o)\left(u^3 + q_j u + \beta\right)}, \qquad \beta = \frac{3}{2} K_p^2 \tilde{H}_p, \qquad j = \begin{cases} l \ \ (\text{longitudinal}), & q_l = \dfrac{c^{*2}}{3}; \\ c \ \ (\text{circumferential}), & q_c = -\dfrac{2c^{*2}}{3}. \end{cases}$$

The denominator of the subintegral function is partially presented in a canonical form and it may be written as

$$f(u) = (u-\beta_o)\left(u^3 + q_j u + \beta\right) = (u-\beta_o)(u-u_1)\left(u^2 + u_1 u + \gamma_j\right), \qquad \gamma_j = q_j + u_1^2$$

where u_1 is only one real radical of the equation

$$u^3 + q_j u + \beta = 0\,;$$

this radical is given as follows

$$u_1 = \left\{-\frac{\beta}{2} + \left[\left(\frac{\beta}{2}\right)^2 + \left(\frac{q_j}{3}\right)^3\right]^{1/2}\right\}^{1/3} + \left\{-\frac{\beta}{2} - \left[\left(\frac{\beta}{2}\right)^2 + \left(\frac{q_j}{3}\right)^3\right]^{1/2}\right\}^{1/3} \approx (-\beta)^{1/3} \quad \text{for} \left(\frac{q_j}{3}\right)^3 \ll \left(\frac{\beta}{2}\right)^2 .$$

Then

$$\tilde{A}(u) = AJ_{01} + BJ_{02} + CJ_{03} + DJ_{04}$$

where

$$J_{01} = \int \frac{du}{u-\beta_o} = \ln\left|u-\beta_o\right|, \qquad J_{02} = \int \frac{du}{u-u_1} = \ln\left|u-u_1\right|,$$

$$J_{03} = \int \frac{du}{M_j} = \begin{cases} \dfrac{2}{\sqrt{\Delta_j}} \arctan \dfrac{2u+u_1}{\sqrt{\Delta_j}}, \\ \dfrac{2}{2u+u_1}, \\ -\dfrac{2}{\sqrt{-\Delta_j}} \operatorname{arctanh} \dfrac{2u+u_1}{\sqrt{-\Delta_j}}, \end{cases} \quad \text{for} \quad \begin{cases} \Delta_j > 0, \\ \Delta_j = 0, \\ \Delta_j < 0, \end{cases} \qquad \Delta j = 4q_j + 3u_1^2\,;$$

$$J_{04} = \int \frac{u\,du}{M_j} = \frac{1}{2}\left(\ln M_j - u_1 J_{03}\right) \qquad M_j = \left(u^2 + u_1 u + \gamma_j\right).$$

The coefficient A, B, C, D are determined from the following equation

$$\frac{1}{(u-\beta_o)(u-u_1)\left(u^2+u_1u+\gamma_j\right)} = \frac{A}{u-\beta_o} + \frac{B}{u-u_1} + \frac{C+Du}{u^2+u_1u+\gamma_j}$$

by the method of multipliers.
The function $\tilde{G}(u)$ is given as follows

$$\tilde{G}(u) = \int \frac{\left(u^5 + \alpha_j u^3 + \beta_1\right)du}{(u-\beta_o)^3\left(u^3 q_j u + \beta\right)^4} = \sum_{i=1}^{3} A_i J_{1i} + \sum_{i=1}^{4} B_i J_{2i} + \sum_{i=1}^{4} C_i J_{3i} + \sum_{i=1}^{4} D_i J_{4i}$$

where

$$\alpha_j = \begin{cases} \alpha_l = \dfrac{10}{9} c^{*2}, \\ \alpha_c = -\dfrac{5}{3} c^{*2}. \end{cases} \qquad \beta_1 = \frac{5}{3} K_p^4 \tilde{H}_p$$

and

$$J_{11}=\int\frac{du}{u-\beta_o}=\ln\left|u-\beta_o\right|,\qquad J_{12}=\int\frac{du}{\left(u-\beta_o\right)^2}=-\frac{1}{u-\beta_o},$$

$$J_{13}=\int\frac{du}{\left(u-\beta_o\right)^3}=-\frac{1}{2\left(u-\beta_o\right)^2},\qquad J_{21}=\int\frac{du}{u-u_1}=\ln\left|u-u_1\right|,$$

$$J_{22}=\int\frac{du}{\left(u-u_1\right)^2}=-\frac{1}{u-u_1},\qquad J_{23}=\int\frac{du}{\left(u-u_1\right)^3}=-\frac{1}{2\left(u-u_1\right)^2},$$

$$J_{23}=\int\frac{du}{\left(u-u_1\right)^4}=-\frac{1}{3\left(u-u_1\right)^3},\qquad J_{31}=\int\frac{du}{M_j}=J_{03},$$

$$J_{32}=\int\frac{du}{M_j^2}=\frac{2u+u_1}{\Delta_j M_j}+\frac{2}{\Delta_j}J_{03},\qquad J_{33}=\int\frac{du}{M_j^3}=\frac{2u+u_1}{\Delta_j}\left(\frac{1}{M_j^2}+\frac{3}{\Delta_j M_j}\right)+\frac{6}{\Delta_j}J_{03},$$

$$J_{34}=\int\frac{du}{M_j^4}=\frac{2u+u_1}{3\Delta_j M_j^3}+\frac{10\left(2u+u_1\right)}{3\Delta_j^2}\left(\frac{1}{2M_j^2}+\frac{3}{\Delta_j M_j}\right)+\frac{20}{\Delta_j^3}J_{03},$$

$$J_{41}=\int\frac{udu}{M_j}=\frac{1}{2}\left(\ln M_j-u_1J_{03}\right),\qquad J_{42}=\int\frac{udu}{M_j^2}=-\frac{2\gamma_j+uu_1}{\Delta_j M_j}+\frac{u_1}{\Delta_j}J_{03},$$

$$J_{43}=\int\frac{udu}{M_j^3}=-\frac{2\gamma_j+uu_1}{2\Delta_j M_j^2}-\frac{3u_1\left(2u+u_1\right)}{2\Delta_j^2 M_j^2}-\frac{3u_1}{\Delta_j^2}J_{03},\qquad J_{44}=\int\frac{udu}{M_j^4}=-\frac{1}{6}\left(\frac{1}{M_j^3}+3u_1J_{34}\right).$$

The coefficient A_i, B_i, C_i, D_i are determined from the following equation

$$\frac{\left(u^5+\alpha_j u^3+\beta_1\right)du}{\left(u-\beta_o\right)^3\left(u^3 q_j u+\beta\right)^4}=\frac{A_1}{u-\beta_o}+\frac{A_2}{\left(u-\beta_o\right)^2}+\frac{A_2}{\left(u-\beta_o\right)^3}+\frac{B_1}{u-u_1}+\frac{B_2}{\left(u-u_1\right)^2}+$$
$$+\frac{B_3}{\left(u-u_1\right)^3}+\frac{B_4}{\left(u-u_1\right)^4}+\frac{C_1+D_1u}{M_j}+\frac{C_2+D_2u}{M_j^2}+\frac{C_3+D_3u}{M_j^3}+\frac{C_4+D_4u}{M_j^4}$$

by the method of multipliers.

The function $\tilde{A}^{(N)}(u)$ is given as follows

$$\tilde{A}^{(N)}(u)=\int\frac{\left(u-\beta_o\right)du}{\left(u-u_1\right)\left(u^2+u_1u+\gamma_j\right)}=\frac{\beta_o-u_1}{\gamma_j+2u_1^2}\left(J_{04}-J_{02}\right)+\left[1+\frac{\left(\beta_o-u_1\right)u_1}{\gamma_j+2u_1^2}\right]J_{03}.$$

For the function $\tilde{G}^{(N)}(u)$ one has the following formula

$$\tilde{G}^{(N)}(u)=\int\frac{\left(u^5+\alpha_j u^3+\beta_1\right)du}{(u-\beta_o)(u-u_1)^4\left(u^2+u_1u+\gamma_j\right)^4}=$$

$$=\sum_{i=1}^{3}A^{(N)}N_{10}+\sum_{i=1}^{4}B_i^{(N)}N_{1i}+\sum_{i=1}^{4}C_i^{(N)}N_{2i}+\sum_{i=1}^{4}D_i^{(N)}N_{3i}$$

where

$$N_{10}=\int\frac{du}{u-\beta_o}=\ln\left|u-\beta_o\right|=J_{11},\qquad N_{11}=\int\frac{du}{u-u_1}=\ln\left|u-u_1\right|=J_{21},$$

$$N_{12}=\int\frac{du}{(u-u_1)^2}=-\frac{1}{u-u_1}=J_{22},\qquad N_{13}=\int\frac{du}{(u-u_1)^3}=-\frac{1}{2(u-u_1)^2}=J_{23},$$

$$N_{14}=\int\frac{du}{(u-u_1)^4}=-\frac{1}{3(u-u_1)^3}=J_{24},\qquad N_{21}=\int\frac{du}{M_j}=J_{31}=J_{03},$$

$$N_{22}=\int\frac{du}{M_j^2}=J_{32},\qquad N_{23}=\int\frac{du}{M_j^3}=J_{33},\qquad N_{24}=\int\frac{du}{M_j^4}=J_{34},$$

$$N_{31}=\int\frac{udu}{M_j}=J_{41},\qquad N_{32}=\int\frac{udu}{M_j^2}=J_{42},\qquad N_{33}=\int\frac{udu}{M_j^3}=J_{43},$$

$$N_{34}=\int\frac{udu}{M_j^4}=J_{44}.$$

The coefficients $A^{(N)}$, $B_i^{(N)}$, $C_i^{(N)}$, $D_i^{(N)}$ are determined from the following equation

$$\frac{u^5+\alpha_j u^3+\beta_1}{(u-\beta_o)(u-u_1)^4\left(u^2+u_1u+\gamma_j\right)^4}=\frac{A^{(N)}}{u-\beta_o}+\frac{B_1^{(N)}}{u-u_1}+\frac{B_2^{(N)}}{(u-u_1)^2}+\frac{B_3^{(N)}}{(u-u_1)^3}+\frac{B_4^{(N)}}{(u-u_1)^4}+$$

$$+\frac{C_1^{(N)}+D_1^{(N)}u}{M_j}+\frac{C_2^{(N)}+D_2^{(N)}u}{M_j^2}+\frac{C_3^{(N)}+D_3^{(N)}u}{M_j^3}+\frac{C_4^{(N)}+D_4^{(N)}u}{M_j^4}$$

by the method of multipliers.

References

[1] Walicka A. (1994): *Micropolar Flow in a Slot between Rotating Surfaces of Revolution.* – Zielona Góra: TU Press.

[2] Walicki E. and Walicka A. (1998): *Mathematical modelling of some biological bearings.* – Smart Materials and Structures, Proc. 4th European and 2nd MiMR Conference, Harrogate, UK, 6-8 July 1998, pp.519-525.

[3] Khonsari M.M. and Dai F. (1992): *On the mixture flow problem in lubrication of hydrodynamic bearing: small solid volume fraction*. – STLE Trib. Trans., vol.35, No.1, pp.45-52.

[4] Lipscomb C.C. and Denn M.M. (1984): *Flow of Bingham fluids in complex geometries*. – J. Non-Newt. Fluid Mech., vol.14, No.3, pp.337-349.

[5] Dorier C. and Tichy J. (1992): *Behaviour of a Bingham-like viscous fluid in lubrication flows*. – J. Non-Newt. Fluid Mech., vol.45, No.3, pp.291-350.

[6] Wada S. and Hayashi H. (1971): *Hydrodynamic lubrication of journal bearings by pseudo-plastic lubricants.* Pt 1, Theoretical studies, Pt 2, Experimental studies. – Bull. JSME, vol.14, No.69, pp.268-286.

[7] Swamy S.T.N., Prabhu B.S. and Rao B.V.A. (1975): *Stiffness and damping characteristics of finite width journal bearing with a non-Newtonian film and their application to instability prediction.* – Wear, vol.32, pp.379-390.

[8] Rajalingham C., Rao B.V.A. and Prabu S. (1978): *The effect of a non-Newtonian lubricant on piston ring lubrication.* – Wear, vol.50, pp.47-57.

[9] Walicka A. (2002): *Rotational Flows of Rheologically Complex Fluids in Thin Channels* (in Russian). – Zielona Góra: University Press.

[10] Walicki E. (2005): *Rheodynamis of Slide Bearings Lubrication* (in Polish). – Zielona Góra: University Press.

[11] Christensen H. (1969-1970): *Stochastic model for hydrodynamic lubrication of rough surfaces*. – Proc. Inst. Mech. Engrs, vol.184, pt 1, pp.1013-1022.

[12] Lin J.-R. (2000): *Surfaces roughness effect on the dynamic stiffness and damping characteristics of compensated hydrostatic thrust bearings*. – Int. J. Machine Tools Manufact., vol.40, pp.1671-1689.

[13] Lin J.-R. (2001): *The effect of couple stresses in the squeeze film behaviour between isotropic rough rectangular plates*. – Int. J. Appl. Mech. Eng., vol.6, No.4, pp.1007-1024.

[14] Bujurke N.M., Kudenatti R.B. and Awati V.B. (2007): *Effect of surface roughness on squeeze film poroelastic bearings with special reference to synovial joints*. – Mathematical Biosciences, vol.209, pp.76-89.

[15] Prakash J. and Tiwari K. (1985): *Effects of surface roughness on the squeeze film between rectangular porous annular disc with arbitrary porous wall thickness*. – Int. J. Mech. Sci., vol.27, No.3, pp.135-144.

[16] Walicka A. (2009): *Surface roughness effects in a curvilinear squeeze film bearing lubricated by a power-law fluid*, Int. J. Appl. Mech. Engng, vol.14, No.1, pp.277-293.

[17] Walicka A. (2012): *Porous curvilinear squeeze film bearing with rough surfaces lubricated by a power-law fluid*. – Journal of Porous Media, vol.15, No.1, pp.29-49.

[18] Walicka A. and Walicki E. (2002): *Surface roughness effect on the pressure distribution in curvilinear thrust bearings*. – Exploitation Problems of Machines, vol.131, No.3, pp.157-167.

[19] Walicka A. and Walicki E. (2002): *Couple stress and surface roughness effects in curvilinear thrust bearings*. – Int. J. Appl. Mech. Engng, vol.7, Spec. Issue: SITC, pp.109-117.

[20] Morgan V.T. and Cameron A. (1957): *Mechanismus of lubrication in porous metal bearings*. – Proc. Conf. on Lubrication and Wear, Inst. Mech. Eng., London 1957, pp.151-157.

[21] Prakash J. and Tiwari K. (1984): *An analysis of the squeeze film between rough porous rectangular plates with arbitrary porous wall thickness*. – Journal of Tribology, Trans. ASME, vol.106, No.2, pp.218-222.

[22] Gururajan K. and Prakash J. (1999): *Surface roughness effects in infinitely long porous journal bearing*. – Journal of Tribology, Trans. ASME, vol.121, No.1, pp.139-147.

[23] Kraemer, E.O. and Williamson, R.V. (1929): *Internal friction and the structure of „solvated" colloids*. – J. Rheology, vol.1, No.1, pp.76-92.

[24] Rabinowitsch, B. (1929): *Über die Viskosität und Elastizität von Solen* (*On the viscosity and elasticity of sols*). – Zeit. Phys. Chem., vol.A145, pp.1-26.

[25] Rotem Z. and Shinnar R. (1961): *Non-Newtonian flow between parallel boundaries in linear movements.* – Chem. Eng. Sie., vol.15, pp.130-143.

[26] Sharma S.C., Jain S.C. and Sah P.L. (2000): *Effect of non-Newtonian behaviour of lubricant and bearing flexibility on the performance of slot-entry journal bearing*. – Tribology Int., vol.33, pp.507-517.

[27] Singh U.P., Gupta R.S. and Kapur V.K. (2011): *On the steady performance of hydrostatic thrust bearing: Rabinowitsch fluid model*. – Tribology Transactions, vol.54, pp.723-729.

[28] Hashimoto H. and Wada S. (1986): *The effects of fluid inertia forces in parallel circular squeeze film bearing lubricated with pseudoplastic fluids*. – J. Tribology, vol.108, pp.282-287.

[29] Lin J.-R. (2012): *Non-Newtonian squeeze film characteristics between annular disks: Rabinowitsch fluid model*. – Tribology Int., vol.52, pp.190-194.

[30] Lin J.-R., Chu L.-M., Hung C.-R., Lu R.-F. and Lin M.-C. (2013): *Effects of non-Newtonian rheology on curved circular squeeze film: Rabinowitsch fluid model*. – Z. Naturforsch., vol.68a, pp.291–299.

[31] Walicka A., Walicki E. and Ratajczak M. (1999): *Pressure distribution in a curvilinear thrust bearing with pseudo-plastic lubricant*. – Appl. Mech. Enging., vol.4 (sp. Issue), pp.81-88.

[32] Walicka A., Walicki E. and Ratajczak M. (2000): *Rotational inertia effects in a pseudo-plastic fluid flow between non-coaxial surfaces of revolution*. – Proc. 4th Minsk Int. Heat Mass Transfer Forum (May 22-27, 2000 Minsk Belarus), pp.19-29.

[33] Ratajczak M., Walicka A. and Walicki E. (2006): *Inertia effects in the curvilinear thrust bearing lubricated by a pseudo-plastic fluid of Rotem-Shinnar*. – Problems of Machines Exploitation, vol.44, pp.159-170.

[34] Walicka A. and Walicki E. (2010): *Performance of the curvilinear thrust bearing lubricated by a pseudo-plastic fluid of Rotem-Shinnar*. – Int. J. Appl. Mech. Enging, vol.15, No.3, pp.895-907.

[35] Walicka A and Jurczak P. (2013): *Pressure distribution in a porous squeeze film bearing lubricated by a Vočadlo fluid*. – Appl. Math. Modelling, vol.37, No.22, pp.9295-9307.

[36] Walicka A. (2017): *Rheology of Fluids in Mechanical Engineering*. – Zielona Góra: University Press.

[37] Hashimoto H. and Wada S. (1986): *The effects of fluid inertia forces in parallel circular squeeze film bearing lubricated with pseudoplastic fluids*. – J. Tribology, vol.108, pp.282-287.

[38] Christensen H. and Tønder K. (1971): *The hydrodynamic lubrication of rough bearing surfaces of finite width*. – ASME, J. Lubric. Technol., vol.93, No.2, pp.324-330.

[39] Christensen H. and Tønder K. (1973): *The hydrodynamic lubrication of rough journal bearings*. – ASME, J. Lubric. Technol., vol.95, No.1, pp.166-172.

[40] Rajalingham C., Rao B.V.A. and Prabu B.S. (1979): *Steady state performance of a hydrodynamic journal bearing with a pseudo-plastic lubricant*. – J. Lubric. Technol., vol.101, pp.497-502.

Received: March 2, 2017
Revised: May 7, 2017