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INFORMATION ACQUISITION AND EFFICIENT MECHANISM DESIGN

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Abstract

We consider a general mechanism design setting where each agent can acquire (covert) information before participating in the mechanism. The central question is whether a mechanism exists which provides the efficient incentives for information acquisition ex-ante and implements the efficient allocation conditional on the private information ex-post.

It is shown that in every private value environment the Vickrey-Groves-Clark mechanism guarantees both ex-ante as well as ex-post efficiency. In contrast, with common values, ex-ante and ex-post efficiency cannot be reconciled in general. Sufficient conditions in terms of sub- and supermodularity are provided when (all) ex-post efficient mechanisms lead to private under- or over-acquisition of information.

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1 Introduction

1.1 Motivation

In most of the literature on mechanism design, the model assumes that a number of economic agents possess a piece of information that is relevant for the efficient allocation of resources. The task of the mechanism designer is to find a game form that induces the agents to reveal their private information. An efficient mechanism is one where the final allocation is efficient given all the private information available in the economy.

In this paper, we take this analysis one step further. We assume that before participating in the mechanism each agent can covertly obtain additional private information at a cost. After the information has been acquired, the mechanism is executed. Hence the primitive notion in our model is an information gathering technology rather than a fixed informational type for each player. It is clear that the properties of the mechanism to be played in the second stage affect the players' incentives to acquire information in the ex ante stage.

The main results in this paper characterize information acquisition in ex post efficient mechanisms. Efficiency of a mechanism in this paper is understood in the same sense as in the original contributions by Vickrey, Clarke and Groves. In particular, we do not impose balanced budget or individual rationality constraints on the mechanism designer. In the independent private values case, we show that the Vickrey-Clarke-Groves (henceforth VCG) mechanism induces efficient information acquisition at the ex ante stage.

The common values case is much less straightforward to analyze. In light of the recent results by Dasgupta & Maskin (1998) and Jehiel & Moldovanu (1998), it is in general impossible to find mechanisms that would result in ex post efficient allocation. Adding an ex ante stage of information acquisition does not alleviate this problem. The two basic requirements for incentive compatibility of the efficient allocation rule are that the signals to the agents be single dimensional and that the allocation rule be monotonic in the signals. Even when these two conditions are met, we show that the efficient mechanisms do not result in ex ante efficient information acquisition. We use ex post equilibrium as our solution concept. An attractive feature of this concept for problems with endogenously determined information is that the mechanisms do not depend on the distributions of the signals. By the revenue equivalence theorem, any allocation rule that can be supported in an ex post equilibrium results in the same expected payoffs to all of the players as the VCG mechanism. But the defining characteristic of the VCG mechanism is that an agent's payoff changes only when the allocation changes due to his announcement of the signal. As a result, the payoffs cannot reflect the direct informational effects on other agents, and hence the private and social incentives will differ in general.

We also investigate the direction in which the incentives to acquire information are distorted. We restrict our attention to the case where the efficient allocation rule can be implemented in an ex post equilibrium and derive new necessary and sufficient conditions for the ex post implementability. It turns out that under our sufficient conditions for implementability, the information acquisition problem also satisfies the conditions for the appropriate multi-agent generalization of a monotone environment as defined in Karlin & Rubin (1956) and Lehmann (1988). As a result, we can expand the scope of our theory beyond signal structures that satisfy Blackwell's order of informativeness to the much larger class of signals ordered according to their effectiveness as defined in Lehmann (1988). We show that in settings with conflicting interests on the state of the world conditional on the allocation rule, any ex post efficient mechanism results in excessive information acquisition. With common interests conditional on the allocation, there is too little investment in information at the ex ante stage.

The paper is organized as follows. The model is laid out in the next section. Section 3 presents the case of a single unit auction as an example of the general theory. The analysis of the independent private values case is given in section 4. Results on efficient ex post implementation are presented in section 5. Section 6 deals with ex ante efficiency in the common values case and section 7 concludes.

1.2 Literature

This paper is related to two strands of literature in mechanism design. It extends the ideas of efficient mechanism design pioneered by Vickrey (1961), Clarke (1971), and Groves (1973) in an environment with fixed private information to an environment with information acquisition.

Our results on ex-post efficient mechanisms in common values environments complement recent work by Dasgupta & Maskin (1998) and Jehiel & Moldovanu (1998). Dasgupta & Maskin (1998) suggest a generalization of the VCG mechanism to obtain an efficient allocation in the context of multi-unit auctions with common values. Jehiel & Moldovanu (1998) analyze the efficient design in a linear setting with multidimensional signals and interdependent allocations. We give necessary conditions as well as weaker sufficient conditions for the efficient design in a general nonlinear environment. The results here are valid for general allocation problems and not only for single or multi-unit auctions.

The existing literature on information acquisition in mechanism design is restricted almost entirely to the study of auctions. For the private value model Hausch & Li (1991) find that first and second price auction give the same incentives to acquire information in a symmetric environment. Tan (1992) considers a procurement model where firms invest in R&D expenditure prior to the bidding stage. The R&D investment leads to a stochastic cost-reduction which is private information to the bidder. In the symmetric equilibrium with decreasing returns to scale, he observes again that revenue equivalence holds between first and second price auction. Stegeman (1996) shows that the second price auction induces efficient information acquisition in the single unit independent private values case. Matthews (1977), (1984) consider endogenous information acquisition in a pure common values auction and analyze the convergence of the value of the winning bid converges to the true value of the object when the number of bidders increases. Hausch & Li (1993) consider a common values model with endogenous entry and information acquisition. Persico (1999) compares the equilibrium incentives of the bidders to acquire information in a first and second price auctions within a general model of affiliated values.

2 Model

2.1 Payoffs

Consider a setting with I agents, indexed by $i \in \mathcal{I} = \{1, ..., I\}$. The agents have to make a collective choice x from a compact set X of possible alternatives. Uncertainty is represented by a set Ω of possible states of the world. An element $\omega \in \Omega$ is a vector $\omega = (\omega_i, \omega_{-i}) = (\omega_1, .., \omega_i, .., \omega_I)$, with:

$$\omega \in \Omega = \mathop{\times}_{i=1}^{I} \Omega_i.$$

and it is assumed that each Ω_i is a finite set. The prior distribution $q(\omega)$ is common knowledge among the players. The marginal distribution over ω_i is denoted by $q(\omega_i)$ and we assume that the prior distribution $q(\omega)$ satisfies independence across i, or:

$$q(\omega) = \prod_i q(\omega_i).$$

(There is some abuse of notation in using the same symbol q for probability distributions over different spaces, but this will lead to no confusion.) We assume that agent i's preferences depend on the choice x, the state of the world ω, and a transfer payment t_i in a quasilinear manner:

$$u_i(x, \omega) - t_i.$$

We also assume that u_i is continuous for all i. The mechanism designer is denoted with a subscript 0, and her utility is assumed to take the following form:

$$\sum_{i=1}^{I} t_i + u_0(x).$$

The model is said to be a *private value model* if for all ω, ω' :

$$\omega_i = \omega_i' \Rightarrow u_i(x, \omega) = u_i(x, \omega') \tag{1}$$

In contrast, if condition (1) is violated, then the model displays *common values.*

2.2 Signals and Posteriors

Agent i can acquire additional information by receiving a noisy signal about the true state of the world. Let S_i be a compact set of possible signal realizations that agent i may observe. Agent i acquires information by choosing a distribution from a family of joint distributions over the space $S_i \times \Omega_i$:

$$\{F^{\alpha_i}(s_i, \omega_i)\}_{\alpha_i \in A_i} \tag{2}$$

parametrized by $\alpha_i \in A_i$. We refer to $F^{\alpha_i}(s_i, \omega_i)$ as the signal and s_i as the signal realization. For brevity, we may sometimes drop realization and simply refer to s_i as signal as well. Each A_i is assumed to be a compact interval in $\mathbb{R}$. We endow $\Delta(S_i \times \Omega_i)$ with the topology of weak convergence and assume that $F^{\alpha_i}(s_i, \omega_i)$ is continuous in α_i in that topology. This ensures that the marginal distributions on S_i are continuous in α_i as well.

Agent i acquires information by choosing α_i. Each fixed α_i corresponds to a statistical experiment, and observing a signal realization $s_i \in S_i$ leads agent i to update her prior belief on ω_i according to Bayes' rule. The resulting posterior belief, $p_i(\omega_i | s_i)$ summarizes the information contained in the signal realization s_i. Considered as a family of distributions on Ω_i parametrized by s_i, we assume that $p_i(\omega_i | s_i)$ is continuous in s_i in the weak topology on Ω_i.[1] The cost of information acquisition is captured in a cost function $c_i(\alpha_i)$ and $c_i(\cdot)$ is assumed to be continuous in α_i for all i.

Let $F^{\alpha_i}(s_i | \omega_i)$ denote the distribution on signals conditional on state of the world ω_i. Posterior beliefs are connected to the prior on Ω_i through the law of iterated expectation for all α_i:

$$q(\omega_i') = \sum_{\omega_i \in \Omega_i} \int_{S_i} p(\omega_i' | s_i) \, dF^{\alpha_i}(s_i | \omega_i) \, q(\omega_i).$$

In many instances, it will be convenient to represent an experiment directly by a joint distribution over ω_i and p_i The signal s_i is then simply equal to the posterior belief, or $s_i = p_i$.

[1] The continuity and compactness assumptions made above are sufficient to guarantee that the choice set of each agent is compact and that the objective function is continuous in the choice variable.

2.3 Efficiency

The ex-ante efficient allocation requires each individual agent i to acquire the efficient amount of information and the allocation x to be optimal conditional on the posterior beliefs of all agents. Since the model has quasilinear utilities, Pareto efficiency is equivalent to surplus maximization.[2] The social utility is defined by

$$u(x,\omega) \triangleq \sum_{i=0}^{I} u_i(x,\omega).$$

The *ex-post efficient* allocation $x(p) \triangleq x(p(\omega))$ maximizes the expected social surplus conditional on the posterior belief $p(\omega)$:

$$u(x,p) \triangleq \sum_{\omega\in\Omega} u(x,\omega)\,p(\omega). \tag{3}$$

Given the assumptions made in the previous subsection, it is clear that a maximizer exists for all p.

Denote the expected social value with posterior belief $p(\omega)$ by $u(x(p),p)$. Similarly, denote by $p_{-i}(\omega)$ the information held by all agents but i, with $p_{-i}(\omega) = (p(\omega_1),...,q(\omega_i),...,p(\omega_I))$, and let $x_{-i}(p_{-i})$ be the allocation that maximizes the expected social value of all agents excluding i:

$$x_{-i}(p_{-i}) \in \arg\max_{x\in X} \sum_{\omega\in\Omega} u_{-i}(x,\omega)\,p_{-i}(\omega), \tag{4}$$

with

$$u_{-i}(x,\omega) \triangleq \sum_{j\neq i} u_j(x,\omega). \tag{5}$$

In this context, notice that in the private value environment, $x_{-i}(p) = x_{-i}(p_{-i})$ as $x_{-i}(p_{-i})$ is independent of $q(\omega_i)$. Let $F^{\alpha}(p)$ be the distribution induced on posteriors by the vector of experiments, where $\alpha = (\alpha_1,...,\alpha_I)$

[2] Recall that the mechanism designer collects all the payments and receives utility from them.

and let $c(\alpha) = \sum_i c_i(\alpha_i)$. An *ex-ante efficient* allocation is a vector of experiments, α^*, and an ex-post efficient allocation $x(p)$ such that α^* solves

$$\max_{\alpha \in A} \int u(x(p), p)\, dF^{\alpha}(p) - c(\alpha). \tag{6}$$

Observe that since we have used the posterior probabilities as arguments in the choice rule, the optimal allocation $x(p)$ does not depend on α. Again, given the continuity and compactness assumptions made in the previous subsection, a solution is guaranteed to exist.

3 Information acquisition in an auction

In this section, we present an example of a single unit auction with two bidders. It is meant to introduce the basic arguments for the private and common values results and to indicate how to extend the logic of the arguments to any number of agents and allocations. A similar example is discussed in Maskin (1992) with a signal space but without an underlying state space.

We begin with a private values model. The set of allocations is the set of possible assignments of the object to bidders, or $x_i = \{i\}$ and $i \in \{1, 2\}$. The value of the object for bidder i is $u_i(x_i, \omega) = u_i(\omega) = 2\omega_i$ and $u_i(x_j, \omega) = 0$ for $i \neq j$. The signal of agent i is simply his posterior belief $p_i = \Pr(\omega_i = 1)$ and his expected utility is $u_i(p) = 2p_i$. The direct VCG mechanism in this setting is the second price auction where bidder i pays the reported valuation of bidder j conditional on obtaining the object. Ex post efficiency dictates that i should get the object if $u_i(p) \geq u_j(p)$, which occurs whenever $p_i \geq p_j$. It follows that the equilibrium utility of bidder i, conditional on obtaining the object, is $u_i(p) - u_j(p)$ which is also equal to his marginal contribution: $u(p) - u_{-i}(p_{-i})$. For an arbitrary fixed realization $p_j = \hat{p}$, the valuations by i and j are depicted in Fig. 1a. The marginal contribution of bidder i, denoted by $MC_i(p)$ has the same slope as $u_i(p)$ for $p_i \geq p_j = \hat{p}$ and is displayed in Fig. 1b.

[INSERT FIGURE 1 HERE]

Consider next information acquisition within this auction. Within a binary state structure a signal is more informative if the posteriors are more concentrated around 0 and 1. Around $\hat{p}$, a local increase in informativeness can be represented as a randomization (with equal probability) over $\hat{p}-\varepsilon$ and $\hat{p}+\varepsilon$ for some $\varepsilon>0$. The convexity of the marginal contribution (see Fig. 1b), implies that information has positive value. More importantly, the private marginal value of signal p_i coincides with the social marginal value. As a result each agent will acquire the socially efficient level of information. The logic of this argument extends to all private value problems as the utility $u_{-i}(x,p)$ of all agents but i is constant in p_i.

To extend the example to a common values environment, redefine $u_i(\omega)=2\omega_i+\omega_j$. The expected valuation is then $u_i(p)=2p_i+p_j$ and under an efficient allocation i gets the object when $p_i\geq p_j$. For a given $p_j=\hat{p}$, the utilities are displayed as functions of p_i in Fig. 2a. The valuation of bidder j now varies with p_i, even though it is less responsive to p_i than the valuation of i. The valuations therefore satisfy a familiar single-crossing condition. However, as the valuation of bidder j varies with p_i, the original VCG mechanism does not induce truth telling anymore. For if we were to apply the mechanism, the equilibrium utility of agent i would be

$$u_i(p)-u_j(p),$$

but for any $p_i>\hat{p}$, bidder i could lower his report to $p_i-\varepsilon$, still get the object, but receive

$$u_i(p)-u_j(p_i-\varepsilon,\hat{p})$$

which would increase his utility. The above argument remains valid until $p_i=\hat{p}$, where a lower report would induce an undesirable change in the allocation. Thus by asking bidder i to pay $u_j(\hat{p},\hat{p})$, incentive compatibility is preserved. The equilibrium utility of agent i is then $u_i(p)-u_j(\hat{p},\hat{p})$ or $u(p)-u_j(\hat{p},\hat{p})$. This differs from the marginal contribution $u(p)-u_j(p_j,q_i)$ insofar as the utility of agent j is evaluated at the pivotal point $\hat{p}$, rather than the prior probability. For this reason, we shall refer to $u(p)-u_j(\hat{p},\hat{p})$ as the pivotal contribution of agent i, or $PC_i(p)$. The discrepancy between marginal and pivotal contribution is depicted in Fig. 2b.

[Insert Figure 2 here]

The divergence has immediate implications for the decision of bidder i to acquire information. As before, more information can be represented locally as a randomization over posteriors around $\hat{p}$. The contrast between pivotal and marginal contribution suggests that in equilibrium bidder i will have excessive incentives to acquire information relative to the socially optimal level. More formally, we observe that $PC_i(p) \geq MC_i(p)$ whenever $\partial u_j(p)/\partial p_i \geq 0$. In words, whenever good news for agent i is also good news for agent j, we observe too much information acquisition by bidder i. Stated in these terms, the result may sound counterintuitive. To see why it must be true, note that in equilibrium, the object is assigned to the bidder with the higher valuation. Incentive compatibility requires bidder i to pay less than the true value of the object for bidder j. Thus the equilibrium utility of agent i overestimates the contribution of bidder i, as the valuation of bidder j is also increasing in p_i.

With a single unit auction and two bidders, the analysis can be made exclusively in terms of the valuation of the object. This raises the question whether similarly intuitive conditions for the nature of the inefficiency can be given for arbitrary sets of agents and allocations. The single crossing condition at $\hat{p}$, necessary for truth telling, implies that the efficient assignment can change only once, and in particular from j to i as p_i is increased. More generally, for a given p_{-i}, the set of efficient allocations must permit a monotone ordering as a function of p_i. Truth telling by bidder i requires (locally) that

$$\frac{\partial u_i(p)}{\partial p_i} \geq 0 \Leftrightarrow \frac{\partial u_i(x_i, p)}{\partial p_i} - \frac{\partial u_i(x_j, p)}{\partial p_i} \geq 0$$

or supermodularity of $u_i(x, p)$ in (x, p_i) under the ordering $x_i \succ x_j$. The condition for under- or overacquisition of information by bidder i, was related to the responsiveness of $u_j(p)$ to p_i. Restating the conditions with the allocations we have

$$\frac{\partial u_j(p)}{\partial p_i} \geq 0 \Leftrightarrow \frac{\partial u_j(x_i, p)}{\partial p_i} - \frac{\partial u_j(x_j, p)}{\partial p_i} \leq 0,$$

or more generally for n allocations and I agents, with $x_{k+1} \succ x_k$:

$$\frac{\partial u_{-i}\left(x_{k+1}, p\right)}{\partial p_i}-\frac{\partial u_{-i}\left(x_k, p\right)}{\partial p_i} \leq 0$$

Thus an almost necessary condition for efficient implementation is that $u_i\left(x, p\right)$ be supermodular in $\left(x, p_i\right)$. If $u_{-i}\left(x, p\right)$ is also supermodular in $\left(x, p_i\right)$, then we expect less than efficient, and if $u_{-i}\left(x, p\right)$ is submodular in $\left(x, p_i\right)$, then we expect more than efficient acquisition of information. In words, if the utility differential between x_{k+1} and x_k responds to an increase in p_i in the same direction for i and $-i$, agent i will acquire insufficient information, but if the differentials respond in opposite directions then i acquires too much information. With congruence in the marginal utilities, agent i has less than socially efficient incentives to make his case, whereas divergence in the marginal utilities leads him to gather more than socially optimal evidence.

Finally notice that in the private value environment,

$$\frac{\partial u_{-i}\left(x_{k+1}, p\right)}{\partial p_i}-\frac{\partial u_{-i}\left(x_k, p\right)}{\partial p_i}=0, \ \forall k, \forall p.$$

4 Private Values

This section considers information acquisition in the context of independent private values. For this environment Vickrey (1961), Clarke (1971) and Groves (1973) showed in increasing generality that the ex-post efficient allocation can be implemented in a direct revelation mechanism.

Definition 1 *A direct revelation mechanism is defined as pair (x, t), where x is an outcome function $x : S \rightarrow X$ and $t : S \rightarrow \mathbb{R}^I$ is a transfer scheme.*

The implementation requires only dominant strategies if the transfer function has the following form for all $i \in \mathcal{I}$:

$$t_i\left(p\right)=h_i\left(p_{-i}\right)-\sum_{j \neq i} u_j\left(x\left(p\right), p_j\right) \tag{7}$$

where $h_i(p_{-i})$ is an arbitrary function of p_{-i}. A special form of the function $h_i(p_{-i})$ gives the *pivotal mechanism*:

$$h_i(p_{-i}) = \sum_{j \neq i} u_j(x_{-i}(p_{-i}), p_j).$$

The transfer function $t_i(p)$ can then be written, using the notation introduced in (5) as:

$$t_i(p) = u_{-i}(x_{-i}(p_{-i}), p_{-i}) - u_{-i}(x(p), p). \tag{8}$$

The pivotal mechanism thus requires a positive transfer from agent i whenever the announcement of his posterior belief changes the allocation relative to what would be ex-post efficient if i were not present. In consequence, the net utility of every agent i in the pivotal mechanism is given by:

$$u(x(p), p) - u_{-i}(x_{-i}(p_{-i}), p_{-i}),$$

which is the contribution of agent i to the social surplus of the agents $\mathcal{I} \backslash i$.

Definition 2 *The* marginal contribution *of agent i is defined as:*

$$MC_i(p) \triangleq u(x(p), p) - u_{-i}(x_{-i}(p_{-i}), p_{-i})$$

We refer to the mechanism which implements the efficient allocation through the transfer function described in (8) as the Vickrey-Clark-Groves (VCG) mechanism. The marginal contribution property suggest that the private and social returns to information acquisition should be equalized in a VCG mechanism.

Definition 3 *A vector of experiments, α, is a local social optimum if for every i and α_{-i}, α_i maximizes*

$$\int_P u(x(p), p)\, dF^{(\alpha_i, \alpha_{-i})}(p) - c(\alpha_i, \alpha_{-i}).$$

Notice that local refers here to the property that α solves the maximization problem for each agent separately, but not necessarily the joint maximization problem when the experimentation levels of all agents are changed simultaneously.

Theorem 1 (Private Values) *With independent private values, every local social optimum can be achieved by the VCG.*

Proof. See appendix. ■

An immediate consequence of Theorem 1 is

Corollary 1 *The ex-ante efficient allocation can be implemented by the VCG mechanism.*

Proof. See appendix. ■

The VCG mechanism may not strongly implement the ex-ante efficient allocation, but any ex-ante allocation which is a local maximum in the information acquisition stage.[3] It follows that the VCG mechanism uniquely implements the ex-ante efficient allocation if there is a unique local and hence global optimum in the information acquisition stage.

The efficiency result can also be generalized to environments where each agent can invest ex ante in technologies that increase their private payoffs. Again, it is the marginal contribution characterization of the VCG mechanism which permits ex-ante efficiency. The efficiency result derived for the VCG mechanism can in fact be extended to any ex-post efficient mechanism by the revenue equivalence theorem. Recall that for every ex-post efficient mechanism with associated transfers $t'(p)$, there exists a set of constants $\{k_i\}_{i\in\mathcal{I}}$ such that, $\forall i, \forall p$, we have

$$\mathbb{E}\left[t'_i(p)\,|p_i\right] = \mathbb{E}\left[t_i(p)\,|p_i\right] + k_i$$

where $t_i(p)$ is the transfer associated with VCG mechanism. Since the constant k_i is independent of p_i, the incentives of each agent to acquire information are not distorted by $t'_i(p)$.

Corollary 2 *The ex-ante efficient allocation can be implemented by every ex-post efficient mechanism.*

[3] Tan (1992) makes a similar observation in the context of ex-ante R&D investments in procurement auctions.

In the current model, information is acquired by all agents simultaneously. However, it is well known in statistical decision theory that a sequential decision procedure may dominate any simultaneous procedure as it economizes on the cost of information acquisition. This observation is valid in the current model as well. In the following we therefore sketch a mechanism that implements the efficient sequential procedure.[4]

Suppose for simplicity that each agent has access to finitely many costly statistical experiments and there is no discounting. The efficient path of information acquisition with perfect observability of the signals can then be solved as a finite time dynamic programming problem. The solution has the property that at most one agent engages in an experiment at any node. The nodes that are not followed by any further information acquisition steps are called terminal nodes. At each terminal node, the efficient allocation conditional on the information along the sample path is selected. This gives an assignment of payoffs to all participants at all terminal nodes.

Consider next the case where the mechanism designer must elicit the information acquired by the agents in an incentive compatible manner. Clearly, this calls for a mechanism that makes each agent accountable for the future information costs incurred by other agents depending on the announced signal. We claim that the following game form results in optimal experimentation and allocation rule. The mechanism designer asks the agents to perform the experiments and report the outcomes from the experiments in the socially optimal sequence under the assumption of truthful revelation. If any other information (e.g. results from experiments that were not supposed to be performed) is reported in any stage, sufficiently large negative transfers are imposed. At every terminal node, the allocation and transfer payments are decided according to the modified VCG mechanism described below.

Consider the incentives of an agent at a penultimate node. If all other agents reported their signals truthfully and the allocations and transfers are determined by the VCG mechanism, then at every penultimate node, the

[4] A formal argument would require a substantial amount of extra notation and is therefore omitted.

agent to move will choose optimally and report truthfully. The decision at the penultimate node is a special case of the situation in Theorem 1 where the signals of all but one agent are degenerate. Hence by Corollary 1, the efficient information acquisition is implemented by this mechanism.

Using a backwards induction argument, we replace the penultimate decision nodes with the payoff vectors (net of the cost of the experiment) resulting from the optimal actions conditional on the outcome in the experiment, and hence transform each penultimate node into a terminal node. This gives rise to a new set of penultimate nodes, and the previous step in the analysis may be repeated until the initial node is reached. By repeating these steps, we effectively include the social costs of the experiments in the final payoffs. Intuitively, by replacing the penultimate nodes with the continuation values, the relevant payoffs become the social values net of the costs of information and as a result, the experimentation costs of agents moving later are internalized by those making the earlier reports. A final and important consequence of the recursive procedure is that the sequential mechanism implements the efficient allocation strongly as every agent acts at every node as if she were maximizing the social value function.

The essential property which allows us to prove ex-ante efficiency with independent private values is the restriction that only agent i can (efficiently) invest in information about his own utility associated with various allocations. The logical next step is therefore to ask whether efficiency can be maintained in environments where the information of agent i is relevant to the utility calculus of agent j. We pursue this question in the context of the independent common value model investigated recently by Dasgupta & Maskin (1998) and Jehiel & Moldovanu (1998). Before we analyze the information acquisition per se, we give a complete characterization of the ex-post efficient allocation and associated equilibrium utilities for each agent in the following section.

5 Common Values: Ex Post Efficiency

This section relies essentially on recent work by Maskin (1992) and Dasgupta & Maskin (1998) in which a generalization of the VCG mechanism to an environment with common values is suggested. More precisely, they show that under certain conditions, a mechanism which shares the main features with the VCG mechanism implements the efficient allocation. The common features in our approach and theirs are: (i) the implementation is independent of the distribution of the signals, (ii) conditional on the allocation, the transfer payment of agent i is independent of the report of agent i, and (iii) the transfer payment of agent i varies with the reports of all other agents and can be interpreted as an "externality payment". The efficiency result is obtained by imposing two conditions on the signal each agent receives: (i) the signal is one-dimensional and (ii) it satisfies a Spence-Mirrlees sorting condition.[5] We adapt their model to our environment with uncertainty about the true state in Subsection 5.1, where we present necessary and sufficient conditions for efficient implementation with a direct revelation mechanism. Similar results are briefly stated for a continuous allocation space in Subsection 5.2. These results allow us to show that the modified mechanism changes the VCG mechanism for private values in the following aspect. Each agent does not receive his marginal contribution anymore, but rather what we call his pivotal contribution. The pivotal contribution differs from the marginal contribution in a systematic manner and we use this relationship subsequently to show why and how ex-ante efficiency necessarily fails to hold with common values.

5.1 Finite Allocation Space

We start by considering a set of finitely many allocations: $X = \left\{x^0, x^1, ..., x^N\right\}$. The expected utility of agent i of an arbitrary allocation x^n with signal s is

[5] Dasgupta & Maskin (1998) actually achieve implementation through an indirect mechanism in which the bidders report their valuations contingent on the reports by the other bidders, but not directly their signals. Jehiel & Moldovanu (1998) present sufficient conditions in a linear model with a direct revelation mechanism.

given by:

$$u_i(x^n, s) = \sum_{\omega \in \Omega} u_i(x^n, \omega) \, p(\omega | s),$$

or, after using independence:

$$u_i(x^n, s) = \sum_{\omega \in \Omega} u_i(x^n, \omega) \prod_{i \in \mathcal{I}} p(\omega_i | s_i).$$

The posterior belief $p(\omega_i | s_i)$ is an element of the simplex generated by the state space Ω_i. The posterior belief is thus a multi-dimensional object, and its dimension is the cardinality of Ω_i minus 1. If we take the signal realization to be directly the posterior belief, then the signal is multidimensional and by the results of Jehiel & Moldovanu (1998), an efficient ex-post implementation does not exist in general. We therefore restrict our attention to an arbitrary class of one-dimensional signals $S_i = [\underline{s}_i, \overline{s}_i] \subset \mathbb{R}$ with the result that the associated posterior beliefs $p(\omega_i | s_i)$ form a one-dimensional manifold in $\Delta(\Omega_i)$. In this section the allocation problem is analyzed exclusively at the ex-post stage. The utilities are therefore written as functions of (x, s) rather than (x, ω) and we assume $u_i(x, s)$ to be continuously differentiable in s for all i.

Next we present necessary and sufficient conditions for efficient implementation in an ex-post equilibrium. An ex-post equilibrium of a mechanism is a set of strategies, such that the strategy of each agent remains a best response if the types of all remaining players were common knowledge. By the revelation principle, we can restrict ourselves to direct mechanisms and truth telling strategies.

Definition 4 *A direct revelation mechanism (x, t) permits implementation in an ex-post equilibrium if* $\forall i, \forall s \in S$:

$$u_i(x(s), s) - t_i(s) \geq u_i(x(\widehat{s}_i, s_{-i}), s) - t_i(\widehat{s}_i, s_{-i}), \ \forall \widehat{s}_i \in S_i.$$

An ex-post equilibrium, while not requiring dominant strategies, remains a Bayesian equilibrium for any prior distribution over types. We shall refer to implementation in ex-post equilibrium also as distribution-free implementation.

For the rest of this section, we fix the realization of the signals s_{-i} for all agents but i. As s_{-i} is held constant, the dependence on s_{-i} is omitted at times for notational ease. Let the set S_i^n be defined as a subset of S_i for which x^n is an efficient allocation:

$$S_i^n = \{s_i \in S_i \,|u\,(x^n, s_i, s_{-i}) \geq u\,(x^m, s_i, s_{-i})\,,\ \forall x_n \neq x_m.\}$$

The sets $\{S_i^n\}_{n=0}^N$ can be taken to form a partition of S_i without loss of generality.

Definition 5 *The collection $\{S_i^n\}_{n=0}^N$ satisfies set convexity if for every n :*

$$s_i, s_i' \in S_i^n \Rightarrow \lambda s_i + (1-\lambda)\, s_i' \in S_i^n,\ \forall \lambda \in [0,1]\,.$$

If set convexity is satisfied, then there exists an optimal policy $x\,(s)$ such that every x^n is employed on a convex set S_i^n and nowhere else. After possibly relabeling the indices, we have the following ordering for the allocation space X:

$$x^0 \prec x^1 \prec ... \prec x^N, \tag{9}$$

such that for all $s_i \in S_i^k$ and $s_i' \in S_i^l$, $k \neq l$,

$$s_i < s_i' \Rightarrow x^k = x\,(s_i, s_{-i}) \prec x\left(s_i', s_{-i}\right) = x^l. \tag{10}$$

We endow the space X with the order defined by (9)-(10) and S_i with the natural order on the real line. It follows that for every two adjacent sets, S_i^{n-1} and S_i^n, the intersection of their closures, $\bar{S}_i^{n-1}$ and $\bar{S}_i^n$, respectively, is given by a single point s_i^n, called a change point:

$$\bar{S}_i^{n-1} \cap \bar{S}_i^n = \{s_i^n\}\,.$$

Define also

$$s^n \triangleq (s_i^n, s_{-i})\,.$$

Every change point s^n has the property that at $s = s^n$:

$$\frac{\partial}{\partial s_i}\left[u\left(x^n, s\right) - u\left(x^{n-1}, s\right)\right] \geq 0.$$

Consider next the truth telling condition for agent i, which is given by

$$u_i\left(x\left(s\right), s\right) - t_i\left(s\right) \geq u_i\left(x\left(\widehat{s}_i, s_{-i}\right), \left(s_i, s_{-i}\right)\right) - t_i\left(\widehat{s}_i, s_{-i}\right), \ \forall \widehat{s}_i \in S_i.$$

As the socially efficient allocation is constant on the set S_i^n, it follows that the transfer payment of agent i has to be constant on S_i^n as well. Denote it by t_i^n. The above inequality can then be written equivalently as:

$$u_i\left(x^n, s_i, s_{-i}\right) - t_i^n \geq u_i\left(x^m, s_i, s_{-i}\right) - t_i^m, \ \forall s_i \in S_i^n, \forall n, m.$$

Proposition 1 *A necessary condition for ex-post implementation is that for* $\forall i, \forall s_{-i}, \forall n$*:*

$$\frac{\partial}{\partial s_i}\left[u_i\left(x^n, s\right) - u_i\left(x^{n-1}, s\right)\right] \geq 0, \ \textit{at } s = s^n. \tag{11}$$

Proof. See appendix. ■

The inequality (11) is a familiar local sorting condition and it implies that the incentive compatible transfers (for all efficient allocations), are uniquely determined (up to a common constant) by:

$$t_i^n - t_i^{n-1} = u_i\left(x^n, s^n\right) - u_i\left(x^{n-1}, s^n\right)$$

or equivalently,

$$t_i^n - t_i^{n-1} = u_{-i}\left(x^{n-1}, s^n\right) - u_{-i}\left(x^n, s^n\right). \tag{12}$$

The mechanism which implements the efficient allocation with the transfers determined by (12) is referred to as the generalized VCG mechanism. For definiteness, we initialize t_i^0 by:

$$t_i^0 \triangleq u_{-i}\left(x_{-i}\left(s_{-i}\right), s_{-i}\right) - u_{-i}\left(x_0, s_0\right), \tag{13}$$

where s_0 is defined as the lowest signal along the i^{th} dimension, or:

$$s_0 \triangleq \left(\underline{s_i}, s_{-i}\right),$$

and s_{-i} represents the private information of all agents but i together with the ex-ante public information $q(\omega_i)$. As the transfer payments t_i^n for every allocation x_n are necessarily determined at the change points, it follows that (generically) every set S_i^n can have at most one change point as otherwise t_i^n would be overdetermined. This is exactly what set convexity guarantees.

Proposition 2 *A generically necessary condition for ex-post implementation is set convexity* $\forall i, \forall s_{-i}$.

Proof. See appendix. ■

The generalized VCG mechanism suggests the following notion of *pivotal contribution* in contrast to the marginal contribution defined earlier. For a given s, let $x(s) = x^n$, and define $s^{n+1} \triangleq s$.[6] The equilibrium utility of agent i can now be equivalently expressed as the pivotal contribution of agent i as follows.

Definition 6 *The* pivotal contribution *of agent i is defined as:*

$$PC_i(s) \triangleq u(x(s), s) - \sum_{k=0}^{n} \left(u_{-i}\left(x^k, s^{k+1}\right) - u_{-i}\left(x^k, s^k\right)\right) - u_{-i}(x_{-i}(s_{-i}), s_{-i}), \tag{14}$$

Definition (6) emphasizes the relationship to the marginal contribution. With private values, the signal s_i may be pivotal for many allocations, but it is irrelevant to the value attached to any allocation for agents other than i, or

$$u_{-i}\left(x^k, s^{k+1}\right) - u_{-i}\left(x^k, s^k\right) = 0, \ \forall k.$$

After noticing that with private values

$$u_{-i}(x_{-i}(s_{-i}), s_{-i}) = u_{-i}(x_{-i}(s), s),$$

it follows that pivotal and marginal contribution coincide for the private value environment. With common values, agent i does not receive his

[6] Recall that we are keeping s_{-i} constant throughout the discussion.

marginal contribution as incentive compatibility requires that agent i compensates the remaining agents for any losses incurred locally by moving from change point s^k to s^{k+1}. From 14, we see that the difference between the private and social value of information is captured in the term $\sum_{k=0}^{n}\left(u_{-i}\left(x^k,s^{k+1}\right)-u_{-i}\left(x^k,s^k\right)\right)$. In the next section, we compare the social and private returns from additional information on Ω_i depending on whether $u_{-i}(x,s)$ is super- or submodular in (x,s_i).

Next we strengthen the local sorting condition to obtain sufficient conditions for ex-post implementation. An obvious necessary as well as sufficient condition is that the equilibrium utility $u_i(x^n,s)-t_i^n$ be single-crossing in (x^n,s_i).[7] Using the characterization of t_i^n as given above, the single-crossing condition can be directly written in terms of the utility functions. For every x^{n-1} and x^n, the difference

$$u_i(x^n,s)-u_i\left(x^{n-1},s\right)+u_{-i}(x^n,s^n)-u_{-i}\left(x^{n-1},s^n\right)$$

has to be single-crossing in s_i, where we observe that the last two terms are constants at $s=s^n$. Thus the local sorting condition, which is a necessary condition, is also sufficient if we strengthen it to a single-crossing condition with the crossing to occur at $s=s^n$.

The next proposition modifies the necessary condition in a different direction, namely by extending the local to a global sorting condition.

Proposition 3 *If set convexity is satisfied and for all i,s_{-i} and n:*

$$\frac{\partial}{\partial s_i}\left[u_i(x^n,s)-u_i\left(x^{n-1},s\right)\right]\geq 0, \tag{15}$$

then an ex-post implementation exists.

Proof. See appendix. ■

Thus if the utility of every agent i displays supermodularity in (x^n,s_i) and set convexity is satisfied, then an ex-post implementation exists. A

[7] A function $f:X\times S\to\mathbb{R}$ satisfies the *single crossing property* if for $x'>x$, $f(x',s)-f(x,s)$, regarded as a function of s crosses zero only once and only from below. A function $f:X\times S\to\mathbb{R}$ is supermodular if for $x'>x$, $f(x',s)-f(x,s)$ is monotone nondecreasing in s.

sufficient condition for set convexity is that the social value $u(x^n, s)$ be single-crossing in (x^n, s_i). We wish to emphasize again that the particular order imposed on the allocation space X may depend on i and s_{-i}, and all that is required is that for every s_{-i}, an order on X can be constructed such that the conditions above for necessity and sufficiency can be met.

It may be noted that set convexity and supermodularity are strictly weaker than the conditions suggested by Dasgupta & Maskin (1998) in the context of a multi-unit auction:

$$u_i\left(x', s\right) > u_i(x, s) \Rightarrow \frac{\partial u_i(x', s)}{\partial s_i} > \frac{\partial u_i(x, s)}{\partial s_i}, \qquad (i)$$

or if agent i prefers x' to x, then an increase in s_i makes his preference even stronger;

$$u_i\left(x', s\right) > u_i(x, s) \ \wedge u\left(x', s\right) = u(x, s) \Rightarrow \frac{\partial u(x', s)}{\partial s_i} > \frac{\partial u(x, s)}{\partial s_i}, \qquad (ii)$$

which states that a marginal increase in the signal s_i has a greater marginal effect on the allocation x' than on x, where x' is preferred to x by i, and finally:

$$u_i\left(x', s\right) > u_i(x, s) \Rightarrow \exists s_i' \text{ s.th. } u\left(x', \left(s_i', s_{-i}\right)\right) > u\left(x, \left(s_i', s_{-i}\right)\right), \qquad (iii)$$

or if x' is preferred to x by i, then there is an s_i' sufficiently high such that x' is also socially preferred to x. Notice in particular that (i) requires that for every s_{-i}, for individual i all strict rankings among allocations are independent of his own signal s_i.

Corollary 3 *Conditions (i)-(iii) imply set convexity and supermodularity.*

Proof. See appendix. ■

In the linear (in the signals) version of the model which is investigated by Jehiel & Moldovanu (1998) with:

$$u_i(x^n) = \sum_{j=1}^{I} u_{ij}(x^n)\, s_j,$$

it follows that necessary and sufficient conditions coincide.

Corollary 4 *A necessary and sufficient condition for efficient implementation in the linear model is*

$$u_{ii}(x^n) \geq u_{ii}(x^m) \Leftrightarrow \sum_{j=1}^{I} u_{ji}(x^n) \geq \sum_{j=1}^{I} u_{ji}(x^m).$$

Proof. See appendix. ■

For completeness, it may be noted that the generalized VCG is essentially the unique mechanism which implements the efficient outcome in an ex-post equilibrium.

Proposition 4 *A mechanism (x, t) implements the efficient allocation only if $t_i(s)$ satisfies (12) for all i and s.*

Proof. See appendix. ■

As the local sorting condition is a necessary condition for ex-post implementation, one might conjecture that Bayesian rather than ex-post implementation might allow for a much wider class of problems to be successfully implemented. In particular, Bayesian implementation might overcome the local failure of the sorting condition provided that the distribution over the signals s_{-i} is sufficiently diverse. The following robustness result for an arbitrary set of utility functions $\{u_i(x, s)\}_{i \in \mathcal{I}}$ shows that if ex-post implementation fails, then it is not possible to find Bayesian implementations for all distributions of signals.

Proposition 5 *The utility functions $\{u_i(x, s)\}_{i \in \mathcal{I}}$ permit Bayesian implementation for all distributions $F_i(s_i)$ if and only if $\{u_i(x, s)\}_{i \in \mathcal{I}}$ permit ex-post implementation for all s.*

Proof. See appendix. ■

5.2 Continuum of Allocations

The sorting and convexity conditions naturally extend to the case of a continuum of allocations. Let $X \subset \mathbb{R}$ be a compact interval of the real line. As before, fix the realization of the signal s_{-i}, and suppose that $x(s)$ is a measurable function of s_i. Set convexity is defined as before. Supposing set convexity, we can impose a complete order, denoted by $\precsim$, on the allocation space X such that the order on X mirrors the order of the signal space by requiring that for all s_i, s_i' :

$$s_i < s_i' \Rightarrow x\left(s_i, s_{-i}\right) \precsim x\left(s_i', s_{-i}\right). \tag{16}$$

For the following we endow X with the complete order defined by (16) and suppose further that $u_i\left(s, x\right)$ is twice continuously differentiable with respect to s and x in the order defined above.

Proposition 6 *Necessary conditions for ex-post implementation are given by:*

1. *set convexity is satisfied* $\forall i, \forall s_{-i}$*;*

2. *local sorting condition:*

$$\frac{u_i\left(x\left(s\right), s\right)}{\partial s_i \partial x} \geq 0,\ \forall i, \forall s.$$

Proof. See appendix. ■

As in the discrete case, the set convexity conditions is generically necessary. If the local sorting condition is extended to a global condition, then we obtain sufficient conditions for implementation.

Proposition 7 *Sufficient conditions for ex-post implementation are given by:*

1. *set convexity,* $\forall i, \forall s_{-i}$

2. *global sorting condition:*

$$\frac{u_i\left(x, s\right)}{\partial s_i \partial x} \geq 0,\ \forall i, \forall s, \forall x.$$

Proof. See appendix. ■

Proposition 7 generalizes an earlier proposition by Jehiel & Moldovanu (1998) from a linear to a nonlinear environment with one-dimensional signals. The transfer payment in the generalized VCG mechanism again permits alternative representations as either:

$$t_i(s) = \int_{\underline{s_i}}^{s_i} \frac{\partial u_i(x(v_i, s_{-i}), (v_i, s_{-i}))}{\partial x} \frac{\partial x(v_i, s_{-i})}{\partial v_i} dv_i + t_i(\underline{s}_i, s_{-i})$$

or

$$t_i(s) = -\int_{\underline{s_i}}^{s_i} \frac{\partial u_{-i}(x(v_i, s_{-i}), (v_i, s_{-i}))}{\partial x} \frac{\partial x(v_i, s_{-i})}{\partial v_i} dv_i + t_i(\underline{s}_i, s_{-i}) \quad (17)$$

For definiteness $t_i(\underline{s}_i, s_{-i})$ is again defined as in (13) earlier.

6 Common Values: Ex-Ante Efficiency

The distinction between marginal and pivotal contribution already hints at a general inefficiency result in the common value environment. In this section, we analyze the consequences of ex-post efficient allocation rules for the process of information acquisition. Such an investigation is naturally constrained by the fact that the permissible signals have to induce ex-post utilities which permit ex-post implementation. In other words, the class of signals to be investigated have to satisfy the sorting and convexity conditions derived earlier to permit an ex-post efficient implementation. As we reformulate the problem in the state space, we have to ask what restrictions on posterior beliefs $p(\omega_i | s_i)$ and utility functions $u_i(x, \omega)$ are required in order to satisfy the monotonicity and sorting conditions on $u_i(x, s)$ for all (x, s). Fortunately, this problem can be adequately addressed in the context of the monotone environment introduced by Karlin & Rubin (1956) for single agent decision problems. The sufficient conditions given in Karlin & Rubin (1956) for decision problems have a natural extension to the game theoretic problem under consideration here. The restriction to a monotone environment comes with an additional benefit with regard to the ordering of information structures. Lehmann (1988) suggested necessary and sufficient conditions to rank the informativeness of different signals for the monotone environment. Lehmann's notion of effectiveness agrees with Blackwell's notion if two signals can be ranked according to Blackwell, but it provides a much more complete ordering in monotone environments. The monotone environment is introduced first in Subsection 6.1 and the informational inefficiency is analyzed in Subsection 6.2. We conclude the section with a public good example in Subsection 6.3.

6.1 Monotone Environment

The monotone environment is characterized by a monotone likelihood ratio condition on posterior beliefs and a single crossing condition on payoff functions. Recall that the posterior belief $p(\omega_i | s_i)$ of agent i is indexed by s_i. The expected utility of i from allocation x at signal s is given by

$$u_i(x,s) = \sum_{\Omega} \prod_{j} p(\omega_j | s_j) u_i(x,\omega). \tag{18}$$

For all i, for all $s_i' > s_i$ and $\omega_i' > \omega_i$, the posterior density function is required to satisfy the monotone likelihood ratio property:

$$p(\omega_i' | s_i') p(\omega_i | s_i) - p(\omega_i' | s_i) p(\omega_i | s_i') \geq 0. \tag{19}$$

As for the payoff functions, we require that for every i, the allocation space X can be endowed with a complete order, denoted by $\succsim$, such that $u_i(x,\omega_i,\omega_{-i})$ and $u(x,\omega_i,\omega_{-i})$ are supermodular in (x,ω_i) for all ω_{-i}. Observe that the ranking of the allocations is allowed to vary with i. The following results are extensions of a result obtained by Karlin & Rubin (1956).

Proposition 8

1. *For every i:*

$$s_i' > s_i \Rightarrow x(s_i', s_{-i}) \succsim x(s_i, s_{-i}), \; \forall s_{-i}.$$

2. *For every i, $u_i(x,s_i,s_{-i})$ and $u(x,s_i,s_{-i})$ are supermodular in (x,s_i) for all s_{-i}.*

Proof. See appendix. ■

In the monotone environment as defined by Karlin & Rubin (1956), the utility function is only assumed to be single crossing. That condition is replaced here with the stronger assumption of supermodularity. The strengthening in the assumption is necessary as we consider a multi-dimensional signal space. As we take the expectations over ω_{-i}, the single-crossing property is not preserved, but supermodularity is. Consider next a family of signals:

$$\{F^{\alpha_i}(s_i, \omega_i)\}_{\alpha_i \in A_i},$$

where we drop for the remainder of this subsection the subscript i as we consider a decision problem in a one-dimensional signal space S_i. (Alternatively, suppose that s_{-i} is given and fixed.) Lehmann (1988) introduced the notion of effectiveness for statistical decision problems in the monotone environment. Let $\mathbf{X}$ be a family of allocation problems with respect to ω and allocations $x \in X$. A signal $F^{\alpha'}$ is said to be more effective than a signal F^{α} with respect to a family of allocation problems $\mathbf{X}$ concerning ω, if for any problem in $\mathbf{X}$, and any allocation rule x for this problem based on F^{α}, there exists an allocation rule x' based on $F^{\alpha'}$ such that

$$\mathbb{E}_{\alpha'}\left[u\left(x', \omega\right)\right] \geq \mathbb{E}_{\alpha}\left[u\left(x, \omega\right)\right].$$

Lehmann (1988) provides a necessary and sufficient condition for $F^{\alpha'}$ to be more effective than F^{α} in monotone decision problems. If all signals $F^{\alpha} \in \{F^{\alpha}(s, \omega)\}_{\alpha \in A}$, satisfy the monotone likelihood ratio, then a necessary and sufficient condition for $F^{\alpha'}$ to be more effective than F^{α} in the class of monotone decision problems is that the function

$$T^{\alpha',\alpha}(s \mid \omega) = \left(F^{\alpha'}(\bullet \mid \omega)\right)^{-1}\left(F^{\alpha}(s \mid \omega)\right)$$

is a nondecreasing function of ω for each s.

6.2 Inefficiency

The structure of the inefficiency in the information acquisition depends ultimately on the differences in the return from information acquisition between the individual and the social utility function. The explicit characterization of the transfer function in the case of the generalized VCG mechanism facilitates such a comparison in the incentives.

For transparency, we state the result first for the single unit auction case in a symmetric environment. In this case an element x in the allocation set X simply represents the assignment of the object to a particular agent. The utility of agent i is then trivially zero for $u_i(x, \omega)$ for all $x \neq \{i\}$. It is therefore sufficient to concentrate on the utility of agent i conditional on obtaining the object, and let

$$u_i(\omega) \triangleq u_i(x, \omega), \text{ for } x = \{i\}.$$

The following theorem then extends the example of a single unit auction presented in Section 3 to an arbitrary number of bidders and arbitrary state space.

Theorem 2 (Inefficiency in Auctions)
Every ex-post efficient single-unit auction

1. *leads to too much information acquisition if for all* j,

$$u_j(\omega_i, \omega_{-i})$$

   is nondecreasing in ω_i.

2. *leads to too little information acquisition if for all* j,

$$u_j(\omega_i, \omega_{-i})$$

   is nonincreasing in ω_i.

Proof. See appendix. ■

The inefficiency in the information acquisition is again to be understood relative to the equilibrium information structure α_{-i}, selected by all agents but i. As we mentioned earlier, a similar resulted is stated in Maskin (1992) with a signal space but without an underlying state space.

This inefficiency results may be compared with results obtained in Persico (1999), where it is shown that the first price auction leads to more information acquisition then the second price auction. His model is one with affiliated signals, and therefore outside our independent common values setting. It should also be pointed out that Persico obtained a ranking of the marginal incentives to acquire information whereas we obtain a global result. The difference is due to the fact that in Persico (1999), under the two auctions, the best allocation policies from the point of view of an individual agent may be different. In the context of efficient implementation, the mechanism designer is implementing the efficient policy and hence the allocation policy is identical for agent i and the social planner who maximizes the social utility.

A similar result applies to general allocation problems. The ex-post efficient mechanism provides insufficient incentives to acquire information when agent i and the complement set $\mathcal{I}\backslash i$ have complementary interest (in their marginal utilities), and excessive incentives when i and $\mathcal{I}\backslash i$ have conflicting incentives regarding the allocative decision x.

Theorem 3 (Inefficiency in Mechanisms)
Every ex-post efficient mechanism

1. *leads to too little information acquisition if* $\forall i, \forall \omega_{-i}$, $u_{-i}(x, \omega_i, \omega_{-i})$ *is supermodular in* (x, ω_i).

2. *leads to too much information acquisition if* $\forall i, \forall \omega_{-i}$, $u_{-i}(x, \omega_i, \omega_{-i})$ *is submodular in* (x, ω_i).

Proof. See appendix. ■

The proof of the theorem compares the private and social values of information on Ω_i. In order to make this comparison, the difference between social and private values is transformed into a comparison of expected payoffs of all other agents but i. By verifying that the transformed problem satisfies the conditions of Theorem 5.1 in Lehmann (1988), we reach our conclusion.

The equilibrium utility function of agent i under allocation x is given by

$$u_i(x, s_i, s_{-i}) - t_i(s_i, s_{-i}).$$

For a fixed s_{-i}, the transfer to be paid by agent i depends only on the allocation but not on his (reported) signal. It follows therefore that the equilibrium utility function of agent i shares the single-crossing and supermodularity property with the utility function before the introduction of the transfers. The difference in the social value and private value for two signal structures, α and α', is given by:

$$\mathbb{E}_{\alpha'}[u(x,s)] - \mathbb{E}_{\alpha}[u(x,s)] \lesseqgtr \mathbb{E}_{\alpha'}[u_i(x,s) - t_i(s)] - \mathbb{E}_{\alpha}[u_i(x,s) - t_i(s)].$$

Let α' be more effective than α. Equivalently this inequality can be expressed as:

$$\mathbb{E}_{\alpha'}[u_{-i}(x,s) + t_i(s)] \lesseqgtr \mathbb{E}_{\alpha}[u_{-i}(x,s) + t_i(s)]$$

But as $t_i(s)$ is constant in s_i conditional on the allocation, the supermodularity property of the term $(u_{-i}(x,s) + t_i(s))$ remains unaffected after dropping the transfer payments $t_i(s)$. Thus we need to sign the following inequality

$$\mathbb{E}_{\alpha'}[u_{-i}(x,s)] \lesseqgtr \mathbb{E}_{\alpha}[u_{-i}(x,s)].$$

In the single unit auction the direction of this inequality can be decided on the basis of the properties stated in Theorem 2. In the case of a general mechanism problem, it is then sufficient to evaluate the supermodularity property of the aggregate utility of all players but i.

6.3 Provision of a Public Good

The general results obtained previously are illustrated in this section by a simple problem of providing a public good. Consider the following location problem for a common resource, say an airport, between agent i and agent

j. The utilities of the location conditional on the true state of the world ω is given by:

$$u_i(x, \omega) = -|x - l_i|(a_{ii}\omega_i + a_{ij}\omega_j)$$

and

$$u_j(x, \omega) = -|x - l_j|(a_{ji}\omega_i + a_{jj}\omega_j).$$

The utility of agent i is determined by the distance between his location $l_i \in [-1, 1]$ and the location of the common resource $x \in [-1, 1]$. The binary state variable $\omega_i = \{0, 1\}$ may represent the uncertain demand for the common resource by agent i and the strength of the preferences are represented by a_{ij}. It is assumed that each agent values the resource positively, or $a_{ii} > 0$, but that additional demand by agent j may have either positive or negative value from the point of view of agent i, or $a_{ij} \lesseqgtr 0$. In the context of the airport example $a_{ij} > 0$ could represent positive spillovers due to higher traffic density and $a_{ij} < 0$ could represent negative spillovers due to increased noise exposure. With a binary state structure, the signal can be taken to be the posterior belief $p_i = \Pr(\omega_i = 1)$. The expected value for agent i is then given by:

$$u_i(x, \omega) = -|x - l_i|(a_{ii}p_i + a_{ij}p_j)$$

For concreteness assume that the agents are located at opposite ends of the interval, and $l_i = -1$ and $l_j = 1$ and that the least expensive location is the center of the interval. The social cost of locating the resource closer to i or j is represented by $c(x) = \frac{1}{2}x^2$. Thus, from a private point of view, each agent prefers the common resource to be as close as possible to his own location if and only if the private expected value of the resource is positive, or $(a_{ii}p_i + a_{ij}p_j) \geq 0$.

The necessary and sufficient conditions for ex-post implementation are verified to be $a_{ii} \geq a_{ji}$ and symmetrically $a_{jj} > a_{ij}$. The ordering of the locations from the point of view of agent i is the reverse of the natural order of x and for agent j it is the natural order of x. The utility of agent j is supermodular in $(-x, \omega_i)$ if $a_{ji} \leq 0$, and submodular in $(-x, \omega_i)$ if $a_{ji} \geq 0$.

The socially efficient policy is to locate the resource at

$$x(p) = (a_{ji} - a_{ii})\, p_i + (a_{jj} - a_{ij})\, p_j$$

provided that $x(p) \in [-1, 1]$.[8] The generalized VCG mechanism is implemented by the transfer functions:

$$\frac{\partial t_i(p)}{\partial p_i} = (a_{ii} p_i - a_{ij} p_j)(a_{ii} - a_{ji})$$

as derived in (17) and symmetrically for j.

Finally, consider the decision of each agent to acquire information before the location decision. To this end, suppose that agent i can choose from the following class of densities and their associated distribution functions of possible posterior realizations on the interval $[0, 1]$:

$$f^{\alpha_i}(p_i) = \begin{cases} \alpha_i + \beta_i p & 0 \le p_i < \frac{1}{2}, \\ \alpha_i + \beta_i - \beta_i p & \frac{1}{2} \le p_i < 1. \end{cases}$$

Note the obvious restriction $\beta_i = 4(1 - \alpha_i)$ and $\alpha_i \in [0, 2]$ for $f^{\alpha_i}(\cdot)$ to be a density function. The density functions in this class are (inversely) tent-shaped, include the uniform density (for $\alpha_i = 1$), and for $0 \le \alpha_i < \alpha_i' \le 2$, the distribution function associated with α_i' is more effective than the one associated with α_i. For simplicity, let the cost of information acquisition be quadratic in $\alpha_i : c_i(\alpha_i) = \frac{1}{2}\alpha_i^2$. The unique equilibrium in information choices is then given by

$$\alpha_i = \frac{1}{48} a_{ii} (a_{ii} - a_{ji})$$

and

$$\alpha_j = \frac{1}{48} a_{jj} (a_{jj} - a_{ij})$$

which should be compared with the efficient level of information acquisition:

$$\alpha_i^* = \frac{1}{48} (a_{ii} - a_{ji})^2$$

[8] For simplicity, we shall neglect the possibility of corner solutions throughout, which is without loss of generality after assuming that $a_{ii} - a_{ji} \le 1$ and $a_{jj} - a_{ij} \le 1$. Note however that corner solutions would modify only the calculus but not the qualitative results.

and

$$\alpha_j^* = \frac{1}{48} (a_{jj} - a_{ij})^2 .$$

It follows directly that agent i acquires more than efficient information if

$$\operatorname{sgn} a_{ii} = \operatorname{sgn} a_{ji},$$

i.e. when the marginal preferences of agent i and j are conflicting. In contrast, agent i acquires less than the efficient amount of information if:

$$\operatorname{sgn} a_{ii} \neq \operatorname{sgn} a_{ji},$$

i.e. when both utility functions are supermodular in $(-x, p_i)$. The actual equilibrium choice of information structures are displayed for sub- and supermodular preferences in Fig. 3.

[Insert Figure 3 here]

7 Conclusion

This paper considers the efficiency of information acquisition in a mechanism design context. In the private values world, any mechanism which implements the efficient allocation, also leads to an efficient level of information acquisition by the agents ex-ante. The efficiency results with private values also extend to a setting where the information is acquired sequentially before a final social allocation is implemented.

In the common values model, the impossibility to obtain both ex-ante as well as ex-post efficiency with common values opens the question of which (second-best) mechanism optimally balances between efficient incentives for ex-ante information acquisition and efficient ex-post implementation. The common value model we investigated here is one where the components ω_i of the state of the world $\omega = (\omega_1, ..., \omega_I)$ are distributed independently. As in mechanism design theory with a fixed information structure, a very different picture emerges with correlated signals. The technique suggested by Cremer and McLean (1985, 1988) for full surplus extraction could be easily adapted to our environment to induce efficient information acquisition if we were concerned with Bayesian implementation. The major difference in the construction of the incentive constraints would be that the expected value of the participation fee conditional on truthful revelation should not be zero but rather reflect the marginal social value of the signal. This would then lead each agent to select the efficient information structure. In consequence, the designer could guarantee the implementation of the efficient information and allocation structure, but would have to cede some of the surplus to induce the adoption of the efficient information structure by the agents. If we insist on ex post implementation, however, mechanisms of the Cremer and McLean type may not work and similar inefficiency results to the ones derived in this paper are likely to emerge.

Finally, this paper considered information acquisition with a fixed number of agents. It may be of interest to investigate the limiting model as the number of agents gets large. Intuitively, one might expect that the problem of each individual agent might be closer to the private value model. If the responsiveness of the marginal utility of all other agents to the signal of agent i declines, then the sub- or supermodularity of $u_{-i}(x, s)$ in (x, s_i) may vanish and yield efficiency in the limit.

8 Appendix

This appendix collects the proofs to the propositions and theorems in the main body of the text.

Proof of Theorem 1. A necessary and sufficient condition for a local social optimum for an ex-ante efficient allocation is that for all i, for any given set of signals α_{-i}, α_i solves

$$\int u\left(x\left(p\right),p\right)dF^{\left(\alpha_i,\alpha_{-i}\right)}\left(p\right)-c(\alpha_i,\alpha_{-i}) \tag{20}$$

By definition of the private value model, the optimal solution of the model when excluding agent i is independent of α_i, or:

$$\int u_{-i}\left(x_{-i}\left(p_{-i}\right),p_{-i}\right)dF^{\left(\alpha_i,\alpha_{-i}\right)}\left(p_{-i}\right)=\int u_{-i}\left(x_{-i}\left(p_{-i}\right),p_{-i}\right)dF^{\alpha_{-i}}\left(p_{-i}\right).$$

Hence the solution to (20) is equivalent to the solution of

$$\max_{\alpha_i}\left\{\begin{array}{c}\int u\left(x\left(p\right),p\right)dF^{\left(\alpha_i,\alpha_{-i}\right)}\left(p\right)-c_i\left(\alpha_i\right)\\ \\ -\int u_{-i}\left(x_{-i}\left(p_{-i}\right),p_{-i}\right)dF^{\alpha_{-i}}\left(p_{-i}\right),\end{array}\right\}$$

and by the definition of the VCG mechanism (see (8)), the last program can be rewritten as

$$\max_{\alpha_i}\int\int\left[u_i\left(x\left(p\right),p\right)-t_i\left(p\right)\right]dF^{\left(\alpha_i,\alpha_{-i}\right)}\left(p\right)-c_i\left(\alpha_i\right),$$

which is precisely the objective function of agent i when deciding on his investment in information.■

Proof of Proposition 1. Suppose not, then there exist some $\varepsilon>0$ s.th.

$$u_i\left(x^{n-1},s^n-\varepsilon\right)-u_i\left(x^n,s^n-\varepsilon\right)<u_i\left(x^{n-1},s^n+\varepsilon\right)-u_i\left(x^n,s^n+\varepsilon\right). \tag{21}$$

But at the same time we require implementation, or

$$u_i\left(x^{n-1},s^n-\varepsilon\right)-t_i^{n-1}\geq u_i\left(x^n,s^n-\varepsilon\right)-t_i^n$$

and

$$u_i\left(x^{n-1}, s^n+\varepsilon\right)-t_i^{n-1} \leq u_i\left(x^n, s^n+\varepsilon\right)-t_i^n.$$

which jointly imply that

$$u_i\left(x^{n-1}, s^n-\varepsilon\right)-u_i\left(x^n, s^n-\varepsilon\right) \geq u_i\left(x^{n-1}, s^n+\varepsilon\right)-u_i\left(x^n, s^n+\varepsilon\right)$$

which leads immediately to a contradiction with (21).■

Proof of Proposition 2. Suppose set convexity fails to hold. Then there exists at least one set S_i^n such that for $s_i, s_i' \in S_i^n$ and for some $\lambda \in (0,1)$, $\lambda s_i+(1-\lambda) s_i' \in S_i^m$ with $m \neq n$. By Proposition 1, the differences $t_i^n-t_i^m$ are uniquely determined by the change points. It follows that if a set S_i^n is not convex, then there are more equations (as defined by the incentive compatibility conditions at the change points) than variables, t_i^n's, and generically, in the payoffs of $u_i(x, s)$, the system of equations has no solution.■

Proof of Proposition 3. By Proposition 1, the transfers are uniquely determined up to a common constant. Consider any adjacent sets S_i^{n-1} and S_i^n:

$$\forall s_i \in S_i^{n-1}: \ u_i\left(x^n, s\right)-u_i\left(x^{n-1}, s\right) \leq t_i^n-t_i^{n-1}$$

and

$$\forall s \in S_i^n: \ u_i\left(x^n, s\right)-u_i\left(x^{n-1}, s\right) \geq t_i^n-t_i^{n-1}.$$

Now consider any arbitrary pair S_i^k and S_i^m ordered so that $x_k \prec x_m$. We want to show that:

$$\forall x_k \prec x_m, \forall s \in S_i^k: \ u_i\left(x^k, s\right)-u_i\left(x^m, s\right) \geq t_i^k-t_i^m. \tag{22}$$

as well as

$$\forall x_m \succ x_k, \forall s_i \in S_i^m: \ u_i\left(x^m, s\right)-u_i\left(x^k, s\right) \geq t_i^m-t_i^k,$$

Consider (22). We can expand the difference on the rhs to

$$u_i\left(x^k, s\right)-u_i\left(x^m, s\right) \geq \sum_{l=k}^{m-1} t_i^l-t_i^{l+1}. \tag{23}$$

Consider the uppermost element of the sum:

$$t_i^{m-1} - t_i^m = u_i\left(x^{m-1}, s^m\right) - u_i\left(x^m, s^m\right)$$

and for all $s < s^m$,

$$t_i^{m-1} - t_i^m \leq u_i\left(x^{m-1}, s\right) - u_i\left(x^m, s\right)$$

or

$$u_i\left(x^m, s\right) - t_i^m \leq u_i\left(x^{m-1}, s\right) - t_i^{m-1} \tag{24}$$

by (15). Replacing the lhs in (24) by the rhs, the inequality (23) becomes a priori harder to satisfy. After this operation, (23) becomes

$$u_i\left(x^k, s\right) - u_i\left(x^{m-1}, s\right) \geq \sum_{l=k}^{m-2} t_i^l - t_i^{l+1},$$

and by repeatedly using the argument in (24), (23) is eventually reduced to

$$u_i\left(x^k, s\right) - u_i\left(x^{k+1}, s\right) \geq t_i^k - t_i^{k+1},$$

which is satisfied by (15), when the transfers are as in (21).■

Proof of Corollary 3. Suppose there is a ranking of allocations $x_0 \prec x_1 \prec ... \prec x_N$ from the point of individual i. Take any point at which the efficient allocation is either k and l. If it is a change point from k to l as s_i increases, denote the point by $s^{k,l}$ otherwise by $s^{l,k}$. Then if l is preferred to k by i, it follows by condition (ii) that

$$\frac{\partial}{\partial s_i}\left[u\left(x^l, s\right) - u\left(x^k, s\right)\right] > 0,$$

and hence it is a point $s^{k,l}$ and by (i) it follows that

$$\frac{\partial}{\partial s_i}\left[u_i\left(x^l, s\right) - u_i\left(x^k, s\right)\right] > 0,$$

as well, which leads to the conclusion that the ordering of the intervals socially is equivalent to the ordering of the marginal returns by i. It remains to show that convexity is implied as well. Suppose that $s, s' \in S_i^k$ and for some $\lambda \in (0,1)$, $\lambda s+(1-\lambda) s' \in S_i^l$. Then there must be at least two points, say s^o and s^{oo} s.th.

$$u\left(x^l, s^o\right) = u\left(x^k, s^o\right)$$

and

$$u\left(x^l, s^{oo}\right) = u\left(x^k, s^{oo}\right),$$

but this violates condition (ii), and hence we obtain a contradiction.■

Proof of Corollary 4. Observe first that with the linear model, the local condition (11) and the global condition (15) coincide. Similarly set convexity requires in the linear model the inequality:

$$\sum_{j=1}^{I} u_{ji}\left(x^l\right) \geq \sum_{j=1}^{I} u_{ji}\left(x^k\right),$$

which concludes the proof.■

Proof of Proposition 4. The proof technique is similar to Theorem 3.2. in Green & Laffont (1979), where the uniqueness of the VCG mechanism is shown for the private value model. For any given s_{-i}, we can write

$$t_i\left(s_i, s_{-i}\right) = \sum_{k=1}^{n}\left(u_{-i}\left(x^{k-1}, s^k\right) - u_{-i}\left(x^k, s^k\right)\right) + h_i\left(s_i, s_{-i}\right) \tag{25}$$

for an arbitrary function $h_i\left(s_i, s_{-i}\right)$. We want to show that if $t_i(s)$ implements the efficient allocation in an ex-post equilibrium for every s, then $h_i\left(s_i, s_{-i}\right)$ must in fact be independent of s_i. Suppose not, than there exist s_i and s_i' such that $h_i\left(s_i, s_{-i}\right) \neq h_i\left(s_i', s_{-i}\right)$. By extension, define s and s' to be $s = \left(s_i, s_{-i}\right)$ and $s' = \left(s_i', s_{-i}\right)$, respectively. Consider first the case that $x(s) = x(s')$. If (x,t) is implemented in ex-post equilibrium, then by Definition (4) we have

$$u_i\left(x(s), s\right) - t_i(s) \geq u_i\left(x\left(s'\right), s\right) - t_i\left(s'\right)$$

as well as

$$u_i\left(x\left(s'\right),s'\right)-t_i\left(s'\right)\geq u_i\left(x\left(s\right),s'\right)-t_i\left(s\right).$$

Since $x(s)=x(s')$, the above two inequalities imply that $t_i(s)=t_i(s')$, and so from (25), we have that $h_i(s)=h_i(s')$, a contradiction.

Consider next that $x(s)\neq x(s')$, and assume without loss of generality that $x(s)=x^{n-1}$ and $x(s')=x^n$. By the argument in the previous step, we may assume that $h_i(s)$ and $h_i(s')$ are constant in s_i and s_i' on the intervals S_i^{n-1} and S_i^n, respectively. But Proposition 1 implies that the equality

$$t_i\left(s'\right)-t_i\left(s\right)=u_{-i}\left(x_{n-1},s_n\right)-u_{-i}\left(x_n,s_n\right)$$

holds, which implies again that $h_i(s)=h_i(s')$, a contradiction.■

Proof of Proposition 5. The 'if' part of the proposition is immediate as ex-post implementation by definition is independent of $F_{-i}(s_{-i})$ and hence if it is feasible, it is feasible for every $F_{-i}(s_{-i})$.

For the 'only if' part, let $F_{-i}(s_{-i})$ be given by a distribution which puts probability 1 on s_{-i} and probability zero on all other realizations $s_{-i}'\in S_{-i}$. The implementation problem then reduces to a single agent implementation problem for which dominant and Bayesian implementation conditions are equivalent, and so are, a fortiori, ex-post and Bayesian conditions.■

Proof of Proposition 6. The necessary conditions with a continuum of allocations can be obtained directly by considering the conditions of the discrete allocation model in the limit as the set of discrete allocation converges to the set of a continuum of allocations. The details are omitted. ■

Proof of Proposition 7. The sufficient conditions with a continuum of allocations can be obtained directly by considering the conditions of the discrete allocation model in the limit as the set of discrete allocation converges to the set of a continuum of allocations. The details are omitted. ■

Proof of Proposition 8. By assumption, $u(x,\omega_i,\omega_{-i})$ is supermodular in (x,ω_i) for every ω_{-i}. The supermodularity property is preserved under expectations:

$$u(x,\omega_i,s_{-i}) = \sum_{\Omega_{-i}} u(x,\omega_i,\omega_{-i}) \prod_{j\neq i} p_j(\omega_j | s_j)$$

and a fortiori $u(x,\omega_i,s_{-i})$ satisfies the single crossing property in (x,ω_i). By Lemma 1 of Karlin & Rubin (1956), it follows that $u(x,s_i,s_{-i})$ satisfies the single crossing property in (x,s_i). A similar argument applies to $u_i(x,\omega_i,s_{-i})$. Furthermore, by Theorem 1 of Karlin & Rubin (1956), it follows that an optimal strategy which is monotone in s_i exists. This proves the first part of the theorem.

If $u(x,\omega_i,s_{-i})$ is supermodular in (x,ω_i) for every s_{-i}, then $u(x,s_i,s_{-i})$ defined as

$$u(x,s_i,s_{-i}) = \sum_{\Omega_i} u(x,\omega_i,s_{-i}) p_i(\omega_i | s_i)$$

is also supermodular in (x,s_i) by Theorem 3.10.1 in Topkis (1998) since $p_i(s_i,\omega_i)$ satisfies the monotone likelihood ratio.■

Proof of Theorem 2. This theorem is a special case of Theorem 3 after introducing the following ranking for the allocations. With a single unit auction, the set of allocations is simply the assignment of the object to a particular bidder. For every i, partition the set of allocation X into x_i and x_{-i} and order the assignments such that $x_i \succ x_{-i}$. (The order among the remaining bidders is irrelevant.) By definition of the single object auction

$$u_i(x_{-i},\omega) = 0.$$

Moreover denote the only nontrivial (in terms of utility) assignment for player i by

$$u_i(\omega) \triangleq u_i(x_i,\omega).$$

To verify the supermodularity property, it is therefore sufficient to examine the behavior of

$$u_i(x_i,\omega) - u_i(x_{-i},\omega)$$

as a function of ω_i. Similarly for $u_{-i}(x,\omega)$. The result is now a direct consequence of Theorem 3.■

Proof of Theorem 3. The proof is written for a continuum of allocations, but all arguments go through with the obvious notational modification for a finite set of allocations. The net utility of agent i under the generalized VCG mechanism is given by

$$v_i(x,s,\widehat{s}) \triangleq u_i(x,s) - t_i(\widehat{s}), \tag{26}$$

where $s=(s_i,s_{-i})$ is true signal and $\widehat{s}=(\widehat{s}_i,s_{-i})$ the reported signal. For a fixed s_{-i}, we can rewrite the transfer $t_i(\widehat{s})$ to be determined directly by x rather than $\widehat{s}$. This is without loss of generality as we recall that $t_i(\widehat{s})$ is constant in $\widehat{s}$ conditional on x. The net utility of agent i can now be written directly as

$$v_i(x,s) \triangleq u_i(x,s) - t_i(x). \tag{27}$$

The transfer $t_i(x)$ for a continuum of allocations is then given by analogy with (17) as

$$t_i(x) = -\int_{\underline{x}}^{x} \frac{\partial u_{-i}(z,s(z))}{\partial z} dz + t_i(\underline{x}), \tag{28}$$

where $s(z)$ defines s_i such that for $s=(s_i,s_{-i})$, z is the optimal allocation. It follows directly from (27) that $v_i(x,s)$ is supermodular in (x,s_i) if and only if $u_i(x,s)$ is supermodular in (x,s_i), which in turn is guaranteed by the supermodularity of $u_i(x,\omega)$ in (x,ω_i), as shown in Proposition 8. By the same token, supermodularity of (x,s_i) is preserved after taking the expectation with respect to the signal $F^{\alpha_{-i}}$ of the remaining agents for $u_i(x,s)$ and $v_i(x,s)$

$$u_i(x,s_i,\alpha_{-i}) \triangleq \mathbb{E}_{\alpha_{-i}}[u_i(x,s_i,s_{-i})]$$

and

$$v_i(x,s_i,\alpha_{-i}) \triangleq \mathbb{E}_{\alpha_{-i}}[v_i(x,s_i,s_{-i})]$$

The expected value of the signal structure F^{α_i} under the optimal allocation policy, which by the ex-post efficiency of the VCG mechanism is identically to $x(s)$ for $u(\cdot)$ and $v_i(\cdot)$, is denoted by:

$$v_i(\alpha) = v_i(\alpha_i, \alpha_{-i}) \triangleq \mathbb{E}_{\alpha_i}[v_i(x(s), s_i, \alpha_{-i})] \tag{29}$$

and likewise for the social utility function:

$$u(\alpha) = u(\alpha_i, \alpha_{-i}) \triangleq \mathbb{E}_{\alpha_i}[u(x(s), s_i, \alpha_{-i})]. \tag{30}$$

Next we show that the incremental returns from a less to a more effective distribution, say α and α' respectively, are larger for the social objective function than for agent i if $u_{-i}(x, \omega)$ is supermodular in (x, ω_i). In other words we want to show that

$$u\left(\alpha'\right) - u(\alpha) \geq v_i\left(\alpha'\right) - v_i(\alpha) \tag{31}$$

when α'_i is more effective than α_i and $\alpha'_{-i} = \alpha_{-i}$. To this end, observe that the inequality (31) can be written as

$$u_{-i}\left(\alpha'\right) + t_i\left(\alpha'\right) \geq u_{-i}(\alpha) + t_i(\alpha) \tag{32}$$

where the utilities $u_{-i}(\cdot)$ and $t_i(\cdot)$ are evaluated for every signal s at $x(s)$. The inequality is then established if we can show that the function $u_{-i}(x, s) + t_i(x)$ is (i) supermodular in (x, s_i) and (ii) achieves a global maximum at $s = s(x)$ for all s. The first property is guaranteed by the same argument as before if $u_{-i}(x, s)$ is supermodular in (x, s_i). The second property is established now. As

$$u_{-i}(x, s) + t_i(x) = u(x, s) - v_i(x, s),$$

it follows that

$$u_{-i}(x, s) + t_i(x)$$

has a stationary point at x for all $s = s(x)$, since by (27):

$$\frac{\partial u_{-i}(x, s)}{\partial x} - \frac{\partial t_i(x)}{\partial x} = \frac{\partial u_{-i}(x, s)}{\partial x} - \frac{\partial u_{-i}(x, s(x))}{\partial x} = 0$$

Notice next that locally at $s = s(x)$, the function is concave as the second derivative with respect to x is given by

$$\frac{\partial^2 u_{-i}(x,s)}{\partial x^2} - \frac{\partial^2 u_{-i}(x,s(x))}{\partial x^2} - \frac{\partial^2 u_{-i}(x,s(x))}{\partial x \partial s}\frac{ds(x)}{dx}$$

as the first two terms cancel at $s = s(x)$, and

$$\frac{\partial^2 u_{-i}(x,s(x))}{\partial x \partial s_i}\frac{ds(x)}{dx} \geq 0$$

by the supermodularity of $u_{-i}(x,s)$ and $u(x,s)$ in (x,s_i). Thus if the local maximum would also be the global maximum, we would have $u_{-i}(x,s) + t_i(x)$ to be a supermodular objective function, with an optimal policy $x = x(s)$ for all s, and hence by Theorem 5.1 of Lehmann (1988) α' would have a higher than α, which in turn would establish (32) and (31). However our standing assumptions don't allow us to conclude that the local maximum is also a global maximum. This final obstacle can be removed by modifying the objective function $u_{-i}(x,s) + t_i(x)$ through the addition of a new function $g(x,s)$ with:

$$G(x,s) \triangleq u_{-i}(x,s) + t_i(x) + g(x,s)$$

such that the following properties are satisfied:

$$g(x(s),s) = 0, \text{ at all } x = x(s); \tag{a}$$

$$G(x,s) \text{ is supermodular in } (x,s_i); \tag{b}$$

and

$$G(x(s),s) \geq G(x,s), \ \forall s,x. \tag{c}$$

If a function $g(x,s)$ exists such that $G(x,s)$ satisfies the properties $(a)-(c)$, then it follows that, using the notation introduced in (29) and (30), that

$$G(\alpha') \geq G(\alpha)$$

and since (b) holds, we can conclude that

$$u_{-i}\left(\alpha'\right) + t_i\left(\alpha'\right) \geq u_{-i}\left(\alpha\right) + t_i\left(\alpha\right)$$

even though the function $u_{-i}(x, s) + t_i(x)$ is evaluated at $x(s)$ for every s, which may not be a global maximum for $u_{-i}(x, s) + t_i(x)$. We therefore conclude by constructing a function $g(x, s)$ which achieves $(a) - (c)$. For every s, define $b(s)$ to be:

$$b(s) \triangleq u(x(s), s) - u_{-i}(x(s), s) - t_i(x(s))$$

and define $g(x, s)$ to be

$$g(x, s) \triangleq u(x, s) - u_{-i}(x, s) - t_i(x) - b(s). \tag{33}$$

It is now easy to verify that $G(x, s)$ shares the supermodularity properties of $u(x, s)$, has a global maximum at $x = x(s)$ for every s, and indeed $g(x(s), s) = 0$, which concludes the proof. The corresponding result for submodularity can be obtained by simply reversing the inequalities.■

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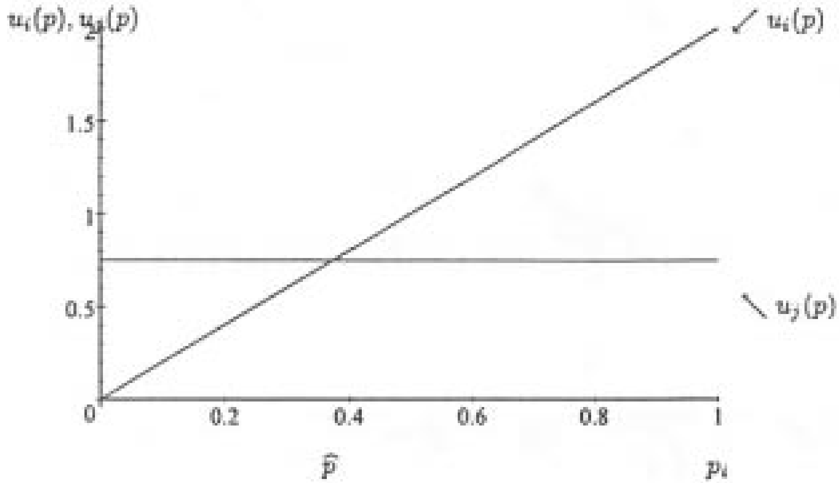

Figure 1a: $u_i(p)$ and $u_j(p)$

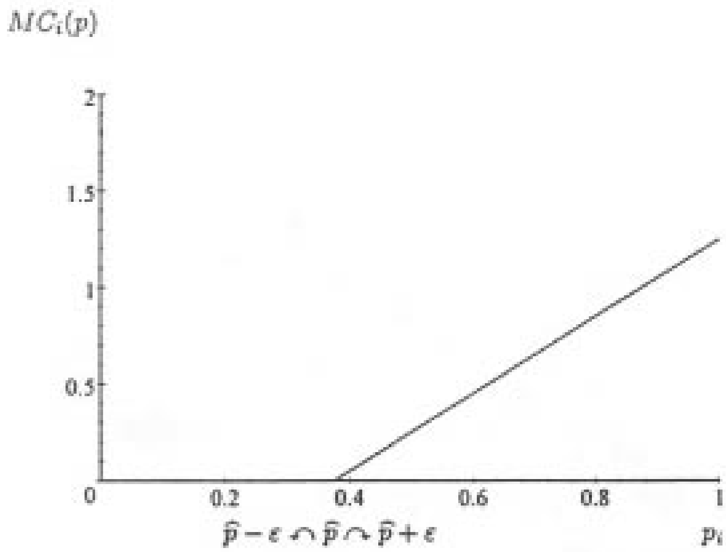

Figure 1b: Marginal contribution $MC_i(p)$

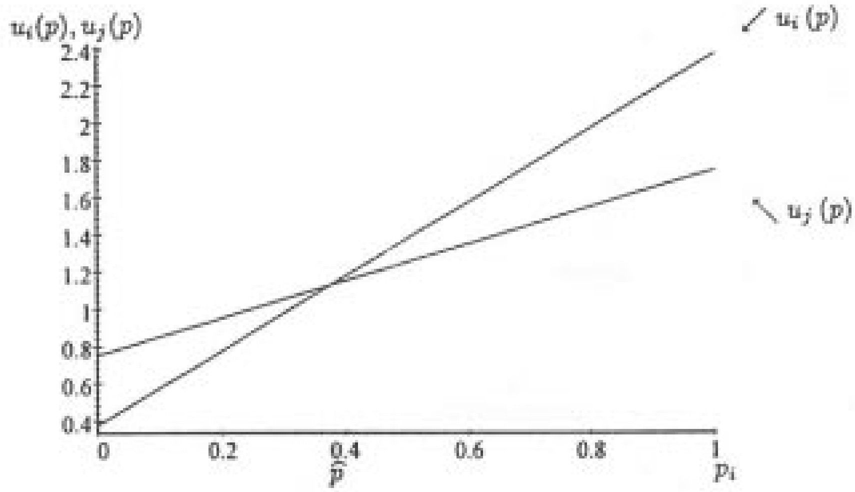

Figure 2a: $u_i(p)$ and $u_j(p)$

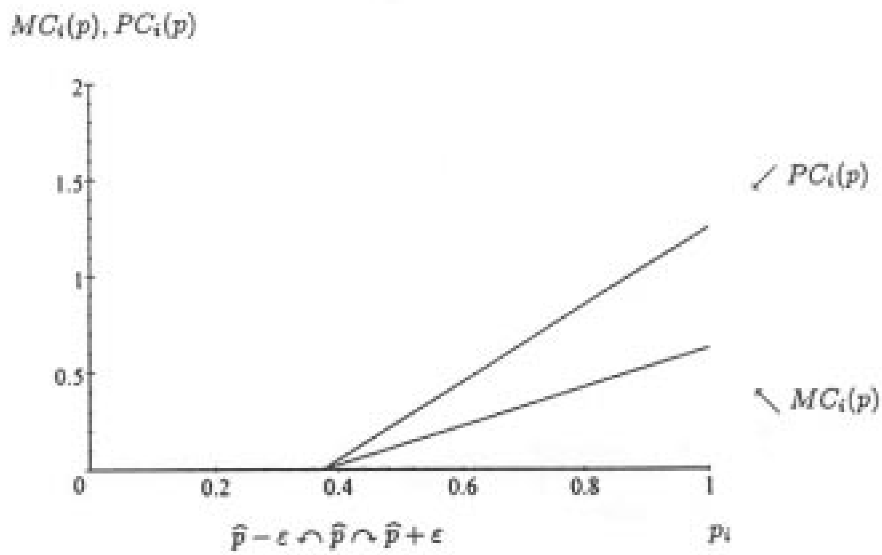

Figure 2b: Marginal and pivotal contribution: $MC_i(p), PC_i(p)$

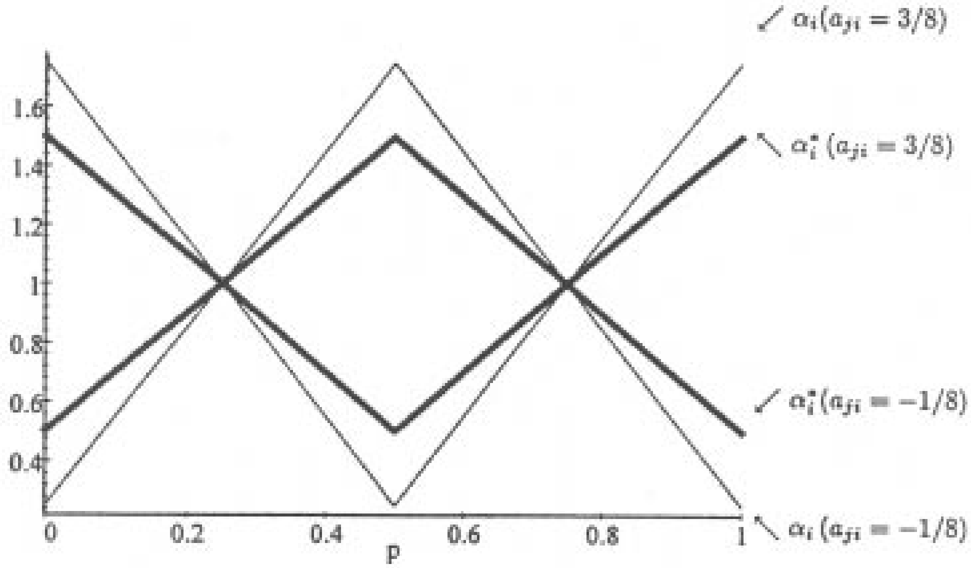

Fig.3: Information Acquisition with a Public Good
($a_{ii} = 3/4,\ c = 1/96$)