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NOTES AND COMMENTS

INFORMATION ACQUISITION IN AUCTIONS

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1. INTRODUCTION

OUR AIM IN THIS PAPER is to study the incentives to acquire information. We consider decision problems where the payoff has the single-crossing property and signals are affiliated with the unknown parameter. We introduce the notion of *risk-sensitivity*, and establish that the value of information is higher in decision problems that are more risk-sensitive. We apply this result to auctions: we are able to show that a first price auction induces more information acquisition than a second price auction.

Consider a decision maker choosing an action a to maximize the expected value of a payoff function $u(v, a)$. The decision maker does not observe v, which he regards as a random variable V. Instead, he observes the realization of a random variable X, a signal which conveys information about v. We assume that the decision maker can make X more informative, at a cost. Making X more informative increases the expected payoff. We analyze the returns to making X more informative.

We focus on problems where X and V are affiliated, and $u(v, a)$ has the weak single-crossing property in $(a; v)$ in the definition of Milgrom and Shannon (1994). See Figure 1 for examples of single-crossing payoff functions $u_I(v, a)$ and $u_{II}(v, a)$.

For these problems we are able to identify a determinant of the value of information. To this end, we introduce the notion of "risk-sensitivity." Let $a^*(v)$ denote the a that maximizes $u(v, a)$ for given v. We define a payoff to be more risk-sensitive than another if, for given v, it decreases more sharply as a moves away from $a^*(v)$. See Figure 1, where u_I is more risk-sensitive than u_{II}.[2] We show that the more risk-sensitive a payoff function, the larger the increase in expected revenue from adopting a more accurate signal. Intuitively, information is more valuable in problem I since inferring the wrong value for v, and hence taking a suboptimal action $a \neq a^*(v)$, results in a larger loss of utility than in problem II. We exploit this property in Theorem 2 to determine which decision problems induce more information acquisition.

We then apply this result to auctions. Suppose that, before an auction, bidders could acquire information about the value V of the object for sale. Would they choose to acquire more information before a first or a second price auction? The answer is: before a first price auction. Indeed, we show that the payoff in a first price auction is more risk-sensitive than in a second price, as in Figure 1. The intuition for why the value of information is higher in a first price auction is as follows. The value of the object V is informative about the opponent's bid, because the opponent's signal is positively correlated with V. When bidder 1 acquires information about V, his signal becomes "more

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[2] Despite the appearance of Figure 1, quasi-concavity of $u(v, a)$ in a is not necessary for our arguments.

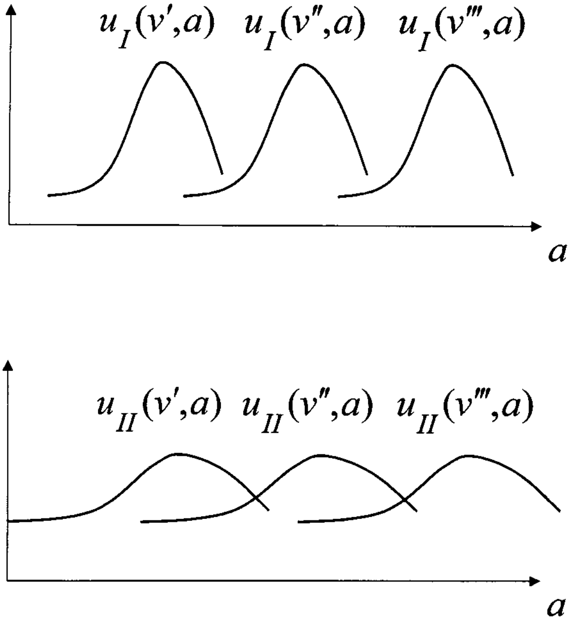

FIGURE 1.—The function u_I is more risk-sensitive than u_{II}. Here $v' < v'' < v'''$.

correlated" with V and bidder 1 ends up bidding closer—in a statistical sense—to bidder 2. Now consider the two mechanisms: in a second price auction the price paid does not depend on the winner's bid; hence, there is no incentive to bid close to the opponent, conditional on winning or losing. The payoff is not very risk-sensitive. However, in a first price auction there is a clear incentive to bid close to the opponent: the winner in a first price auction could save money by bidding just above the second highest bidder. The payoff is very risk-sensitive. As a result, correlation with the opponent's bid is more valuable in a first price auction. Since information about the object provides correlation with the opponent's bid, the result follows.

The paper proceeds as follows. Section 2 analyzes decision problems. First, the model and the assumptions are presented. Section 2.3 introduces accuracy, a somewhat novel definition of informativeness of signals. This notion is more general than Blackwell's sufficiency for the kind of decision problems we consider. Section 2.4 presents the result on the marginal revenue of information. Section 3 applies this result to auctions. The model is introduced in Section 3.1, and in Section 3.2 first and second price auctions are compared. Section 4 discusses related literature and concludes.

2. AFFILIATED DECISION PROBLEMS

2.1. *The Decision Problem*

A *payoff function* is a function

$$u(v,a): \mathscr{V}\times\mathscr{A} \to \mathbb{R}.$$

The real number $v \in \mathscr{V}$ is an unknown parameter, seen as the realization of a random variable V. Let $g(v)$ be the prior density for V, with c.d.f. $G(v)$. The real number $a \in \mathscr{A}$ is the action that the decision maker takes.

The decision maker cannot observe v, but can observe a *signal* X^η that he chooses from a *family of signals* $\{X^\eta\}_{\eta\in E}$, where E is an interval of the real line. X^η is a random variable with conditional density $f^\eta(x|v)$ and c.d.f. $F^\eta(x|v)$. Denote the support of X^η by $\mathscr{X}^\eta$, and let $\mathscr{X} := \bigcup_{\eta\in E}\mathscr{X}^\eta$. We use the term *statistical structure* to denote a prior $G(v)$ together with a family of signals.

A payoff function together with a signal X^η and a prior $G(v)$ give rise to the *decision problem*

$$\max_{a\in\mathscr{A}}\int_{\mathscr{V}} u(v,a)\,dG^\eta(v|x),$$

where $G^\eta(v|x)$ denotes the density for V conditional on observing $X^\eta = x$.

Let $a^\eta(x)$ be an optimal action upon observing $X^\eta = x$,

$$a^\eta(x) \in \operatorname{argmax}_{a\in\mathscr{A}}\int_{\mathscr{V}} u(v,a)\,dG^\eta(v|x).$$

Let

$$R(\eta) := \int_{\mathscr{V}}\int_{\mathscr{X}} u(v,a^\eta(x))\,dF^\eta(x|v)\,dG(v)$$

denote the expected payoff of a decision maker endowed with signal X^η.

A decision maker choosing his signal X^η from $\{X^\eta\}_{\eta\in E}$ will balance $R(\eta)$ with the cost $C(\eta)$ of acquiring that signal. We call the optimization problem

$$\max_{\eta\in E} R(\eta) - C(\eta)$$

an *information acquisition problem*. In the case where $R(\eta)$ is differentiable, let

$$MR(\eta) := \frac{\partial}{\partial\eta}R(\eta)$$

denote the marginal revenue to the decision maker from increasing η.

We restrict attention to the class of payoffs that have the single-crossing property in $(a;v)$ (see Athey (1997), Milgrom and Shannon (1994), and Figure 1 for pictures of single-crossing functions). The next definition is instrumental in defining the single-crossing property.

DEFINITION 1 (Karamardian and Schaible (1990)): A function $H(v)$ is *quasi-monotone* if $v' > v$ and $H(v) > 0$ imply $H(v') \geq 0$.

A quasi-monotone function $H(v)$ crosses the line $y \equiv 0$ at most once, and from below, as v increases.

DEFINITION 2: A function $u(v,a)$ has the *single crossing property in* $(a;v)$ if for any pair $a' > a$, $u(v,a') - u(v,a)$ is quasi-monotone in v.

Milgrom and Shannon (1994) call this the *weak single-crossing property*. Suppose $u(v,a)$ is differentiable in a. By Theorem 3 in Milgrom and Shannon (1994), if $u(v,a)$ has the single-crossing property in $(a;v)$, then $\partial u(v,a)/\partial a$ is quasi-monotone as a function of v.

DEFINITION 3: Given two functions $u_I(v,a)$ and $u_{II}(v,a)$, we say that u_I is *more risk-sensitive than* u_{II} (and we write $u_I \succeq u_{II}$) if $u_I - u_{II}$ has the single-crossing property in $(a;v)$. When $u_I \succeq u_{II}$ and $u_{II} \succeq u_I$, we say that u_I and u_{II} have the same degree of risk-sensitivity (and we write $u_I \sim u_{II}$).

Suppose u_I and u_{II} are differentiable in a. Then $u_I \succeq u_{II}$ means that $\partial u_I(v,a)/\partial a$ crosses $\partial u_{II}(v,a)/\partial a$ at most once, and from below, as v increases; see Figure 1.

The class of single-crossing payoff functions is naturally connected with the notion of affiliation. When a single-crossing payoff function is coupled with a signal X^η that is affiliated with V, the resulting optimal strategy $a^\eta(x)$ is nondecreasing in x (see Athey (1997), Jewitt (1987)).

DEFINITION 4 (Milgrom and Weber (1982)): Two random variables X and V with joint density $f(x,v)$ are *affiliated* when

$$x' > x,\ v' > v \Rightarrow f(x',v')f(x,v) \geq f(x,v')f(x',v).$$

Affiliation implies that the ratio $F(x|v)/f(x|v)$ is nonincreasing in v (see Milgrom and Weber (1982)). Affiliation between two random variables is equivalent to the monotone likelihood ratio property. A high realization of x is "good news" in the terminology of Milgrom (1981), in that it is associated with higher values of V. When X and V are affiliated, we use the terms *affiliated signals* and *affiliated decision problem* in the obvious way.

2.2. *Assumptions*

We focus on smooth decision problems, where signals vary continuously in the index η as defined in the following two assumptions.

A1: *The payoff $u(v,a)$ is differentiable in a, and the optimal action $a^\eta(x)$ is a differentiable function of x and η.*

A2: *For all (x,v) the function $F^\eta(x|v)$ is differentiable with respect to η on E, and is continuous in v.*

Assumption A1 is a restriction on the payoff and on the optimal strategy: it requires some knowledge about the solution of the decision problem. In particular the optimal strategy must be a smooth function of the signal. Many important economic models satisfy A1, including those in this paper.[3] Assumption A2 requires smoothness in the way the family of signals depends on η; see Persico (1996a) for examples of families of signals satisfying A2.[4]

[3] This assumption is relaxed in Athey and Levin (1998).

[4] For example, A2 is satisfied if X^η is distributed as a Uniform, or as a Normal, with mean v and variance $1/\eta$.

2.3. *Information*

The standard notion of informativeness of a signal is Blackwell's sufficiency. However, there are very few pairs of signals that are ranked in terms of sufficiency, including some that *cannot* be ranked despite one signal appearing intuitively to be more informative than the other. Thus, a more general (and easier to check) notion of informativeness of signals is useful.

Here we use a notion of informativeness which we call accuracy. Given two information structures, verifying that one is more accurate than another is straightforward. In addition, whenever a signal is sufficient for another then it is also more accurate, but not conversely.[5] Accuracy, which has been proposed in the statistics literature (Lehmann (1988)), also arises naturally in other economic contexts such as principal-agent problems (see Jewitt (1997) and Kim (1995)). Accuracy is simpler to verify than sufficiency, and more intuitive. Section 2.4 below shows that accuracy is the appropriate concept for understanding the marginal value of information in affiliated decision problems.

Definition 5: Given two signals X^{η} and X^{θ}, we say that X^{θ} is *more accurate* than X^{η} if

$$T_{\eta,\theta,v}(x) := F^{\theta^{-1}}(F^{\eta}(x|v)|v)$$

is nondecreasing in v, for every x.

We say that the family of signals $\{X^{\eta}\}_{\eta\in E}$ is *A-ordered* if a signal with a higher index is more accurate than a signal with a lower index.

In order to understand better the concept of accuracy, note that from the definition of $T_{\eta,\theta,v}(x)$ it follows that

$$(1) \qquad T_{\eta,\theta,v}(X^{\eta}|v) \quad \text{is distributed as} \quad X^{\theta}|v.$$

Thus, a more accurate signal can be obtained by subjecting a less accurate signal to the $T_{\eta,\theta,v}$ transformation. The fact that $T_{\eta,\theta,v}(x)$ is increasing in v means that the $T_{\eta,\theta,v}(\cdot)$ transformation varies together with v: the new signal obtained by applying this transformation is higher than the old signal when v is high, and lower when v is low. Hence, the notion of a more accurate signal can be interpreted as one of a signal that is more correlated with the random variable V; the $T_{\eta,\theta,v}(\cdot)$ transformation imposes this additional correlation on the original signal.[6]

[5] For instance, consider a signal distributed as a Uniform with mean v. Decreasing the variance of the uniform distribution makes the signal more accurate, but it does *not* make it more informative in Blackwell's sense. For details concerning the concept of accuracy and its relationship to sufficiency, see Persico (1996a). See also Athey and Levin (1998) for related concepts.

[6] Applying accuracy to hypothesis testing may be illuminating. Consider a case with two states of the world, $v_1 < v_2$. Let X^{η} be an information structure affiliated with V. Any most powerful test of v_1 versus v_2 has the form "accept v_2 if and only if $X^{\eta} > x^*$." The probability of type I error is then $F^{\eta}(x^*|v_2)$, and the probability of type II error is $1 - F^{\eta}(x^*|v_1)$. Let X^{θ} be more accurate than X^{η}. Using information structure X^{θ} we can design a test that is more powerful than the above test. Indeed, choose x^{**} such that $F^{\theta}(x^{**}|v_2) = F^{\eta}(x^*|v_2)$. Then the test "accept v_2 if and only if $X^{\theta} > x^{**}$" has the same probability of type I error as the previous test. Furthermore, since X^{θ} is more accurate than X^{η}, $x^{**} \geq F^{\theta^{-1}}(F^{\eta}(x^*|v_1)|v_1)$. This means that the test based on X^{θ} has lower probability of type II error; see Lehmann (1988).

The next theorem formalizes that A-order is a necessary and sufficient condition for "better information," when X^θ, X^η are affiliated with V and the payoff satisfies the single crossing property.

THEOREM 1: *Suppose X^θ, X^η are affiliated with V. Then*

$$\{X^\theta \text{ is more accurate than } X^\eta\} \Leftrightarrow \left\{\begin{array}{l}\text{For all payoffs } u(v,a) \text{ having the} \\ \text{single-crossing property}, R(\theta) \geq \\ R(\eta).\end{array}\right\}.$$

PROOF: See Lehmann (1988).

2.4. *The Marginal Value of Information*

Consider two decision problems with the same statistical structure. Which of the two problems has the highest marginal return to information? Theorem 2 below says that it is the more risk-sensitive. For pictorial insight refer to Figure 1 where $u_I \succeq u_{II}$: intuitively, a marginal increase in the accuracy of information is more valuable in problem I since inferring the wrong value for v, and hence taking a suboptimal action, results in a larger marginal loss of utility than in problem II. Once this is established, Theorem 2, part (iii) tells which of the two decision problems will yield greater information acquisition. Note that Theorem 2 holds a fortiori if acquiring information is taken to be in the sense of Blackwell, because if two signals are ranked according to sufficiency they are necessarily ranked according to accuracy.

THEOREM 2: *Given $\mathscr{V}$ and $\mathscr{A}$, consider two payoff functions u_I and u_{II} associated to the same A-ordered statistical structure, giving rise to two information acquisition problems satisfying A1 and A2. If $u_I(v, a_I^\eta(x)) \succeq u_{II}(v, a_{II}^\eta(x))$, then:*

(i) $MR_I(\eta) \geq MR_{II}(\eta)$;

(ii) *if $u_I(v, a_I^\eta(x)) \sim u_{II}(v, a_{II}^\eta(x))$, then $MR_I(\eta) = MR_{II}(\eta)$;*

(iii) *the set of optimal accuracies in problem I is higher, in the strong set order,*[7] *than that of problem II.*

PROOF: For part (i), notice that

$$\begin{aligned} &MR_I(\eta) - MR_{II}(\eta) \\ &\quad = \frac{d}{d\theta} \int_{\mathscr{V}} \int_{\mathscr{X}} [u_I(v, a_I^\theta(x)) - u_{II}(v, a_{II}^\theta(x))] \, dF^\theta(x|v) \, dG(v) \bigg|_{\theta=\eta}. \end{aligned}$$

Denoting

$$u^\theta(v,x) := u_I(v, a_I^\theta(x)) - u_{II}(v, a_{II}^\theta(x)),$$

[7] Given A and B subsets of the real line, we say that *A is higher than B in the strong set order* when, for every $a \in A$ and $b \in B$ we have $\max\{a,b\} \in A$ and $\min\{a,b\} \in B$.

we wish to show that

$$\frac{d}{d\theta}\int_{\mathscr{V}}\int_{\mathscr{X}} u^{\theta}(v,x)\, dF^{\theta}(x|v)\, dG(v)\bigg|_{\theta=\eta} \geq 0.$$

Expression (1) on page 139 suggests a change of variable: we can rewrite $MR_I(\eta) - MR_{II}(\eta)$ as

$$\text{(2)} \qquad \begin{aligned} &\frac{d}{d\theta}\int_{\mathscr{V}}\int_{\mathscr{X}} u^{\theta}(v, T_{\eta,\theta,v}(x))\, dF^{\eta}(x|v)\, dG(v)\bigg|_{\theta=\eta} \\ &= \frac{d}{d\theta}\int_{\mathscr{X}}\int_{\mathscr{V}} u^{\theta}(v, T_{\eta,\theta,v}(x))\, dG^{\eta}(v|x)\, dF^{\eta}(x)\bigg|_{\theta=\eta} \\ &= \int_{\mathscr{X}}\int_{\mathscr{V}} \frac{d}{d\theta} u^{\theta}(v, T_{\eta,\theta,v}(x))\, dG^{\eta}(v|x)\, dF^{\eta}(x)\bigg|_{\theta=\eta}. \end{aligned}$$

Differentiability of the above expression is guaranteed by A1, A2. The inner integral can be computed as

$$\text{(3)} \qquad \begin{aligned} &\int_{\mathscr{V}}\left[\frac{\partial}{\partial a} u_I(v, a_I^{\eta}(x))\right]\left[\frac{\partial}{\partial\theta} a_I^{\theta}(x)\bigg|_{\theta=\eta} + a_I^{\eta\prime}(x)\frac{\partial}{\partial\theta} T_{\eta,\theta,v}(x)\bigg|_{\theta=\eta}\right] dG^{\eta}(v|x) \\ &- \int_{\mathscr{V}}\left[\frac{\partial}{\partial a} u_{II}(v, a_{II}^{\eta}(x))\right]\left[\frac{\partial}{\partial\theta} a_{II}^{\theta}(x)\bigg|_{\theta=\eta} + a_{II}^{\eta\prime}(x)\frac{\partial}{\partial\theta} T_{\eta,\theta,v}(x)\bigg|_{\theta=\eta}\right] \\ &\times dG^{\eta}(v|x). \end{aligned}$$

The first order conditions for problems I and II are

$$\int_{\mathscr{V}}\left[\frac{\partial}{\partial a} u_m(v, a_m^{\eta}(x))\right] dG^{\eta}(v|x) = 0 \quad \text{for} \quad m = I, II.$$

Thus, equation (3) simplifies to

$$\text{(4)} \qquad \int_{\mathscr{V}} \frac{\partial}{\partial x} u^{\eta}(v,x) \frac{\partial}{\partial\theta} T_{\eta,\theta,v}(x)\bigg|_{\theta=\eta} dG^{\eta}(v|x).$$

Now we want to apply Lemma 1 in Appendix A to conclude that expression (4), and hence (2), are nonnegative; this will conclude the proof. To this end we note that $\partial u^{\eta}(v,x)/\partial x$ is quasi-monotone by assumption. By the first-order conditions

$$\int_{\mathscr{V}} \frac{\partial}{\partial x} u^{\eta}(v,x)\, dG^{\eta}(v|x) = 0.$$

It remains to show that $\partial T_{\eta,\theta,v}(x)/\partial\theta$ is nondecreasing in v; write

$$\frac{\partial}{\partial\theta} T_{\eta,\theta,v}(x)\bigg|_{\theta=\eta} = \lim_{\theta\downarrow\eta} \frac{T_{\eta,\theta,v}(x) - T_{\eta,\eta,v}(x)}{\theta - \eta}.$$

The term $T_{\eta,\eta,v}(x) \equiv x$ is independent of v. Since $T_{\eta,\theta,v}(x)$ is nondecreasing in v by assumption, we obtain that $\partial T_{\eta,\theta,v}(x)/\partial\theta$ is nondecreasing in v as required.[8]

[8] Reasoning along the lines of this proof, it is possible to obtain a "marginal version" of Theorem 1. See Persico (1996a) for details.

Part (ii) is straightforward.

Part (iii) follows from the fact that the function $R_m(\eta) - C(\eta)$ (where $m = I, II$ and $II < I$) is seen to have the single-crossing property in $(\eta; m)$. This is easily proved using part (i) above. Then Theorem 4 by Milgrom and Shannon (1994) on monotone comparative statics yields the results. *Q.E.D.*

The intuition for this result is the following. In view of expression (1), acquiring a more accurate signal is equivalent to observing $y = T_{\eta,\theta,v}(x)$ instead of x. Because $T_{\eta,\theta,v}(\cdot)$ is increasing in v, y will be larger than x if v is high, and smaller if v is low. Thus, for each v the action $a^{\eta}(y)$ is closer than $a^{\eta}(x)$ to $\text{argmax}_a u(a,v)$. This increases the payoff to the decision maker (these last two steps make use of the fact that the payoff is single-crossing: see Figure 1). How much the payoff increases, however, depends on how steeply $u(a,v)$ increases as a moves towards $\text{argmax}_a u(a,v)$. The steepness of this increase is precisely the risk-sensitivity of the payoff.

In general, checking that $u_I(v, a_I^{\eta}(x)) \succeq u_{II}(v, a_{II}^{\eta}(x))$, as required by Theorem 2, requires some knowledge of the optimal action in decision problems I and II. However, this is not the case in the application to auctions, as we see in the next section.

3. AUCTIONS

In this section we use the results developed above to discuss information acquisition in first and second price auctions.

3.1. *The Auction Model*

There are two players, 1 and 2,[9] and an object to be auctioned whose value to player i is $u_i = u(V_i, V_j)$. The function $u(v_i, v_j)$ is increasing in v_i and nondecreasing in v_j. V_1 and V_2 are random variables unobserved by the players; players share a prior $g(v_1, v_2)$ on their distribution, with $g(\cdot,\cdot)$ a symmetric affiliated density function.

Player i chooses a signal X_i^{η} from a family $\{X_i^{\eta}\}_{\eta \in E}$ at a cost $C(\eta)$. X_i^{η} has a density $f_{X_i}^{\eta}(x_i|v_i)$ whose c.d.f. satisfies A2. Thus, X_i^{η} conveys information about v_i. X_i^{η} is affiliated with V_i. The family of signals is A-ordered, and $E = [\underline{\eta}, \infty)$. We assume $C(\underline{\eta}) = 0$. The interpretation is that agents receive a signal of accuracy $\underline{\eta}$ for free, and may choose to improve the accuracy of their signal at a cost $C(\eta)$. For each η_1, η_2 chosen by the bidders, the joint distribution of signals and value is

$$f^{\eta_1\eta_2}(x_1, x_2, v_1, v_2) = f_{X_1}^{\eta_1}(x_1|v_1) f_{X_2}^{\eta_2}(x_2|v_2) g(v_1, v_2).$$

Given this statistical structure, X_1, X_2, V_1, and V_2 are affiliated (see Theorem 1 (ii) in Milgrom and Weber (1982)). Let $\tilde{u}(v_i, x_j) = E(u(V_i, V_j)|V_i = v_i,\ X_j = x_j)$.

Two special cases of this model are, independent signals when $g(v_1, v_2) = g(v_1)g(v_2)$, and the mineral rights model when $V_1 \equiv V_2 \equiv V$.

The *m information acquisition game* consists of two stages:

(i) player i chooses η_i, independently from, and simultaneously with, his opponent.

(ii) after observing the realization of $X_i^{\eta_i}$, but not the opponent's choice of η_j, players compete for the object in mechanism m.

[9] This is for expositional ease: all the results carry over to the n-bidders case.

A pure strategy in the m information acquisition game is a pair $(\eta_m, b_m(\cdot))$ listing the accuracy choice in stage (i) and the strategy in the subsequent bidding stage. A symmetric pure strategy combination for the m information acquisition game is a pure strategy combination of the form: in the first stage, both players acquire a signal of accuracy η_m; in the second stage, both players play strategy $b_m(\cdot)$ in mechanism m. We model information acquisition as a *covert* activity, which means that in stage (ii) players do not observe the opponent's accuracy choice before bidding. This allows us to apply the results in the previous section to characterize the equilibrium of the game.

We introduce some notation about the marginal benefit from increasing accuracy. First, we define the marginal benefit from increasing accuracy starting from η, when the opponent has accuracy θ and moreover (wrongly) thinks the situation is symmetric at θ. Let

$AMR_m(\theta, \eta) :=$ player 1's marginal revenue from increasing η in mechanism m when (a) player 1 has accuracy η, and (b) is best responding to a player 2 who has accuracy θ and plays the symmetric equilibrium strategy as if both players had accuracy θ.

Let now

$$MR_m(\eta) := AMR_m(\eta, \eta).$$

$MR_m(\eta)$ is the marginal revenue from increasing accuracy when both bidders have accuracy η. We denote with $MC(\eta)$ the marginal cost of increasing accuracy.

3.2. *First vs. Second Price Auction*

We first show how the auction model of Section 3.1 relates to the analysis of Section 2. Consider a second price auction. Let us check that a bidder faces an affiliated decision problem when choosing his bid. Bidders 1 and 2 receive signals X_1 and X_2 respectively, and bid for an object of value V. Suppose that the joint density for X_1, X_2, V_1, and V_2 is

$$f(x_1|v_1)f(x_2|v_2)g(v_1, v_2).$$

The problem of bidder 1 with signal x_1 in a second price auction is

$$\max_b \int_{-\infty}^{b_S^{-1}(b)} [v(x_1, x_2) - b_S(x_2)] f(x_2|x_1)\, dx_2,$$

where

$$\begin{aligned} v(x_1, x_2) &= E(u(V_1, V_2)|X_1 = x_1, X_2 = x_2) \\ &= \int_{-\infty}^{+\infty} \tilde{u}(v_1, x_2) f(v_1|x_1, x_2)\, dv_1, \end{aligned}$$

and $b_S(x_2)$ is bidder 2's strategy. Substituting for $v(x_1, x_2)$ yields

$$\begin{aligned} &\max_b \int_{-\infty}^{b_S^{-1}(b)} \left[\int_{-\infty}^{+\infty} (\tilde{u}(v_1, x_2) - b_S(x_2)) f(v_1|x_1, x_2)\, dv_1 \right] f(x_2|x_1)\, dx_2 \\ &\quad = \max_b \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{b_S^{-1}(b)} (\tilde{u}(v_1, x_2) - b_S(x_2)) \right] \\ &\qquad \times f(x_2|x_1) f(v_1|x_1, x_2)\, dx_2\, dv_1. \end{aligned}$$

Since $f(x_2|x_1)f(v_1|x_1,x_2)=f(x_2|v_1)f(v_1|x_1)$, the previous expression can be written as

$$\max_b \int_{-\infty}^{+\infty} u_S(v_1,b)f(v_1|x_1)\,dv_1,$$

where

$$u_S(v_1,b)=\int_{-\infty}^{b_S^{-1}(b)}[\tilde{u}(v_1,x_2)-b_S(x_2)]\,dF(x_2|v_1).$$

A similar analysis can be performed on the first price auction, to get

$$u_F(v_1,b)=\int_{-\infty}^{b_F^{-1}(b)}[\tilde{u}(v_1,x_2)-b]\,dF(x_2|v_1),$$

where b_F denotes bidder 2's strategy. As is well known (see, for example, Athey (1997)), $u_S(v_1,b)$ and $u_F(v_1,b)$ have the single-crossing property in $(b;v_1)$. Thus, the bidders' decision problems in first and second price auctions are of the type analyzed in Section 2. Then, in view of Theorem 1, accuracy is the appropriate concept of "better information" for a first or second price auction: increasing the accuracy of one's information is beneficial, irrespective of the opponent's strategy and accuracy level. This observation is formalized in the following Fact.

FACT 1: Consider the first and second price information acquisition games described in Section 3.1. Let the index $m=F,S$ denote a first and second price auction, respectively. Suppose player 2 has accuracy θ and plays any increasing strategy $b_m^\theta(\cdot)$. Suppose player 1 has accuracy η and plays his best response to $b_m^\theta(\cdot)$. Then for all (η,θ), increasing η is beneficial to player 1. Formally, $AMR_F(\theta,\eta), AMR_S(\theta,\eta)>0$.

The next proposition proves that at a symmetric equilibrium the marginal return to information is higher in a first price auction than in a second price.

PROPOSITION 1: *Consider the first and second price information acquisition games described in Section* 3.1. *Then* $MR_F(\eta)\geq MR_S(\eta)$ *for all* η.

PROOF: In view of Theorem 2, it is enough to show that $u_F(v,b_F^\eta(x_1))\succeq u_S(v,b_S^\eta(x_1))$. For a second price auction we have, at a symmetric equilibrium,

$$u_S(v,b_S^\eta(x_1))=\int_{-\infty}^{x_1}[\tilde{u}(v_1,y)-b_S^\eta(y)]f_{X_2}^\eta(y|v_1)\,dy.$$

For a first price auction we have

$$u_F(v,b_F^\eta(x_1))=\int_{-\infty}^{x_1}[\tilde{u}(v_1,y)-b_F^\eta(x_1)]f_{X_2}^\eta(y|v_1)\,dy, \tag{5}$$

whence

$$u_F(v,b_F^\eta(x_1))-u_S(v,b_S^\eta(x_1))=\int_{-\infty}^{x_1}[b_S^\eta(y)-b_F^\eta(x_1)]f_{X_2}^\eta(y|v_1)\,dy.$$

We need to show that

$$\frac{\partial}{\partial x_1}[u_F(v, b_F^\eta(x_1)) - u_S(v, b_S^\eta(x_1))]$$

is quasi-monotone. This expression reads

$$\left[-b_F^\eta(x_1) - b_F^{\eta\prime}(x_1)\frac{F_{X_2}^\eta(x_1|v_1)}{f_{X_2}^\eta(x_1|v_1)} + b_S^\eta(x_1)\right] f_{X_2}^\eta(x_1|v_1). \tag{6}$$

Since X_2 and V_1 are affiliated, $F_{X_2}^\eta(x_1|v_1)/f_{X_2}^\eta(x_1|v_1)$ is nonincreasing in v_1. Thus, expression (6) is quasi-monotone in v_1. *Q.E.D.*

The proof of Proposition 1 hinges on the properties of $F(x|v)/f(x|v)$. This ratio can be interpreted as an index of the "money left on the table" by the winner in a first-price auction. When $F(x|v)/f(x|v)$ is very large, the probability that your opponent is bidding close to you is small. Then, in a first price auction you could save money by reducing your bid, and still be winning with almost the same probability; in other words, lots of money is left on the table. By contrast, in a second price auction this deviation is not profitable because you pay the other player's bid: no money is left on the table in a second price auction. An additional bit of correlation with v (information) is more useful in a first price auction because it allows to correlate more closely your bid with the opponent's, and thus to reduce the money left on the table.[10]

We next translate Proposition 1 into a statement about symmetric pure strategy equilibria of the information acquisition games. A necessary condition for a symmetric equilibrium at (η_m^*, η_m^*) is that $MC(\cdot)$ intersects $MR_m(\cdot)$ from below in η_m^*. In view of Proposition 1 the following is straightforward:

FACT 2: Suppose a symmetric pure strategy equilibrium exists for a first and second price information acquisition game described in Section 3.1. If $MC(\cdot)$ only intersects $MR_S(\cdot)$ once, then the equilibrium accuracy in the first price auction η_F^* will be higher than or equal to η_S^*, the accuracy in a second price auction.

The question then is which cost functions satisfy the conditions of Fact 2. A cost function of the form $C(\eta) = (\eta - \underline{\eta})^\alpha$ will certainly do for α sufficiently high. Indeed, if α is large enough we are guaranteed that MC crosses the MR_m functions only once, and from below, at η_m^*; then Fact 2 applies. To check that an equilibrium exists at η_m^*, we must also check that $MC(\eta) < AMR_m(\eta_m^*, \eta)$ if and only if $\eta < \eta_m^*$. This is guaranteed by choosing α sufficiently large, because by Fact 1 $AMR_F(\eta_F^*, \eta), AMR_S(\eta_S^*, \eta) > 0$. These observations are collected in the following corollary.

COROLLARY 1: *Take a cost function $C(\eta) = (\eta - \underline{\eta})^\alpha$ and an α sufficiently high. Then for the first and second price information acquisition game described in Section* 3.1, *a pure strategy symmetric equilibrium exists and is unique. Equilibrium accuracy is higher in the first than in the second price game.*

[10] In the limiting case where V_1 and V_2 are independent, acquiring information about v_1 does not make player 1 more correlated with player 2's bid. In this case $F_{X_2}^\eta(x_1|v_1)/f_{X_2}^\eta(x_1|v_1)$ is independent of v_1 and thus $MR_F = MR_S$.

What about the revenue to the auctioneer? An important result in auction theory states that—when the information structure is exogenously fixed—a second price auction gives the auctioneer a higher expected profit than a first price auction (Milgrom and Weber (1982)). However, when information acquisition is allowed as in our model, it is possible that a first price auction yields more revenue than a second price. See Persico (1996b) for an example.

4. DISCUSSION AND RELATED LITERATURE

We obtain an explicit expression for the marginal value of information in the class of decision problems where the payoff function exhibits the single-crossing property and the statistical structure is affiliated. We show that the value of information in such decision problems is determined by the degree of risk-sensitivity of the payoff.

We use this result to demonstrate how auction formats can be ranked in terms of bidders' incentives to acquire information about the object for sale. Different auctions use winning and losing bids in different ways to determine payoffs. Since learning about the object for sale conveys information about the opponent's bid, information about the object is more valuable in auctions where information about the opponent's bid is more important. Our work shows how bidders' information acquisition behavior varies across auction formats. We find that a first price auction is more risk-sensitive than a second price, and thus information is more valuable in the former. This is because in a first price, more information allows the winning bidder to reduce the money left on the table. This effect is absent in a second price auction: there is no money left on the table there. This makes information more valuable in a first price.

In contrast to most results on the value of information in oligopoly games, we analyze a very general model of information acquisition without resorting to special functional forms. Our results should be compared with the existing literature on information acquisition in auctions. In an unpublished work, Matthews (1977) compares a first and a second price auction: the two auction forms are found to give the same incentives to acquire information. Similarly, Hausch and Li (1991) find no difference in the incentives to acquire information, among all independent private-values auction mechanisms. Our work makes clear that this is due to their choice of statistical structure.[11] In a recent work independent of ours, Gaier (1995) takes up Matthews' primitives and finds that a first price auction gives stronger incentives to acquire information than a royalty rate auction. This result can be couched in the terms of the present paper, and is discussed in Persico (1996b). The techniques presented here are used in Persico (1996b) to compare other auction forms in terms of marginal incentives to acquire information.

The ideas developed in this paper translate immediately to modeling research and development. Consider the following more general specification for the value to player i: $u_i = u(V_i, V_j, X_i, \eta_i)$. Here, increasing accuracy may have a value-enhancing (or cost-reducing) effect, and receiving a high signal may be good for the payoff. These features allow us to reinterpret increasing accuracy as R&D activity, and all the results in the present work apply. Tan (1992) presents a model where, prior to a procurement auction, firms invest in cost-reducing R&D. Tan finds that first and second price are equivalent in terms of incentives to invest. Using the intuition developed here helps to interpret that result. In Tan's model, the R&D activity is one of *independent* stochastic cost reduction,

[11] The statistical structures of Matthews (1977, 1984) and Hausch and Li (1991) share the feature that $F_{X_2}(x_1|v_1)/f_{X_2}(x_1|v_1)$ is independent of v_1.

in that a firm's decision to reduce costs does not make its cost function statistically more correlated with the other firm's. Thus, once again an independence assumption is key to the result. Our work shows that if this assumption is relaxed a first price auction induces more investment in R&D than a second price.

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APPENDIX

The following lemma is used in the literature to establish comparative statics results in setups with the monotone likelihood ratio and the single-crossing property. The lemma is also widely used in the literature on comparative statics of risk (see Eeckhoudt and Gollier (1994)).

LEMMA 1: *Let (c,d) be an interval of the real line, $J(\cdot)$ a nondecreasing function, $H(\cdot)$ a quasi-monotone function. Assume that for some measure μ on $\Re$ we have*

$$\int_c^d H(v)\, d\mu(v) = 0. \tag{7}$$

Then $\int_c^d H(v)J(v)\, d\mu(v) \geq 0$.

PROOF: Because H is quasi-monotone and satisfies (7), there must be a $v_0 \in [c,d]$ such that H is nonpositive for $v < v_0$, and nonnegative for $v > v_0$. Denote $\tilde{J}(v) := J(v) - J(v_0)$. Since $J(\cdot)$ is nondecreasing, $\tilde{J}(v)$ is nonpositive for $v < v_0$, and nonnegative for $v > v_0$. Then

$$\begin{aligned}\int_c^d H(v)J(v)\, d\mu(v) &= \int_c^d H(v)\tilde{J}(v)\, d\mu(v)\\ &= \int_c^{v_0} H(v)\tilde{J}(v)\, d\mu(v) + \int_{v_0}^d H(v)\tilde{J}(v)\, d\mu(v) \geq 0,\end{aligned}$$

where the first equality uses (7) and the inequality follows from H and $\tilde{J}$ having the same sign on (c, v_0) and (v_0, d). *Q.E.D.*

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