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**Information and Entropy  
Econometrics —  
A Review and Synthesis**

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# Information and Entropy Econometrics — A Review and Synthesis

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## Information and Entropy Econometrics — A Review and Synthesis\*

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This work is dedicated to Marge and George Judge

### Abstract

The overall objectives of this review and synthesis are to study the basics of information-theoretic methods in econometrics, to examine the connecting theme among these methods, and to provide a more detailed summary and synthesis of the sub-class of methods that treat the observed sample moments as stochastic. Within the above objectives, this review focuses on studying the inter-connection between information theory, estimation, and inference. To achieve these objectives, it provides a detailed survey of information-theoretic concepts and quantities used within econometrics. It also illustrates

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the use of these concepts and quantities within the subfield of information and entropy econometrics while paying special attention to the interpretation of these quantities. The relationships between information-theoretic estimators and traditional estimators are discussed throughout the survey. This synthesis shows that in many cases information-theoretic concepts can be incorporated within the traditional likelihood approach and provide additional insights into the data processing and the resulting inference.

*Keywords:* Empirical likelihood; entropy, generalized entropy; information; information theoretic estimation methods; likelihood; maximum entropy; stochastic moments.

*JEL codes:* C13, C14, C49, C51

## Preface

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This review and synthesis is concerned with information and entropy econometrics (IEE). The overall objective is to summarize the basics of information-theoretic methods in econometrics and the connecting theme among these methods. The sub-class of methods that treat the observed sample moments as stochastic is discussed in greater detail. Within the above objective, we restrict our attention to study the interconnection between information theory, estimation, and inference. We provide a detailed survey of information-theoretic concepts and quantities used within econometrics and then show how these quantities are used within IEE. We pay special attention for the interpretation of these quantities and for describing the relationships between information-theoretic estimators and traditional estimators.

In Section 1, an introductory statement and detailed objectives are provided. Section 2 provides a historical background of IEE. Section 3 surveys some of the basic quantities and concepts of information theory. This survey is restricted to those concepts that are employed within econometrics and that are used within that survey. As many of these concepts may not be familiar to many econometricians and economists, a large number of examples are provided. The concepts discussed



include entropy, divergence measures, generalized entropy (known also as Cressie Read function), errors and entropy, asymptotic theory, and stochastic processes. However, it is emphasized that this is not a survey of information theory. A less formal discussion providing interpretation of information, uncertainty, entropy and ignorance, as viewed by scientists across disciplines, is provided at the beginning of that section. In Section 4, we discuss the classical maximum entropy (ME) principle (both the primal constrained model and the dual concentrated and unconstrained model) that is used for estimating under-determined, zero-moment problems. The basic quantities discussed in Section 3, are revisited again in connection with the ME principle. In Section 5, we discuss the motivation for information-theoretic (IT) estimators and then formulate the generic IT estimator as a constrained optimization problem. This generic estimator encompasses all the estimators within the class of IT estimators. The rest of this section describes the basics of specific members of the IT class of estimators. These members compose the sub-class of methods that incorporate the moment restrictions within the generic IT-estimator as (pure) zero moments' conditions, and include the empirical likelihood, the generalized empirical likelihood, the generalized method of moments and the Bayesian method of moments. The connection between each one of these methods, the basics of information theory and the maximum entropy principle is discussed. In Section 6, we provide a thorough discussion of the other sub-class of IT estimators: the one that views the sample moments as stochastic. This sub-class is also known as the generalized maximum entropy. The relevant statistics and information measures are summarized and connected to quantities studied earlier in the survey. We conclude with a simple simulated example. In Section 7, we provide a synthesis of likelihood, ME and other IT estimators, via an example. We study the interconnections among these estimators and show that though coming from different philosophies they are deeply rooted in each other, and understanding that interconnection allows us to understand our data better. In Section 8, we summarize related topics within IEE that are not discussed in this survey.

Readers of this survey need basic knowledge of econometrics, but do not need prior knowledge of information theory. Those who are familiar

with the concepts of IT should skip Section 3, except Section 3.4 which is necessary for the next few sections. Those who are familiar with the ME principle can skip parts of Section 4, but may want to read the example in Section 4.7. The survey is self contained and interested readers can replicate all results and examples provided. No detailed proofs are provided, though the logic behind some less familiar arguments is provided. Whenever necessary the readers are referred to the relevant literature.

This survey may benefit researchers who wish to have a fast introduction to the basics of IEE and to acquire the basic tools necessary for using and understanding these methods. The survey will also benefit applied researchers who wish to learn improved new methods, and applications, for extracting information from noisy and limited data and for learning from these data.

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# 1

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## Introductory Statement, Motivation, and Objective

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All learning, information gathering and information processing, is based on limited knowledge, both *a priori* and data, from which a larger “truth” must be inferred. To learn about the true state of the world that generated the observed data, we use statistical models that represent these outcomes as functions of unobserved structural parameters, parameters of priors and other sampling distributions, as well as complete probability distributions. Since we will never know the true state of the world, we generally focus, in statistical sciences, on recovering information about the complete probability distribution, which represents the ultimate truth in our model. Therefore, all estimation and inference problems are translations of limited information about the probability density function (pdf) toward a greater knowledge of that pdf. However, if we knew all the details of the true mechanism then we would not need to resort to the use of probability distributions to capture the perceived uncertainty in outcomes that results from our ignorance of the true underlying mechanism that controls the event of interest.

Information theory quantities, concepts, and methods provide a unified set of tools for organizing this learning process. They provide a

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discipline that at once exposes more clearly what the existing methods do, and how we might better accomplish the main goal of scientific learning. This review first studies the basic quantities of information theory and their relationships to data analysis and information processing, and then uses these quantities to summarize (and understand the connection among) the improved methods of estimation and data processing that compose the class of entropy and information-theoretic methods. Within that class, the review concentrates on methods that use conditional and unconditional stochastic moments.

It seems natural to start by asking what is information, and what is the relationship between information and econometric, or statistical analysis. Consider, for example, Shakespeare's "Hamlet," Dostoevsky's "The Brothers Karamazov," your favorite poem, or the US Constitution. Now think of some economic data describing the demand for education, or survey data arising from pre-election polls. Now consider a certain speech pattern or communication among individuals. Now imagine you are looking at a photo or an image. The image can be sharp or blurry. The survey data may be easy to understand or extremely noisy. The US Constitution is still being studied and analyzed daily with many interpretations for the same text, and your favorite poem, as short as it may be, may speak a whole world to you, while disliked by others.

Each of these examples can be characterized by the amount of information it contains or by the way this information is conveyed or understood by the observer — the analyst, the reader. But what is information? What is the relationship between information and econometric analysis? How can we efficiently extract information from noisy and evolving complex observed economic data? How can we guarantee that only the relevant information is extracted? How can we assess that information? The study of these questions is the subject of this survey and synthesis.

This survey discusses the concept of *information* as it relates to econometric and statistical analyses of data. The meaning of "information" will be studied and related to the basics of Information Theory (IT) as is viewed by economists and researchers who are engaged in deciphering information from the (often complex and evolving) data,

while taking into account what they know about the underlying process that generated these data, their beliefs about the (economic) system under investigation, and nothing else. In other words, the researcher wishes to extract the available information from the data, but wants to do it with minimal *a priori* assumptions. For example, consider the following problem taken from Jaynes's famous Brandeis lectures (1963). We know the empirical mean value (first moment) of, say one million tosses of a six-sided die. With that information the researcher wishes to predict the probability that in the next throw of the die we will observe the value 1, 2, 3, 4, 5 or 6. The researcher also knows that the probability is proper (sum of the probabilities is one). Thus, in that case, there are six values to predict (six unknown values) and two observed (known) values: the mean and the sum of the probabilities. As such, there are more unknown quantities than known quantities, meaning there are infinitely many probability distributions that sum up to one and satisfy the observed mean. In somewhat more general terms, consider the problem of estimating an unknown discrete probability distribution from a finite and possibly noisy set of observed (sample) moments. These moments (and the fact that the distribution is proper — summing up to one) are the only available information. Regardless of the level of noise in these observed moments, if the dimension of the unknown distribution is larger than the number of observed moments, there are infinitely many proper probability distributions satisfying this information (the moments). Such a problem is called an under-determined problem. Which one of the infinitely many solutions should one use? In all the IEE methods, the one solution chosen is based on an information criterion that is related to Shannon's information measure — entropy.

When analyzing a linear regression, a jointly determined system of equations, a first-order Markov model, a speech pattern, a blurry image, or even a certain text, if the researcher wants to understand the data but without imposing a certain structure that may be inconsistent with the (unknown) truth, the problem may become inherently under-determined. The criterion used to select the desired solution is an information criterion which connects statistical estimation and inference with the foundations of IT. This connection provides us with an



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IT perspective of econometric analyses and reveals the deep connection among these “seemingly distinct” disciplines. This connection gives us the additional tools for a better understanding of our limited data, and for linking our theories with real observed data. In fact, information theory and data analyses are the major thread connecting most of the scientific studies trying to understand the true state of the world with the available, yet limited and often noisy, information.

Within the econometrics and statistical literature the family of IT estimators composes the heart of IEE. It includes the Empirical (and Generalized Empirical) Likelihood, the Generalized Method of Moments, the Bayesian Method of Moments and the Generalized Maximum Entropy among others. In all of these cases the objective is to extract the available information from the data with minimal assumptions on the data generating process and on the likelihood structure. The logic for using minimal assumptions in the IEE class of estimators is that the commonly observed data sets in the social sciences are often small, the data may be non-experimental noisy data, the data may be arising from a badly designed experiment, and the need to work with nonlinear (macro) economic models where the maximum likelihood estimator is unattractive as it is not robust to the underlying (unknown) distribution. Therefore, (i) such data may be ill-behaved leading to an ill-posed and/or ill-conditioned (not full rank) problem, or (ii) the underlying economic model does not specify the complete distribution of the data, but the economic model allows us to place restrictions on this (unknown) distribution in the form of population moment conditions that provide information on the parameters of the model. For these estimation problems and/or small and non-experimental data it seems logical to estimate the unknown parameters with minimum *a priori* assumptions on the data generation process, or with minimal assumptions on the likelihood function. Without a pre-specified likelihood, other non maximum likelihood methods must be used in order to extract the available information from the data. Many of these methods are members of the class of Information-Theoretic (IT) methods.

This survey concentrates on the relationship between econometric analyses, data and information with an emphasis on the philosophy leading to these methods. Though, a detailed exposition is provided

here, the focus of this survey is on the sub-class of IT estimators that view the observed moments as stochastic. Therefore, the detailed formulations and properties of the other sub-class of estimators that view the observed moments as (pure) zero-moment conditions will be discussed here briefly as it falls outside the scope of that review and because there are numerous recent reviews and texts of these methods (e.g., Smith, 2000, 2005, 2007; Owen, 2001; Hall, 2005; Kitamura, 2006). However, the connection to IT and the ME principle, and the inter-relationships among the estimators, is discussed here as well.

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