Information and Strategic Political Polarization *

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Abstract

We develop a model of electoral competition in which two opportunistic candidates select their policy position and invest in quality. Policy positions are observed and, during the campaign, the press reveals some information about quality. We demonstrate that when information is imperfect, the Black-Downs median voter theorem fails to hold. For intermediate information levels, the unique equilibrium is such that candidates propose policies different from the median voter's bliss point. By contrast, convergence to the median occurs when quality is (almost) always or (almost) never revealed. We also show that a profit-maximizing press may collect more information than socially optimal.

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During the 1995 electoral campaign in Belgium, the VLD –a traditionally right-wing party– decided to "go median". In an initiative they called "Het Grote Referendum," they elicited the position preferred by voters on a number of important issues, and committed to follow popular will if elected. Their attempt failed spectacularly. In 1999, the VLD reversed its strategy and proposed what could be considered an excessively rightist platform from the median voter's viewpoint. Yet, this latter strategy proved successful and the VLD's front-runner became Prime Minister. The Belgian case is suggestive but by no means exceptional. In 1997, Tony Blair received unprecedented support for a Labour candidate, not because his program was perceived as being closest to the median voter, but because he convinced the electorate that his "third way" policy proposals were better.

This perception that bringing voters to platforms is a better strategy than bringing platforms to voters is ubiquitous. According to *The Economist*, the lack of a moderate policy is not the main weakness of the Tories in the UK. Instead, the party "needs think-tanks [...] to channel [...] ideas and potential supporters towards the party" (Oct. 15, 2005, p15). Politics in the US is no different. In their article very aptly titled "Swing ideas, not swing voters" the Democrat strategists Baer and Cherny (2006) claim that the current political advantage of the Republican Party stems from the ability of its candidates to develop "signature ideas". This strategy, labelled as a "coherent public philosophy," is rewarded even when the electorate has ideological reservations. Similarly, Galston and Kamarck (2005, p63) argue that "[c]andidates who say only what they think others want to hear cannot display strength."

All these examples show that, to attract a majority of votes, parties cannot simply try to appear "moderate" or "median". Quite the contrary. In sharp contrast with the predictions of the median voter theorem, winning an election is generally about crafting a convincing philosophy, that the electorate will view as superior to that of the opponents. This was decisive for the VLD in 1999 and for the Labour Party in 1997; it is the present challenge of the Tories in the UK and of the Democrats in the US. To win, a platform must be "trusted" by the electorate. The question is: what makes a political platform appear trustworthy?

This paper proposes a two-candidate model that sheds light on this mechanism of trust, and on the link between quality, trust, and policy positions. To analyze this problem, our model incorporates two dimensions. First, the usual horizontal, Downsian, policy dimension along which politicians locate themselves. Second, a vertical dimension that we call "quality". Introducing such a vertical dimension is a recurrent theme in the literature. Stokes (1963) coined the term "valence" to capture a set of attributes orthogonal to policy (or ideology) that are valued by the voters. More recently, the Political Economics literature recognised that the actions of a politician can also influence this vertically differentiating variable (see e.g. Persson and Tabellini 2000, ch. 4, for a broad review). Our quality dimension is introduced in the latter way.

The novelty of our approach lies in the combination of two factors. First, in contrast to the literature on valence (see e.g. Groseclose 2001 or Aragonés and Palfrey 2002), we assume that politicians are perfectly identical *ex ante*. To increase the quality of their platform, candidates have to undertake some costly and unobservable action, under the anticipation that quality helps winning the election.[1] Second, we analyze the role of information on the candidates' strategic choices. We take into account the fact that,

[1] As usual in moral hazard, an unobservable action directed to improve quality should be broadly interpreted (resources spent to find a suitable set of advisors, time dedicated to understand the needs of citizens, etc). See Ferejohn (1986), Caillaud and Tirole (1999, 2002) and Persson et al. (1997, 1998) for related examples.

before casting their ballot, citizens *may* remain uncertain about the relative quality of the two platforms. If, during the campaign, voters become informed about qualities, they use that information in their voting decision. If voters remain in doubt, they must use other elements of information (policy in our model) to decide which candidate they should trust. Trustworthiness is thus an endogenous outcome in our analysis.

Using this approach, we identify interesting interactions between the information revealed during the electoral campaign, and the equilibrium competition among candidates. For the two extreme cases already analysed in the literature, namely when voters either (almost) always or (almost) never learn the quality of the platforms before the election, the Median Voter Theorem holds. In the policy dimension, opportunistic parties locate at the median voter's preferred position (from now on, the "centrist" position). Then, if voters (almost) always observe quality, parties "invest" in quality to try and dominate their opponent. If voters (almost) never observe quality, they neglect that dimension. In both cases, ideology and quality are two independent and orthogonal dimensions.

More surprizing, the Median Voter Theorem no longer holds if information about platform quality *may* be revealed during the election race. Why would an opportunistic party deliberately deviate from the median voter's preferred policy? The point we make is that voters want candidates to invest resources in quality but candidates cannot commit to spend these resources when their actions are not observable.[2] The value of polarization is then strategic: a candidate who offers a non-centrist policy position is handicapping himself. His only chance of winning is now to be sufficiently better in the

[2] Clearly, candidates use all means available to try and convince voters that their platform is better than that of their opponent. However, in the absence of hard information, voters correctly consider such discourses as uninformative cheap talk. Our working assumption is to focus on the case in which cheap talk cannot give an initial advantage to either candidate.

vertical dimension (so as to compensate for his worse policy position). The key is that, by adopting such a non-centrist policy, the candidate manipulates his own marginal return to investment and thus implicitly commits to spend more resources on quality. When this happens in equilibrium, the extreme candidate benefits from the *trust* of the electorate whenever voters remain uncertain about platforms quality at the time of the election. Our results show that this mechanism works only if the information revealed prior to the election is "intermediate". When the information revealed is "too small", incentives to invest in quality are too weak. When the information revealed is "too high", the non-centrist candidate almost never benefits from the trust of the electorate, which is his main reason for selecting a polarized stance in the first place. Convergence to the median voter's favourite policy is therefore a result that holds in extreme cases (no or full revelation of quality) but not in more general settings (imperfect information).

Several lessons can be drawn from this simple result. First, our theory explains why even purely opportunistic and ex ante identical parties can appear ideologically polarized, and why polarization may actually get reinforced by improved information.[3] This is in sharp contrast with the standard predictions of the literature (see the discussion below). Second, and going back to the motivating examples, the paper offers a plausible explanation of why the VLD's strategy of promising to blindly follow popular desires did not receive the support of the Belgian electorate: voters perceived that a party without ideological commitment cannot have developed a valuable political program. Third, the quality premium implied by the polarization of a party sometimes offsets its less desirable ideology, which means that the welfare of voters can be greater under polarization than under centrist policies.

[3] Empirically, Poole and Rosenthal (1991,1997) and McCarty et al. (2006) show that parties and candidates in the US tend to systematically fall in ideologically differentiated "camps", and that polarization has been increasing over the last thirty years.

Our paper thus sheds light on the effects of information on political competition. The accuracy of information can be seen as the result of an investment by the press to learn the quality of platforms and sell it to the electorate. Under this interpretation, our model measures the inefficiencies due to information being collected and provided by a privately interested press rather than a social welfare maximizer. We show that the press may undersupply or, more surprisingly, oversupply information, inducing parties to choose inefficient platforms and/or excessively low investments in quality.

Related Literature

Since Black (1958) and Downs (1957), the theoretical benckmark is that parties ought to be "median" to win. Yet, polarization is the rule more than the exception. To account for that reality, the extant literature generally introduces an exogenous ideological motivation in the parties' objective function. As the results of Calvert (1985), Wittman (1983) or Roemer (2001) demonstrate, however, polarization then requires that there is sufficient uncertainty about the position of the median voter.[4] Hence, according these theories, the improved polling methods over the last decades should have reduced polarization, while the opposite was observed in the US (see e.g. McCarty et al. 2006 for a recent reference).

Aragonés and Palfrey (2002), Castanheira (2003), or Kartik and McAfee (2006) show that purely opportunistic parties may choose a polarized ideology even if they have no policy preference. In Aragonés and Palfrey, this requires one party to have an exogenous valence advantage.[5] In Castanheira, the argument relies on signalling by

[4] Other important references are Bernhardt and Ingberman (1985), Alesina and Rosenthal (1995, ch. 2), Besley and Coate (1997), Roemer (1997), and Rivière (2000).

[5] Groseclose (2001) combines preference for ideology and an exogenous valence advantage. He shows that the advantaged party moves towards the center while the disadvantaged party moves away from the center.

voters, and repeated elections. Yet, the position of the median voter still needs to be sufficiently uncertain for the result to hold, otherwise all candidates select the median voter's preferred policy. In Kartik and McAfee, this requires that "good" candidates have an exogenously assigned, non-median, ideology. Other candidates are then forced to adopt non-median platforms to mimic the behaviour of high-quality ones.[6] In this paper, we show that polarization does not hinge on either voter uncertainty, ex ante differences, or repeated game effects.

The moral hazard aspects of political competition are also analysed in Caillaud and Tirole (1999, 2002) and Castanheira et al. (2005). However, their focus is on how *intra*-party tensions affect political support for the party. Here, we assume such conflicts away. In our model, candidates and parties are one and the same; these two labels are thus used interchangeably.

The remainder of the paper is organised as follows. Section 1 lays out the model. Section 2 characterizes the equilibria, including the choice of platform positions and the amount of resources invested in quality, as a function of the information revealed to the public. It also endogenises the role of the press in collecting and diffusing information. Section 3 discusses the robustness of our results. Finally, Section 4 concludes.

1 The model

1.1 Players, ideologies and platform qualities

There are two candidates indexed by $i \in \{A, B\}$, and a homogeneous electorate with single-peaked preferences. Voters care both about the *position* (or *ideology*) x_i and the

[6] In section 4.1., we show how assigning an exogenously non-median ideology to high-quality platforms adds a signalling component to our moral hazard setup.

quality v_i of each party's platform. Denoting by $V(x_i, v_i)$ the voters' utility if party i is elected, we have:

$$V(x_i, v_i) = \lambda\, f(x_i) + v_i \qquad \text{with } \lambda > 0, \tag{1}$$

where λ represents the weight of policy position (ideology) relative to quality. To limit the number of cases, we assume that party A can only choose between a 'leftist' (L) and a 'centrist' (C) policy, whereas party B chooses between a 'centrist' (C) and a 'rightist' (R) policy: $x_A \in \{L, C\}$ and $x_B \in \{C, R\}$.[7] Platform C is the most preferred by voters, and platforms L and R are positioned symmetrically around C, that is, $f(C) > f(L) = f(R)$.[8] The relevant variable that we will use from now on is:

$$\Delta \equiv \lambda \Big[f(C) - f(L) \Big] \quad (> 0), \tag{2}$$

which corresponds to the voters' utility gain when a party adopts a centrist instead of an extremist policy (left or right), weighed by the importance of ideology relative to quality. It is important to note that, in our formulation, (i) higher quality is always valued by voters ($\partial V/\partial v_i > 0$), and (ii) its marginal effect is independent of the policy proposed by the candidate ($\partial V(L, v_i)/\partial v_i = \partial V(C, v_i)/\partial v_i = \partial V(R, v_i)/\partial v_i$). This rules out exogenous reasons for polarization such as an extremist policy being preferred by voters despite its less desirable ideology only because quality is more valued at that position.

To also abstract from other exogenous motivations for polarization, we assume that candidates are ex ante identical and purely opportunistic, i.e. without any ideology or intrinsic preference over policy positions. For the sake of simplicity, the realised quality

[7] Section 3.2 discusses the implications of generalizing the platform space.

[8] This framework is formally equivalent to the case in which voters have heterogeneous preferences but the position of the median voter is known and equal to $x_m = C$. Our results also extend to the case of a median voter randomly located around $x_m = C$ (see section 4.2).

of each platform can only take two values: $v_i \in \{0, 1\}$. A candidate can obtain a low-quality platform ($v_i = 0$) at no cost. By undertaking a costly and unobservable action a^i, he increases the probability of obtaining a high-quality platform ($v_i = 1$). We denote by $\beta(a^i) \equiv \Pr(v^i = 1 \,|\, a^i)$ this probability, and assume that $\beta' > 0$ and $\beta'' \leqslant 0$. The cost of action a^i is $\nu(a^i)$ with $\nu' > 0$ and $\nu'' > 0$.

Action a^i represents for example the amount of time, effort, and resources invested by the candidate to find a policy that voters will endorse, to select experienced and competent advisors or to engage in any other costly activity that positively affects his appeal to voters. Another interpretation could be that refusing bribes is somehow costly, but accepting them may result in a political scandal, in which case voters would attribute a low quality to the politician's platform ($-a^i$ is the amount of bribes accepted, and the more bribes are accepted, the higher is the probability that $v_i = 0$).

Given action a^i, the *probability of obtaining a high quality* or, in short, the *expected quality* of a candidate's platform is thus continuous, and given by $q^i = \beta(a^i)$. Since there is a one-to-one mapping between action a^i and expected quality q^i, we will for the rest of the paper define the candidates' problem in terms of optimal choice of expected quality rather than optimal choice of action. From the previous formulation, the cost of obtaining an expected quality q^i is $c(q^i) \equiv \nu(\beta^{-1}(q^i))$, with $c'(q^i) > 0$ and $c''(q^i) > 0$ for all $q^i > 0$. In addition, we assume that $c(0) = 0$ and $c'(0) = 0$. Note that, since the cost function $c(\cdot)$ is identical for both parties, none of them has an exogenous quality advantage.

So, following (1), the *expected* utility of voters when candidate i is elected can be rewritten as a function of his policy position x_i and his probability of obtaining a high quality q^i:

$$\mathsf{E}V(x_i, q^i) = \lambda\, f(x_i) + q^i. \tag{3}$$

Since candidates are purely opportunistic and have to spend a cost $c(q^i)$ to achieve high quality with probability q^i, we can express their expected utility as:

$$U^i_{x_A x_B}(q^A, q^B) = \pi^i_{x_A x_B}(q^A, q^B) - c(q^i), \tag{4}$$

where $\pi^i_{x_A x_B}(\cdot)$ is the (ex ante) probability that candidate i is elected given platform positions (x_A, x_B). This formalization accounts for the fact that (i) candidates are only interested in winning the election, (ii) policy choices are costless, and (iii) achieving a high quality is costly. In the Industrial Organization jargon, candidates can differentiate horizontally (policy) and vertically (quality). The equilibrium values of q^A and q^B are determined in the next section.

It is important to note that, by design, we have ruled out all exogenous rationales for polarization. From the viewpoint of voters and as already discussed, we have assumed that their valuation of quality is independent of the policy location –see (1) and (3). Moreover, candidates are ex ante identical in the mind of voters. Since voters have single-peaked preferences, they have an unambiguous predilection for a moderate position ($x_i = 0$) and a high quality ($v_i = 1$).[9] Therefore, and despite the separation between the policy and quality dimensions, our model is not about non-transitive preferences: for any given quality, centrist policies are always Condorcet winners. From the viewpoint of a candidate, his cost of achieving an expected quality q^i is independent of platform positions and of the resources spent by the rival. This avoids ad-hoc trade-offs by candidates between, for example, how desirable voters find a given policy position

[9] In the previous literature, the incentives to polarize are triggered either by an intrinsic preference for extremist positions or by the exogenous valence disadvantage of one party. When the weight of this preference or the valence disadvantage goes to zero, so does the degree of polarization.

vs. how costly it is to reach high expected quality in that position.

1.2 Information

As mentioned in the introduction, incorporating vertical differentiation in political economy models is not new. Yet, a crucial though generally overlooked aspect of elections is that, at the time of casting their ballots, voters may still be uncertain about the relative merits of platforms. To introduce this uncertainty in the model, we assume that the electoral campaign generates *noisy information* about the relative quality of candidates. We model imperfect revelation of information in its simplest possible form: with exogenous and publicly known probability p voters become perfectly informed about the quality of both candidates. With probability $1 - p$, they remain ignorant.[10] When qualities are learnt, no uncertainty remains and voters elect the candidate that yields highest utility –see (1). By contrast, when no information is revealed during the electoral campaign, voters must form beliefs about the investment in quality by candidates and "trust" the one that yields highest *expected* utility –see (3).[11] In section 2.3, we explain how p would be chosen if it were controlled by a profit-maximizing press.

The novelty of our analysis thus lies in this imperfect observability of quality. In other words, our work embraces two well-known extreme situations: the case in which the quality dimension is not considered (formally, $p = 0$), and the case in which both quality and ideology matter but cannot interact (formally, $p = 1$). Our contribution is to unveil the endogenous interactions between these two dimensions when $p \in (0, 1)$.

[10] Caillaud and Tirole (1999, 2002) have a similar formalization, except that they consider only one party.

[11] This extreme (all or nothing) information revelation assumption and the fact that voters cannot learn the quality of just one party is not necessary for the results. In fact, what matters is that, in some states of nature, the electorate at large is better informed about the valence of parties than in others.

Admittedly, parties also remain vague (deliberately or not) about their ideological position. However, we believe that the uncertainty is greater in quality than in ideology. Among other things, the ideological position of parties is determined over long periods of time, whereas the quality of candidates (or, more exactly, the voters' valuation of the relative qualities) varies substantially across elections. Voters thus have comparatively better information to judge whether a party is left-wing, centrist, or right-wing, than to judge whether the reforms proposed for tax, education, or social security are the appropriate ones.

1.3 Timing

We can now summarize the timing of the game. First, candidates simultaneously select the ideological position of their platform (x_i), which become public knowledge. Second, candidates simultaneously select the amount of investment in quality for the forthcoming election. This determines their probability of achieving high quality (q^i). Third, the electoral campaign takes place: qualities are realised, and nature chooses whether they become public or not. Fourth, voters elect one candidate, given the candidates' ideological positions and the information they have on qualities (if any). This timing is depicted in Figure 1.

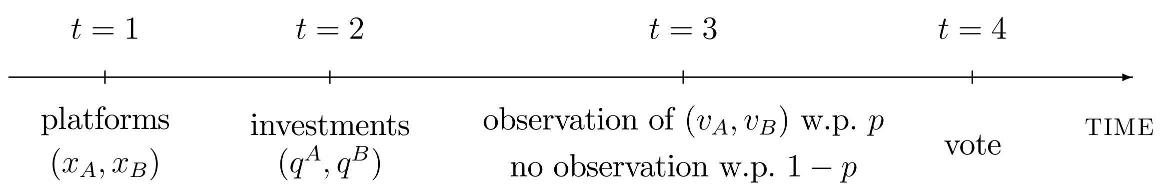

Fig. 1. TIMING.

This timing seems the most natural for two main reasons. First, because ideology is less malleable than quality. Each party inherits a history of platform choices. Changes in ideological location are possible but they require intra-party coordination, compromises, debates and other costly activities. It seems unrealistic to think that changes of this sort are mostly unveiled shortly before the election. By contrast, the content of the platform and the arguments chosen to publicize its quality dimension are typically kept secret until the electoral campaign starts.

Second and most importantly, because quality is often platform specific. Advisors and think-tanks generally develop new policy proposals tailored to the ideology of the candidate. It is thus difficult to believe that parties can develop ideology-free reforms, and give them an ideological bent only at the time of the campaign. In other words, parties must first decide which ideology they will be proposing in order to determine where to look for quality improvements. A candidate who adopts a left-wing policy based, for example, on an increase in public expenditures must then determine the optimal allocation of those resources between education, health care, infrastructure, etc. A party adopting a right-wing position with anti-terrorism as a major policy concern must then propose efficient and constitutionally viable measures that increase homeland security.

Despite our confidence in the proposed timing, we still feel that the effects highlighted in the paper are relevant only if they hold after some modifications in the sequence of events. In section 3.1, we discuss the equilibrium choices of ideology and quality under three alternative timings: simultaneous choice of quality and ideology, existence of a fall back ideology, and possibility of platform relocation after the observation of quality.[12] We show that our results are robust to these changes, even though new effects may emerge.

1.4 Strategy space

Given that each party decides between two platforms, there are four potential pairs of positions: both candidates located at the median voter's preferred platform (C, C), which we will call full convergence; both candidates at the extremes (L, R), which we will call full or symmetric polarization; and one extremist and one centrist party (L, C) or (C, R), which we will call partial or asymmetric polarization.

To focus on the interesting situation, we assume that the quality dimension is sufficiently important, so that voters prefer a high-quality party with an extremist platform rather than a low-quality party with a centrist platform. This is summarized as follows:

Assumption 1 $\Delta < 1$.

Were this assumption violated, a party adopting an extreme position would automatically lose the election. Hence, platforms L or R could never be of potential interest, and the unique equilibrium would always imply policies (C, C).[13]

[12] We thank an anonymous referee for suggesting these extensions.

[13] Assumption 1 is automatically satisfied if we consider a continuous policy space. Section 3.2 explains how our extend to more general strategy spaces.

2 Characterization of the equilibria

2.1 Parties' investment, quality, and payoff

As usual, we solve the model backwards. The election takes place at $t = 4$. Depending on Nature's move at $t = 3$, voters may or may not be informed about realised qualities. When qualities are observed, voters make an informed choice. By contrast, when qualities are not observed, voters must rely on expected utilities –see (3). To assess the expected utility of a platform, one has to determine the quality choice q^i made by candidates at $t = 2$, given (i) the observable pair of platform positions (x_A, x_B) selected at $t = 1$, (ii) the existing uncertainty about the future revelation of information that will occur at $t = 3$, and (iii) the equilibrium voting behavior at $t = 4$.

Denote by $q^i_{x_A x_B}$ the equilibrium probability that party i obtains a high quality when ideological positions are (x_A, x_B). Similarly, denote by $\mathsf{E}q^i_{x_A x_B}$ the voters' *perceived* probability that party i obtains a high quality when locations are (x_A, x_B), and no information is revealed at $t = 3$. From now on, and by abuse of language, we will for short refer to $q^i_{x_A x_B}$ as "quality" and to $\mathsf{E}q^i_{x_A x_B}$ as "anticipated quality".[14]

For the subgame where no information is revealed at $t = 3$, we summarize voting behaviour with the function $\kappa^i_{x_A x_B}(\mathsf{E}q^A_{x_A x_B}, \mathsf{E}q^B_{x_A x_B})$. This function results from the equilibrium (possibly mixed) strategy of the voters, and it represents the probability that they elect party i when qualities remain unobserved and platforms are located at x_A and x_B. We shall interpret this function as a measure of trust by voters on party i when no information is revealed.

Given $q^i_{x_A x_B}$ and $\mathsf{E}q^i_{x_A x_B}$, we can compute the optimal quality chosen by candidates

[14] Obviously, in a rational expectations equilibrium voters cannot be fooled, so $\mathsf{E}q^i_{x_A x_B} = q^i_{x_A x_B}$.

at $t = 2$ for any pair of locations, as well as their corresponding expected payoffs. That is, we can solve for equilibrium investments in quality for the subgame in which platform positions have already been determined.

Case 1. Symmetric positions: full convergence (C, C) or full polarization (L, R)

For any symmetric pair of platforms, each candidate's probability of being elected is given by:

$$\pi_S^A = p\left[q^A\left(1-q^B\right)+\tfrac{q^Aq^B+\left(1-q^A\right)\left(1-q^B\right)}{2}\right]+(1-p)\,\kappa_S^A(\mathsf{E}q_S^A,\mathsf{E}q_S^B) \quad (=1-\pi_S^B) \tag{5}$$

$$\pi_S^B = p\left[q^B\left(1-q^A\right)+\tfrac{q^Aq^B+\left(1-q^A\right)\left(1-q^B\right)}{2}\right]+(1-p)\,\kappa_S^B(\mathsf{E}q_S^A,\mathsf{E}q_S^B) \quad (=1-\pi_S^A) \tag{6}$$

where subscript S denotes symmetric locations. Intuitively, if both candidates locate symmetrically around the median voter and their realised qualities are observed, the party with highest quality is elected. If both qualities are equal, each party is elected with probability one-half. This is summarized by the first term in (5) and (6). If realised qualities are not observed, voters must rely on anticipated qualities. This is summarized by the second term in (5) and (6). This voting behavior holds both under (C, C) and (L, R).

From (4), (5), (6) and taking first-order conditions, we deduce that the optimal quality q_S chosen by each party when platforms are symmetric neither depends on $\mathsf{E}q_S^i$ nor on the quality level chosen by the other party. It is unique and given by:

$$c'(q_S) = \frac{p}{2}. \qquad \textbf{(C1)}$$

Thus, candidates have stronger incentives to invest time and resources in increasing quality when the result of their investment is more frequently observed by voters (p large), and when the marginal cost of increasing quality is lower (c' small).

Note also that, if qualities are not observed, voters prefer the candidate whose *anticipated quality* $\mathsf{E}q_S^i$ is highest. That is, $\kappa_S^A = 1$ if $\mathsf{E}q_S^A > \mathsf{E}q_S^B$, $\kappa_S^A = 1/2$ if $\mathsf{E}q_S^A = \mathsf{E}q_S^B$ and $\kappa_S^A = 0$ if $\mathsf{E}q_S^A < \mathsf{E}q_S^B$. From **(C1)** and given rational anticipation of investment levels, $\mathsf{E}q_S^i = q_S$ for all i. Therefore, $\kappa_S^A = \kappa_S^B = 1/2$. This implies the following expected equilibrium utility for both candidates:

$$U_S^A(q_S, q_S) = U_S^B(q_S, q_S) = \frac{1}{2} - c(q_S). \tag{7}$$

Each party is elected with probability one-half in equilibrium, but is trapped into spending a cost $c(q_S)$. As already mentioned, this optimal value q_S holds for any symmetric pair of platforms. This implies that, *ceteris paribus*, symmetric polarization (L, R) is unambiguously detrimental to the median voter (we elaborate on this point below) whereas revelation of information is unambiguously beneficial.

Case 2. Asymmetric polarization: (L, C) **or** (C, R)

Suppose that party A is more extreme than party B, i.e. $x_A = L$ and $x_B = C$ (the case $x_A = C$ and $x_B = R$ is identical). Each candidate's probability of being elected is given by:

$$\pi_{LC}^A = p\, q^A(1 - q^B) + (1 - p)\, \kappa_{LC}^A(\mathsf{E}q_{LC}^A, \mathsf{E}q_{LC}^B) \quad (= 1 - \pi_{LC}^B), \tag{8}$$

$$\pi_{LC}^B = p\left[1 - q^A(1 - q^B)\right] + (1 - p)\, \kappa_{LC}^B(\mathsf{E}q_{LC}^A, \mathsf{E}q_{LC}^B) \quad (= 1 - \pi_{LC}^A). \tag{9}$$

Given Assumption 1, if qualities are observed, the extremist party wins the election if and only if he obtains a high quality and his opponent a low one. Otherwise, he loses (first term in (8) and (9)). If qualities are not observed then, just like before, voters have to make their choice based on platform positions and the anticipation of the candidates' qualities (second term in (8) and (9)).

From (4), (8), (9) and taking first-order conditions, we find that if candidates adopt asymmetric positions at $t = 1$, then the optimal quality chosen by each party is independent of the anticipated qualities $\mathsf{E}q^i_{LC}$. However, and contrary to the symmetric case, it now depends on the rival's choice of quality. Formally:[15]

$$c'(q_X) = p\,(1 - q_M)\,, \tag{\textbf{C2}}$$

$$c'(q_M) = p\,q_X, \tag{\textbf{C3}}$$

where subscripts X and M denote the equilibrium levels of the variable for the "extremist" and the "moderate" party respectively (i.e., $q_X = q^A_{LC}$ and $q_M = q^B_{LC}$, $\pi_X = \pi^A_{LC}$, $\kappa_X = \kappa^A_{LC}$, and so on). Note that, as long as Assumption 1 holds, equilibrium qualities are independent of Δ, the relative distance between the two platforms.

Conditions **(C2)** and **(C3)** show that the extremist party views qualities as strategic substitutes whereas the moderate party views qualities as strategic complements. Yet, *ceteris paribus*, an increase in the amount of information p makes both candidates more aggressive, just like in the symmetric case. As a result, the equilibrium choice of quality is always increasing in p for the moderate party, whereas it can be increasing or decreasing in p for the extremist party. Figure 2 below illustrates this effect in a quadratic cost example. It displays the reaction functions of the two candidates for low, intermediate and high values of p: the higher is p, the higher is the equilibrium value of q_M. The expected quality of the extremist party, instead, is maximal for the

[15] Call $q^*_X(q_M)$ the reaction function of the extremist party and $q^*_M(q_X)$ the reaction function of the moderate one. From **(C2)**, note that $dq^*_X/dq_M < 0$, $q^*_X(0) > 0$ and $q^*_X(1) = 0$. From **(C3)**, $dq^*_M/dq_X > 0$, $q^*_M(0) = 0$ and $q^*_M(1) > 0$. It is therefore immediate that, for every p, q_X and q_M exist and are unique.

intermediate value of p.

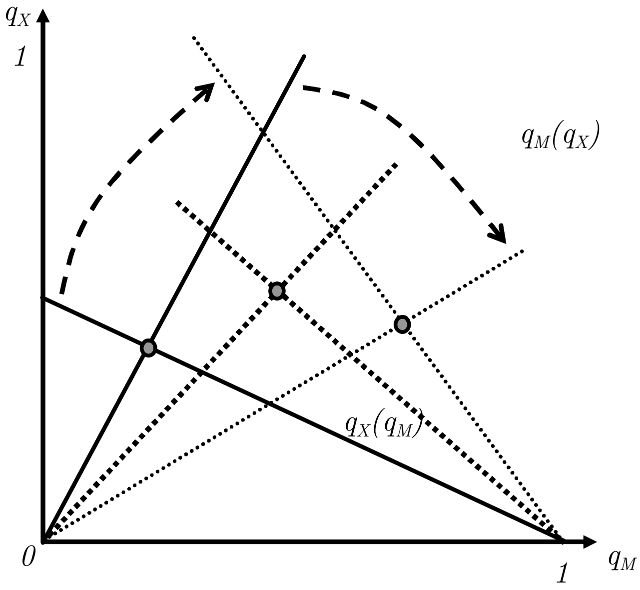

Fig. 2. Effect of an increase in p on the parties' reaction functions when platforms are asymmetric.

Overall, **(C1)**, **(C2)**, **(C3)** show that (i) the choice of a policy position affects the candidates' incentives to invest in quality; and (ii) the amount of information p is essential to pinpoint the equilibrium expected quality of the candidates. Therefore, to determine the equilibrium policy positions, we have to investigate the relationship between the amount of information p and the incentives to increase quality. The following lemma treats this issue:

Lemma 1 *There exists a non-empty open interval* $\mathcal{P}_X = (0, p_X)$ *such that the extremist party has strictly higher quality than the moderate party* ($q_X > q_M$) *if and only if* $p \in \mathcal{P}_X$.

Proof. See Appendix A1. □

This result stresses the idea that there always exist values of p for which an extremist party *endogenously* chooses to offer a better platform than a moderate party. The intuition directly results from **(C2)**, **(C3)** and Figure 2. When $p = 0$, no party has incentives to invest in quality ($q_X = q_M = 0$). We have argued (see Figure 2) that the moderate party's incentives to invest in quality are monotonically increasing in p. By contrast, the extremist party's incentives are hump-shaped. This hump-shaped relationship results from the combination of two opposite forces. For low values of p, a direct effect dominates. Namely, both parties' incentives to invest are increasing in p, but this force is steeper for the extremist: he needs to obtain a high quality in order to win when qualities are revealed. For higher values of p, an indirect effect dominates: the moderate party invests heavily in quality, and this discourages the extremist from investing –qualities are strategic substitutes from his viewpoint (see Figure 2). The combination of increasing and hump-shaped functions leads to Lemma 1. Note, still, that for some functions $c(\cdot)$ it may well be the case that $\mathcal{P}_X = (0, 1]$.

Since voters benefit from investments, high observed quality is always appreciated and, in equilibrium, rewarded with election. Furthermore, if the anticipated quality of the extremist party is "substantially larger" than that of the moderate party, voters will trust and elect the former whenever qualities are not revealed ($\kappa_X = 1$). Conversely, if the anticipated quality differential is small or negative, voters will, in the absence of information, trust and elect the moderate party ($\kappa_X = 0$). Our next objective is to define which anticipated quality differentials induce voters to trust the extremist, despite its less desirable ideology.

Definition 1 *For a given* Δ*, we denote by* $\mathcal{P}$ *the set of probabilities* p *satisfying* $\Delta < q_X - q_M$*. Formally, if* $p \in \mathcal{P}$ *then* $\kappa_X = 1$ *and* $\kappa_M = 0$*. (Naturally,* $\mathcal{P} \subset \mathcal{P}_X$*).*

In words, $\mathcal{P}$ is the set of probabilities p such that if (i) one candidate proposes a centrist and the other an extremist platform, and (ii) voters do not observe qualities, then the extremist candidate wins the election. From Lemma 1, when $p \in \mathcal{P}_X$, the extremist has a higher anticipated quality than the moderate, but the difference may not compensate for his less appealing ideology. In the subset $\mathcal{P}$ ($\subset \mathcal{P}_X$) instead, the quality differential does offset the loss due to polarization, and voters strictly prefer to trust the extreme policy.

Note that the extremist party will never be trusted if information revelation is below a certain threshold. Given **(C2)** and **(C3)**, when qualities are seldom observed, incentives to invest are very weak for both candidates. Therefore, the (slightly) higher quality of the extremist does not compensate for his less desirable ideology (formally, there exists $\underline{p}$ such that if $p \in [0, \underline{p}]$, then $0 \leqslant q_X - q_M < \Delta$). Also, since q_X and q_M are independent of Δ, the set $\mathcal{P}$ shrinks as Δ increases (in particular, $\mathcal{P} = \varnothing$ if $\Delta \to 1$ and $\mathcal{P} = \mathcal{P}_X$ if $\Delta \to 0$).

Given Definition 1, we are finally in a position to determine the expected utility of candidates in the asymmetric position case:

$$U_X(q_M, q_X) = \begin{cases} p\, q_X(1 - q_M) + (1 - p) - c(q_X) & \text{if } p \in \mathcal{P} \\ p\, q_X(1 - q_M) - c(q_X) & \text{if } p \notin \mathcal{P} \end{cases} \quad (10)$$

$$U_M(q_M, q_X) = \begin{cases} p\Big[1 - q_X(1 - q_M)\Big] - c(q_M) & \text{if } p \in \mathcal{P} \\ p\Big[1 - q_X(1 - q_M)\Big] + (1 - p) - c(q_M) & \text{if } p \notin \mathcal{P} \end{cases} \quad (11)$$

These will be used to determine the platform positioning equilibria of the game.

2.2 *The determinants of strategic polarization*

In section 2.1 we have determined the voting strategy of the electorate ($t = 4$) for every resolution of uncertainty ($t = 3$), and the optimal level of investment by candidates for every pair of platforms ($t = 2$). Working by backward induction, we can now determine the policy optimally selected by candidates ($t = 1$). This step will complete the analysis of the game.

Recall that (i) candidates are purely opportunistic and bear a cost of investing, (ii) voters dislike distance from the centrist platform, and (iii) the cost of improving quality is independent of the platform position adopted. Therefore, it seems natural to expect that candidates will always select the platform most preferred by the median voter, that is $x_A = x_B = C$, and then compete in quality. Yet, the existence of imperfect (albeit symmetric) information endogenously affects the marginal return to investing in quality at the different platform positions. This in turn has an impact on the strategic choice of policy. Our first result provides a sufficient condition for the median voter theorem to hold.

Proposition 1 ***(Sufficient condition for Median Platforms)***

A sufficient condition for (C, C) to be the unique equilibrium in the choice of ideology by candidates is $p \notin \mathcal{P}_X$. Hence, $q_X > q_M$ is a necessary condition for polarization.

Proof. See Appendix A2. □

According to Lemma 1, the reasons for selecting a non-centrist policy could be of two different natures. If $p \in \mathcal{P}_X$, a deviation from (C, C) constitutes an implicit commitment to increase investment (formally, $q_X > q_M$). We call this case "deviation for quality". If $p \notin \mathcal{P}_X$, then a deviation from (C, C) induces the party to reduce his quality, with the corresponding savings on investment cost (formally, $q_X < q_M$). We call this case "deviation for laziness". Proposition 1 shows that *only deviation for quality may occur.* When a party engages in a deviation for laziness, the benefits of a lower investment never compensate for the smaller probability of being elected. Hence, Proposition 1 shows that if we ever observe one extreme and one moderate platform position, then we know for sure that the extremist party is trying harder to increase quality than the moderate one.

In light of Proposition 1, it only remains to determine the platform positions selected by candidates when $q_X > q_M$. In order to restrict the number of cases to analyze, we introduce the following technical assumption that will be maintained for the rest of the paper:

Assumption 2 $c'''(q) \geqslant 0$ *for all* $q > 0$*.*

Imposing convexity of the marginal cost of investment is just a convenient way of keeping all equilibrium qualities within a certain range. In particular, this assumption rules out a situation in which q_M and q_S are close to 0 and q_X is close to 1.[16]

We are now in a position to offer a complete characterization of the policy adopted by candidates at $t = 1$ as a function of the probability p that qualities become public. Once platforms are determined, the investments chosen by candidates at $t = 2$ are simply given by **(C1-C2-C3)** and the corresponding expected utilities are determined

[16] As explained in section 3.3, most of the results still hold when Assumption 2 is violated. Note that restrictions on the rate of convexity of the cost function, although difficult to interpret economically, are quite frequent in contract theory as technical devices to avoid non-convexities in the overall maximization problem (see e.g. the classical papers by Guesnerie and Laffont (1984) or Laffont and Tirole (1986)).

by (7), (10), and (11).

Proposition 2 ***(Extended Median Voter Theorem)***

There exist two non-empty sets $\mathcal{P}_1$ and $\mathcal{P}_2$ such that:

- (C, C) *is an equilibrium if and only if* $p \in \mathcal{P}_1 \cap \mathcal{P}$ *or* $p \notin \mathcal{P}$*;*
- (L, R) *is an equilibrium if and only if* $p \in \mathcal{P}_2 \cap \mathcal{P}$*;*
- (L, C) *and* (C, R) *are both equilibria if and only if* $p \in \mathcal{P} \setminus (\mathcal{P}_1 \cup \mathcal{P}_2)$*.*

Proof. See Appendix A3. □

A first glance at Proposition 2 immediately reveals that the median voter theorem does not generally hold under imperfect observability of quality. Furthermore, convergence to (C, C) is the unique outcome either when quality is almost never observable (p close to zero) or when quality cannot interact with the platform choice (p close to one):

Corollary 1
There exist $(\underline{p}, \overline{p}) \in (0, 1)^2$ *such that* (C, C) *is the only equilibrium if* $p \leq \underline{p}$ *or* $p \geq \overline{p}$*.*

Importantly, the reasons for convergence in these two extreme cases are of very different nature. We have shown that when $p \in \mathcal{P}_X$, asymmetric positions imply that the extreme party invests more than the moderate. Nevertheless, if quality is rarely observed, the difference in incentives is very weak, see **(C2)** and **(C3)**. For that reason, voters are not willing to trust an extremist candidate. In other words, when p is small enough, voters prefer centrist candidates because the loss due to polarization always offsets the gain of a slightly higher expected quality (technically, if $p < \underline{p}$ then $p \notin \mathcal{P}$). The case $p \to 0$ thus corresponds to the standard Hotelling model in which the quality dimension is absent.

The case $p \to 1$ corresponds to another standard model studied in the literature. Recall from Proposition 1 that a party may only be willing to adopt an extreme position if it serves as an implicit incentive to increase quality. This "deviation for quality" is profitable when qualities do not become public, because it allows the extremist candidate to benefit from the voters' trust. If p is high, the likelihood $(1-p)$ that voters need to trust a party due to a lack of information becomes too low (technically, if $p > \overline{p}$ then $p \in \mathcal{P}_1$). Hence, when $p \to 1$, quality does matter but convergence occurs because candidates anticipate that they will hardly ever benefit from the voters' trust. Note the paradoxical effect of platform polarization: a candidate regards the adoption of an extremist stance as an implicit commitment to invest heavily in quality, but then hopes that the results of his endeavour do not become public.

To sum up, as p increases, two effects operate in opposite directions. First, voters are more willing to support extremist candidates when qualities are not revealed. Second, candidates are less willing to deviate from centrist positions because they have a lower probability of taking advantage of this trust.

A second corollary of Proposition 2 is that, for Δ sufficiently small, median platforms cannot be an equilibrium if the quality of information is neither too small nor too high:

Corollary 2

For Δ sufficiently small, there always exists some p such that (L, R) is the only equilibrium.

For intermediate values of p, the endogenous interaction between ideology and quality induces candidates to select polarized positions if Δ is not too large. Which situation prevails in equilibrium (partial or full polarization) will crucially depend on the amount of information p and on the shape of the cost function $c(\cdot)$. Note also that multiple equilibria may exist: by symmetry, whenever (L, C) is an equilibrium, (C, R) is another one. Also, if $\mathcal{P} \cap \mathcal{P}_1 \cap \mathcal{P}_2 \neq \varnothing$ then both (C, C) and (L, R) are equilibria for some p. Last, polarization is not just a theoretical curiosity: as the above corollary shows, if the degree of polarization Δ can be relatively small, then it always occurs for some levels of information revelation.

It is interesting to notice the difference between the *ex ante* and the *interim* value of information. Increases in p may trigger full polarization. This creates a time-inconsistency problem for voters. At time 0, citizens would like to commit not to pay too much attention to the information about quality in order to avoid platform polarization. However, at time 1, once policies are chosen, information will never be disregarded. More surprisingly, the candidates may also face a time-inconsistency problem. Under asymmetric polarization, both candidates may ex ante prefer p to be reduced, since it allows them to decrease investment. However, once investments are sunk, the moderate candidate can only win when qualities are revealed, so he *ex post* prefers p to be as high as possible.

Now that we have studied the conditions for polarization, it is straightforward to determine the welfare impact of the different choices of political platforms:

Corollary 3 *(i)* (C, C) *always Pareto dominates* (L, R).

(ii) (L, C) *and* (C, R) *sometimes Pareto dominate* (C, C).

Recall that the investments in quality depend exclusively on the candidates' *relative* extremism. When the incentives of candidates to deviate are too strong, both offer extremist positions (L, R). This decreases the welfare of voters (since qualities are the same as under full convergence), without affecting the welfare of candidates. By contrast, under asymmetric positions, the extremist may be induced to increase quality and the moderate to reduce it relative to the symmetric case. When the higher quality of the extremist offsets the joint costs of his less desirable ideology and of the lower quality of the moderate, voters benefit from partial polarization. Moreover, if an equilibrium with asymmetric positioning exists, then the two candidates are necessarily better off under that pair of platforms than either under full convergence or under full polarization.[17] This idea that both candidates will strictly prefer (L, C) or (C, R) to (C, C) or (L, R) when $p \in \mathcal{P} \backslash (\mathcal{P}_1 \cup \mathcal{P}_2)$ implies that the equilibrium with asymmetric positions is very robust: even if candidates could collude or communicate at the platform selection stage, they would still choose partial polarization.

Remark. Our model shares features with the career-concerns literature (see Holmström, 1999). As in Holmström's work, the effort of agents (here, investment by candidates) stochastically affects output (here, quality), which is noisily observed by the principal (here, the median voter). As in a rat race, candidates are trapped by the investment anticipated by voters. The key novelty of our paper is that the candidates can choose the level of investment in which they trap themselves. By selecting an (observable) policy platform, candidates implicitly commit to spend a given amount of resources, and therefore to reach a certain expected quality. This quality is rationally anticipated by citizens and affects their voting strategy.

[17] The formal argument is simple. Suppose that (C, R) is an equilibrium of the game. By construction, $U^B_{CR}(q^A_{CR}, q^B_{CR}) > U^B_{CC}(q_S, q_S) = U^B_{LR}(q_S, q_S)$ and $U^A_{CR}(q^A_{CR}, q^B_{CR}) > U^A_{LR}(q_S, q_S) = U^A_{CC}(q_S, q_S)$.

2.3 Endogenous information quality and the role of the press

If we think of p as representing the *quality of the press*, the results presented so far show how the press affects the political competition both in the policy and the quality dimensions. It is then possible, within our model, to endogenize the role of the press. Suppose that, by spending costly resources at $t = 0$, an independent and profit-maximizing press can increase its likelihood p of learning the quality of candidates at $t = 3$. This information is then used to extract resources from the electorate (e.g., through newspaper sales).

There are (at least) two reasons why the press will not spontaneously maximize social welfare. First, the press is subject to the classical hold-up problem. Under symmetric positions, for example, a higher investment in information by the press always induces a higher investment in quality by the candidates, see **(C1)**. These increased expected qualities benefit voters even if realizations remain unknown. Instead, the press can only extract rents from the voters if it obtains information on the candidates' realised qualities. Second, due to this same hold-up problem, the press will not either internalize all the effects of p on the candidates' choice of ideology. For example, the press will extract the same rents under full convergence as under full polarization, despite the Pareto dominance of the former over the latter.

Overall, these arguments show that the private value of the information obtained by the press differs from its social value. Therefore, one can immediately deduce that the equilibrium choices of both quality and ideology will be different from those obtained if the voters themselves could invest in collecting the information.

Our main and most surprising finding in this variation of the hold-up problem is that the press will sometimes invest excessively in information. The intuition is the following. Consider the case of asymmetric polarization. From the voters' viewpoint, an increase in the quality of the extremist candidate is always very valuable, because that candidate is elected with probability one when qualities remain unobserved. However, we know that the quality of the extremist can be decreasing in p (see Figure 2). Thus, the press may end up collecting a level of information so high that it discourages the extremist to invest in quality, with the resulting adverse effect on the welfare of voters.[18]

2.4 An example

In this section, we illustrate the results obtained in Proposition 2 with a functional example. Let $c(q) = \alpha\, q^2/2$, where α is a constant. In equilibrium, and for values of α such that interior solutions exist, we have:

$$q_S = \frac{p}{2\alpha}; \quad q_X = \frac{p\,\alpha}{\alpha^2 + p^2}; \quad q_M = \frac{p^2}{\alpha^2 + p^2}.$$

Notice that $q_X > q_M \;\Leftrightarrow\; \alpha > p$, and therefore $\mathcal{P}_X = (0, \alpha)$. The necessary condition for polarization ($p \in \mathcal{P}$ or, equivalently, $\Delta < q_X - q_M$) is then:

$$\Delta < \frac{p\,(\alpha - p)}{\alpha^2 + p^2}.$$

The sets $\mathcal{P}_1$ and $\mathcal{P}_2$ as defined in Proposition 2 are such that $\mathcal{P}_1 = [p_1, 1]$ and $\mathcal{P}_2 = [0, p_2]$ with $p_1 < p_2$ for all α (see Appendix A3). Since $\mathcal{P}_1 \cup \mathcal{P}_2 = [0, 1]$, candidates never adopt asymmetric positions. Figure 3 depicts the boundaries of the sets $\mathcal{P}$, $\mathcal{P}_1$, $\mathcal{P}_2$ in the (p, α) space, for two values of Δ (0.05 and 0.15).

[18] A formalization of this result (and some others) as well as its proof can be found in our earlier working paper. See also Strömberg (2004a) for an empirical analysis of the links between press and political competition, and Larcinese (2007) for an analysis of the voter's politically-motivated demand for newspapers.

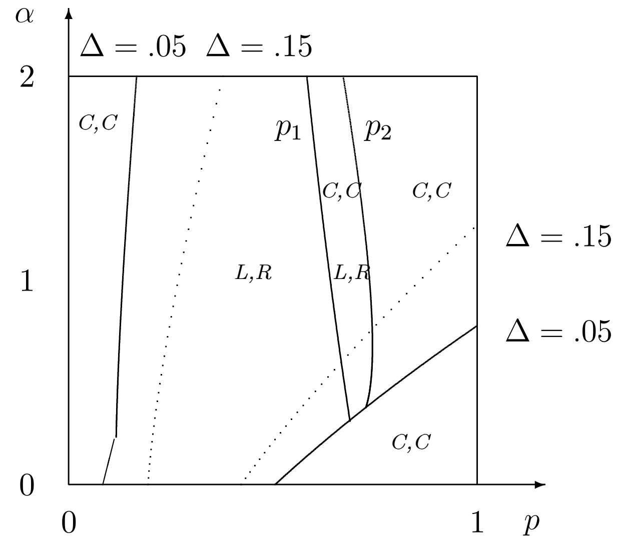

Fig. 3. Positioning equilibria as a function of (p, α)

The bounds of $\mathcal{P}$ are given by the left-most and right-most solid (resp. dotted) curves when $\Delta = 0.05$ (resp. $\Delta = 0.15$). Note that $\mathcal{P}$ shrinks as Δ increases. The left and lower-right areas thus correspond to the equilibrium (C, C) as, in these regions, $p \notin \mathcal{P}$. Next, the left solid curve inside $\mathcal{P}$ displays p_1 as a function of α and the right solid curve represents p_2 as a function of α. Hence, within $\mathcal{P}$ and according to Proposition 2, in the area to the right of p_2 the unique equilibrium is (C, C), in the area to the left of p_1 the unique equilibrium is (L, R) In the region between p_1 and p_2 both (C, C) and (L, R) are equilibria.

Finally, Figure 4 displays the voters' expected utility as a function of p when $\alpha = 1.5$ and $\alpha = 3$ (the dotted lines capture the two levels of welfare when (C, C) and (L, R) coexist). For given platform positions, the welfare of voters is increasing in p and decreasing in the cost of investment α (both measures affect positively the quality of candidates). However, the endogeneity of platform positions implies that a marginal increase in p or a marginal reduction in α may trigger the decision of candidates to diverge and end up being detrimental to voters.

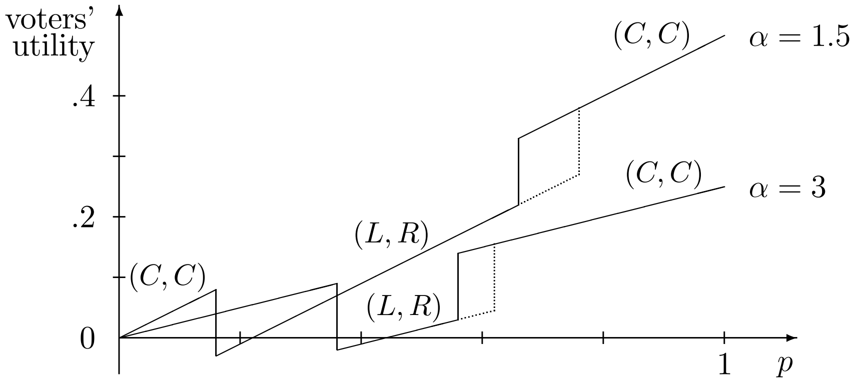

Fig. 4. Voters' expected welfare ($\Delta = 0.1$)

3 Robustness analysis

The model presented above emphasizes tractability at the expense of generality. Section 3.1 explores how the equilibria in Propositions 1 and 2 are affected when the timing is modified to introduce more interconnected decisions between investments in quality, ideology choices and ideology shifts. Sections 3.2 discusses the robustness of our results when the platform space is enlarged. Finally, 3.3 examines what happens when other assumptions are relaxed.

3.1 Timing

Often in the Industrial Organization literature, timing has a crucial impact in the outcome of a game (see Figure 1). In our setup, however, the essence of our results remains valid when the timing is modified, as long as moral hazard and imperfect information are elements of the political game. To demonstrate this, we propose three modifications in the sequence of events. Each modification delivers new insights. However, in all of them, divergence as an implicit commitment for quality remains an equilibrium outcome if and only if the amount of information revealed to voters is neither too high nor too low (the formal treatment of these extensions is relegated to the appendix).

1. Simultaneous choice of quality and ideology. One may argue that the process of designing a "good" policy cannot be separated from the choice of an ideological position. A straightforward way to introduce this joint decision in the baseline model is to suppose that candidates *simultaneously* select their platform and investment in quality. This modifies the information available: the choice of investment must be made before observing the ideology selected by the rival.

In Appendix A4, we prove that the logic behind the sequential game studied in Section 2 applies equally to the simultaneous game. The idea is simple. By the very definition of Nash and Subgame Perfect equilibria, whenever a pure strategy equilibrium of the simultaneous game with positions (x_A, x_B) exists, candidates choose the same qualities (q^A, q^B) as in the sequential game. As a result, the difference between the sequential and the simultaneous cases exclusively stems from deviations. In the sequential game, if a candidate deviates to a different platform position at $t = 1$, both candidates adjust qualities at $t = 2$. In the simultaneous game, only the deviating candidate adjusts his quality. This modified timing thus multiplies the number of cases to be considered and affects the cutoff probabilities that separate the different regions. However, the essence of the argument remains intact: convergence is still the unique equilibrium when p is close to 0 or p is close to 1 whereas (partial or full) polarization occurs for intermediate values of p.

2. Tying quality and ideology. While the above extension shows that simultaneity *per se* does not affect the essence of the argument, one may wish to capture a closer relationship between quality and ideology, on the lines of the "character" model of Kartik and McAfee (2006). Assume for instance that think-tanks associated with parties L and R can only produce left-wing and right-wing political innovations, respectively. Assume also that candidates learn their quality before announcing their platform: position C then becomes for both parties their no-innovation, fall-back option. How does this setup modify the candidates' incentive to invest in quality? As we develop in Appendix A5, this alternative model introduces a signalling mechanism that was absent previously. Here, voters may use ideology as a direct signal of quality. Therefore, a low-quality candidate will be tempted to adopt an extreme platform, only to pool with a high-quality candidate. We show that a separating equilibrium is obtained if the quality of information is sufficiently high. In that case, low-quality candidates (those who did not invest or those whose investment failed) always propose their fall back option C, and voters know that an extremist has high quality with probability 1. Overall, we are back to the baseline model, except that the incentive to exert effort is now given by $c'(q_S) = 1/2$ instead of $c'(q_S) = p/2$.

3. Signalling and platform relocation. The relation between ideology and quality unveiled in the previous extension suggests that modifications in the sequence of events may add a signalling dimension to the model. In fact, reversing the baseline timing (investment followed by platform choice) transforms our moral hazard problem into an adverse selection, signaling game in the spirit of Rogoff (1990). To illustrate this point, suppose that candidates choose effort, observe the resulting quality and then choose their ideology. A candidate who obtains a high quality, may then be tempted to adopt suboptimal policies (excessive deficits in Rogoff's model; extreme platforms in the present case), to signal their quality. Although interesting, this money-burning result is somewhat contrived in our setting, as it requires that qualities be "portable" across ideologies. It is hard to believe that a well-suited, say, centrist or left-wing solution to unemployment can be moved to a right-wing position for signalling purposes.

Still, one may envisage a richer setup that incorporates signalling into the moral hazard framework of our baseline model. Consider the timing described in Figure 1 and suppose that candidates can modify their platform after having observed the outcome of their investment, but that ideologies are not portable. Since relocation nullifies the outcome of past effort, any move between $t = 2$ and $t = 3$ implies for sure a low quality. In that setup, relocation from an extreme to a moderate platform after observing a high quality or relocation from a moderate to an extreme platform are both dominated strategies. We are thus left with one possible relocation: that of an extremist who realises his efforts were unsuccessful. Appendix A6 shows under which conditions this extremist prefers to maintain its ideological stance in equilibrium. The basic conclusion is twofold. First, the possibility of relocation enlarges the set of equilibrium choices: polarization as a commitment to increase quality, followed by relocation in case of investment failure can be an outcome of the game. Second, for some intermediate values of p, and only for those values, polarization without relocation is sustainable. In that case, the incentives of candidates remain identical to those in the baseline model.

3.2 Platform space

4. Sharing extremist platforms. In our model, (L, R) is strategically equivalent to (L, L) and to (R, R). Hence, if we assumed that $x_A \in \{L, C, R\}$ and/or $x_B \in \{L, C, R\}$, we could reach the counter-intuitive equilibrium situation where $x_m = C$ and parties offer (L, L) or (R, R). However, it is easy to show that, if we slightly extend our basic model and assume that (i) the median voter is randomly and symmetrically located around C, and (ii) voters' utility is concave around their bliss policy, then an equilibrium in which both candidates share a common extremist platform is no longer sustainable. (This is true even if the probability of an extremist median voter is arbitrarily small.)

5. *Enlarging the set of platform policies.* It would be interesting, although technically challenging, to generalize the choice set of candidates to a continuous (or discrete but large) number of ideologies. Our model provides some preliminary insights in that direction. First, adding the possibility of arbitrarily small moves in the ideology space is formally equivalent to letting $\Delta \to 0$ in our case. By Corollary 2, polarization thus necessarily occurs for intermediate values of p. So, our main result still holds in this more general setting. Second and most interestingly, a pure strategy equilibrium in the location game is unlikely to exist if parties can choose any desired degree of polarization. Our conjecture is that parties will randomize between a set of platforms, anticipating higher levels of investment in more extremist platforms. Moreover, the highest degree of extremism chosen in the mixed-strategy equilibrium becomes endogenous but bounded, much like in the literature with exogenous valence (Aragonés and Palfrey (2002) or Groseclose (2001)).

3.3 Other generalizations

6. *Cost and quality functions.* Assumption 2 is a sufficient condition to show that (L, R) is not an equilibrium when $p \to 1$. The idea is the following. Suppose that q_S and q_M are close to 0 and q_X is close to 1 (which can only occur if Assumption 2 is violated).

If parties expect (L, R) to be the equilibrium, then no party is willing to moderate his platform. This deviation would trigger a tremendous increase in investment by the opponent, which would wipe out all the benefits of moderation. Note, however, that such an argument heavily relies on the binary structure of platform positions and qualities (see point 4 above), as well as on the sequentiality of the timing (see point 1). This is why we want to avoid it in a first place.

7. Increasing the number of candidates. It is well known that the median voter theorem does not hold with more than two candidates (Palfrey, 1984). Hence, the contribution of our model is more limited in that context. However, there is one interesting feature in a three-party model that we wish to stress. Suppose that one party has a fixed centrist position, whereas the other two parties can choose their ideology. If parties diverge symmetrically (i.e., they propose platforms (L, C, R)), they will invest more resources than if they all keep moderate platforms (C, C, C). As a result, when qualities remain unobserved, the probability of election of one of the parties with endogenous location decreases from 1/2 in the first case to 1/3 in the second one. The general implication is that the welfare value of polarization tends to be underestimated when we focus on a two-party case: voters always prefer (C, C) to (L, R) but they might prefer (L, C, R) to (C, C, C).

4 Conclusions

This paper proposed a model in which political candidates can affect the quality of their platform (e.g., by investing costly resources) but the result of their investment is imperfectly observed by voters. We argued that endogenous interactions arise between their incentives to invest in quality and their strategic choice of policy positions. More specifically, we showed that opportunistic candidates may prefer to propose extreme policies as a commitment to increase their investment in quality. Moreover, voters can end up benefitting from such polarization.

One may wonder whether platform polarization is a more adequate commitment device to invest in quality than other possible handicapping mechanisms. As mentioned in the introduction, political practitioners do see ideology as a major dimension, one that must be used, to convince voters that one's platform has a high *quality*. Also, we have assumed that candidates are opportunistic to better highlight the incentive motives for polarization. In reality, however, candidates have intrinsic preferences over platform positions. This makes the ideology dimension an even more natural candidate for commitment. Last but not least, the costs of polarization need not be excessively high: parties can offer platforms that are different, yet close to the median voter's position. They can also propose non-centrist ideas only in dimensions that have a strong ideological component (e.g., abortion and death penalty in the US or joining the Euro area in UK). This can be a relatively inexpensive way to convincingly display one's differences on a few issues and, at the same time, swing moderate voters by increasing the overall level of investment in quality. In turn, it also explains why a rational electorate is reluctant to support a party who does not exhibit commitment to some ideology. Quoting Downs (1957): *"lack of information creates a demand for ideologies"*.

These results also shed light on several existing puzzles in the literature. First, it is surprisingly difficult for a new and moderate party to challenge the lead of existing, non-moderate, parties. And yet, in the absence of moral hazard considerations, a centrist party should have a substantial advantage over polarized ones. We argue that its lower popularity stems from its lower implicit incentives to invest in quality (see also Caillaud and Tirole (1999) for an alternative explanation based on intra-party competition). Interestingly, it is easy to see that voters would then value even more candidates who have an intrinsic ideological motivation. Such candidates value victory more highly, and therefore invest even more in quality.[19] Thus, whenever our polarization results hold, voters would not only demand more ideological candidates, but also ones that hold non-median views.

Next, the extant literature does not shed much light on why polarization increased over time, despite improved polling methods that allow parties to better track voters' preferences. In contrast with that literature, we showed that uncertainty about the preferences of voters may not be the key element that induces polarization. If voters are on average ill-informed, parties have incentives to adopt polarized platforms in order to earn their trust.[20] In that case, better information can actually be the cause of stronger polarization. Last, our model allows for a novel look at the role of the press in determining political polarization and the quality of proposed policies.[21] Given the candidates' moral hazard problem, we found that a profit-maximizing press would not be sufficient to maximize social welfare. Interestingly, it may overinvest in information, inducing candidates to choose excessively low levels of investment in quality and, possibly, inefficient platforms.

[19] This implies that our results reveal the upper bound of the welfare costs of polarization.

[20] This result is also consistent with the fact that, in order to achieve a moderate policy, voters may prefer to "split tickets" across two opposite extreme parties (see Alesina and Rosenthal, 1995) rather than voting for a centrist party.

[21] Strömberg (2004b) focuses on a different type of information. In his model, the press informs the voters about the location of parties. He shows that parties tend to move away from uninformed voters.

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Appendix

A1. Proof of Lemma 1

The set of possible rankings among the different qualities is:

$$\begin{array}{llll} 1/2 > q_X > q_S > q_M & \text{or} & q_X > 1/2 > q_M > q_S & \text{if } p \in \mathcal{P}_X \\ q_M \geqslant q_S \geqslant q_X \geqslant 1/2 & \text{or} & q_S \geqslant q_M \geqslant 1/2 \geqslant q_X & \text{if } p \notin \mathcal{P}_X \end{array}$$

To see this, suppose first that $q_X > q_M$. Then by **(C2)**, **(C3)** and $c'' > 0$ we have $1 - q_M > q_X$. Hence: $q_X + q_M < 1$, and thus $q_M < 1/2$. By **(C1)** and **(C2)**, this implies that $q_X > q_S$. Overall, either $q_X < 1/2$ in which case $1/2 > q_X > q_S > q_M$ or $q_X > 1/2$ in which case $q_X > 1/2 > q_M > q_S$. A similar reasoning when $q_X \leqslant q_M$ results in the other two inequalities.

Showing that $\mathcal{P}_X$ (the set where $q_X > q_M$) is an open interval requires a more elaborated proof. Differentiating **(C2)** and **(C3)** with respect to p we get:

$$c''(q_X)\frac{dq_X}{dp} = (1 - q_M) - p\frac{dq_M}{dp}, \tag{12}$$

$$c''(q_M)\frac{dq_M}{dp} = q_X + p\frac{dq_X}{dp}. \tag{13}$$

As $p = 0$ implies $q_M = q_X = 0$, we can infer that $\left.\frac{dq_X}{dp}\right|_{p=0} > \left.\frac{dq_M}{dp}\right|_{p=0} = 0$. Therefore, $q_X > q_M$ when $p \to 0$. As a result, it is sufficient to prove that q_X and q_M intersect at most once in order to conclude that $\mathcal{P}_X$ is an open interval. From **(C2)** and **(C3)**, we know that $q_X = q_M \Leftrightarrow q_X = q_M = 1/2$. Using this relationship, and computing both the sum and the difference between (12) and (13), we get:

$$c''(1/2)\left[\left.\frac{dq_M}{dp}\right|_{q_X=q_M} + \left.\frac{dq_X}{dp}\right|_{q_X=q_M}\right] = 1 - p\left[\left.\frac{dq_M}{dp}\right|_{q_X=q_M} - \left.\frac{dq_X}{dp}\right|_{q_X=q_M}\right]$$

$$c''(1/2)\left[\left.\frac{dq_M}{dp}\right|_{q_X=q_M} - \left.\frac{dq_X}{dp}\right|_{q_X=q_M}\right] = p\left[\left.\frac{dq_M}{dp}\right|_{q_X=q_M} + \left.\frac{dq_X}{dp}\right|_{q_X=q_M}\right],$$

which necessarily implies that:

$$\left.\frac{dq_M}{dp}\right|_{q_X=q_M} - \left.\frac{dq_X}{dp}\right|_{q_X=q_M} > 0$$

This is sufficient to prove that q_X and q_M intersect at most once and therefore that $\mathcal{P}_X$ is an open interval. □

A2. Proof of Proposition 1

From Lemma 1, we know that if $p \notin \mathcal{P}_X$ then $q_M > q_X$, and therefore $\kappa_M = 1$ and $\kappa_X = 0$. Hence, the utility of the extremist and moderate party when $p \notin \mathcal{P}_X$ are:

$$U_X = p\, q_X(1 - q_M) - c(q_X) \quad \text{and} \quad U_M = p\Big[1 - q_X(1 - q_M)\Big] + (1 - p) - c(q_M).$$

Convergence to (C, C) is the unique equilibrium if $U_X < U_S = \frac{1}{2} - c(q_S) < U_M$, where the first inequality ensures that it is not profitable to deviate from (C, C) and the second one ensures that (L, R) is not sustainable. Using the above expressions of U_X and U_M and rearranging terms, we get:

$$p\, q_X\,(1 - q_M) + c\,(q_S) - c\,(q_X) < \tfrac{1}{2}, \qquad \textbf{(B1)}$$

$$p\, q_X\,(1 - q_M) + c\,(q_M) - c\,(q_S) < \tfrac{1}{2}, \qquad \textbf{(B2)}$$

as the necessary and sufficient conditions for (C, C) to be the unique equilibrium when $p \notin \mathcal{P}$. Given $q \geqslant 0$, $c(q) \geqslant 0$ and $c''(q) > 0$ for all q, we have that for any pair $(q, \tilde{q})$:

$$(q - \tilde{q}) \cdot c'(\tilde{q}) \;<\; c(q) - c(\tilde{q}) \;<\; (q - \tilde{q}) \cdot c'(q). \qquad (14)$$

Using (14), **(C1)** and **(C3)**, we obtain the following inequalities:

$$c(q_S) - c(q_X) \quad < \quad (q_S - q_X) \cdot c'(q_S) = \tfrac{1}{2}\, p\,(q_S - q_X), \qquad (15)$$

$$c(q_M) - c(q_S) \quad < \quad (q_M - q_S) \cdot c'(q_M) = p\, q_X\,(q_M - q_S). \qquad (16)$$

Given (15) and (16), then sufficient conditions for **(B1)** and **(B2)** to hold are respectively:

$$p\, q_X\,(1 - q_M) + \tfrac{1}{2}\, p\;(q_S - q_X) < \tfrac{1}{2} \quad \Leftrightarrow \quad p\,\Big(q_X\,(1 - 2q_M) + q_S\Big) < 1, \qquad (17)$$

$$p\, q_X\,(1 - q_M) + p\, q_X\,(q_M - q_S) < \tfrac{1}{2} \quad \Leftrightarrow \quad p\, q_X\,(1 - q_S) < \tfrac{1}{2}. \qquad (18)$$

Last and again from Lemma 1, we know that when $p \notin \mathcal{P}_X$, then both $q_M \geq 1/2$ (which ensures that (17) holds) and $q_S \geq 1/2$ (which ensures that (18) holds). □

A3. Proof of Proposition 2 and corollaries

Recall from Definition 1 that $p \in \mathcal{P} \;\Leftrightarrow\; \kappa_X = 1$ and $\kappa_M = 0$. Let us define $\mathcal{P}_1$ as the set of probabilities such that, if $p \in \mathcal{P}$, then a deviation from (C, C) is dominated.

Similarly, we define $\mathcal{P}_2$ as the set of probabilities such that, if $p \in \mathcal{P}$, then a deviation from (L, R) is dominated. Formally,

$$\begin{aligned}\mathcal{P}_1 &= \{p : \kappa_X = 1 \Rightarrow U_S > U_X\} \\ \mathcal{P}_2 &= \{p : \kappa_X = 1 \Rightarrow U_S > U_M\}\end{aligned}$$

The proof consists of the following steps. First, we show that (C, C) is the unique equilibrium of the game if $p \notin \mathcal{P}$. Second, we show that the sets $\mathcal{P}_1$ and $\mathcal{P}_2$ are not empty. Third, we show that if $p \in \mathcal{P} \backslash (\mathcal{P}_1 \cup \mathcal{P}_2)$, only asymmetric positions can be equilibria of the game. Fourth, we show that (C, C) is the unique equilibrium when $p \to 1$. Last, we show that (L, R) is the unique equilibrium when $p \to 0$ and $\Delta \to 0$.

Step 1. (C, C) is the unique positioning equilibrium for any $p \notin \mathcal{P}$.

Let $\bar{\mathcal{P}} = [0, 1] \setminus \mathcal{P}$. When $p \in \bar{\mathcal{P}}$, we must have $\kappa_X \leq 1/2$ which implies:

$$\pi_X \leq p\, q_X \,(1 - q_M) + \frac{1-p}{2} \tag{19}$$

$$\pi_M \geq p\,[1 - q_X \,(1 - q_M)] + \frac{1-p}{2} \tag{20}$$

Two situations must be considered separately: $p \in \bar{\mathcal{P}} \cap \bar{\mathcal{P}}_X$ and $p \in \bar{\mathcal{P}} \cap \mathcal{P}_X$. From Proposition 1 we know that (C, C) is the only equilibrium if $p \in \bar{\mathcal{P}}_X$. Using Lemma 1, only two cases remain to be considered for $p \in \bar{\mathcal{P}} \cap \mathcal{P}_X$:

- $1/2 > q_X > q_S > q_M$. In this case, $\pi_X < 1/2$, $\pi_S = 1/2$ and $\pi_M > 1/2$. Noting that $c(q_X) > c(q_S) > c(q_M)$ immediately proves that both $U_X < U_S$ and $U_M > U_S$ hold. Therefore, (C, C) is the unique equilibrium of the game.

- $q_X > 1/2 > q_M > q_S$. Using (19), the inequality $U_X < U_S$ becomes:

$$p\left[\frac{1}{2} - q_X \,(1 - q_M)\right] > c\,(q_S) - c\,(q_X)\,.$$

From (15), $c(q_S) - c(q_X) < \frac{1}{2}\,p\,(q_S - q_X)$. Therefore, $U_X < U_S$ is necessarily satisfied if:

$$1 - 2q_X\,(1 - q_M) > (q_S - q_X) \;\Leftrightarrow\; q_S + q_X(1 - 2q_M) < 1.$$

Given that $q_X < 1$ and $q_S < q_M$, we know that $q_S + q_X(1 - 2q_M) < 1 - q_M$, which means that the above inequality always holds. Last, using (20), $U_M > U_S$ becomes:

$$p\left[\frac{1}{2} - q_X \,(1 - q_M)\right] > c(q_M) - c(q_S).$$

From (16), $c(q_M) - c(q_S) < p\, q_X\, (q_M - q_S)$. Therefore, $U_M > U_S$ is satisfied if:

$$p\left[\frac{1}{2} - q_X\,(1-q_M)\right] > p\, q_X\,(q_M - q_S) \;\Leftrightarrow\; \frac{1}{2} > q_X\,(1-q_S)$$

which always holds given that $q_S < 1/2$ and, from Assumption 2, $q_X < 2q_S$.

Step 2. The sets $\mathcal{P}_1$ and $\mathcal{P}_2$ are not empty.

When $\kappa_X = 1$, we have:

$$U_X = 1 - p\,[1 - q_X\,(1-q_M)] - c(q_X) \qquad \text{and} \qquad U_M = p\,[1 - q_X\,(1-q_M)] - c(q_M)$$

Therefore, $\mathcal{P}_1$ is the set of p for which the following inequality holds:

$$p > f_1(p) = \frac{\frac{1}{2} - c(q_X) + c(q_S)}{1 - q_X(1-q_M)}, \tag{21}$$

and $\mathcal{P}_2$ is the set of p for which the following condition holds:

$$p < f_2(p) = \frac{\frac{1}{2} + c(q_M) - c(q_S)}{1 - q_X(1-q_M)}. \tag{22}$$

When $p \to 0$, then $q_M \to 0$, $q_S \to 0$ and $q_X \to 0$, so $f_2(p) \to 1/2$ and therefore $\mathcal{P}_2$ cannot be empty. Next, using (15), the inequality $p > f_1(p)$ necessarily holds if:

$$p > \frac{\frac{1}{2} - \frac{1}{2}\,p\,(q_X - q_S)}{1 - q_X\,(1-q_M)} \quad \Leftrightarrow \quad p\left[1 - \frac{1}{2}\,q_X(1-2q_M) - \frac{1}{2}\,q_S\right] > \frac{1}{2}.$$

From Lemma 1, two cases are possible: (i) $q_S < q_M < 1/2$, and (ii) $q_M < q_S < q_X < 1/2$. In both of them, the term in brackets is greater than $1/2$ and therefore the inequality always holds when $p \to 1$. Hence, $\mathcal{P}_1$ cannot be empty either.

Step 3. (C,R) and (L,C) are the only possible equilibria for all $p \in \mathcal{P} \setminus (\mathcal{P}_1 \cup \mathcal{P}_2)$.

This is immediate since, by definition, asymmetric positioning dominates (C,C) when $p \in \mathcal{P} \cap \bar{\mathcal{P}}_1$ and asymmetric positioning dominates (L,R) when $p \in \mathcal{P} \cap \bar{\mathcal{P}}_2$.

Corollary 1. (C,C) is the only possible equilibrium for $p \to 0$ or $p \to 1$.
When $p \to 0$, $q_M \to 0$, $q_S \to 0$ and $q_X \to 0$. Therefore, $p \notin \mathcal{P}_X$, and (C,C) is thus the only equilibrium by Proposition 1. Next, we know that if $p \to 1$, then $p \in \mathcal{P}_1$. Thus, it is sufficient to show that $p \notin \mathcal{P}_2$ when $p \to 1$ to conclude that (C,C) is the only

equilibrium for $p \to 1$. Note that $p \in \mathcal{P}_2 \;\Leftrightarrow\; p < f_2(p)$. Using (16), we know that $c(q_M) - c(q_S) < p\, q_X\,(q_M - q_S)$. Hence, a necessary condition for $p \in \mathcal{P}_2$ is:

$$p < \frac{\frac{1}{2} + p\, q_X\,(q_M - q_S)}{1 - q_X(1 - q_M)} \quad \Leftrightarrow \quad p\Big[1 - q_X(1 - q_S)\Big] < \frac{1}{2}.$$

However, the above inequality cannot hold for $p = 1$ given that, by Assumption 2, $q_X < 2q_S$.

Corollary 2. We show here that (L, R) is the only possible equilibrium when $p = \varepsilon$, with $\varepsilon \to 0^+$ and $\Delta \to 0$.

When $\Delta \to 0$, then $\mathcal{P} \to \mathcal{P}_X$. Hence, from Lemma 1 and Step 2 above, small but positive values of p are such that $p \in \mathcal{P} \cap \mathcal{P}_2$ and $p \notin \mathcal{P}_1$. □

A4. The simultaneous case

Suppose that candidates choose location and investment simultaneously. In the timing of Figure 1, this amounts to merging $t = 1$ and $t = 2$. It is immediate to verify that Proposition 1 extends to the simultaneous case, that is, only deviations for quality may occur.[22] Also, for each p, if a pure strategy Nash equilibrium with platform choices (x_A, x_B) exists, the equilibrium qualities (q^A, q^B) must be the same as in the sequential game. Therefore, for each pair of platforms, we need to check the conditions on p such that candidates do not have incentives to deviate from the qualities (q_S, q_X, q_M) determined in section 2.

Case 1: (C, C). Given **(C2)**, if a candidate deviates, it sets a quality $\tilde{q}_{SX}$ such that:

$$c'(\tilde{q}_{SX}) = p(1 - q_S) \qquad \textbf{(C2}'\textbf{)}$$

where subscript "SX" denotes a candidate who deviates from symmetric to extremist. Since only a deviation for quality can be profitable (Proposition 1), (C, C) is necessarily an equilibrium if $p \notin \tilde{\mathcal{P}}_{SX}$, where:

$$\tilde{\mathcal{P}}_{SX} \;=\; \{p: \;\Delta < \tilde{q}_{SX} - q_S\}$$

Note that there always exist a threshold $\underline{p}'$ (> 0) such that $[0, \underline{p}'] \not\subset \tilde{\mathcal{P}}_{SX}$. When $\Delta < \tilde{q}_{SX} - q_S$, (C, C) is still an equilibrium if and only if:

$$\tilde{U}_{SX}(\tilde{q}_{SX}, q_S) < U_S(q_S, q_S).$$

[22]Naturally, as we show below, the set of probabilities such that a deviation results in a quality increase are different.

Using (7) and (10), this condition can be rewritten as:

$$p\,\tilde{q}_{SX}(1-q_S)+(1-p)-c(\tilde{q}_{SX})<\frac{1}{2}-c(q_S) \quad\Leftrightarrow\quad p>\tilde{f}_1(p)\equiv\frac{\frac{1}{2}-c(\tilde{q}_{SX})+c(q_S)}{1-\tilde{q}_{SX}(1-q_S)}.$$

Using (14), the inequality $p>\tilde{f}_1(p)$ necessarily holds if:

$$p>\frac{\frac{1}{2}-\frac{1}{2}\,p\,(\tilde{q}_{SX}-q_S)}{1-\tilde{q}_{SX}\,(1-q_S)} \quad\Leftrightarrow\quad p\left[1-\frac{1}{2}\Big(\tilde{q}_{SX}+q_S-2\tilde{q}_{SX}\,q_S\Big)\right]>\frac{1}{2}.$$

Since the term in brackets is greater than 1/2 for all $(\tilde{q}_{SX},q_S)\in(0,1)^2$, the inequality $p>\tilde{f}_1(p)$ holds when $p\to 1$. Overall, (C,C) is an equilibrium either if $p\notin\tilde{\mathcal{P}}_{SX}$ or if $p\in\tilde{\mathcal{P}}_{SX}\cap\tilde{\mathcal{P}}_1$, where the sets $\tilde{\mathcal{P}}_{SX}$ and $\tilde{\mathcal{P}}_1$ are defined as:

$$\begin{aligned}\tilde{\mathcal{P}}_{SX} &= \{p:\ \Delta<\tilde{q}_{SX}-q_S\}\\ \tilde{\mathcal{P}}_1 &= \{p:\ p>\tilde{f}_1(p)\}\end{aligned}$$

Note in particular that, just like in the sequential case, there exist $(\underline{p}',\overline{p}')\in(0,1)^2$ such that (C,C) is an equilibrium if $p\leq\underline{p}'$ or $p\geq\overline{p}'$.

Case 2: (L,R). Given **(C3)**, if a candidate deviates, it sets a quality $\tilde{q}_{SM}$ such that:

$$c'(\tilde{q}_{SM})=p\,q_S \qquad \textbf{(C3')}$$

where subscript "SM" denotes a candidate who deviates from symmetric to moderate. (L,R) is an equilibrium if and only if:

$$\tilde{U}_{SM}(\tilde{q}_{SM},q_S)<U_S(q_S,q_S). \qquad (23)$$

By Proposition 1, a necessary condition for this inequality to hold is that: $\Delta<q_S-\tilde{q}_{SM}$. That is, voters must trust the extremist candidate more than the moderate. Subject to $\Delta<q_S-\tilde{q}_{SM}$, by (7) and (11), condition (23) can be rewritten as:

$$p\Big[1-q_S(1-\tilde{q}_{SM})\Big]-c(\tilde{q}_{SM})<\frac{1}{2}-c(q_S) \quad\Leftrightarrow\quad p<\tilde{f}_2(p)\equiv\frac{\frac{1}{2}-c(q_S)+c(\tilde{q}_{SM})}{1-q_S(1-\tilde{q}_{SM})}.$$

Thus, (L,R) is an equilibrium if $p\in\tilde{\mathcal{P}}_{SM}\cap\tilde{\mathcal{P}}_2$ where $\tilde{\mathcal{P}}_{SM}$ and $\tilde{\mathcal{P}}_2$ are defined as:

$$\begin{aligned}\tilde{\mathcal{P}}_{SM} &= \{p:\ \Delta<q_S-\tilde{q}_{SM}\}\\ \tilde{\mathcal{P}}_2 &= \{p:\ p<\tilde{f}_2(p)\}\end{aligned}$$

Case 3: (L, C) or (C, R). We know from the sequential case that a necessary condition for asymmetric polarization to be an equilibrium is:

$$\Delta < q_X - q_M.$$

We need to check potential deviations by the extremist and by the moderate.

• An extremist who deviates chooses quality $\tilde{q}_{XS} \equiv q_S$ which, by **(C1)**, is independent of the rival's choice (the subscript "XS" denotes a candidate who deviates from extremist to symmetric). If $q_S > q_M$, the deviating candidate is elected with probability 1 whenever platform qualities remain unobserved. This makes the deviation profitable. Formally, $\tilde{U}_{XS}(q_S, q_M) > \tilde{U}_{XS}(q_X, q_M) > U_X(q_X, q_M)$ for all $q_S > q_M$.

If $q_S < q_M$, the deviating candidate loses the support of voters when qualities are not disclosed. He does not have an incentive to deviate if and only if:

$$\tilde{U}_{XS}(q_S, q_M) < U_X(q_X, q_M),$$

which, using (7) and (10), can be rewritten as:

$$\begin{aligned} p\left[\frac{1 + q_S - q_M}{2}\right] - c(q_S) &< p\, q_X(1 - q_M) + (1 - p) - c(q_X) \\ \Leftrightarrow \quad p < \tilde{f}_3(p) &\equiv \frac{1 - c(q_X) + c(q_S)}{1 - q_X(1 - q_M) + \frac{1}{2}(1 + q_S - q_M)} \end{aligned}$$

• A moderate candidate who deviates will also choose quality $\tilde{q}_{MS} \equiv q_S$ (the subscript "MS " denotes a candidate who deviates from moderate to symmetric). By Lemma 1, we know that $q_X > q_M \Rightarrow q_X > q_S$, that is, the moderate still does not gain the support of voters after deviation. As a result, he does not have an incentive to deviate. Formally, $\tilde{U}_{MS}(q_S, q_X) < U_M(q_S, q_X) < U_M(q_M, q_X)$ for all $q_M < q_X$.

To sum up, (L, C) and (C, R) are equilibria if $p \in \mathcal{P} \cap \tilde{\mathcal{P}}_{MS} \cap \tilde{\mathcal{P}}_3$ where the sets $\mathcal{P}$, $\tilde{\mathcal{P}}_{MS}$ and $\tilde{\mathcal{P}}_3$ are defined as:

$$\begin{aligned} \mathcal{P} &= \{p : \ \Delta < q_X - q_M\} \\ \tilde{\mathcal{P}}_{MS} &= \{p : \ q_M - q_S > 0\} \\ \tilde{\mathcal{P}}_3 &= \{p : \ p < \tilde{f}_3(p)\} \end{aligned}$$

A5. Tying Quality and Ideology

Suppose that whenever a candidate finds a high-quality platform, he is constrained by the ideology L or R contained in that platform. Working by backward induction, we begin by identifying the conditions under which high-quality candidates will prefer to maintain an extreme ideology in $t = 3$, while low-quality candidates will prefer remain in C. To this end, first note that, in equilibrium, the probabilities of election associated with each ideological position are:

$$\begin{cases} \pi_R(q_R = 1) = & \frac{q_L}{2} + (1 - q_L) = 1 - \frac{q_L}{2}; \\ \pi_L(q_L = 1) = & 1 - \frac{q_R}{2}; \\ \pi_C(q_R = 0) = & \pi_C(q_L = 0) = 0 \times q_L + \frac{1-q_L}{2} = \frac{1-q_L}{2}. \end{cases} \tag{24}$$

These read as follows: a high-quality candidate who adopts the high-quality platform is certain to win if he faces a low-quality candidate with a centrist position, and he wins with probability 1/2 if he faces a high-quality candidate with an extreme position. Conversely, a low-quality candidate with a centrist platform is only elected with probability 1/2 when he faces another low-quality candidate. Note that the quality of information p is absent from these election probabilities, since platform positions perfectly reveal quality in such an equilibrium.

Incentive condition for high-quality candidates. The only possible deviation for a high-quality candidate is to revert to a centrist location, thereby proposing a low-quality platform. This brings him an election probability $\pi_C(q = 0)$, which is always strictly smaller than $\pi_R(q_R = 1)$. Hence, this incentive condition is never binding in equilibrium.

Incentive condition for low-quality candidates. The only possible deviation for a low-quality candidate is to adopt an extreme position, despite having low quality. This deviation allows him to be mistaken for a high-quality candidate when voters do not observe platform quality. However, it also implies that he loses the election when qualities are observed:

$$\pi_R(q_R = 0) = (1 - p) \times \pi_R(q_R = 1) + p \times 0.$$

Comparing this probability with $\pi_C(q_R = 0)$ shows that:

$$p \geq (2 - q_L)^{-1}$$

is a necessary and sufficient condition for this deviation to be a dominated strategy.

Hence, $p \geq (2 - q_L)^{-1}$ is a necessary and sufficient condition to obtain a separating equilibrium, and obtain (24)

A6. Relocation

This appendix does not provide a full characterization of equilibria given the possibility of relocation. Instead, we limit ourselves to the continuation game where at least one party has chosen an extremist position. In that subgame, we determine the conditions such that relocation is suboptimal. These conditions are sufficient for the equilibria found in Proposition 2 to hold. (Note that investments in quality are sunk at the relocation stage. Thus, continuation utilities depend exclusively on the probability of election.)

Case 1: continuation game given platforms (L, R).

Given (5), the continuation utility of a party that has obtained a low quality is:

$$p \, \frac{1 - q_S}{2} + (1 - p)\frac{1}{2} \tag{25}$$

If the party relocates to C, its continuation utility becomes:

$$\begin{array}{ll} p(1 - q_S) & \text{if} \quad \Delta < q_S \\ p(1 - q_S) + 1 - p & \text{if} \quad \Delta > q_S \end{array} \tag{26}$$

The party that relocates is elected if the rival's quality is revealed to be low (because of a more desirable ideology). When the rival's quality remains unknown, its expected quality is q_S. Voters again support the relocating party if the gain in the ideology dimension Δ compensates for the loss in expected quality $q_S - 0$. Overall, and given (25) and (26), relocation from full polarization is suboptimal in the continuation game if and only if $p \in \hat{\mathcal{P}}_{LR}$, where

$$\hat{\mathcal{P}}_{LR} \; = \; \{ \, p : \; \Delta < q_S \; \text{ and } \; p < \frac{1}{2 - q_S} \, \}.$$

Thus, given Proposition 2, a sufficient condition for (L, R) followed by no relocation to be an equilibrium of the game is $p \in \hat{\mathcal{P}}_{LR} \cap \mathcal{P}_2 \cap \mathcal{P}$.

Case 2: continuation game given platforms (L, C) or (C, R).

Assume $p \in \mathcal{P}$, which we know is a necessary condition for asymmetric polarization (see Proposition 2). Given (10), the continuation utility of an extremist party with low quality is:

$$1 - p, \tag{27}$$

because it benefits from the trust of the electorate when qualities remain unobserved. If the extremist relocates to C, its continuation utility becomes:

$$p\,\frac{1 - q_M}{2}. \tag{28}$$

The party that relocates the platform loses the trust of the electorate but wins with probability 1/2 if the quality of the rival is revealed to be low. Given (27) and (28), relocation from asymmetric polarization to full convergence is suboptimal in the continuation game if and only if, conditional on $p \in \mathcal{P}$, then $p \in \hat{\mathcal{P}}_{LC}$, where:

$$\hat{\mathcal{P}}_{LC} \;=\; \{\; p: \; p < \frac{2}{3 - q_M} \;\}$$

Thus, given Proposition 2, a sufficient condition for (L, C) and (C, R) followed by no relocation to be an equilibrium of the game is $p \in \hat{\mathcal{P}}_{LC} \cap \mathcal{P} \backslash (\mathcal{P}_1 \cup \mathcal{P}_2)$.