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INFORMATION COSTS, DURATION OF SEARCH, AND TURNOVER: THEORY AND APPLICATIONS

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1. INTRODUCTION

Several years ago Philip Nelson observed that certain problems arise in extending search theory to deal with nonhomogeneous goods. In particular, he noted that "[i]nformation about quality differs from information about price because the former is usually more expensive to buy than the latter" [1970, p. 311]. To analyze the implications of this observation, Nelson divided goods into two classes, search goods and experience goods. Search goods are those for which utility is assessed before purchase by actual inspection. Experience goods are those for which utility is assessed after purchase by actual consumption.

These definitions turn out to be very strong. So strong, in fact, that they lead to some difficulties. Consider, for example, experience goods. For these goods utility is assessed after purchase by actual consumption. But at least the price is observed before purchase. Since for experience goods this cannot, by definition, affect the choice of which brand to buy, Nelson was forced to assume that "consumers either sample at random from among all brands or from among those brands in the price range the consumer deems appropriate for himself" [1970, p. 313]. These assumptions require that consumers either ignore prices completely or have perfect information regarding prices, neither of which seems likely.

The problem with Nelson's definitions is that goods generally possess a number of characteristics which can differ in their degree of observability. Thus a good might possess some "search" characteristics and some "experience" characteristics. Furthermore, whether a particular characteristic is a search characteristic or an experience characteristic ought to be endogenously determined by the consumer.

In Wilde [1977] I analyze a model in which goods are described by precisely two characteristics, price and quality. The market distribution of price and quality is given by a joint p.d.f., $\phi(p,q)$. The cost of drawing a sample of one from ϕ is given by c_S, where $c_S \geq 0$. Once an observation is drawn from ϕ, price is observed costlessly. Quality, however, can never be observed before purchase. In this case quality is an extreme example of an experience characteristic, one for which the cost of observation prior to purchase is infinite. The present paper extends this model to allow quality to be observed at some finite cost, c_T, where $c_T \geq 0$. The purpose of the exercise is to explicate the relationships between information costs, duration of search, and turnover.[2]

The model developed in Wilde [1977] applies to a number of cases discussed by Nelson. For example, it formalizes his prototypic experience good, canned tuna fish. Nelson suggested that "[t]o evaluate brands of canned tuna fish, the consumer would almost certainly purchase brands of tuna fish for consumption. He could, then, determine from several purchases which brand he preferred. For tuna fish there is no effective search alternative open. At the low price of experience, there is insufficient demand for specialized establishments selling tastes of various brands of tuna fish" [1970, p. 312]. In this case, Nelson seems to be suggesting that the "price of experience" is low because the price of the good is low. However, the price of experience can be low for other reasons as well. For example, if there is little variance in quality then the price of experience is low because there is little chance of purchasing a low-quality good. In fact, it will turn out that by allowing quality to be observed before purchase, at some cost, an explicit expression for the price of experience can be derived. Utilizing this expression, this paper will also explicate the relationship between information costs and the price of experience.

The focus of the model developed in section 2 is again on an imperfectly informed consumer who is interested in maintaining a one unit per period flow of consumption of a good which is described by price and quality. The market offers various combinations of price and quality but the consumer cannot costlessly observe them; by paying a search cost the consumer can sample a good from the market, but only price is observed. Quality can be observed either before purchase by actual inspection (at some additional cost) or after purchase by actual consumption. Whether quality is observed before purchase (herein called inspection) or after purchase (herein called evaluation), the consumer can return to the market and resample if the observed quality level is too low. The initial problem is to characterize the optimal strategy for a consumer in such an environment. For a fixed utility function, joint distribution of price and quality, and cost of search, three possibilities arise depending on the cost of inspection:

(1) If the cost of inspection is low enough, inspection will be the optimal strategy for low prices and drawing a new observation will be the optimal strategy for high prices.

(2) If the cost of inspection is of an intermediate amount, evaluation will be the optimal strategy for low prices, inspection will be the optimal strategy for intermediate prices, and drawing a new observation will be the optimal strategy for high prices.

(3) If the cost of inspection is high enough, evaluation will be the optimal strategy for low prices and drawing a new observation will be the optimal strategy for high prices.

The formal results of the paper relate to the characterization of the consumer's optimal strategy which is summarized by (1)-(3) above. However, these results have broad implications. Three important observations emerge from the analysis.

First, as mentioned above, the price of experience can be defined analytically. This is important because it allows one to differentiate between the direct, short-run benefits of purchasing a good which are derived from its consumption and the indirect, long-run benefits of purchasing a good which are derived from evaluation of its quality attributes.

Second, it will be demonstrated that in some cases quality will be a pure search characteristic (#1 above), in some cases it will be neither a pure search characteristic nor a pure experience characteristic (#2 above), and in some cases it will be a pure experience characteristic (#3 above). Hence, not only is determining whether a particular good is an experience good or a search good a complex matter, but so is determining whether a particular characteristic of that good is an experience characteristic or a search characteristic. Since these distinctions have become very popular in the literature it is important to understand their limitations.

Finally, the comparative statics associated with the characterization of the consumer's optimal strategy will show that the theoretical foundations of much of the empirical work on duration of search and turnover in labor markets and marriage markets is unsound.

This paper is organized as follows. Section 2 introduces the basic model. Section 3 considers the case in which quality is always observed before purchase, section 4 considers the case in which quality is observed before purchase for some prices but is observed after purchase for other prices, and section 5 considers the case in which quality is always observed after purchase. Section 6 discusses the empirical implications of the model in the labor market and the marriage market. Section 7 provides a brief conclusion.

2. THE MODEL: NOTATION, DEFINITIONS, AND ASSUMPTIONS

In this section the basic model will be developed and an explicit expression for the price of experience given. Sections 3, 4 and 5 provide comparative statics for the three cases mentioned in the introduction.

Assume the good which is sought by the consumer has a lifetime of one period. Let $U(p,q)$ be the total net value to the consumer of purchasing and consuming the good characterized by price p and quality q, where U is differentiable and bounded on $R_+ \times R_+$ with $\partial U/\partial p < 0$ and $\partial U/\partial q > 0$. Let $\phi(p,q)$ be the consumer's subjective estimate of the market density of P and Q. For mathematical convenience, assume ϕ is strictly positive on $R_+ \times R_+$. Define $f(p)$ as the marginal density of P and $g(q|p)$ as the conditional density of Q given $P = p$, both based on ϕ.

The cost of drawing an observation at random from ϕ is c_S, where $c_S \geq 0$. The cost of observing the true value of Q prior to purchase is c_T, where $c_T \geq 0$ (both c_S and c_T are measured in the same units as U).

The consumer can sample as many observations as desired from ϕ at the beginning of each period. Any number of inspections are also allowed. However, the consumer demands precisely one unit of the good each period. Search and inspection are assumed to be timeless in order to avoid confounding the direct costs of these activities with the opportunity cost of delaying the purchase decision.

The consumer's objective is to maximize expected discounted utility of consumption net of search costs. Sampling is assumed to be without recall, the horizon is infinite and the discount rate is β, where $0 < \beta < 1$.

Now suppose the consumer has drawn an observation of p from f. Three reactions are possible: sample again without observing quality; inspect quality and then either buy the good forever or sample again; or evaluate quality and then either buy the good forever or sample again. Let v(p) be the expected value of drawing an observation of p and then proceding optimally. By the principle of optimality,

$$v(p) = -c_S + \max\{V, B(p), T(p)\}, \tag{1}$$

where V is the expected value of v(p) taken with respect to f, B(p) is the expected value of buying the good without observing quality and then proceding optimally, and T(p) is the expected value of testing quality prior to purchasing the good and then proceding optimally.

To analyze (1), observe first that once quality is known, the value of the optimal policy is the same whether quality is observed via inspection or evaluation. Define this value as k(p). Then

$$k(p) = VG[q^*(p)|p] + \int_{q^*(p)}^{\infty} \frac{U(p,q)}{1-\beta}\, g(q|p)dq \tag{2}$$

where $q^*(p)$ is that quality level which makes the consumer indifferent between consuming the good characterized by $\{p, q^*(p)\}$ and searching again for a new good from ϕ. That is, $q^*(p)$ is defined by

$$U(p, q^*(p)) = V(1-\beta).$$

The logic of (2) is that if $q < q^*(p)$, then the consumer rejects the good and samples again. This happens with probability $G[q^*(p)|p]$. If $q \geq q^*(p)$, then the good is acceptable and the consumer receives

the conditional expected value of $U(p,q)/(1-\beta)$, given $q \geq q^*(p)$.[3]

Using k(p), both B(p) and T(p) are defined straightforwardly:

$$B(p) = EU(p,Q) + \beta k(p) \tag{3}$$

$$T(p) = -c_T + k(p). \tag{4}$$

It is convenient at this point to assume that the functional equation (1) has a unique, bounded solution. In this case V is unique and well-defined.[4]

In analyzing the optimal policy it will also be convenient to make a transformation of variables in the definition of k(p). Since it is ultimately final utility which matters to the consumer, the focus of (2) can be shifted from the conditional distribution of quality given price to the conditional distribution of utility given price. That is, let $\Psi(w|p)$ be the conditional distribution of utility given $P = p$. Then k(p) becomes

$$k(p) = V\Psi[V(1-\beta)|p] + \int_{V(1-\beta)}^{\bar{\bar{z}}(p)} \frac{w}{1-\beta}\, \psi(w|p)dw \tag{5}$$

where $\bar{\bar{z}}(p) = \lim_{q\to\infty} U(p,q)$. Integrating (5) by parts gives

$$k(p) = (1-\beta)^{-1}[\bar{\bar{z}}(p) - \int_{V(1-\beta)}^{\bar{\bar{z}}(p)} \Psi(w|p)dw]. \tag{6}$$

Finally, noting that B(p) can be rewritten as

$$B(p) = -c_B(p) + k(p) \tag{7}$$

where

$$c_B(p) = (1-\beta)k(p) - EU(p,Q),$$

one can see that $c_B(p)$ is the implicit cost of observing quality by actual consumption. Using (5) it is easy to show that

$$c_B(p) = \int_{\overline{z}(p)}^{V(1-\beta)} \Psi(w|p)\,dw, \tag{8}$$

where $\overline{z}(p) = U(p,0)$.

Equation (8) provides an analytical expression for Nelson's "price of experience" which is directly comparable to c_T. Furthermore, it has a natural interpretation. Recall that k(p) is the expected value of an optimal policy once quality is known, given the observed price is p. When quality is observed via actual consumption this value is not obtained for one period (since in this model the good lasts for precisely one period). Hence $(1-\beta)k(p)$ is the gross opportunity cost of consuming the good given quality is unknown. But consumption of the good yields utility, in this case EU(p,Q). The net opportunity cost of consuming the good given quality is unknown is the difference between these two quantities.

It is enormously useful to express the price of experience in this form. Analytically, it makes the comparison between B(p) and T(p) easier. Conceptually, it helps identify factors which might effect the

decision whether to observe quality before purchase or after purchase. For example, suppose the cost of search, c_S, increases. Then surely the value of an optimal policy will fall, i.e. $\partial V/\partial c_S < 0$. Equation (8) suggests that the price of experience will then fall as well. These and other results will be formalized in the following sections. First, however, a few more preliminary assumptions will be needed.

Using the definition of $c_B(p)$ introduced above in equation (7), the functional equation (1) can be rewritten as

$$v(p) = -c_S + \max\{V, k(p) - c_B(p),\ k(p) - c_T\}. \tag{9}$$

The next step in characterizing an optimal policy is to compare V, $k(p) - c_B(p)$, and $k(p) - c_T$. Unfortunately, without more structure on ϕ, and thus on Ψ, any number of things can happen. Nelson recognized this problem as well, stating

> Prior to using [a] brand, all the consumer knows is its price. But this knowledge provides only the roughest sort of guide to choice, for the consumer must assume a generally positive relationship between price and quality. In the absence of any other information, the consumer would not know if he were better off experimenting with low- or high-priced brands. [1970, p. 373].

To get around this problem, Nelson converts the joint distribution of price and quality to a distribution of net utility and proceeds under the assumption that evaluation is <u>always</u> used to observe quality. In the present analysis, since the decision whether to observe quality before purchase or after purchase is endogenous, some formal structure must be placed directly on Ψ. The standard assumption is that

$\partial\Psi(w|p)/\partial p > 0$. This assumption implies that, on average, higher price is associated with lower utility even though higher price may well be associated with lower utility. Because this assumption is discussed at length in Wilde [1977], it will be assumed to hold here without further rationalization.[5]

Several implications follow directly from the assumption that $\partial\Psi(w|p)/\partial p > 0$. First, it implies the price of experience is increasing in the price of the good. To see this simply take the derivative of (8) with respect to p:

$$c_B'(p) = \int_{\bar{z}(p)}^{V(1-\beta)} \partial\Psi(w|p)/\partial p \, dw - \Psi(\bar{z}(p)|p)\bar{z}'(p)$$

$$= \int_{\bar{z}(p)}^{V(1-\beta)} \partial\Psi(w|p)/\partial p \, dw$$

since $\Psi(z(p)|p) = 0$ by definition. Second, $\partial\Psi(w|p)/\partial p > 0$ implies that observing quality before purchase becomes a less desirable alternative to sampling again as the observed price increases. That is, using (6),

$$T'(p) = k'(p) = -\int_{V(1-\beta)}^{\bar{\bar{z}}(p)} \partial\Psi(w|p)/\partial p \, dw \leq 0. \tag{10}$$

Moreover, $B'(p) = k'(p) - c_B'(p) \leq T'(p)$ as $c_B'(p) \geq 0$, whence observing quality after purchase also becomes a less desirable alternative to sampling again as the observed price increases and it does so at an even faster rate than observing quality before purchase.

This completes the preliminary analysis of the model. It turns out that three qualitatively distinct forms of the optimal policy are possible. If c_T is low enough then inspection always dominates evaluation. If c_T is somewhat higher then evaluation dominates inspection for one set of prices and inspection dominates evaluation for another set of prices, and if c_T is high enough then evaluation always dominates inspection. These three cases are analyzed in the next three sections.

3. THE OPTIMAL POLICY, CASE A: INSPECTION ONLY

In comparing the expected value of observing quality before purchase to the expected value of observing quality after purchase, the crucial variables are $c_B(p)$ and c_T. It is clear that

$$B(p) \gtreqless T(p) \quad \text{as} \quad c_T \gtreqless c_B(p).$$

But $c_B(p)$ is increasing in p so that $c_B(0) \geq c_T$ implies $c_B(p) \geq c_T$ for all $p \geq 0$. Hence the expected value of observing quality before purchase will always be greater than the expected value of observing quality after purchase when $c_B(0) > c_T$. This is obviously most likely to be the case when $c_B(0)$ is large and c_T is small. Equation (8) suggests $c_B(0)$ is most likely to be large when $V(1-\beta)$ is significantly greater than $\bar{z}(0)$. But $V(1-\beta) = U(0,q^*(0))$ and $\bar{z}(0) = U(0,0)$. Hence $c_B(0)$ is most likely to be large when $q^*(0)$ is high. In other words, inspection is likely to dominate evaluation for all prices when the cost of inspection is low or when few quality levels are acceptable even at low prices. The latter might be the case, for example, when the cost of drawing observations from ϕ is low or the variation in utility due to quality is high relative to the variation in utility due to price.[6]

In the remainder of this section it will be assumed that $c_B(0) \geq c_T$; i.e., it will be assumed that

$$\int_{\bar{z}(0)}^{V(1-\beta)} \Psi(w|0)\,dw \geq c_T.$$

This implies $B(p) \leq T(p)$ for all $p \geq 0$, in which case $B(p)$ can be ignored completely; characterizing the optimal policy reduces to comparing $T(p)$ and V. Two possibilities arise. In the first there exists a unique finite price, say p_T^*, such that observing quality prior to purchase is optimal for $p \leq p_T^*$ and sampling again is optimal for $p \geq p_T^*$ (see figure 1). The critical price is defined by $V = T(p_T^*)$.[7] In the second V is strictly less than $T(p)$ for all $p \geq 0$ so that observing quality prior to purchase is always optimal.[8]

Assume for the remainder of this section that p_T^* exists and is finite. Then p_T^* and q^* partition $R_+ \times R_+$ into three sets (see figure 2). In region I, $p > p_T^*$ so the good is rejected outright. In region II, $p \leq p_T^*$ but $q < q^*(p)$ so that quality is observed prior to purchase but the good is subsequently rejected (i.e., not purchased). In region III, $p \leq p_T^*$ and $q \geq q^*(p)$ so that quality is observed prior to purchase and the good is subsequently accepted (i.e., purchased).

How do changes in c_S and c_T effect this partition? To answer this question one needs to know how changes in c_S and c_T effect p_T^* and $q^*(p)$. The following results are straightforward, but tedious, and can be found in appendix 1 of this paper. It is shown there that

$$\frac{dp_T^*}{dc_S} > 0 \quad \text{and} \quad \frac{dp_T^*}{dc_T} < 0.$$

Of course V falls as either c_S or c_T rises. Since $U(p,q^*(p)) = V(1-\beta)$ by definition and U is increasing in q, this implies

$$\frac{dq^*(p)}{dc_S} < 0 \quad \text{and} \quad \frac{dq^*(p)}{dc_T} < 0.$$

In other words, an increase in the cost of drawing observations from ϕ will make inspection an optimal strategy for more prices while an increase in the cost of inspection will make inspection an optimal strategy for less prices. An increase in either cost will make more quality levels acceptable for any given price.

Next, consider how changes in c_S and c_T effect the number of observations which must be drawn from ϕ (whether or not quality is inspected) before an acceptable good is found. Define

$$I_S = \int_0^{p_T^*} [1 - G(q^*(p)|p)]f(p)\,dp$$

and

$$I_F = \int_0^{p_T^*} G(q^*(p)|p)f(p)\,dp.$$

Here I_F is the probability that a random price-quality combination will fall in region II and I_S is the probability that a random price quality combination will fall in region III. It is shown in appendix 1 that the following hold:

$$\frac{\partial I_S}{\partial c_S} > 0 \quad \text{and} \quad \frac{\partial I_F}{\partial c_S} \gtreqless 0$$

while

$$\frac{\partial I_S}{\partial c_T} \gtreqless 0 \quad \text{and} \quad \frac{\partial I_F}{\partial c_T} < 0.$$

Consider first an increase in the cost of drawing observations from ϕ. Since p_T^* increases, inspection becomes an optimal strategy for more prices. Furthermore, since $q^*(p)$ decreases, more quality levels are acceptable for any given price. Hence the expected number of observations which must be drawn from ϕ before an acceptable good is found $(1/I_S)$ falls. The effect of an increase in c_S on I_F is ambiguous, however, because the decrease in $q^*(p)$ counteracts the increase in p_T^* rather than reinforcing it.

Precisely the opposite happens when the cost of inspection increases. Since p_T^* falls, inspection becomes an optimal strategy for less prices. Furthermore, since $q^*(p)$ still decreases, fewer quality levels are acceptable for any given price. Hence I_F falls. Since the effect on I_S is ambiguous, it is impossible to assert that an increase in the cost of inspection reduces the expected number of observations which must be drawn from ϕ before an acceptable good is found.

4. THE OPTIMAL POLICY, CASE B: INSPECTION AND EVALUATION

It was assumed throughout section 3 that $c_B(0) \geq c_T$ so that inspection dominated evaluation for all prices. Recall that

$$c_B(0) = \int_{\bar{z}(0)}^{V(1-\beta)} \Psi(w|0)\,dw.$$

It is immediate from this equation that $\partial c_B(0)/\partial c_S < 0$ and $\partial c_B(0)/\partial c_T < 0$ since V is decreasing in either cost and $\Psi(w|0) > 0$ for w close to $\bar{z}(0)$. Hence increases in either c_S or c_T make it less likely that $c_B(0) \geq c_T$.

Assume for the remainder of this paper that $c_B(0) < c_T$. Then two possibilities arise. In the first, inspection is optimal for one set of prices and evaluation is optimal for another set of prices. In the second, inspection is never optimal. This section will analyze the first possibility and the next section will analyze the second possibility.[9]

In order for inspection to be optimal for one set of prices and evaluation to be optimal for another set of prices two conditions must be met. Assuming $c_B(0) < c_T$, the first can be stated as follows.

> Condition 1: The cost of inspection must not be so great that evaluation dominates inspection for all prices.

When condition 1 holds, since $B'(p) \leq T'(p) \leq 0$, there exists a finite price p_{BT}^* such that evaluation dominates inspection for $p \leq p_{BT}^*$ and inspection dominates evaluation for $p \geq p_{BT}^*$. The critical price is defined by $T(p_{BT}^*) = B(p_{BT}^*)$ (see figure 3).

Assuming condition 1 holds, the second condition can be stated as follows.

Condition 2: The expected return to search must be low enough that for some prices inspection dominates drawing another observation from ϕ.

Condition 2 requires that for some $p > p^*_{BT}$, $T(p) > V$. However as in section 3, it might be that $T(p) > V$ for all $p \geq p^*_{BT}$. Assume this isn't the case. Then there exists a finite price p^*_T such that inspection is optimal for $p \in [p^*_{BT}, p^*_T]$ and drawing another observation from ϕ is optimal for $p \geq p^*_T$. Again, as in section 3, the critical price is defined by $T(p^*_T) = V$ (see figure 3).[10]

Overall the situation dealt with in this section is the most interesting of the model since it shows that quality can be a search characteristic for some prices and an experience characteristic for other prices. That is, the configuration of utility, search costs, inspection costs, and the joint distribution of price and quality are such that inspection, evaluation, and drawing another observation from ϕ are all optimal strategies for various prices. In general, p^*_T, p^*_{BT}, and q^* partition $R_+ \times R_+$ into five regions. As in section 3 of this paper, Region I includes prices for which rejecting the good outright is optimal. In region II, $p^*_{BT} < p \leq p^*_T$ but $q < q^*(p)$ so that quality is observed prior to purchase but the good is subsequently rejected and in region III, $p^*_{BT} < p \leq p^*_T$ but $q \geq q^*(p)$ so that quality is observed prior to purchase and the good is subsequently accepted. There are two additional regions, though. In region IV, $p \leq p^*_{BT}$ and $q < q^*(p)$ so that the good is purchased with-

out quality having been observed but is not repurchased. In region V, $p \leq p^*_{BT}$ and $q \geq q^*(p)$ so that the good is purchased without quality having been observed and is repurchased in all subsequent periods. Figure 4 illustrates these regions.

How do changes in c_S and c_T effect this partition? As before, to answer this question we need to know how changes in c_S and c_T effect p^*_{BT}, p^*_T and $q^*(p)$. The following results are derived in appendix 2. As in section 3,

$$\frac{dp^*_T}{dc_S} > 0 \quad \text{and} \quad \frac{dp^*_T}{dc_T} < 0.$$

Also,

$$\frac{dp^*_{BT}}{dc_S} > 0 \quad \text{and} \quad \frac{dp^*_{BT}}{dc_T} > 0.$$

Finally,

$$\frac{dq^*(p)}{dc_S} < 0 \text{ and } \frac{dq^*(p)}{dc_T} < 0.$$

An increase in the cost of drawing observations from ϕ will increase the set of prices for which evaluation is an optimal strategy. Furthermore, it will increase the set of prices for which either inspection or evaluation is an optimal strategy. An increase in the cost of inspection will increase the set of prices for which evaluation is an optimal strategy, will decrease the set of prices for which inspection is an optimal strategy, and will decrease the set of prices for which either inspection or evaluation is an optimal strategy. Finally, an increase in either c_S or c_T will make more quality levels acceptable at any given price.

It is again useful to consider how increases in c_S and c_T effect the probability a random price-quantity combination will fall in any given region. Define

$$E_S = \int_0^{p^*_{BT}} [1 - G(q^*(p)\,|\,p)]f(p)dp$$

$$E_F = \int_0^{p^*_{BT}} G(q^*(p)\,|\,p)f(p)dp$$

$$I_S = \int_{p^*_{BT}}^{p^*_T} [1 - G(q^*(p)\,|\,p)]f(p)dp$$

and

$$I_F = \int_{p^*_{BT}}^{p^*_T} G(q^*(p)\,|\,p)f(p)dp.$$

Here I_F, I_S, E_F and E_S are the probabilities a random price-quality combination will fall in region II, III, IV or V, respectively. It is shown in appendix 2 that the following hold.

$$\frac{\partial I_S}{\partial c_S} \gtreqless 0,\ \frac{\partial I_F}{\partial c_S} \gtreqless 0,\ \frac{\partial E_S}{\partial c_S} > 0,\ \text{and}\ \frac{\partial E_F}{\partial c_S} \gtreqless 0.$$

Furthermore,

$$\frac{\partial I_S}{\partial c_T} \gtreqless 0,\ \frac{\partial I_F}{\partial c_T} < 0,\ \frac{\partial E_S}{\partial c_T} > 0,\ \text{and}\ \frac{\partial E_F}{\partial c_T} \gtreqless 0.$$

Finally,

$$\frac{\partial(I_S + E_S)}{\partial c_S} > 0 \quad \text{and} \quad \frac{\partial(I_S + E_S)}{\partial c_T} \gtreqless 0.$$

While the majority of these partial derivatives are ambiguous in sign, a number of interesting observations can still be made. First, the probability that a random price-quality combination will be acceptable (whether quality is inspected or evaluated) is given by $I_S + E_S$. Hence the expected number of observations needed to locate an acceptable good is $1/(I_S + E_S)$. As before, this quantity is decreasing in c_S and ambiguous in c_T. Second, both $\partial E_S/\partial c_S > 0$ and $\partial E_S/\partial c_T > 0$. That is, the probability that a random price-quality combination will be purchased without quality having been observed and subsequently repurchased is increasing in either cost. This is because when either c_S or c_T increases, the set of prices for which evaluation is optimal increases and the set of quality levels which are acceptable for any given price also increases. However, this necessarily implies $\partial E_F/\partial c_S$ and $\partial E_F/\partial c_T$ are ambiguous in sign. This last observation is important. When a price-quality combination falls in region IV the good is purchased without quality having been observed, but is not subsequently repurchased. This "brand disloyalty" is analogous to a job-quit in the labor market or a divorce in the marriage market. It is crucial to recognize that the likelihood of these events does not appear to be systematically related to either search costs or inspection costs. This point will be discussed in more detail in section 6.

5. THE OPTIMAL POLICY, CASE C: EVALUATION ONLY

Suppose that $c_B(0) < c_T$ but that either condition 1 or condition 2 of section 4 does not hold.[11] Then evaluation will always dominate inspection for any relevant price.[12] As always, two further possibilities arise. In the first there exists a unique finite price, p_B^* such that evaluation is optimal for $p \le p_B^*$ and drawing another observation from ϕ is optimal for $p \ge p_B^*$. The critical price is defined by $B(p_B^*) = V$ (see figure 5). In the second $B(p) > V$ for all $p \ge 0$ so that evaluation is always optimal.

Assume for the remainder of this section that p_B^* exists and is finite.[13] Then p_B^* and q^* partition $R_+ \times R_+$ into three regions. As in sections 3 and 4, region I includes prices for which rejecting the good outright is optimal. In region IV, $p \le p_B^*$ and $q < q^*(p)$ so the good is purchased without quality having been observed and is subsequently rejected (i.e, not repurchased). In region V, $p \le p_B^*$ but $q \geq q^*(p)$ so the good is purchased without quality having been observed and is subsequently accepted (i.e., repurchased). Figure 6 illustrates these regions.

Of course c_T has no effect on this partition since $T(p) \le \max\{B(p),V\}$ for all $p \ge 0$. With respect to c_S, the following results are established in appendix 3.

$$\frac{dp_B^*}{dc_S} > 0 \quad \text{and} \quad \frac{dV}{dc_S} < 0.$$

As one might expect, an increase in the cost of drawing observations from ϕ increases the set of prices for which evaluation is optimal.

In this case the expected number of observations which must be drawn from ϕ before an acceptable good is found is given by $1/E_S$ where

$$E_S = \int_0^{p_B^*} [1 - G(q^*(p)\,|\,p)] f(p)\,dp.$$

Appendix 3 also shows that $\partial E_S/\partial c_S > 0$ so fewer observations from ϕ are needed to find an acceptable good as c_S rises. Finally, define E_F as

$$E_F = \int_0^{p_B^*} G(q^*(p)\,|\,p) f(p)\,dp.$$

As in section 4, E_F is the probability that a random price-quality combination will be purchased without quality having been observed and subsequently rejected (i.e., not repurchased). Appendix 3 shows $\partial E_F/\partial c_S \gtreqless 0$.

6. APPLICATIONS

The model analyzed in this paper has obvious analogues in the labor market and the marriage market. The product market has been used as the setting up to this point because much of the relevant literature deals with consumer behavior (e.g., Nelson [1970], Wilde [1977], Lippman and McCall [1979a], and Hey and McKenna [1979]). However, many of the important qualitative implications of the model emerge more sharply in the labor market than in the product market.

The labor market analogue concerns an unemployed worker searching for a job. This individual pays a search cost in order to sample vacancies, but only the wage rate is observed. Nonwage characteristics can be observed either by paying an inspection cost or by taking the job.

Two aspects of the labor market analogue are of primary interest, unemployment and the quit rate. Unemployment is related to the expected duration of search. The quit rate is related to the probability that a job which is accepted will subsequently be rejected.

Suppose that the distributions of net utility associated with jobs, conditional on the wage rate, are stochastically decreasing in the wage rate. Then in the most interesting case there will be a range of low wages for which renewing search is optimal, a range of intermediate wages for which inspection is optimal, and a range of high wages for which evaluation is optimal (see section 4 above).

The effects of an increase in information costs on the duration of search seem straightforward. An increase in the cost of search makes both inspection and evaluation more desirable alternatives. Hence the duration of search should fall. An increase in the cost of inspection will likely have ambiguous effects on the duration of search since it makes evaluation a more desirable alternative but it also makes inspection a less desirable alternative.

The effects of an increase in information costs on the quit rate are less obvious. The argument would seem to go as follows: A quit requires that two events occur: (1) a wage rate is observed for which it is optimal to take the job without observing its nonwage component first, and (2) the nonwage component turns out to be too low so that it is optimal to quit and renew search once it is

observed. But, an increase in either search costs or inspection costs makes evaluation optimal for more wages. In particular, there are lower wages for which evaluation is now optimal. These jobs must have higher nonwage components in order to be acceptable; i.e., for them the probability of turnover is higher. Hence the overall probability of a quit should increase as either cost increases.

Unfortunately these heuristics are incomplete because neither considers the fact that an increase in either c_S or c_T will decrease the return to search, making lower values of the nonwage component acceptable at any given wage. While this reinforces the argument regarding the relationship between information costs and the duration of search, it weakens the argument regarding the relationship between information costs and the quit rate. In fact an increase in either cost could either increase or decrease the quit rate.[14]

To justify these assertions formally the model of section 4 must be used. Consider first the duration of search. There are really two measures which are of interest. First, $E_S + I_S$ gives the probability that a random price-quality observation will ultimately be acceptable (regardless of how quality is observed). Second, $E_F + E_S + I_S$ gives the probability that a random price observation will cause search to cease (although perhaps only temporarily). It was shown in section 4 that

$$\frac{\partial(E_S + I_S)}{\partial c_S} > 0 \quad \text{and} \quad \frac{\partial(E_S + I_S)}{\partial c_T} \gtreqless 0.$$

It is also the case that

$$\frac{\partial(E_F + E_S + I_S)}{\partial c_S} > 0 \quad \text{and} \quad \frac{\partial(E_F + E_S + I_S)}{\partial c_T} \gtreqless 0.$$

Purchases of goods for which quality has not been observed are really part of the search process. Hence $1/(E_S + I_S)$, which might be called the "pure duration of search", is the proper expression for the expected duration of search. However, empirically it would often be impossible to differentiate between observations which fall in region IV and observations which fall in region V. In other words, the observed duration of search would often correspond to $1/(E_F + E_S + I_S)$, which might be called the "effective duration of search."

Similar problems arise with respect to turnover. In fact, there are three measures of turnover embedded in this model, one ex ante and two ex poste. The ex ante measure is simply E_F; it gives the probability that a random price-quality observation will be purchased once and only once. One ex post measure is what might be called the "pure failure rate for evaluation," $\hat{E}_F = E_F/(E_F + E_S)$; it gives the conditional probability that a good will be rejected given that it is purchased without quality having been observed. However, empirically it would often be hard to differentiate between observations which fall in region III and observations which fall in region V. Hence

the other ex post measure is what might be called the "effective failure rate for evaluation," $\hat{\hat{E}}_F = E_F/(E_F + E_S + I_S)$; it gives the conditional probability that a good will be rejected given that it is purchased (regardless of whether quality is observed before purchase or after purchase). It turns out that none of these measures is systematically related to either search costs or inspection costs (see Appendix 2).

The third application of this theoretical framework is to the marriage market. The marriage market analogue concerns an unwed individual searching for a marriage partner. This individual pays a search cost in order to sample potential partners, but only some characteristics are observed. Other characteristics can be observed either by paying an inspection cost or by getting married.

The aspect of the marriage market analogue which is of primary interest is dissolution. The most complete analysis of the relationship between information costs and probability of dissolution is provided by Becker, Landes and Michael [1977]. These authors consider two cases, one in which remarriage is impossible and one in which the remarriage market is identical to the marriage market.

Consider first the case in which remarriage is impossible. When remarriage is impossible, the value of dissolution is a constant. In the model analyzed in this paper an analogous assumption is that the value of <u>not</u> repurchasing a good is a constant, say $\bar{V}$. Then instead of (3), B(p) would be defined as

$$B(p) = E[U(p,Q)] + \beta\bar{k}(p) \tag{3'}$$

where

$$\bar{k}(p) = \bar{V}G[\bar{q}^*(p)|p] + \int_{\bar{q}^*(p)}^{\infty} \frac{U(p,q)}{1-\beta} g(q|p)dq$$

and $\bar{q}^*(p)$ is defined by

$$U(p,\bar{q}^*(p)) = \bar{V}(1-\beta).$$

The definition of T(p) would remain as in (4). This modification affects the comparative statics of the model in a straightforward way; for $p \le p^*_{BT}$, $d\bar{q}^*(p)/dc_S = 0 = d\bar{q}^*(p)/dc_T$. Hence as before,

$$\frac{\partial E_S}{\partial c_S} > 0 \quad \text{and} \quad \frac{\partial E_S}{\partial c_T} > 0$$

but now

$$\frac{\partial E_F}{\partial c_S} > 0 \quad \text{and} \quad \frac{\partial E_F}{\partial c_T} > 0.$$

Furthermore, as before,

$$\frac{\partial \hat{E}_S}{\partial c_S} < 0 \quad \text{and} \quad \frac{\partial \hat{E}_F}{\partial c_T} > 0$$

but now

$$\frac{\partial \hat{E}_S}{\partial c_T} < 0 \quad \text{and} \quad \frac{\partial \hat{E}_F}{\partial c_T} > 0.$$

However, it remains true, even when the value of <u>not</u> repurchasing a good is constant, that $\hat{\hat{E}}_S$ and $\hat{\hat{E}}_F$ bear no systematic relationship to information costs.

These results are, on the surface, consistent with those of Becker, Landes and Michael. Those authors assert that because "the probability of entering a mismatch" would be greater, "an increase in the cost of intensive or extensive search would increase the probability of a dissolution" [1977, p. 1150].[15] The definition of "probability of a dissolution" these authors use to arrive at this theoretical conclusion is apparently $\hat{E}_F$. The problem is that $\hat{E}_F$ is unobservable, it is $\hat{\hat{E}}_F$ which is observed, and even when remarriage is impossible, there is no systematic relationship between $\hat{\hat{E}}_F$ and either c_S or c_T!

The situation is even more difficult when the remarriage market is identical to the marriage market. Here, just as in the labor market analogue, none of the partial derivatives relating information costs to turnover can be signed.

7. CONCLUSION

This paper has established a number of strong results which go against the grain of the extant literature. These results obtain because goods are viewed as multi-characteristic composites in which individual characteristics have specific informational properties. Under these circumstances consumers have an incentive to pursue very complicated information acquisition strategies. In order to analyze this possiblity, I have assumed that consumers are quite sophisticated. It might be objected, however, that actual consumers do not use optimizing strategies as complicated as those studied in this paper. Rather, consumers use various satisficing strategies. There is much to be said for this point of view, but satisficing strategies are even less well understood than optimizing strategies. Furthermore, most empirical work on the duration of search and turnover is based, implicitly or explicitly, on simpler versions of the optimizing model developed in this paper (e.g., Becker, Landes, and Michael [1977]). Thus, if my characterization of goods is accurate, resolution of the optimizing versus satisficing controversy is more important than is commonly supposed.

FOOTNOTES

1. This paper is a substantially revised version of "More on Inspection and Evaluation in Product Markets," unpublished manuscript, California Institute of Technology, 1978. I would like to thank Steve Lippman and Alan Schwartz for very helpful comments on an earlier draft.

2. The formal analysis in this paper will be set in the product market. In this case turnover is a nonrepeat sale. Section 6 will discuss applications to the labor market and the marriage market. In the labor market turnover is a quit and in the marriage market turnover is a divorce.

3. Since $\partial U(p,q)/\partial q > 0$, $q^*(p)$ will be unique with $q \geq q^*(p)$ an acceptable quality level and $q < q^*(p)$ an unacceptable quality level.

4. Existence of a bounded solution to (1) is straightforward. Uniqueness can be established along traditional lines if search and inspection are not assumed to be timeless (see Wilde [1979]) or by appealing to MacQueen and Miller [1960] if the support of ϕ is assumed to be compact. More recently, Robbins [1970]

shows that under the other assumptions of this paper, $EU(P,Q) < \infty$ is sufficient for uniqueness.

5. Even in the case where c_T is infinite, some formal structure must be placed on Ψ. See Wilde [1977] or Hey and McKenna [1979] for more details.

6. There is no guarantee that $c_B(p) > 0$. In particular, it might be the case that $c_B(0) = 0$. It can be shown, however, that there exists $\varepsilon > 0$ such that $c_B(0) > 0$ if $c_S < \varepsilon$.

7. Since $\partial\Psi(w|p)/\partial p > 0$ only implies $T'(p) \leq 0$, it is possible that the equation $T(p) = V$ does not have a unique solution. However, in this case $\{p|T(p) = V\} \equiv [p_T^*, \infty)$ where $p_T^* = \inf\{p|T(p) = V\}$, so that p_T^* defined in this fashion satisfies the formal requirements stated in the text.

8. Formal proofs of these assertions have been omitted since they are trivial. It is immediate, however, that since $c_B(0) \geq 0$, there is always a small enough value of c_T(possible 0) such that $B(p) \leq T(p)$ for all $p \geq 0$.

9. Again, formal proofs that ranges of c_S and c_T exist such that both possibilities occur have been omitted. The text following identifies two necessary conditions for inspection to be optimal for one set of prices and evaluation to be optimal for another

set of prices. There seems to be little value in making these more formal.

10. Again, since $\partial\Psi(w|p)/\partial p > 0$ only implies $B'(p) \leq 0$, the solution to the equation $T(p) = B(p)$ may not be unique. In this case $\{p|T(p) = B(p)\} \equiv [p_{BT}^*, \infty)$ where $p_{BT}^* = \inf\{p|T(p) = B(p)\}$, so that p_{BT}^* defined in this fashion satisfies the formal requirements stated in the text.

11. This will certainly be the case when c_T is large enough since U is bounded.

12. That is, $B(p) \geq T(p)$ for any p such that $V < \max\{B(p), T(p)\}$.

13. Define $p_B^* = \inf\{p|B(p) = V\}$. Then if p_B^* is not unique $\{p|B(p) = V\} \equiv [p_B^*, \infty)$, so that p_B^* defined in this fashion satisfies the formal requirements stated in the text.

14. See Lippman and McCall [1979b] for an extensive discussion of these points in a related model.

15. Intensive search corresponds to inspection and extensive search corresponds to drawing another observation from ϕ. Hence the cost of intensive search is c_T and the cost of extensive search is c_S.

REFERENCES

Becker, Gary; Elisabeth Landes and Robert Michael, "An Economic Analysis of Marital Instability," Journal of Political Economy 85 (December 1977), pp. 1141-1188.

Hey, J.D., and C.J. McKenna, "Consumer Search with Uncertain Product Quality," Discussion Paper No. 33, I.S.E.R./Department of Economics, University of York (1979). Forthcoming in Journal of Political Economy.

Lippman, S., and J.J. McCall, "The Economics of Belated Information," Working Paper No. 286, Western Management Science Institute, University of California at Los Angeles (1979a).

__________, "Search Unemployment: Mismatches, Layoffs, and Unemployment Insurance," Working Paper No. 297, Western Management Science Insitute, University of California at Los Angeles, (1979b).

MacQueen, J., and R.G. Miller, Jr., "Optimal Persistence Policies," Operations Research 8 (1960), pp. 362-80.

Nelson, P., "Information and Consumer Behavior," Journal of Political Economy 78 (March 1970), pp. 311-329.

Robbins, H., "Optimal Stopping," American Mathematical Monthly 77 (1970), pp. 333-343.

Wilde, L.L., "On the Formal Theory of Inspection and Evaluation in Product Markets," Social Science Working Paper No. 146, California Institute of Technology, July 1977. Forthcoming in Econometrica (February 1980).

__________, "An Information-Theoretic Approach to Job Quits," in Studies in the Economics of Search, S. Lippman and J. J. McCall (eds.), Amsterdam: North-Holland, 1979.

APPENDIX 1

This appendix analyzes the partial derivatives presented in section 3. In that section it was assumed that $c_B(0) \geq c_T$ so that $T(p) \geq B(p)$ for all $p \geq 0$. Hence the functional equation (1) can be written as

$$V(p) = -c_S + \max\{T(p), V\}.$$

Taking the expectation on both sides with respect to f(p), we get

$$V = -c_S + \int_0^{p_T^*} T(p) f(p) dp + \int_{p_T^*}^{\infty} V f(p) dp$$

or

$$0 = \int_0^{p_T^*} [T(p) - V] f(p) dp - c_S \ .$$

Furthermore $T(p_T^*) - V = 0$. Define

$$h(p_T^*, V, c_S, c_T) = \int_0^{p_T^*} [T(p) - V] f(p) dp - c_S$$

and

$$i(p_T^*, V, c_S, c_T) = T(p_T^*) - V.$$

Taking the derivatives of h and i with respect to c_S we get

$$\begin{bmatrix} h_1 & h_2 \\ i_1 & i_2 \end{bmatrix} \begin{bmatrix} dp_T^*/dc_S \\ dV/dc_S \end{bmatrix} = \begin{bmatrix} -h_3 \\ -i_3 \end{bmatrix} .$$

But using $T(p) = k(p) - c_T$ we have the following:

$$h_1 = \partial h/\partial p_T^* = 0$$

$$h_2 = \partial h/\partial V = \int_0^{p_T^*} [\Psi(V(1-\beta)|p) - 1] f(p) dp < 0$$

$$h_3 = \partial h/\partial c_S = -1 < 0$$

$$i_1 = \partial i/\partial p_T^* = \partial T(p_T^*)/\partial p = k'(p_T^*) < 0$$

$$i_2 = \partial i/\partial V = \Psi(V(1-\beta)|p_T^*) - 1 < 0$$

$$i_3 = \partial i/\partial c_S = 0 \ .$$

Hence

$$\begin{bmatrix} dp_T^*/dc_S \\ dV/dc_S \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} i_2 & -h_2 \\ -i_1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

where $\Delta = h_1 i_2 - i_1 h_2 = -i_1 h_2$. Thus,

$$\frac{dp_T^*}{dc_S} = \frac{-i_2}{i_1 h_2} ,$$

and

$$\frac{dV}{dc_S} = \frac{1}{h_2} \quad .$$

Hence $dp_T^*/dc_S > 0$ and $dV/dc_S < 0$.

Similarly

$$\begin{bmatrix} dp_T^*/dc_T \\ \\ dV/dc_T \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} i_2 & -h_2 \\ \\ -i_1 & 0 \end{bmatrix}\begin{bmatrix} -h_4 \\ \\ -i_4 \end{bmatrix}$$

where

$$h_4 = \partial h/\partial c_T = -F(p_T^*) < 0$$

$$i_4 = \partial i/\partial c_T = -1 < 0.$$

Thus

$$\frac{dp_T^*}{dc_T} = \frac{i_2 h_4 + h_2}{i_2 h_2}$$

$$= \frac{1}{i_1 h_2}\left[\int_0^{p_T^*}[\Psi(V(1-\beta)\,|\,p) - 1]f(p)dp - F(p_T^*)\Psi(V(1-\beta)\,|\,p_T^*)\right]$$

$$= \frac{1}{i_1 h_2}\left[\int_0^{p_T^*}[\Psi(V(1-\beta)\,|\,p) - \Psi(V(1-\beta)\,|\,p_T^*)]f(p)dp - F(p_T^*)\right]$$

and

$$\frac{dV}{dc_T} = \frac{1}{h_2} \quad .$$

Hence $dp_T^*/dc_T < 0$ and $dV/dc_T < 0$.

Now consider I_S and I_F. Recall

$$I_S = \int_0^{p_T^*}[1 - G(q^*(p)\,|\,p)]f(p)dp = \int_0^{p_T^*}[1 - \Psi(V(1-\beta)\,|\,p)]f(p)dp.$$

Hence

$$\frac{\partial I_S}{\partial c_S} = [1 - \Psi(V(1-\beta)\,|\,p_T^*)]f(p_T^*)\frac{dp_T^*}{dc_S} + \int_0^{p_T^*}-\psi(V(1-\beta)\,|\,p)(1-\beta)\frac{dV}{dc_S}f(p)dp.$$

But

$$I_F = \int_0^{p_T^*}G(q^*(p)\,|\,p)f(p)dp = \int_0^{p_T^*}\Psi(V(1-\beta)\,|\,p)f(p)dp$$

so

$$\frac{\partial I_F}{\partial c_S} = \Psi(V(1-\beta)\,|\,p_T^*)f(p_T^*)\frac{dp_T^*}{dc_S} + \int_0^{p_T^*}\psi(V(1-\beta)\,|\,p)(1-\beta)\frac{dV}{dc_S}f(p)dp.$$

Thus $\partial I_S/\partial c_S > 0$ but $\partial I_F/\partial c_S$ cannot be signed. Similarly,

$$\frac{\partial I_S}{\partial c_T} = [1 - \Psi(V(1-\beta)\,|\,p_T^*)]f(p_T^*)\frac{dp_T^*}{dc_T} + \int_0^{p_T^*}-\psi(V(1-\beta)\,|\,p)(1-\beta)\frac{dV}{dc_T}f(p)dp$$

and
$$\frac{\partial I_F}{\partial c_T} = \Psi(V(1-\beta)|\, p_T^*) f(p_T^*) \frac{dp_T^*}{dc_T} + \int_0^{p_T^*} \psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_T} f(p)dp.$$

Thus $\partial I_F/\partial c_T < 0$ but $\partial I_S/\partial c_T$ cannot be signed. Finally, recall $\hat{I}_S = I_S/F(p_T^*)$ so that

$$\frac{\partial \hat{I}_S}{\partial c_S} = \frac{(\partial I_S/\partial c_S)F(p_T^*) - I_S f(p_T^*)(dp_T^*/dc_S)}{F(p_T^*)^2}$$

$$= F(p_T^*)^{-2} \cdot \{f(p_T^*)\frac{dp_T^*}{dc_S}\int_0^{p_T^*}[\Psi(V(1-\beta)|p) - \Psi(V(1-\beta)|p_T^*)]f(p)dp$$

$$- \int_0^{p_T^*}\psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_S} f(p)dp \cdot F(p_T^*)\}.$$

Similarly

$$\frac{\partial \hat{I}_S}{\partial c_T} = F(p_T^*)^{-2} \cdot \{f(p_T^*)\frac{dp_T^*}{dc_T}\int_0^{p_T^*}[\Psi(V(1-\beta)|p) - \Psi(V(1-\beta)|p_T^*)]f(p)dp$$

$$- \int_0^{p_T^*}\psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_T} f(p)dp \cdot F(p_T^*)\}.$$

Since $\partial\Psi(w|p)/\partial p > 0$, $\partial\hat{I}_S/\partial c_S$ is ambiguous in sign but $\partial\hat{I}_S/\partial c_T > 0$. Furthermore $\hat{I}_F = 1 - \hat{I}_S$ so $\partial\hat{I}_F/\partial c_S$ is ambiguous in sign but $\partial\hat{I}_F/\partial c_T < 0$.

APPENDIX 2

This appendix analyzes the partial derivatives presented in section 4. In that section it is assumed that $c_B(0) < c_T$ but $T(p) > V$ for some p such that $B(p) < T(p)$ so that both evaluation and inspection are optimal strategies for some prices. Here

$$V = -c_S + \int_0^{p_T^*} B(p)f(p)dp + \int_{p_{BT}^*}^{p_T^*} T(p)f(p)dp + \int_{p_T^*}^{\infty} Vf(p)dp$$

or

$$0 = \int_0^{p_{BT}^*}[B(p) - V]f(p)dp + \int_{p_{BT}^*}^{p_T^*}[T(p) - V]f(p)dp - c_S \ .$$

Furthermore $T(p_T^*) - V = 0$ and $T(p_{BT}^*) - B(p_{BT}^*) = 0$.

Define

$$h(p_T^*, p_{BT}^*, V, c_S, c_T) = \int_0^{p_{BT}^*}[B(p) - V]f(p)dp + \int_{p_{BT}^*}^{p_T^*}[T(p) - V]f(p)dp - c_S$$

$$i(p_T^*, p_{BT}^*, V, c_S, c_T) = T(p_T^*) - V,$$

and

$$j(p_T^*, p_{BT}^*, V, c_S, c_T) = T(p_{BT}^*) - B(p_{BT}^*).$$

Taking the derivatives of h, i, and j with respect to c_S we get

$$\begin{bmatrix} h_1 & h_2 & h_3 \\ i_1 & i_2 & i_3 \\ j_1 & j_2 & j_3 \end{bmatrix} \begin{bmatrix} dp_T^*/dc_S \\ dp_{BT}^*/dc_S \\ dV/dc_S \end{bmatrix} = \begin{bmatrix} -h_4 \\ -i_4 \\ -j_4 \end{bmatrix} \qquad (*)$$

where

$$h_1 = 0$$

$$h_2 = 0$$

$$h_3 = \int_0^{p_{BT}^*} [\frac{\partial B}{\partial V} - 1] f(p)dp + \int_{p_{BT}^*}^{p_T^*} [\frac{\partial T}{\partial V} - 1] f(p)dp \ < 0$$

$$h_4 = 1$$

$$i_1 = T'(p_T^*) = k'(p_T^*) \ < 0$$

$$i_2 = 0$$

$$i_3 = \frac{\partial k}{\partial V} - 1 = \Psi(V(1-\beta)|p_T^*) - 1 \ < 0$$

$$i_4 = 0$$

$$j_1 = 0$$

$$j_2 = T'(p_{BT}^*) - B'(p_{BT}^*) = \int_{\bar{z}(p)}^{V(1-\beta)} \partial\Psi(w|p)/\partial p \, dw \ > 0$$

$$j_3 = \frac{\partial k}{\partial V} - \beta \frac{\partial k}{\partial V} = (1-\beta)\Psi(V(1-\beta)|p_{BT}^*) \ > 0$$

$$j_4 = 0$$

Hence the system (*) can be written

$$\begin{bmatrix} 0 & 0 & h_3 \\ i_1 & 0 & i_3 \\ 0 & j_2 & j_3 \end{bmatrix} \begin{bmatrix} dp_T^*/dc_S \\ dp_{BT}^*/dc_S \\ dV/dc_S \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Inverting the matrix yields

$$\begin{bmatrix} dp_T^*/dc_S \\ d_{BT}^*/dc_S \\ dV/dc_S \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} -j_2 i_3 & j_2 h_3 & 0 \\ -i_1 j_3 & 0 & i_1 h_3 \\ i_1 j_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

where $\Delta = h_3 i_1 j_2$. Thus

$$\frac{dp_T^*}{dc_S} = \frac{-i_3}{h_3 i_1}$$

$$\frac{dp_{BT}^*}{dc_S} = \frac{-j_3}{h_3 j_2}$$

$$\frac{dV}{dc_S} = \frac{1}{h_3}$$

Hence $dp_T^*/dc_S > 0$, $dp_{BT}^*/dc_S > 0$, and $dV/dc_S < 0$.

Similarly

$$h_5 = \int_{p^*_{BT}}^{p^*_T} \frac{\partial T}{\partial c_T} f(p)dp = F(p^*_{BT}) - F(p^*_T) < 0$$

$$i_5 = \frac{\partial T}{\partial c_T} = -1 < 0$$

$$j_5 = \frac{\partial T}{\partial c_T} = -1 < 0 ,$$

so that

$$\begin{bmatrix} dp^*_T/dc_T \\ dp^*_{BT}/dc_T \\ dV/dc_T \end{bmatrix} = \frac{1}{\Delta}\begin{bmatrix} -j_2 i_3 & j_2 h_3 & 0 \\ -i_1 j_3 & 0 & i_1 h_3 \\ i_1 j_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} -h_5 \\ 1 \\ 1 \end{bmatrix}$$

where again $\Delta = h_3 i_1 j_2$. Thus

$$\frac{dp^*_T}{dc_T} = \frac{i_3 h_5 + h_3}{h_3 i_1}$$

$$\frac{dp^*_{BT}}{dc_T} = \frac{h_5 j_3 + h_3}{h_3 j_2}$$

$$\frac{dV}{dc_T} = \frac{-h_5}{h_3}$$

Now

$$i_3 h_5 + h_3 = [\Psi(V(1-\beta)|p^*_T) - 1][F(p^*_{BT}) - F(p^*_T)]$$

$$+ \int_0^{p^*_{BT}} [\beta\Psi(V(1-\beta)|p) - 1]f(p)dp + \int_{p^*_{BT}}^{p^*_T} [\Psi(V(1-\beta)|p) - 1]f(p)dp$$

$$= \int_0^{p^*_{BT}} [\beta\Psi(V(1-\beta)|p) - 1]f(p)dp + \int_{p^*_{BT}}^{p^*_T} [\Psi(V(1-\beta)|p) - \Psi(V(1-\beta)|p^*_T)]f(p)dp.$$

Therefore, since $\partial\Psi(w|p)/\partial p > 0$, $i_3 h_5 + h_3 < 0$. Thus $dp^*_T/dc_T < 0$. Furthermore $dp^*_{BT}/dc_T > 0$ and $dV/dc_T < 0$.

Now, consider E_S, E_F, I_S and I_F. Recall

$$E_S = \int_0^{p^*_{BT}} [1 - G(q^*(p)|p)]f(p)dp = \int_0^{p^*_{BT}} [1 - \Psi(V(1-\beta)|p)]f(p)dp.$$

Hence

$$\frac{\partial E_S}{\partial c_S} = [1 - \Psi(V(1-\beta)|p^*_{BT})]f(p^*_{BT})\frac{dp^*_{BT}}{dc_S} + \int_0^{p^*_{BT}} -\psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_S} f(p)dp$$

and

$$E_F = \int_0^{p^*_{BT}} G(q^*(p)|p)f(p)dp = \int_0^{p^*_{BT}} \Psi(V(1-\beta)|p)f(p)dp$$

so that

$$\frac{\partial E_F}{\partial c_S} = \Psi(V(1-\beta)|p^*_{BT})f(p^*_{BT})\frac{dp^*_{BT}}{dc_S} + \int_0^{p^*_{BT}} \psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_S}f(p)dp.$$

Thus $\partial E_S/\partial c_S > 0$ but $\partial E_F/\partial c_S$ cannot be signed. Next, recall

$$I_S = \int_{p^*_{BT}}^{p^*_T} [1 - G(q^*(p)|p)]f(p)dp = \int_{p^*_{BT}}^{p^*_T} [1 - \Psi(V(1-\beta)|p)]f(p)dp.$$

Hence

$$\frac{\partial I_S}{\partial c_S} = [1 - \Psi(V(1-\beta)|p^*_T)]f(p^*_T)\frac{dp^*_T}{dc_S} - [1 - \Psi(V(1-\beta)|p^*_{BT})]f(p^*_{BT})\frac{dp^*_{BT}}{dc_S}$$

$$+ \int_{p^*_{BT}}^{p^*_T} -\psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_S}f(p)dp,$$

and

$$I_F = \int_{p^*_{BT}}^{p^*_T} G(q^*(p)|p)f(p)dp = \int_{p^*_{BT}}^{p^*_T} \Psi(V(1-\beta)|p)f(p)dp$$

so that

$$\frac{\partial I_F}{\partial c_S} = \Psi(V(1-\beta)|p^*_T)f(p^*_T)\frac{dp^*_T}{dc_S} - \Psi(V(1-\beta)|p^*_{BT})f(p^*_{BT})\frac{dp^*_{BT}}{dc_S}$$

$$+ \int_{p^*_{BT}}^{p^*_T} \psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_S}f(p)dp.$$

Thus neither $\partial I_S/\partial c_S$ nor $\partial I_F/\partial c_S$ can be signed. Similarly,

$$\frac{\partial E_S}{\partial c_T} = [1 - \Psi(V(1-\beta)|p^*_{BT})]f(p^*_{BT})\frac{dp^*_{BT}}{dc_T} + \int_0^{p^*_{BT}} -\psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_T}f(p)dp,$$

$$\frac{\partial E_F}{\partial c_T} = \Psi(V(1-\beta)|p^*_{BT})f(p^*_{BT})\frac{dp^*_{BT}}{dc_T} + \int_0^{p^*_{BT}} \psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_T}f(p)dp,$$

$$\frac{\partial I_S}{\partial c_T} = [1 - \Psi(V(1-\beta)|p^*_T)]f(p^*_T)\frac{dp^*_T}{dc_T} - [1 - \Psi(V(1-\beta)|p^*_{BT})]f(p^*_{BT})\frac{dp^*_{BT}}{dc_T}$$

$$+ \int^{p^*_T} -\psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_T}f(p)dp,$$

and

$$\frac{\partial I_F}{\partial c_T} = \Psi(V(1-\beta)|p^*_T)f(p^*_T)\frac{dp^*_T}{dc_T} - \Psi(V(1-\beta)|p^*_{BT})f(p^*_{BT})\frac{dp^*_{BT}}{dc_T}$$

$$+ \int_{p^*_{BT}}^{p^*_T} \psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_T}f(p)dp.$$

Finally, consider $\hat{E}_S$, $\hat{E}_F$, $\hat{I}_S$, and $\hat{I}_F$. Recall $\hat{E}_S = E_S/F(p^*_{BT})$. Hence

$$\frac{\partial \hat{E}_S}{\partial c_S} = \frac{(\partial E_S/\partial c_S)F(p^*_{BT}) - E_S f(p^*_{BT})(dp^*_{BT}/dc_S)}{F(p^*_{BT})^2}$$

$$= F(p^*_{BT})^{-2}\{f(p^*_{BT})\frac{dp^*_{BT}}{dc_S}\int_0^{p^*_{BT}} [\Psi(V(1-\beta)|p) - \Psi(V(1-\beta)|p^*_{BT})]f(p)dp$$

$$+ \int_0^{p^*_{BT}} \psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_S} f(p)dp \cdot F(p^*_{BT})\}.$$

Similarly,

$$\frac{\partial \hat{E}_S}{\partial c_T} = F(p^*_{BT})^{-2} \; f(p^*_{BT})\frac{dp^*_{BT}}{dc_T}\int_0^{p^*_{BT}} [\Psi(V(1-\beta)|p) - \Psi(V(1-\beta)|p^*_{BT})]f(p)dp$$

$$- \int_0^{p^*_{BT}} \psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_S} f(p)dp \cdot F(p^*_{BT})\} .$$

Again $\partial\Psi(w|p)/\partial p > 0$, so that $\partial\hat{E}_S/\partial c_S > 0$. But $\partial\hat{E}_S/\partial c_T$ is ambiguous in sign. Furthermore, $\hat{E}_F = 1 - \hat{E}_S$ so $\partial E_F/\partial c_S < 0$ but $\partial\hat{E}_F/\partial c_T$ is ambiguous in sign. And, at last, we have

$$\frac{\partial \hat{I}_S}{\partial c_S} = \frac{(\partial I_S/\partial c_S)[F(p^*_T) - F(p^*_{BT})] - I_S[f(p^*_T)\frac{dp^*_T}{dc_S} - f(p^*_{BT})\frac{dp^*_{BT}}{dc_S}]}{[F(p^*_T) - F(p^*_{BT})]^2}$$

$$= [F(p^*_T) - F(p^*_{BT})]^{-2}\{f(p^*_T)\frac{dp^*_T}{dc_S}\int_{p^*_{BT}}^{p^*_T}[\Psi(V(1-\beta)|p) - \Psi(V(1-\beta)|p^*_T)]f(p)dp$$

$$- f(p^*_{BT})\frac{dp^*_{BT}}{dc_S}\int_{p^*_{BT}}^{p^*_T}[\Psi(V(1-\beta)|p) - \Psi(V(1-\beta)|p^*_{BT})]f(p)dp$$

$$- [F(p^*_T) - F(p^*_{BT})]\int_{p^*_{BT}}^{p^*_T}\Psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_S} f(p)dp\}$$

Similarly,

$$\frac{\partial I_S}{\partial c_T} = [F(p^*_T) - F(p^*_{BT})]^{-2}\{f(p^*_T)\frac{dp^*_T}{dc_T}\int_{p^*_{BT}}^{p^*_T}[\Psi(V(1-\beta)|p) - \Psi(V(1-\beta)|p^*_T)]f(p)dp$$

$$- f(p^*_{BT})\frac{dp^*_{BT}}{dc_T}\int_{p^*_{BT}}^{p^*_T}[\Psi(V(1-\beta)|p) - \Psi(V(1-\beta)|p^*_{BT})]f(p)dp$$

$$- [F(p^*_T) - F(p^*_{BT})]\int_{p^*_{BT}}^{p^*_T}\Psi(V(1-\beta)|p)(1-\beta)\frac{dV}{dc_T} f(p)dp\}$$

Alas, none of these partial derivatives can be signed either.

APPENDIX 3

This appendix analyzes the partial derivatives presented in section 5. In that section it is assumed that $c_B(0) < c_T$ but $T(p) < V$ for all p such that $B(p) < T(p)$. Here

$$0 = \int_0^{p_B^*} [B(p) - V] f(p) dp.$$

Furthermore, $B(p_B^*) = T$. Define

$$h(p_B^*, V, c_S, c_T) = \int_0^{p_B^*} [B(p) - V] f(p) dp$$

and

$$i(p_B^*, V, c_S, c_T) = B(p_B^*) - V.$$

Taking the derivatives of h and i with respect to c_S,

$$\begin{bmatrix} h_1 & h_2 \\ i_1 & i_2 \end{bmatrix} \begin{bmatrix} dp_B^*/dc_S \\ dV/dc_S \end{bmatrix} = \begin{bmatrix} -h_3 \\ -i_3 \end{bmatrix}.$$

But using $B(p) = k(p) - c_B(p)$,

$$h_1 = 0$$

$$h_2 = \int_0^{p_B^*} [\Psi(V(1-\beta)|p) - 1] f(p) dp < 0$$

$$h_3 = -1$$

$$i_1 = k'(p_B^*) < 0$$

$$i_2 = \Psi(V(1-\beta)|p_B^*) - 1 < 0$$

$$i_3 = 0 .$$

Hence

$$\begin{bmatrix} dp_B^*/dc_S \\ dV/dc_S \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} i_2 & -h_2 \\ -i_1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

where $\Delta = -i_1 h_2$. Thus

$$\frac{dp_B^*}{dc_S} = \frac{-i_2}{i_1 h_2}$$

$$\frac{dV}{dc_S} = \frac{1}{h_2} .$$

Hence $\frac{dp_B^*}{dc_S} > 0$ and $\frac{dV}{dc_S} < 0$.

Since $T(p)$ is irrelevant in this case, $\frac{dp_B^*}{dc_T} = 0 = \frac{dV}{dc_T}$.

Now consider E_S and E_F. Recall

$$E_S = \int_0^{p_B^*} [1 - G(q^*(p)|p)] f(p) dp = \int_0^{p_B^*} [1 - \Psi(V(1-\beta)|p) f(p) dp.$$

Hence

$$\frac{\partial E_S}{\partial c_S} = [1 - \Psi(V(1-\beta)\,|\,p_B^*)]f(p_B^*)\frac{dp_B^*}{dc_S}, - \int_0^{p_B^*}\psi(V(1-\beta)\,|\,p)(1-\beta)\frac{dV}{dc_S}f(p)dp.$$

But

$$E_F = \int_0^{p_B^*} G(q^*(p)\,|'p)f(p)dp = \int_0^{p_B^*}\Psi(V(1-\beta)\,|\,p)f(p)dp$$

so

$$\frac{\partial E_F}{\partial c_S} = \Psi(V(1-\beta)\,|\,p_B^*)f(p_B^*)\frac{dp_B^*}{dc_S} + \int_0^{p_B^*}\psi(V(1-\beta)\,|\,p)(1-\beta)\frac{dV}{dc_S}f(p)dp.$$

Thus $\partial E_S/\partial c_S > 0$ but $\partial E_F/\partial c_S$ cannot be signed. Moreover, $\hat{E}_S = E_S/F(p_B^*)$ so

$$\frac{\partial \hat{E}_S}{\partial c_S} = \frac{(\partial E_S/\partial c_S)F(p_B^*) - E_S\, f(p_B^*)\,(dp_B^*/dc_S)}{F(p_B^*)^2}$$

$$= F(p_B^*)^{-2}\ f(p_B^*)\frac{dp_B^*}{dc_S}\int_0^{p_B^*}[\Psi(V(1-\beta)\,|\,p) - \Psi(V(1-\beta)\,|\,p_B^*)]f(p)dp$$

$$- \int_0^{p_B^*}\psi(V(1-\beta)\,|\,p)(1-\beta)\frac{dV}{dc_S}f(p)dp \cdot F(p_B^*)\} .$$

Since $\partial\Psi(w|p)/\partial p > 0$, $\partial\hat{E}_S/\partial c_S$ is ambiguous in sign.

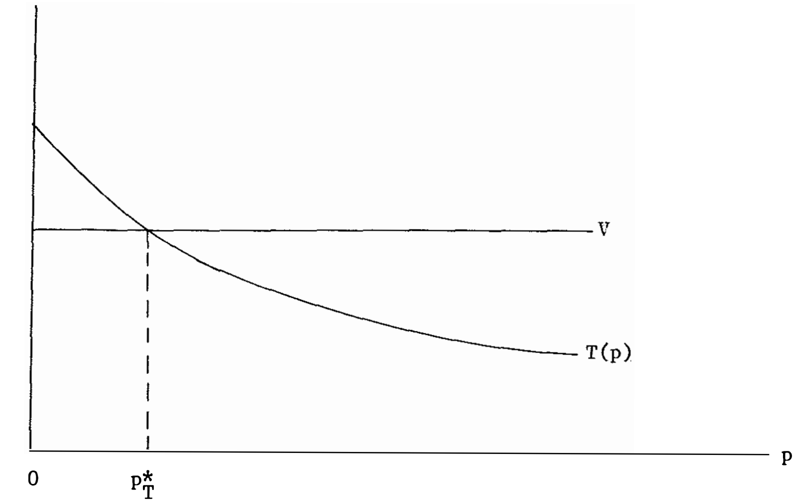

Figure 1: Definition of p_T^*

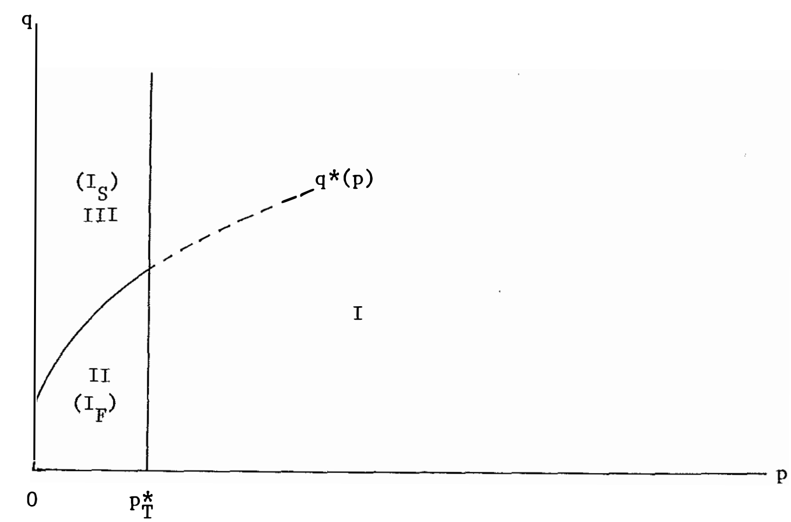

Figure 2: Price-quality combinations, Case A: I = reject outright, II = inspect and subsequently reject, III = inspect and subsequently accept.

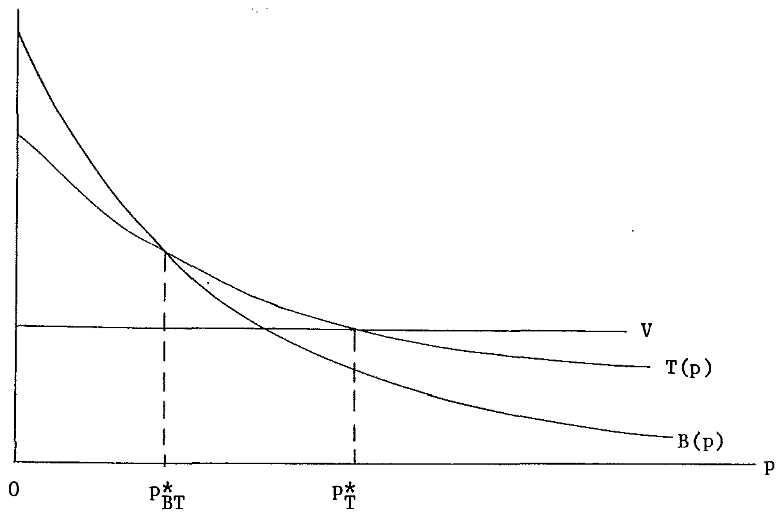

Figure 3: Definition of p^*_{BT} and p^*_T

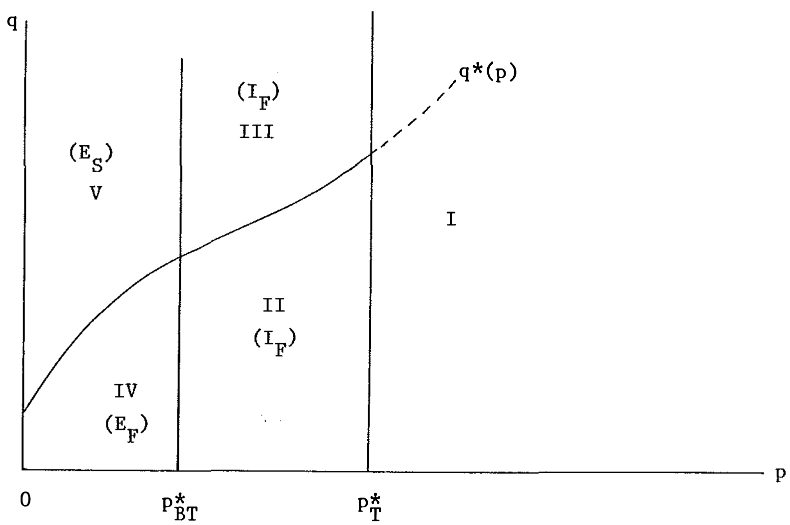

Figure 4: Price-quality combinations, Case B: I = reject outright, II = inspect and ultimately reject, III = inspect and ultimately accept, IV = purchase once and only once, V = purchase once and forever more.

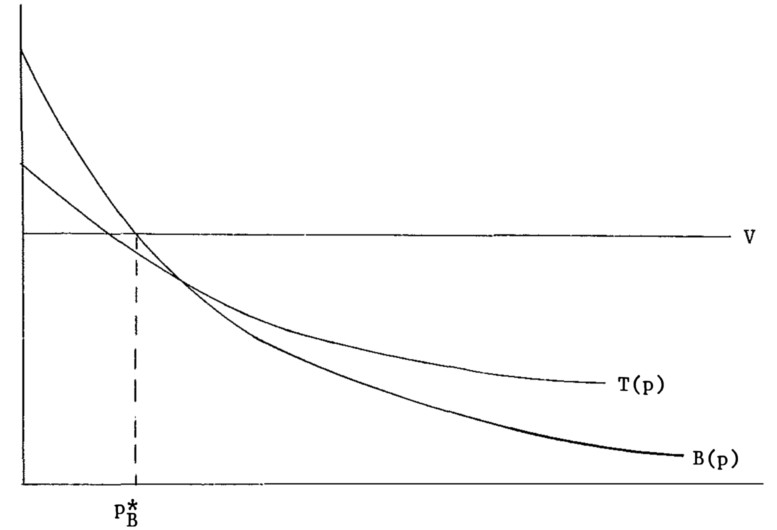

Figure 5: Definition of p^*_B

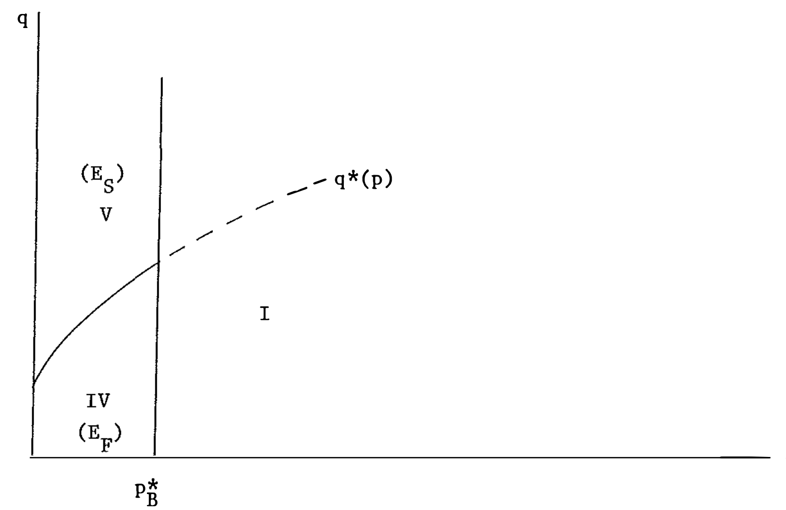

Figure 6: Price-quality combinations; Case C: I = reject outright, IV = purchase once and only once, V = purchase once and forever more.